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Research Article

Finite Groups Whose Certain Subgroups of Prime Power Order Are S-Semipermutable

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Let G be a finite group. A subgroup H of G is said to be S-semipermutable in G if H permutes with every Sylow p-subgroup of G with $(p, |H|) = 1$. In this paper, we study the influence of S-permutability property of certain abelian subgroups of prime power order of a finite group on its structure.

1. Introduction

All groups considered in this paper will be finite. Two subgroups H and K of a group G are said to permute if $HK = KH$. It is easily seen that H and K permute if and only if HK is a subgroup of G. We say, following Kegel [1], that a subgroup of G is S-quasinormal in G if it permutes with every Sylow subgroup of G. Chen [2] introduced the following concept: a subgroup H of G is said to be S-semipermutable in G if H permutes with every Sylow p-subgroup of G with $(p, |H|) = 1$. Obviously, every S-quasinormal subgroup of G is an S-semipermutable subgroup of G. In contrast to the fact that every S-quasinormal subgroup of G is a subnormal subgroup of G (see [1]), it does not hold in general that every S-semipermutable subgroup of G is a subnormal subgroup of G. It suffices to consider the alternating group of degree 4.

Several authors have investigated the structure of a finite group when some information is known about some subgroups of prime power order in the group. Huppert [3] proved that a finite group G is solvable provided that all subgroups of prime order are normal in G. Buckley [4], proved that a group G of odd order is supersolvable provided that all subgroups of prime order are normal in G. Srinivasan [5], and proved that a finite group G is supersolvable if the maximal subgroups of every Sylow subgroup of G are normal in G.

Developing the result of Srinivasan, Ramadan [6] proved that if G is a solvable group and the maximal subgroups of every Sylow subgroup of the Fitting subgroup $F(G)$ of G are normal in G, then G is supersolvable.

For a finite p-group P, we denote

$$\Omega(P) = \Omega_1(P) \quad \text{if } p > 2, \qquad \Omega(P) = \langle \Omega_1(P), \Omega_2(P) \rangle \quad \text{if } p = 2, \tag{1.1}$$

where $\Omega_i(P) = \langle x \in P : |x| = p^i \rangle$.

Of late there has been a considerable interest to investigate the influence of the abelian subgroups of largest possible exponent of prime power order (we call such subgroups ALPE-subgroups) on the structure of the group. Asaad et al. [7] proved that if G is a group such that for every prime p and every Sylow p-subgroup G_p of G, the ALPE-subgroups of G_p (resp., $\Omega(G_p)$) are normal in G, then G is supersolvable. Ramadan [8] proved the following two results. (1) Let G be a group such that for every prime p and every Sylow p-subgroup G_p of G, the ALPE-subgroups of G_p (resp., $\Omega(G_p)$) are S-quasinormal in G, then G is supersolvable. (2) Let K be a normal subgroup of G such that G/K is supersolvable. If for every prime p and every Sylow p-subgroup K_p of K, the ALPE-subgroups of K_p (resp., $\Omega(K_p)$) are S-quasinormal in G, then G is supersolvable.

In this paper, we study the structure of a finite group under the assumption that certain subgroups of prime power order are S-semipermutable in the group. We focus our attention on S-semipermutability property of the ALPE-subgroups of a fixed ALPE-subgroup having maximal order of the Sylow subgroups of a finite group. Furthermore, we improve and extend the above-mentioned results by using the concept of S-semipermutability.

2. Preliminaries

In this section, we give some results which will be useful in the sequal.

Lemma 2.1 (see [2, Lemmas 1 and 2]). *Let G be a group.*

(i) *If H is a S-semipermutable subgroup of G and K is a subgroup of G such that $H \leq K \leq G$, then H is S-semipermutable in K.*

(ii) *Let π be a set of primes, N a normal π'-subgroup of G, and H a π-subgroup of G. If H is S-semipermutable in G, then HN/N is S-semipermutable in G/N.*

Lemma 2.2 (see [9, Lemma A]). *Let H be a p-subgroup of G; for some prime p. Then H is S-quasinormal in G if and only if $O^p(G) \leq N_G(H)$, where $O^p(G)$ is the normal subgroup of G generated by all p'-elements of G.*

