

Research Article

On a Quasi-Neutral Approximation of the Incompressible Navier-Stokes Equations

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This paper considers a pressureless Euler-Poisson system with viscosity in plasma physics in the torus \mathbf{T}^3 . We give a rigorous justification of its asymptotic limit toward the incompressible Navier-Stokes equations via quasi-neutral regime using the modulated energy method.

1. Introduction

We will consider the following system:

$$\begin{aligned}\partial_t \mathbf{u}^\epsilon + (\mathbf{u}^\epsilon \cdot \nabla) \mathbf{u}^\epsilon &= \mu \Delta \mathbf{u}^\epsilon + \nabla \mathcal{U}^\epsilon, \\ \partial_t n^\epsilon + \operatorname{div} (n^\epsilon \mathbf{u}^\epsilon) &= 0, \\ \Delta \mathcal{U}^\epsilon &= \frac{n^\epsilon - 1}{\epsilon^2},\end{aligned}\tag{1.1}$$

for $x \in \mathbf{T}^3$ and $t > 0$, $n^\epsilon \in \mathbb{R}$, $\mathbf{u}^\epsilon \in \mathbb{R}^2$. ϵ is small parameter and $\mu > 0$ is a constant viscosity coefficient. To solve uniquely the Poisson equation, we add the $\int_{\mathbf{T}^3} n^\epsilon dx = 1$. Passing to the limit when $\epsilon \rightarrow 0$, it is easy to see, at least at a very formal level, that $(n^\epsilon, \mathbf{u}^\epsilon)$ tends to $(n^{\text{NS}}, \mathbf{u}^{\text{NS}})$, where $n^{\text{NS}} = 1$ and

$$\begin{aligned}\partial_t \mathbf{u}^{\text{NS}} + (\mathbf{u}^{\text{NS}} \cdot \nabla) \mathbf{u}^{\text{NS}} &= \mu \Delta \mathbf{u}^{\text{NS}} + \nabla \mathcal{U}^{\text{NS}}, \\ \operatorname{div} \mathbf{u}^{\text{NS}} &= 0.\end{aligned}\tag{1.2}$$

In other words, \mathbf{u}^{NS} is a solution of the incompressible Navier-Stokes equations. The aim of this paper is to give a rigorous justification to this formal computation.

The Euler-Poisson system with viscosity (1.1) is a physical model involving dissipation see [1], which here could be regarded as a viscous approximation of Euler-Poisson. Formally, it is a kind of new approximation of the incompressible Navier-Stokes equations of viscous fluid in real world.

It should be pointed out that there have been a lot of interesting results about the topic on the quasi-neutral (or called zero-Debye length) limit, for the readers to see [2–5] for isentropic Euler-Poisson system, [6, 7] for nonisentropic Euler-Poisson system, [8–10] for Vlasov-Poisson system, [11, 12] for drift-diffusion system, [13] for Euler-Maxwell equations, and therein references. We also mention that the above limit has been studied in [14, 15]. But in this present paper, the convergence result and the method of its proof is different from that of [14, 15].

The main focus in this paper is on the use of modulated energy techniques and div-curl for studying incompressible fluids. And for that, we assume that $n^\epsilon(x, \cdot)$ has total mass equal to 1 and the mean values of \mathbf{u}^ϵ vanish, that is, $\mathbf{m}(\mathbf{u}^\epsilon) = (1/(2\pi)^3) \int_{\mathbf{T}^3} \mathbf{u}^\epsilon dx = 0$. We also restrict ourselves to the case of well-prepared initial data and the case of periodic torus. Indeed, the quasi-neutral limit is much more difficult without these assumptions.

In this note, we will use some inequalities in Sobolev spaces, such as basic Moser-type calculus inequalities, Young inequality, and Gronwall inequality.

The paper is organized as follows. In Section 2 we state our main result. Estimates and proofs are given in Section 3.

