

Research Article

Traveling-Wave Solution of Modified Liouville Equation by Means of Modified Simple Equation Method

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We construct the traveling wave solutions involving parameters of modified Liouville equation by using a new approach, namely the modified simple equation method. The proposed method is direct, concise, and elementary and can be used for many other nonlinear evolution equations.

1. Introduction

The investigation of the traveling-wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Several direct methods for finding the explicit traveling-wave solutions to nonlinear partial differential equations have been proposed, such as the tanh-function method and its various extension [1], the Jacobi elliptic function expansion method [2], the homogeneous balance method [3, 4], the F-expansion method and its extension [5], the variational iteration method [6], (G'/G) -expansion method [7], and so on. More recently, a new method, named modified simple equation method [8, 9], has been proposed to construct more explicit traveling-wave solutions of modified Liouville equation.

2. Description of the Modified Simple Equation Method

Suppose that a nonlinear equation, say in two independent variables x and t is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0, \quad (2.1)$$

where $u = u(x, t)$ is an unknown function, P is a polynomial in $u = u(x, t)$ and its various partial derivatives, in which the highest-order derivatives and nonlinear terms are involved. In the following, the main steps of the modified simple equation method are given.

Step 1. The traveling-wave variable

$$u(x, t) = u(\xi), \quad \text{where } \xi = Ax + Bt \quad (2.2)$$

permits us reducing (2.1) to an ODE for $u = u(\xi)$ in the form

$$P(u, -Vu', u', V^2u'', -Vu'', u'', \dots) = 0. \quad (2.3)$$

Step 2. Suppose that the solution of ODE (2.1) can be expressed by a polynomial in (ψ'/ψ) as follows:

$$u(\xi) = \sum_{i=0}^n \alpha_i \left(\frac{\psi'}{\psi} \right)^i, \quad (2.4)$$

where α_i are arbitrary constants to be determined such that $\alpha_n \neq 0$, while $\psi(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer n by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (2.3).

Step 4. We substitute (2.4) into (2.3), we calculate all the necessary derivatives u', u'', \dots , and then we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of ψ'/ψ and its derivatives. In this polynomial, we equate with zero all the coefficients of it. This operation yields a system of equations which can be solved to find α_i and $\psi(\xi)$. Consequently, we can get the exact solution of (2.1).

3. Application of the Method

In this section, we would like to use our method to obtain new and more general exact traveling wave solutions of the modified Liouville equation

$$w_{tt} = a^2 w_{xx} + b e^{\beta w}, \quad (3.1)$$

where a, b , and β are arbitrary constants.

Suppose $e^{\beta w} = u(x, t)$, where the traveling-wave transformation is

$$u(x, t) = u(\xi), \quad \xi = Ax + Bt. \quad (3.2)$$

By using the traveling-wave variable (3.2), (3.1) is converted into an ODE for $u = u(\xi)$

$$uu'' - u'^2 + ku^3 = 0, \quad \text{where } k = \frac{b\beta}{a^2 A^2 - B^2}. \quad (3.3)$$

Suppose that the solution of the ODE (3.3) can be expressed by a polynomial in (ψ'/ψ) as follows:

$$u(\xi) = \sum_{i=0}^n \alpha_i \left(\frac{\psi'}{\psi} \right)^i, \quad (3.4)$$

where α_i are arbitrary constants provided $\alpha_n \neq 0$.

Considering the homogeneous balance between the highest order derivatives and the nonlinear terms in (3.3), we get $n = 2$ and hence the solution takes the following form:

$$u(\xi) = \alpha_0 + \alpha_1 \left(\frac{\psi'}{\psi} \right) + \alpha_2 \left(\frac{\psi'}{\psi} \right)^2, \quad (3.5)$$

where $\alpha_2 \neq 0$. On substituting (3.5) into the ODE (3.3) and equating all the coefficients of $\psi^{-1}, \psi^{-2}, \psi^{-3}, \psi^{-4}, \psi^{-5}, \psi^{-6}$ to zero, we, respectively, obtain

$$k\alpha_0^3 = 0, \quad (3.6)$$

$$3k\alpha_0^2\alpha_1\psi' + \alpha_0\alpha_1\psi''' = 0, \quad (3.7)$$

$$\begin{aligned} 2\alpha_0\alpha_2\psi''^2 - \alpha_1^2\psi''^2 - 3\alpha_0\alpha_1\psi''\psi' + 3k\alpha_0^2\alpha_2\psi'^2 + 3k\alpha_0\alpha_1^2\psi'^2 \\ + 2\alpha_0\alpha_2\psi'\psi''' + \alpha_1^2\psi'\psi''' = 0, \end{aligned} \quad (3.8)$$

$$\begin{aligned} k\alpha_1^3\psi'^3 + 2\alpha_0\alpha_1\psi'^3 - 2\alpha_1\alpha_2\psi'\psi''^2 - \alpha_1^2\psi'^2\psi'' + 3\alpha_1\alpha_2\psi'^2\psi''' \\ - 10\alpha_0\alpha_2\psi'^2\psi'' + 6k\alpha_0\alpha_1\alpha_2\psi'^3 = 0, \end{aligned} \quad (3.9)$$

$$\begin{aligned} -2\alpha_2^2\psi'^2\psi''^2 + 2\alpha_2^2\psi'^3\psi''' + \alpha_1^2\psi'^4 + 3k\alpha_0\alpha_2^2\psi'^4 + 3k\alpha_1^2\alpha_2\psi'^4 \\ + 6\alpha_0\alpha_2\psi'^4 - 5\alpha_1\alpha_2\psi'^3\psi'' = 0, \end{aligned} \quad (3.10)$$

$$3k\alpha_1\alpha_2^2\psi'^5 - 2\alpha_2^2\psi'^4\psi'' + 4\alpha_1\alpha_2\psi'^5 = 0, \quad (3.11)$$

$$k\alpha_2^3\psi'^6 + 2\alpha_2^2\psi'^6 = 0. \quad (3.12)$$

Equations (3.6), (3.8), and (3.12) give $\alpha_0 = 0$, $\alpha_1 = 0$, $\alpha_2 = -2/k$ ($\psi' \neq 0$, otherwise it is the trivial case); those satisfy (3.7) and (3.9), and (3.10), (3.11), respectively, yields

$$\psi''^2 - \psi'\psi''' = 0, \quad (3.13)$$

$$\psi'' = 0. \quad (3.14)$$

Equation (3.14) gives $\psi'' = 0$. Integrating $\psi'' = 0$ with respect to ξ , we get $\psi = C_1 + C_2\xi$ and the solution of ODE (3.3) takes the following form:

$$u(\xi) = \frac{-2}{k} \left(\frac{C_1}{C_1 + C_2\xi} \right)^2 = \frac{2(B^2 - a^2A^2)}{b\beta} \left(\frac{C}{Ax + Bt + C} \right)^2, \quad \text{where } C = \frac{C_1}{C_2}. \quad (3.15)$$

And finally the traveling-wave solution of (3.1) is

$$w = \frac{1}{\beta} \ln \left[\frac{2(B^2 - a^2 A^2)}{b\beta} \left(\frac{C}{Ax + Bt + C} \right)^2 \right]. \quad (3.16)$$

4. Conclusion

On comparing this method with the other methods via the tanh-function method, homogeneous balance method, and the (G'/G) -expansion method used in [1, 4, 7], we see that the modified simple equation method is much more simpler than these methods because these methods have used the computer programs, while the modified simple equation method has not used these programs. Also we deduce that the modified simple equation method is effective and standard which allows us to solve complicated nonlinear evolution equations in the mathematical physics.

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