Lemma 2.3. *Let H be a p-subgroup of G, p is a prime. Then the following statements are equivalent:*

(i) *H is S-quasinormal in G;*

(ii) *$H \leq O_p(G)$ and H is S-semipermutable in G.*

Proof. (i) $\Rightarrow$ (ii): Suppose that H is S-quasinormal in G. So it follows by [1, Satz 1, page 209] that H is subnormal in G and then by [10, Lemma 8.6(a), page 28] that $H \leq O_p(G)$. Since H is S-quasinormal in G, obviously, it is S-semipermutable in G. Thus (ii) holds.

(ii) $\Rightarrow$ (i): Since H is S-semipermutable in G, then $HG_q = G_qH$ for every Sylow q-subgroup G_q of G with $(q, |H|) = 1$. Clearly, $H = O_p(G) \cap HG_q$ is normal in HG_q and so $G_q \leq N_G(H)$. Thus $O^p(G) \leq N_G(H)$. Applying Lemma 2.2, we have that H is S-quasinormal in G. Thus (i) holdes. □

Lemma 2.4 (see [7, Theorem 4, page 253]). *Let P be a normal p-subgroup of G. If the ALPE-subgroups of P are normal in G, then P is supersolvably embedded in G.*

Lemma 2.5 (see [11, Lemma 3.8, page 2245]). *Let p be the smallest prime dividing the order of a group G, and let G_p be a Sylow p-subgroup of G. If $\Omega(G_p) \leq \mathrm{genz}_\infty(G)$, then G is p-nilpotent.*

Lemma 2.6 (see [12, Lemma 2.6]). *Let N be a nontrivial normal subgroup of a group G. If $N \cap \Phi(G) = 1$, then the Fitting subgroup $F(N)$ of N is the direct product of minimal normal subgroups of G which are contained in $F(N)$.*

Lemma 2.7 (see [13, Lemma 3.3.1, page 23]). *Suppose that G_p is a normal Sylow p-subgroup of G and that $\Omega(G_p)K$ is supersolvable, where K is a p'-Hall subgroup of G. Then G is supersolvable.*

3. Main Results

Theorem 3.1. *Let p be the smallest prime dividing the order of a group G, and let G_p be a Sylow p-subgroup of G. Fix an ALPE-subgroup P of G_p having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is p-nilpotent.*

Proof. Suppose that the theorem is false, and let G be a counterexample of minimal order. We prove the following steps.

(1) If $P \leq M < G$, Then M Is p-Nilpotent

It is clear to see by Lemma 2.1 that the ALPE-subgroups of P are S-semipermutale in M, so that M satisfies the hypothesis of the theorem. Thus, the minimality of G yields that M is p-nilpotent.

(2) $N_G(P)$ Is p-Nilpotent

Suppose that P is normal in G. Let H be an ALPE-subgroup of P (in particular, we may take $H = P$). By hypothesis, H is S-semipermutable in G and so by Lemma 2.3, we have that H is S-quasinormal in G. Hence HG_q is a subgroup of G, where G_q is a Sylow q-subgroup of G with $q \neq p$. Clearly, H is a subnormal Hall subgroup of HG_q. Thus H is normal in HG_q and hence H is normal in PG_q as P is abelian. Thus P is supersolvably embedded in PG_q by Lemma 2.4 and so $P \leq Q_\infty(PG_q)$. Since $Q_\infty(PG_q) \leq \mathrm{genz}_\infty(PG_q)$ by [14, page 34], it follows by Lemma 2.5 that PG_q is p-nilpotent. Thus $PG_q = P \times G_q$. Hence $G_q \leq C_G(P)$, so that $O^p(G) \leq C_G(P)$. If $C_G(P) < G$, then $C_G(P)$ is p-nilpotent by (1). Thus $O^p(G)$ is p-nilpotent and so G is p-nilpotent: a contradiction. Thus we may assume that $C_G(P) = G$. Then $P \leq Z(G)$, in particular, $P \leq Z(G_p)$. So, $P = G_p$ by the maximality of P and we have by [15, Theorem 4.3, page 252] that G is p-nilpotent: a contradiction. Thus we may assume that $N_G(P) < G$. According to (1), we have that $N_G(P)$ is p-nilpotent.