2. Main Result

Throughout the paper, we will denote by C a number independent of ϵ , which actually may change from line to line. Moreover (\cdot, \cdot) and $\|\cdot\|$ stand for the usual L^2 scalar product and norm, $\|\cdot\|_s$ is the usual H^s Sobolev norm, and $\|\cdot\|_{s,\infty}$ is the usual $W^{s,\infty}$ norm.

The study of the asymptotic behavior of the sequence $(\mathbf{u}^\epsilon, n^\epsilon)$, as ϵ goes to zero, leads to the statement of our main result.

Theorem 2.1. *Let \mathbf{u}^{NS} be a solution of the incompressible Euler equations (1.2) such that $\mathbf{u}^{\text{NS}} \in ([0, T], H^{s+3}(\mathbf{T}^3))$ and $\int_{\mathbf{T}^3} \mathbf{u}^{\text{NS}} dx = 0$ for $s > (5/2)$. Assume that $(n_0^\epsilon, \mathbf{u}_0^\epsilon)$ be a sequence of initial data such that $\int_{\mathbf{T}^3} n_0^\epsilon dx = 1$, $\int_{\mathbf{T}^3} \mathbf{u}_0^\epsilon dx = 0$ and*

$$\begin{aligned} \left\| \mathbf{u}_0^\epsilon - \mathbf{u}_0^{\text{NS}} \right\|_{s+1} &\leq C\epsilon, \\ \left\| n_0^\epsilon - 1 \right\|_s &\leq C\epsilon^2 \end{aligned} \tag{2.1}$$

with $\mathbf{u}_0^{\text{NS}} = \mathbf{u}^{\text{NS}}|_{t=0}$. Then there is a sequence $(n^\epsilon, \mathbf{u}^\epsilon) \in C([0, T], H^s \times H^{s+1}(\mathbf{T}^3))$ of solutions to (1.1) with initial data $(n_0^\epsilon, \mathbf{u}_0^\epsilon)$ belonging to $C([0, T_\epsilon], H^s \times H^{s+1}(\mathbf{T}^3))$ with $\liminf_{\epsilon \rightarrow 0} T_\epsilon \geq T$. Moreover for any $T_1 < T$ and ϵ small enough,

$$\begin{aligned} \left\| \mathbf{u}^\epsilon(t) - \mathbf{u}^{\text{NS}}(t) \right\|_s &\leq C\epsilon, \\ \left\| n^\epsilon(t) - 1 \right\|_s &\leq C\epsilon^2, \end{aligned} \tag{2.2}$$

for any $0 \leq t \leq T_1$.

3. Proof of the Theorem

If $(\mathbf{u}^\epsilon, n^\epsilon)$ is a solution to system (1.1), we introduce

$$\begin{aligned}\mathbf{u}^\epsilon &= \mathbf{u}^{\text{NS}} + \epsilon \mathbf{u}, \\ n^\epsilon &= 1 + \epsilon^2 \left(n + \Delta \mathcal{V}^{\text{NS}} \right), \\ \mathcal{V}^\epsilon &= \mathcal{V}^{\text{NS}} + \mathcal{V}.\end{aligned}\quad (3.1)$$

Since the pressure \mathcal{V}^{NS} in the incompressible Navier-Stokes equation is given by

$$\Delta \mathcal{V}^{\text{NS}} = \nabla \mathbf{u}^{\text{NS}} : \nabla \mathbf{u}^{\text{NS}}, \quad (3.2)$$

where, $\nabla \mathbf{u} : \nabla \mathbf{v} = \sum_{i,j=1}^3 (\partial_{x_i} \mathbf{u} / \partial_{x_j}) (\partial_{x_j} \mathbf{v} / \partial_{x_i})$. Then the vector $(\mathbf{u}^1, n^1, \mathcal{V}^1)$ solves the system

