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Research Article

On a Quasi-Neutral Approximation of the Incompressible Navier-Stokes Equations

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This paper considers a pressureless Euler-Poisson system with viscosity in plasma physics in the torus $\mathbf{T}^3$. We give a rigorous justification of its asymptotic limit toward the incompressible Navier Stokes equations via quasi-neutral regime using the modulated energy method.

1. Introduction

We will consider the following system:

$$\begin{gathered}
\partial_t \mathbf{u}^\epsilon + (\mathbf{u}^\epsilon \cdot \nabla)\mathbf{u}^\epsilon = \mu \Delta \mathbf{u}^\epsilon + \nabla \mathcal{V}^\epsilon, \\
\partial_t n^\epsilon + \operatorname{div}(n^\epsilon \mathbf{u}^\epsilon) = 0, \\
\Delta \mathcal{V}^\epsilon = \frac{n^\epsilon - 1}{\epsilon^2},
\end{gathered} \tag{1.1}$$

for $x \in \mathbf{T}^3$ and $t > 0$, $n^\epsilon \in \mathbb{R}$, $\mathbf{u}^\epsilon \in \mathbb{R}^2$. ϵ is small parameter and $\mu > 0$ is a constant viscosity coefficient. To solve uniquely the Poisson equation, we add the $\int_{\mathbf{T}^3} n^\epsilon dx = 1$. Passing to the limit when $\epsilon \to 0$, it is easy to see, at least at a very formal level, that $(n^\epsilon, \mathbf{u}^\epsilon)$ tends to $(n^{\mathrm{NS}}, \mathbf{u}^{\mathrm{NS}})$, where $n^{\mathrm{NS}} = 1$ and

$$\begin{gathered}
\partial_t \mathbf{u}^{\mathrm{NS}} + \left(\mathbf{u}^{\mathrm{NS}} \cdot \nabla\right)\mathbf{u}^{\mathrm{NS}} = \mu \Delta \mathbf{u}^{\mathrm{NS}} + \nabla \mathcal{V}^{\mathrm{NS}}, \\
\operatorname{div} \mathbf{u}^{\mathrm{NS}} = 0.
\end{gathered} \tag{1.2}$$

In other words, $\mathbf{u}^{\mathrm{NS}}$ is a solution of the incompressible Navier-Stokes equations. The aim of this paper is to give a rigorous justification to this formal computation.

The Euler-Poisson system with viscosity (1.1) is a physical model involving dissipation see [1], which here could be regarded as a viscous approximation of Euler-Poisson. Formally, it is a kind of new approximation of the incompressible Navier-Stokes equations of viscous fluid in real world.

It should be pointed out that there have been a lot of interesting results about the topic on the quasi-neutral (or called zero-Debye length) limit, for the readers to see [2–5] for isentropic Euler-Poisson system, [6, 7] for nonisentropic Euler-Poisson system, [8–10] for Vlasov-Poisson system, [11, 12] for drift-diffusion system, [13] for Euler-Maxwell equations, and therein references. We also mention that the above limit has been studied in [14, 15]. But in this present paper, the convergence result and the method of its proof is different from that of [14, 15].

The main focus in this paper is on the use of modulated energy techniques and div-curl for studying incompressible fluids. And for that, we assume that $n^{\epsilon}(x,\cdot)$ has total mass equal to 1 and the mean values of $\mathbf{u}^{\epsilon}$ vanish, that is, $\mathbf{m}(\mathbf{u}^{\epsilon}) = (1/(2\pi)^3)\int_{\mathbf{T}^3} u^{\epsilon} dx = 0$. We also restrict ourselves to the case of well-prepared initial data and the case of periodic torus. Indeed, the quasi-neutral limit is much more difficult without these assumptions.

In this note, we will use some inequalities in Sobolev spaces, such as basic Moser-type calculus inequalities, Young inequality, and Gronwall inequality.

The paper is organized as follows. In Section 2 we state our main result. Estimates and proofs are given in Section 3.