(3) $O_{p'}(G)=1$

If $O_{p'}(G)\neq 1$, we consider the quotient group $G/O_{p'}(G)$. Clearly, $G_pO_{p'}(G)/O_{p'}(G)$ is a Sylow p-subgroup of $G/O_{p'}(G)$ and $PO_{p'}(G)/O_{p'}(G)$ is an ALPE-Subgroup of $G_pO_{p'}(G)/O_{p'}(G)$ having maximal order. By Lemma 2.1, the hypotheses are inherited over $G/O_{p'}(G)$. Thus, the minimality of G implies that $G/O_{p'}(G)$ is p-nilpotent, hence G is p-nilpotent, which is a contradiction.

(4) $G=G_pG_q$, Where G_q Is a Sylow q-Subgroup of G with $q\neq p$

Since G is not p-nilpotent by [15, Theorem 4.5, page 253], there exists a subgroup H of G_p such that $N_G(H)$ is not p-nilpotent. But $N_G(G_p)$ is p-nilpotent by a similar argument of the proof of the step (2). Thus we may choose a subgroup H of G_p such that $N_G(H)$ is not p-nilpotent but $N_G(K)$ is p-nilpotent for every subgroup K of G_p with $H<K\leq G_p$. It is easy to see that $N_G(G_p)\leq N_G(H)\leq G$. If $N_G(H)<G$, it follows by (1) that $N_G(H)$ is p-nilpotent: a contradiction. Thus $N_G(H)=G$. This leads to $O_p(G)\neq 1$ and $N_G(K)$ is p-nilpotent for every subgroup K of G_p with $O_p(G)<K\leq G_p$. Now, by [15, Theorem 4.5, page 253] again, we see that $G/O_p(G)$ is p-nilpotent and therefore that G is p-solvable. Since G is p-solvable, for any $q\in\pi(G)$ with $q\neq p$, there exists a Sylow q-subgroup G_q of G such that $G_pG_q\leq G$ by [15, Theorem 3.5, page 229]. If $G_pG_q<G$, then G_pG_q is p-nilpotent by (1) and hence $O_p(G)G_q$ is p-nilpotent. Thus $O_p(G)G_q=O_p(G)\times G_q$. This leads to $G_q\leq C_G(O_p(G))\leq O_p(G)$ by [15, Theorem 3.2, page 228] as $O_{p'}(G)=1$ by (3), which is a contradiction. Thus $G=G_pG_q$.

(5) The Final Contradiction

Let N be a minimal normal subgroup of G such that $N\leq O_p(G)$. Clearly, $N\cap Z(G_p)\neq 1$ and so $Z(G_p)\leq P$ by the maximality of P. Hence $1\neq N\cap Z(G_p)\leq N\cap P$. By hypothesis, $PG_q\leq G$ for any Sylow q-subgroup G_q of G with $(q,|P|)=1$. It is easy to see that $N\cap P=N\cap PG_q\lhd PG_q$. Thus $O^p(G)\leq N_G(N\cap P)$. If $N_G(N\cap P)<G$, then by (1), $N_G(N\cap P)$ is p-nilpotent. Hence $O^p(G)$ is p-nilpotent and so also does G: a contradiction. Thus we may assume that $N_G(N\cap P)=G$. By the minimality of N and since $N\cap P\neq 1$, we have that $N\cap P=N$ and so $N\leq P$. If $PG_q<G$, then PG_q is p-nilpotent by (1) and hence NG_q is p-nilpotent. Thus $NG_q=N\times G_q$ and so $G_q\leq C_G(N)$. Thus by (4), $G/C_G(N)$ is a p-group and so by [14, Theorem 6.3, page 221], $N\leq Z_\infty(G)$. Since $Z_\infty(G)\leq Q_\infty(G)$, we have that $N\leq Q_\infty(G)$ which implies that N is supersolvably embedded in G and so clearly that $|N|=p$. Thus, it is easy to see that the quotient group G/N satisfies the hypothesis of the theorem by Lemma 2.1. Now, by the minimality of G, we see that G/N is p-nilpotent. Since the class of all p-nilpotent groups is a saturated formation, we have that N is the unique minimal normal subgroup of G and $N\nleq\Phi(G)$. Thus $\Phi(G)=1$ and hence $N=O_p(G)$ by Lemma 2.6 and so $F(G)=O_p(G)=N$ by (3). Hence $G_q\leq C_G(F(G))$. Since G is solvable, it follows by [15, Theorem 2.6, page 216] that $C_G(F(G))\leq F(G)=O_p(G)$: a contradiction. Thus we must have $G=PG_q$. Let G_q^* be a Sylow q-subgroup of $N_G(P)$. By (2), we have that $G_q^*\lhd N_G(P)$. Hence $N_G(P)=PG_q^*=P\times G_q^*$. Thus $P\leq Z(N_G(P))$, and, therefore, G is p-nilpotent by [15, Theorem 4.3, page 252]: a final contradiction. □