$$\begin{aligned}\partial_t \mathbf{u} + \mathbf{u}^{\text{NS}} \cdot \nabla \mathbf{u} &= \frac{\nabla \mathcal{V}}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{u}^{\text{NS}} + \mu \Delta \mathbf{u}, \\ \partial_t n + \mathbf{u}^{\text{NS}} \cdot \nabla n &= -\frac{\text{div } \mathbf{u}}{\epsilon} - \epsilon \text{div} \left((n + \Delta \mathcal{V}^{\text{NS}}) \mathbf{u} \right) - \partial_t \Delta \mathcal{V}^{\text{NS}} - \mathbf{u}^{\text{NS}} \cdot \nabla \Delta \mathcal{V}^{\text{NS}}, \\ \Delta \mathcal{V} &= n.\end{aligned}\quad (3.3)$$

As in [16], we make the following change of unknowns:

$$d = \text{div } \mathbf{u}, \quad \mathbf{c} = \text{curl } \mathbf{u}. \quad (3.4)$$

By using the last equation and taking the curl and the divergence of the first equation in (3.5), we get the following system:

$$\begin{aligned}\partial_t d + \mathbf{u}^{\text{NS}} \cdot \nabla d &= \frac{n}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) d - \epsilon \nabla \mathbf{u} : \nabla \mathbf{u} - \nabla \mathbf{u} : \nabla \mathbf{u}^{\text{NS}} + \mu \Delta d, \\ \partial_t \mathbf{c} + \mathbf{u}^{\text{NS}} \cdot \nabla \mathbf{c} &= -\epsilon (\mathbf{u} \cdot \nabla) \mathbf{c} - \epsilon (\mathbf{c} \cdot \nabla) \mathbf{u} - (\mathbf{c} \cdot \nabla) \mathbf{u}^{\text{NS}} \\ &\quad + \text{curl} \left(\nabla \mathbf{u}^{\text{NS}} \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u}^{\text{NS}} \right) - \epsilon d \mathbf{c} + \mu \Delta d, \\ \partial_t n + \mathbf{u}^{\text{NS}} \cdot \nabla n &= -\frac{d}{\epsilon} - \epsilon (\mathbf{u} \cdot \nabla) n - \epsilon (n + \Delta \mathcal{V}^{\text{NS}}) d - \mathbf{u} \cdot \nabla \Delta \mathcal{V}^{\text{NS}} - (\partial_t + \mathbf{u}^{\text{NS}} \cdot \nabla) \Delta \mathcal{V}^{\text{NS}}.\end{aligned}\quad (3.5)$$

This last system can be written as a singular perturbation of a quasilinear symmetrizable hyperbolic system. Setting $\mathbb{W}^\epsilon = (d, \mathbf{c}, n)^T$ yields

$$\partial_t \mathbb{W}^\epsilon + A(t, x, \partial_x) \mathbb{W}^\epsilon = \frac{1}{\epsilon} K \mathbb{W}^\epsilon - \epsilon B(t, x, \partial_x) \mathbb{W}^\epsilon + S(\mathbb{W}^\epsilon) + \mu N(\mathbb{W}^\epsilon) + R, \quad (3.6)$$

where

$$\begin{aligned}
A(t, x, \partial_x) &= \text{diag}(\mathbf{u}^{\text{NS}} \cdot \nabla, \mathbf{u}^{\text{NS}} \cdot \nabla I_3, \mathbf{u}^{\text{NS}} \cdot \nabla), B(t, x, \partial_x) = \text{diag}(\mathbf{u} \cdot \nabla, \mathbf{u} \cdot \nabla I_3, \mathbf{u} \cdot \nabla), \\
K &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad N(\mathbb{W}^\epsilon) = \begin{pmatrix} \Delta d \\ \Delta \mathbf{c} \\ 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ 0 \\ -(\partial_t + \mathbf{u}^{\text{NS}} \cdot \nabla) \Delta \mathcal{U}^{\text{NS}} \end{pmatrix}, \\
S(\mathbb{W}^\epsilon) &= \begin{pmatrix} -\epsilon \nabla \mathbf{u} : \nabla \mathbf{u} - \nabla \mathbf{u} : \nabla \mathbf{u}^{\text{NS}} \\ -\epsilon(\mathbf{c} \cdot \nabla) \mathbf{u} - (\mathbf{c} \cdot \nabla) \mathbf{u}^{\text{NS}} + \text{curl}(\nabla \mathbf{u}^{\text{NS}} \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u}^{\text{NS}}) - \epsilon d \mathbf{c} \\ -\epsilon(n + \Delta \mathcal{U}^{\text{NS}})d - \mathbf{u} \cdot \nabla \Delta \mathcal{U}^{\text{NS}} \end{pmatrix}.
\end{aligned} \tag{3.7}$$