2. Main Result

Throughout the paper, we will denote by C a number independent of ϵ, which actually may change from line to line. Moreover $(\cdot,\cdot)$ and $\|\cdot\|$ stand for the usual L^2 scalar product and norm, $\|\cdot\|_s$ is the usual H^s Sobolev norm, and $\|\cdot\|_{s,\infty}$ is the usual $W^{s,\infty}$ norm.

The study of the asymptotic behavior of the sequence $(\mathbf{u}^{\epsilon}, n^{\epsilon})$, as ϵ goes to zero, leads to the statement of our main result.

Theorem 2.1. *Let $\mathbf{u}^{NS}$ be a solution of the incompressible Euler equations* (1.2) *such that $\mathbf{u}^{NS} \in ([0,T], H^{s+3}(\mathbf{T}^3))$ and $\int_{\mathbf{T}^3} \mathbf{u}^{NS} dx = 0$ for $s > (5/2)$. Assume that $(n_0^{\epsilon}, \mathbf{u}_0^{\epsilon})$ be a sequence of initial data such that $\int_{\mathbf{T}^3} n_0^{\epsilon}\, dx = 1$, $\int_{\mathbf{T}^3} \mathbf{u}_0^{\epsilon}\, dx = 0$ and*

$$\begin{gathered}\left\|\mathbf{u}_0^{\epsilon} - \mathbf{u}_0^{NS}\right\|_{s+1} \le C\epsilon, \\ \left\|n_0^{\epsilon} - 1\right\|_s \le C\epsilon^2\end{gathered} \tag{2.1}$$

with $\mathbf{u}_0^{NS} = \mathbf{u}^{NS}\,|_{t=0}$. Then there is a sequence $(n^{\epsilon}, \mathbf{u}^{\epsilon}) \in C([0,T], H^s \times H^{s+1}(\mathbf{T}^3))$ of solutions to (1.1) *with initial data $(n_0^{\epsilon}, \mathbf{u}_0^{\epsilon})$ belonging to $C([0,T_{\epsilon}], H^s \times H^{s+1}(\mathbf{T}^3))$ with $\liminf_{\epsilon\to 0} T_{\epsilon} \ge T$. Moreover for any $T_1 < T$ and ϵ small enough,*

$$\begin{gathered}\left\|\mathbf{u}^{\epsilon}(t) - \mathbf{u}^{NS}(t)\right\|_s \le C\epsilon, \\ \|n^{\epsilon}(t) - 1\|_s \le C\epsilon^2,\end{gathered} \tag{2.2}$$

for any $0 \le t \le T_1$.

3. Proof of the Theorem

If $(\mathbf{u}^\epsilon, n^\epsilon)$ is a solution to system (1.1), we introduce

$$\mathbf{u}^\epsilon = u^{\mathrm{NS}} + \epsilon \mathbf{u},$$
$$n^\epsilon = 1 + \epsilon^2 \left(n + \Delta \mho^{\mathrm{NS}} \right), \tag{3.1}$$
$$\mho^\epsilon = \mho^{\mathrm{NS}} + \mho.$$

Since the pressure $\mho^{\mathrm{NS}}$ in the incompressible Navier-Stokes equation is given by

$$\Delta \mho^{\mathrm{NS}} = \nabla \mathbf{u}^{\mathrm{NS}} : \nabla \mathbf{u}^{\mathrm{NS}}, \tag{3.2}$$

where, $\nabla \mathbf{u} : \nabla \mathbf{v} = \sum_{i,j=1}^{3} (\partial_{x_i} \mathbf{u} / \partial_{x_j})(\partial_{x_j} \mathbf{v} / \partial_{x_i})$. Then the vector $(\mathbf{u}^1, n^1, \mho^1)$ solves the system