We need the following result.

Theorem 3.2. *Let $\mathfrak{F}$ be a saturated formation containing the class of supersolvable groups $\mathcal{U}$. Let G_p be a normal Sylow p-subgroup of a group G such that $G/G_p \in \mathfrak{F}$. Fix an ALPE-subgroup P of G_p having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then $G \in \mathfrak{F}$.*

Proof. We treat the following two cases.

Case 1. $O_{p'}(G) \neq 1$.

Clearly, $G_pO_{p'}(G)/O_{p'}(G)$ is a normal Sylow p-subgroup of $G/O_{p'}(G)$ and $PO_{p'}(G)/O_{p'}(G)$ is an ALPE-subgroup of $G_pO_{p'}(G)/O_{p'}(G)$ having maximal order. By hypothesis and Lemma 2.1, the ALPE-subgroups of $PO_{p'}(G)/O_{p'}(G)$ are S-semipermutable in $G/O_{p'}(G)$. Clearly,

$$\frac{(G/G_p)}{(G_pO_{p'}(G)/G_p)} \cong \frac{G}{G_pO_{p'}(G)} \cong \frac{(G/O_{p'}(G))}{(G_pO_{p'}(G)/O_{p'}(G))} \in \mathfrak{F}. \tag{3.1}$$

Thus, our hypothesis carries over to $G/O_{p'}(G)$ and so $G/O_{p'}(G) \in \mathfrak{F}$ by induction on the order of G. Therefore, $G/(O_{p'}(G) \cap G_p) \cong G \in \mathfrak{F}$.

Case 2. $O_{p'}(G) = 1$.

Let H be an ALPE-subgroup of P. Then H is S-quasinormal in G by Lemma 2.3 and hence $O^p(G) \leq N_G(H)$ by Lemma 2.2. Let $T = PO^p(G)$. Then H is normal in T. Thus Lemma 2.4 implies that P is supersolvably embedded in T. Then, $T/C_T(P)$ is supersolvable by [14, Lemma 7.15, page 35]. Clearly, $T_p = G_p \cap T \lhd T$, where T_p is a Sylow p-subgroup of T. Let Q be a p'-subgroup of $C_T(P)$. Then $QP = Q \times P$ is a group of automorphisms of $T_p = O_p(T)$. But $C_{T_p}(P) = P$, and consequently, Q acts trivially on $C_{T_p}(P)$. Then Q acts trivially on T_p by [15, Theorem 3.4, page 179], that is, $Q \leq C_T(T_p)$. It is easy to see that T is subnormal in G and so $O_{p'}(T) \leq O_{p'}(G) = 1$. Hence $F(T) = T_p$. Since T is solvable, it follows by [15, Theorem 2.6, page 216] that $Q \leq C_T(F(T)) \leq F(T) = T_p$: a contradiction. Hence $C_T(P)$ must be a p-group and so $C_T(P) = P$. Thus, $T/C_T(P) = T/P$ is supersolvable which implies that T is supersolvable by [16, Theorem 4]. Thus $O^p(G)$ is supersolvable and therefore, $G = G_pO^p(G)$ is supersolvable by [17, Exercise 7.2.23, page 159]. Hence, $G \in \mathcal{U} \subseteq \mathfrak{F}$.