For $|\alpha| \leq s$ with $s > d/2$, we set

$$\begin{aligned}
\mathcal{E}_{\alpha, s}^\lambda(t) &= \frac{1}{2} \left(\|\partial_x^\alpha d\|^2 + \|\partial_x^\alpha \mathbf{c}\|^2 + \|\partial_x^\alpha n\|^2 \right), \\
\mathcal{E}_s^\lambda(t) &= \sum_{|\alpha| \leq s} \mathcal{E}_{\alpha, s}^\lambda(t).
\end{aligned} \tag{3.8}$$

Before performing the energy estimate, we apply the operator ∂_x^α for $\alpha \in \mathbb{N}^3$ with $|\alpha| \leq s$ to (3.6), to obtain

$$\begin{aligned}
\partial_t \partial_x^\alpha \mathbb{W}^\epsilon + A(t, x, \partial_x) \partial_x^\alpha \mathbb{W}^\epsilon &= \frac{1}{\epsilon} K \partial_x^\alpha \mathbb{W}^\epsilon - \epsilon B(t, x, \partial_x) \partial_x^\alpha \mathbb{W}^\epsilon + \partial_x^\alpha S(\mathbb{W}^\epsilon) + \mu \partial_x^\alpha N(\mathbb{W}^\epsilon) \\
&+ [\partial_x^\alpha, A(t, x, \partial_x)] \mathbb{W}^\epsilon - \epsilon [\partial_x^\alpha, B(t, x, \partial_x)] \mathbb{W}^\epsilon + \partial_x^\alpha R.
\end{aligned} \tag{3.9}$$

Now, we proceed to perform the energy estimates for (3.9) in a classical way by taking the scalar product of system (3.9) with $\partial_x^\alpha \mathbb{W}^\epsilon$.

Let us start the estimate of each term. First, since $A(t, x, \partial_x)$ is symmetric and $\text{div } \mathbf{u}^{\text{NS}} = 0$, we have that

$$(A(t, x, \partial_x) \partial_x^\alpha \mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) = - \int_{\mathbb{T}^3} \text{div } \mathbf{u}^{\text{NS}} \left(|\partial_x^\alpha d|^2 + |\partial_x^\alpha \mathbf{c}|^2 + |\partial_x^\alpha n|^2 \right) dx = 0. \tag{3.10}$$

Next, since K is skew-symmetric, we have that

$$\frac{1}{\epsilon} (K \partial_x^\alpha \mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) = 0. \tag{3.11}$$

By integration by parts, we have

$$-\epsilon (B(t, x, \partial_x) \partial_x^\alpha \mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) = \epsilon \int_{\mathbb{T}^3} \text{div } \mathbf{u} |\partial_x^\alpha \mathbb{W}^\epsilon|^2 dx \leq \|\text{div } \mathbf{u}\|_{0, \infty} \mathcal{E}_s^\epsilon(t) \leq (\mathcal{E}_s^\epsilon(t))^{3/2}. \tag{3.12}$$

For later estimates in this paper, we recall some results on Moser-type calculus inequalities in Sobolev spaces [17, 18].