$$\begin{gathered}
\partial_t \mathbf{u} + \mathbf{u}^{\mathrm{NS}} \cdot \nabla \mathbf{u} = \frac{\nabla \mho}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{u}^{\mathrm{NS}} + \mu \Delta \mathbf{u}, \\
\partial_t n + \mathbf{u}^{\mathrm{NS}} \cdot \nabla n = -\frac{\operatorname{div} \mathbf{u}}{\epsilon} - \epsilon \operatorname{div}\left(\left(n + \Delta \mho^{\mathrm{NS}} \right) \mathbf{u} \right) - \partial_t \Delta \mho^{\mathrm{NS}} - \mathbf{u}^{\mathrm{NS}} \cdot \nabla \Delta \mho^{\mathrm{NS}}, \\
\Delta \mho = n.
\end{gathered} \tag{3.3}$$

As in [16], we make the following change of unknowns:

$$d = \operatorname{div} \mathbf{u}, \qquad \mathbf{c} = \operatorname{curl} \mathbf{u}. \tag{3.4}$$

By using the last equation and taking the curl and the divergence of the first equation in (3.5), we get the following system:

$$\begin{gathered}
\partial_t d + \mathbf{u}^{\mathrm{NS}} \cdot \nabla d = \frac{n}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) d - \epsilon \nabla \mathbf{u} : \nabla \mathbf{u} - \nabla \mathbf{u} : \nabla \mathbf{u}^{\mathrm{NS}} + \mu \Delta d, \\
\partial_t \mathbf{c} + \mathbf{u}^{\mathrm{NS}} \cdot \nabla \mathbf{c} = -\epsilon (\mathbf{u} \cdot \nabla) \mathbf{c} - \epsilon (\mathbf{c} \cdot \nabla) \mathbf{u} - (\mathbf{c} \cdot \nabla) \mathbf{u}^{\mathrm{NS}} \\
+ \operatorname{curl}\left(\nabla \mathbf{u}^{\mathrm{NS}} \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u}^{\mathrm{NS}} \right) - \epsilon d \mathbf{c} + \mu \Delta d, \\
\partial_t n + \mathbf{u}^{\mathrm{NS}} \cdot \nabla n = -\frac{d}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) n - \epsilon \left(n + \Delta \mho^{\mathrm{NS}} \right) d - \mathbf{u} \cdot \nabla \Delta \mho^{\mathrm{NS}} - \left(\partial_t + \mathbf{u}^{\mathrm{NS}} \cdot \nabla \right) \Delta \mho^{\mathrm{NS}}.
\end{gathered} \tag{3.5}$$

This last system can be written as a singular perturbation of a quasilinear symmetrizable hyperbolic system. Setting $\mathbb{W}^\epsilon = (d, \mathbf{c}, n)^T$ yields

$$\partial_t \mathbb{W}^\epsilon + A(t, x, \partial_x) \mathbb{W}^\epsilon = \frac{1}{\epsilon} K \mathbb{W}^\epsilon - \epsilon B(t, x, \partial_x) \mathbb{W}^\epsilon + S(\mathbb{W}^\epsilon) + \mu N(\mathbb{W}^\epsilon) + R, \tag{3.6}$$