□

As an immediate consequence of Theorem 3.2, we have the following theorem.

Corollary 3.3. *Let G_p be a normal Sylow p-subgroup of a group G such that G/G_p is supersolvable. Fix an ALPE-subgroup P of G_p having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

We now prove the following theorem.

Theorem 3.4. *Let G be a group. For every prime p and every Sylow p-subgroup G_p of G, fix an ALPE-subgroup P of G_p having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

Proof. By repeated applications of Theorem 3.1, the group G has a Sylow tower of supersolvable type. Hence G has a normal Sylow p-subgroup G_p, where p is the largest prime dividing the order of G. By Lemma 2.1, our hypothesis carries over to G/G_p. Thus G/G_p is supersolvable by induction on the order of G. Now, it follows from Corollary 3.3 that G is supersolvable. □

As an immediate consequence of Theorem 3.4, we have the following corollary.

Corollary 3.5 (Asaad et al. [7]). *If G is a group such that the ALPE-subgroups of every Sylow subgroup of G are normal in G, then G is supersolvable.*

Corollary 3.6 (Ramadan [8]). *If G is a group such that the ALPE-subgroups of every Sylow subgroup of G are S-quasinormal in G, then G is supersolvable.*

We need the following Lemma.

Lemma 3.7. *Let K be a normal p-subgroup of a group G such that G/K is supersolvable. Fix an ALPE-subgroup P of K having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

Proof. Let G_p be a Sylow p-subgroup of G. We treat the following two cases.

Case 1. $K = G_p$.

Then by Corollary 3.3, G is supersolvable.

Case 2. $K < G_p$.

Put $\pi(G) = \{p_1, p_2, \ldots, p_n\}$, where $p_1 > p_2 > \cdots > p_n$. Since G/K is supersolvable, it follows by [18, Theorem 5, page 5] that G/K possesses supersolvable subgroups M/K and L/K such that $|G/K : M/K| = p_1$ and $|G/K : L/K| = p_n$. Since M/K and L/K are supersolvable, it follows that M and L are supersolvable by induction on the order of G. Since $|G : M| = |G/K : M/K| = p_1$ and $|G : L| = |G/K : L/K| = p_n$, it follows again by [18, Theorem 5, page 5] that G is supersolvable. □

Now, we can prove the following theorem.

Theorem 3.8. *Let K be a normal subgroup of G such that G/K is supersolvable. For every prime p dividing the order of K and every Sylow p-subgroup K_p of K, fix an ALPE-subgroup P of K_p having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

Proof. By Lemma 2.1, the ALPE-subgroups of P are S-semipermutable in K. Hence K is supersolvable by Theorem 3.4. Thus K has a normal Sylow p-subgroup K_p, where p is the largest prime dividing the order of K. Since K_p is characteristic in K and $K \lhd G$, we have that $K_p \lhd G$. Clearly, $(G/K_p)/(K/K_p) \cong G/K$ is supersolvable. By Lemma 2.1, our hypothesis carries over to G/K_p and hence G/K_p is supersolvable by induction on the order of G. Now, it follows from Lemma 3.7 that G is supersolvable. □

As an immediate consequence of Theorem 3.8, we have the following corollary.

Corollary 3.9 (Ramadan [8]). *Let K be a normal subgroup of a group G such that G/K is supersolvable. If the ALPE-subgroups of every Sylow subgroup of K are S-quasinormal in G, then G is supersolvable.*

4. Similar Results

Following similar arguments to those of Theorem 3.1, it is possible to prove the following result.

Theorem 4.1. *Let p be the smallest prime dividing the order of a group G and let G_p be a Sylow p-subgroup of G. Fix an ALPE-subgroup P of $\Omega(G_p)$ having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is p-nilpotent.*

We prove the following lemma.

Lemma 4.2. *Let K be a normal p-subgroup of a group G such that G/K is supersolvable. Fix an ALPE-subgroup P of $\Omega(K)$ having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

Proof. Let G_p be a Sylow p-subgroup of G. We treat the following two cases.