where

$$A(t,x,\partial_x)=\operatorname{diag}\left(\mathbf{u}^{\mathrm{NS}}\cdot\nabla,\mathbf{u}^{\mathrm{NS}}\cdot\nabla I_3,\mathbf{u}^{\mathrm{NS}}\cdot\nabla\right),B(t,x,\partial_x)=\operatorname{diag}(\mathbf{u}\cdot\nabla,\mathbf{u}\cdot\nabla I_3,\mathbf{u}\cdot\nabla),$$
$$K=\begin{pmatrix}0&\mathbf{0}&1\\ \mathbf{0}&\mathbf{0}&\mathbf{0}\\ -1&\mathbf{0}&0\end{pmatrix},\qquad N(\mathbb{W}^\epsilon)=\begin{pmatrix}\Delta d\\ \Delta\mathbf{c}\\ 0\end{pmatrix},\qquad R=\begin{pmatrix}0\\ \mathbf{0}\\ -\left(\partial_t+\mathbf{u}^{\mathrm{NS}}\cdot\nabla\right)\Delta\mho^{\mathrm{NS}}\end{pmatrix},\tag{3.7}$$
$$S(\mathbb{W}^\epsilon)=\begin{pmatrix}-\epsilon\nabla\mathbf{u}:\nabla\mathbf{u}-\nabla\mathbf{u}:\nabla\mathbf{u}^{\mathrm{NS}}\\ -\epsilon(\mathbf{c}\cdot\nabla)\mathbf{u}-(\mathbf{c}\cdot\nabla)\mathbf{u}^{\mathrm{NS}}+\operatorname{curl}\left(\nabla\mathbf{u}^{\mathrm{NS}}\cdot\mathbf{u}-\mathbf{u}\cdot\nabla\mathbf{u}^{\mathrm{NS}}\right)-\epsilon d\mathbf{c}\\ -\epsilon\left(n+\Delta\mho^{\mathrm{NS}}\right)d-\mathbf{u}\cdot\nabla\Delta\mho^{\mathrm{NS}}\end{pmatrix}.$$

For $|\alpha|\le s$ with $s>d/2$, we set

$$\mathcal{E}^\lambda_{\alpha,s}(t)=\frac{1}{2}\left(\|\partial_x^\alpha d\|^2+\|\partial_x^\alpha\mathbf{c}\|^2+\|\partial_x^\alpha n\|^2\right),$$
$$\mathcal{E}^\lambda_s(t)=\sum_{|\alpha|\le s}\mathcal{E}^\lambda_{\alpha,s}(t).\tag{3.8}$$

Before performing the energy estimate, we apply the operator ∂_x^α for $\alpha\in\mathbb{N}^3$ with $|\alpha|\le s$ to (3.6), to obtain

$$\begin{aligned}\partial_t\partial_x^\alpha\mathbb{W}^\epsilon+A(t,x,\partial_x)\partial_x^\alpha\mathbb{W}^\epsilon&=\frac{1}{\epsilon}K\partial_x^\alpha\mathbb{W}^\epsilon-\epsilon B(t,x,\partial_x)\partial_x^\alpha\mathbb{W}^\epsilon+\partial_x^\alpha S(\mathbb{W}^\epsilon)+\mu\partial_x^\alpha N(\mathbb{W}^\epsilon)\\&\quad+[\partial_x^\alpha,A(t,x,\partial_x)]\mathbb{W}^\epsilon-\epsilon[\partial_x^\alpha,B(t,x,\partial_x)]\mathbb{W}^\epsilon+\partial_x^\alpha R.\end{aligned}\tag{3.9}$$

Now, we proceed to perform the energy estimates for (3.9) in a classical way by taking the scalar product of system (3.9) with $\partial_x^\alpha\mathbb{W}^\epsilon$.

Let us start the estimate of each term. First, since $A(t,x,\partial_x)$ is symmetric and $\operatorname{div}\mathbf{u}^{\mathrm{NS}}=0$, we have that

$$(A(t,x,\partial_x)\partial_x^\alpha\mathbb{W}^\epsilon,\partial_x^\alpha\mathbb{W}^\epsilon)=-\int_{\mathbf{T}^3}\operatorname{div}\mathbf{u}^{\mathrm{NS}}\left(|\partial_x^\alpha d|^2+|\partial_x^\alpha\mathbf{c}|^2+|\partial_x^\alpha n|^2\right)dx=0.\tag{3.10}$$

Next, since K is skew-symmetric, we have that

$$\frac{1}{\epsilon}(K\partial_x^\alpha\mathbb{W}^\epsilon,\partial_x^\alpha\mathbb{W}^\epsilon)=0.\tag{3.11}$$

By integration by parts, we have

$$-\epsilon(B(t,x,\partial_x)\partial_x^\alpha\mathbb{W}^\epsilon,\partial_x^\alpha\mathbb{W}^\epsilon)=\epsilon\int_{\mathbf{T}^3}\operatorname{div}\mathbf{u}|\partial_x^\alpha\mathbb{W}^\epsilon|^2dx\le\|\operatorname{div}\mathbf{u}\|_{0,\infty}\mathcal{E}^\epsilon_s(t)\le(\mathcal{E}^\epsilon_s(t))^{3/2}.\tag{3.12}$$

For later estimates in this paper, we recall some results on Moser-type calculus inequalities in Sobolev spaces [17, 18].

Lemma 3.1. *Let $s \geq 1$ be an integer. Suppose $u \in H^s(\mathbf{T}^3)$, $\nabla u \in L^\infty(\mathbf{T}^3)$, and $v \in H^{s-1}(\mathbf{T}^3) \cap L^\infty(\mathbf{T}^3)$. Then for all multi-indexes $|\alpha| \leq s$, one has $(\partial_x^\alpha(uv) - u\partial_x^\alpha v) \in L^2(\mathbf{T}^3)$ and*

$$\|\partial_x^\alpha(uv) - u\partial_x^\alpha v\| \leq C_s\Big(\|\nabla u\|_{0,\infty}\Big\|D^{|\alpha|-1}v\Big\| + \Big\|D^{|\alpha|}u\Big\|\|v\|_{0,\infty}\Big), \tag{3.13}$$

where

$$\Big\|D^h u\Big\| = \sum_{|\alpha|=h}\|\partial_x^\alpha u\|, \quad \forall h \in \mathbb{N}. \tag{3.14}$$

Moreover, if $s \geq 3$, then the embedding $H^{s-1}(\mathbf{T}^3) \hookrightarrow L^\infty(\mathbf{T}^3)$ is continuous and one has

$$\|uv\|_{s-1} \leq C_s\|u\|_{s-1}\|v\|_{s-1}, \qquad \|\partial_x^\alpha(uv) - u\partial_x^\alpha v\| \leq C_s\|u\|_s\|v\|_{s-1}. \tag{3.15}$$

By using basic Moser-type calculus inequalities and Sobolev's lemma, we have

$$(\partial_x^\alpha S(\mathbb{W}^\epsilon), \partial_x^\alpha \mathbb{W}^\epsilon) \leq C\mathcal{E}_s^\epsilon(t) + C\epsilon(\mathcal{E}_s^\epsilon(t))^{3/2}. \tag{3.16}$$

After a a direct calculation, one gets

$$\mu(\partial_x^\alpha N(\mathbb{W}^\epsilon), \partial_x^\alpha \mathbb{W}^\epsilon) = -\mu\int_{\mathbf{T}^3}\Big(|\nabla\partial_x^\alpha d|^2 + |\nabla\partial_x^\alpha \mathbf{c}|^2\Big)dx. \tag{3.17}$$

To estimate the commutator, we have

$$\begin{aligned}
&([\partial_x^\alpha, A(t,x,\partial_x)]\mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) \\
&\quad = \int\Big(\Big[\partial_x^\alpha, \mathbf{u}^{\mathrm{NS}}\cdot\nabla\Big]d\partial_x^\alpha d + \Big[\partial_x^\alpha, \mathbf{u}^{\mathrm{NS}}\cdot\nabla\Big]\mathbf{c}\partial_x^\alpha \mathbf{c} + \Big[\partial_x^\alpha, \mathbf{u}^{\mathrm{NS}}\cdot\nabla\Big]n\partial_x^\alpha n\Big)dx \\
&\quad \leq C\Big(\Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_s\|\nabla d\|_{0,\infty} + \Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_{0,\infty}\|\nabla d\|_{s-1}\Big)\|d\|_s \\
&\qquad + C\Big(\Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_s\|\nabla \mathbf{c}\|_{0,\infty} + \Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_{0,\infty}\|\nabla \mathbf{c}\|_{s-1}\Big)\|\mathbf{c}\|_s \\
&\qquad + C\Big(\Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_s\|\nabla n\|_{0,\infty} + \Big\|\mathbf{u}^{\mathrm{NS}}\Big\|_{0,\infty}\|\nabla n\|_{s-1}\Big)\|n\|_s \\
&\quad \leq C\mathcal{E}_s(t).
\end{aligned} \tag{3.18}$$