Case 1. By [15, Theorem 2.1, page 221], there exists a p'-Hall subgroup T, which is a complement to G_p in G. Hence $G/G_p \cong T$ is supersolvable. Since $\Omega(G_p)$ is characteristic in G_p and $G_p \lhd G$, we have that $\Omega(G_p) \lhd G$. Clearly, $\Omega(G_p)T/\Omega(G_p) \cong T$ is supersolvable. Thus, our hypothesis and Corollary 3.3 imply that $\Omega(G_p)T$ is supersolvable. Therefore, G is supersolvable by Lemma 2.7.

Case 2. Put $\pi(G) = \{p_1, p_2, \dots, p_n\}$, where $p_1 > p_2 > \cdots > p_n$. Since G/K is supersolvable, it follows by [18, Theorem 5, page 5] that G/K possesses supersolvable subgroups M/K and L/K such that $|G/K : M/K| = p_1$ and $|G/K : L/K| = p_n$. Since M/K and L/K are supersolvable, it follows that M and L are supersolvable by induction on the order of G. Since $|G : M| = |G/K : M/K| = p_1$ and $|G : L| = |G/K : L/K| = p_n$, it follows again by [18, Theorem 5, page 5] that G is supersolvable.

□

By a similar proof to the proof of Theorem 3.4, we can prove the following theorem.

Theorem 4.3. *Let G be a group. For every prime p and every Sylow p-subgroup G_p of G, fix an ALPE-subgroup P of $\Omega(G_p)$ having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

As an immediate consequence of Theorem 4.3, we have the following corollary.

Corollary 4.4 (Asaad et al. [7]). *If G is a group such that for every prime p and every Sylow p-subgroup G_p, the ALPE-subgroups of $\Omega(G_p)$ are normal in G, then G is supersolvable.*

Corollary 4.5 (Ramadan [8]). *If G is a group such that for every prime p and every Sylow p-subgroup G_p, the ALPE-subgroups of $\Omega(G_p)$ are S-quasinormal in G, then G is supersolvable.*

We can now prove the following corollary.

Corollary 4.6. *Let K be a normal subgroup of G such that G/K is supersolvable. For every prime p dividing the order of K and every Sylow p-subgroup K_p of K, fix an ALPE-subgroup P of $\Omega(K_p)$ having maximal order. If the ALPE-subgroups of P are S-semipermutable in G, then G is supersolvable.*

Proof. By Lemma 2.1, the ALPE-subgroups of P are S-semipermutable in K. Hence K is supersolvable by Theorem 4.3. Thus K has a normal Sylow p-subgroup K_p, where p is the largest prime dividing the order of K. Since K_p is characteristic in K and $K \lhd G$, we have that $K_p \lhd G$. Clearly, $(G/K_p)/(K/K_p) \cong G/K$ is supersolvable. By Lemma 2.1, the hypothesis of our theorem carries over to G/K_p. Thus G/K_p is supersolvable by induction on the order of G and it follows that G is supersolvable by Lemma 4.2. □

Remarks 4.7. (a) The converse of Theorem 3.4 is not true. For example, set $G = S_3 \times Z_3$, where $S_3 = \langle x, y \mid x^3 = y^2 = 1,\ yx = x^2y \rangle$ and $Z_3 = \langle z \mid z^3 = 1 \rangle$. Clearly, G is supersolvable and G has an abelian Sylow 3-subgroup of exponent 3. It is easy to check that G contains a subgroup $\langle xz \rangle$ of order 3 which fails to be S-semipermutable in G.

(b) Theorem 4.3 is not true when the smallest prime dividing the order of G is even and $\Omega(G_p) = \Omega_1(G_p)$, where G_p is a Sylow p-subgroup of G. For example, if Q is the quaternion group $\langle a, b \mid a^4 = 1,\ b^2 = a^2,\ b^{-1}ab = a^{-1} \rangle$, C_9 is a cyclic group of order 9 with generator c, and the action of C_9 on Q is given by $a^c = b,\ b^c = ab$, then the semidirect product of Q by C_9 is a group of even order in which every subgroup of prime order is S-semipermutable. Clearly, the semidirect product of Q by C_9 is not supersolvable (see Buckley [4, Examples (ii)]).

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