Also, we have

$$
\begin{aligned}
&-\epsilon([\partial_x^\alpha, B(t,x,\partial_x)]\mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) \\
&\quad = -\epsilon \int ([\partial_x, \mathbf{u}\cdot\nabla] d\partial_x^\alpha d + [\partial_x, \mathbf{u}\cdot\nabla]\mathbf{c}\partial_x^\alpha \mathbf{c} + [\partial_x, \mathbf{u}\cdot\nabla] n\partial_x^\alpha n) dx \\
&\quad \le C\epsilon\big(\|\mathbf{u}\|_s\|\nabla d\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty}\|\nabla d\|_{s-1}\big)\|d\|_s \\
&\qquad + C\epsilon\big(\|\mathbf{u}\|_s\|\nabla \mathbf{c}\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty}\|\nabla \mathbf{c}\|_{s-1}\big)\|\mathbf{c}\|_s \\
&\qquad + C\epsilon\big(\|\mathbf{u}\|_s\|\nabla n\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty}\|\nabla n\|_{s-1}\big)\|n\|_s \\
&\quad \le C\epsilon(\mathcal{E}_s^\epsilon(t))^{3/2}.
\end{aligned}
\tag{3.19}
$$

Here, we have used the inequality

$$
\|\mathbf{u}\|_s \le C\|\nabla \mathbf{u}\|_{s-1} \le C(\|d\|_{s-1} + \|\mathbf{c}\|_{s-1}). \tag{3.20}
$$

Finally, the Young inequality gives

$$
(\partial_x^\alpha R, \partial_x^\alpha \mathbb{W}^\epsilon) \le \left\| \partial_t \Delta \mho^{\mathrm{NS}} + \mathbf{u}^{\mathrm{NS}} \cdot \nabla \Delta \mho^{\mathrm{NS}} \right\|_s \|n\|_s \le C(1 + \mathcal{E}_s^\epsilon(t)). \tag{3.21}
$$

Notice that, to get the last line, we have used (3.2).

Now, we collect all the previous estimates (3.10)–(3.21) and we sum over α to find

$$
\frac{d}{dt}\mathcal{E}_s^\epsilon(t) \le C\Big(1 + \mathcal{E}_s^\epsilon(t) + \varepsilon(E_s^\epsilon)^{3/2}(t)\Big). \tag{3.22}
$$

We can conclude using a standard Gronwall's lemma, that if the solution $(\mathbf{u}^{\mathrm{NS}}, \mho^{\mathrm{NS}})$ of Navier-Stokes equations (1.2) is smooth on the time interval $[0,T]$, for any $T_1 < T$ there exists ϵ_0 such that the sequence $(\mathbb{W}^\epsilon)_{\epsilon<\epsilon_0}$ is bounded in $C([0,T_1], H^s(\mathbf{T}^3))$. Then we have

$$
\begin{aligned}
\mathbb{W}^\epsilon &= (\operatorname{div}\mathbf{u}, \operatorname{curl}\mathbf{u}, n), \\
\mathbf{u}^\epsilon &= \mathbf{u}^{\mathrm{NS}} + \epsilon\mathbf{u}, \\
n^\epsilon &= 1 + \epsilon^2\Big(n + \Delta\mho^{\mathrm{NS}}\Big).
\end{aligned}
\tag{3.23}
$$

The assumptions that we have made on the initial data imply that $(1/\epsilon)(\mathbf{u}^\epsilon - \mathbf{u}^{\mathrm{NS}})$, $(1/\epsilon^2)(n-1)$ is bounded. This proves Theorem 2.1.

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