Lemma 3.1. *Let $s \geq 1$ be an integer. Suppose $u \in H^s(\mathbf{T}^3)$, $\nabla u \in L^\infty(\mathbf{T}^3)$, and $v \in H^{s-1}(\mathbf{T}^3) \cap L^\infty(\mathbf{T}^3)$. Then for all multi-indexes $|\alpha| \leq s$, one has $(\partial_x^\alpha(uv) - u\partial_x^\alpha v) \in L^2(\mathbf{T}^3)$ and*

$$\|\partial_x^\alpha(uv) - u\partial_x^\alpha v\| \leq C_s \left(\|\nabla u\|_{0,\infty} \|D^{|\alpha|-1}v\| + \|D^{|\alpha|}u\| \|v\|_{0,\infty} \right), \quad (3.13)$$

where

$$\|D^h u\| = \sum_{|\alpha|=h} \|\partial_x^\alpha u\|, \quad \forall h \in \mathbb{N}. \quad (3.14)$$

Moreover, if $s \geq 3$, then the embedding $H^{s-1}(\mathbf{T}^3) \hookrightarrow L^\infty(\mathbf{T}^3)$ is continuous and one has

$$\|uv\|_{s-1} \leq C_s \|u\|_{s-1} \|v\|_{s-1}, \quad \|\partial_x^\alpha(uv) - u\partial_x^\alpha v\| \leq C_s \|u\|_s \|v\|_{s-1}. \quad (3.15)$$

By using basic Moser-type calculus inequalities and Sobolev's lemma, we have

$$(\partial_x^\alpha S(\mathbb{W}^\epsilon), \partial_x^\alpha \mathbb{W}^\epsilon) \leq C \mathcal{E}_s^\epsilon(t) + C\epsilon (\mathcal{E}_s^\epsilon(t))^{3/2}. \quad (3.16)$$

After a direct calculation, one gets

$$\mu(\partial_x^\alpha N(\mathbb{W}^\epsilon), \partial_x^\alpha \mathbb{W}^\epsilon) = -\mu \int_{\mathbf{T}^3} \left(|\nabla \partial_x^\alpha d|^2 + |\nabla \partial_x^\alpha c|^2 \right) dx. \quad (3.17)$$

To estimate the commutator, we have

$$\begin{aligned} & ([\partial_x^\alpha, A(t, x, \partial_x)] \mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) \\ &= \int \left([\partial_x^\alpha, \mathbf{u}^{\text{NS}} \cdot \nabla] d \partial_x^\alpha d + [\partial_x^\alpha, \mathbf{u}^{\text{NS}} \cdot \nabla] \mathbf{c} \partial_x^\alpha \mathbf{c} + [\partial_x^\alpha, \mathbf{u}^{\text{NS}} \cdot \nabla] n \partial_x^\alpha n \right) dx \\ &\leq C \left(\|\mathbf{u}^{\text{NS}}\|_s \|\nabla d\|_{0,\infty} + \|\mathbf{u}^{\text{NS}}\|_{0,\infty} \|\nabla d\|_{s-1} \right) \|d\|_s \\ &\quad + C \left(\|\mathbf{u}^{\text{NS}}\|_s \|\nabla \mathbf{c}\|_{0,\infty} + \|\mathbf{u}^{\text{NS}}\|_{0,\infty} \|\nabla \mathbf{c}\|_{s-1} \right) \|\mathbf{c}\|_s \\ &\quad + C \left(\|\mathbf{u}^{\text{NS}}\|_s \|\nabla n\|_{0,\infty} + \|\mathbf{u}^{\text{NS}}\|_{0,\infty} \|\nabla n\|_{s-1} \right) \|n\|_s \\ &\leq C \mathcal{E}_s(t). \end{aligned} \quad (3.18)$$

Also, we have

$$\begin{aligned}
& -\epsilon([\partial_x^\alpha B(t, x, \partial_x)]\mathbb{W}^\epsilon, \partial_x^\alpha \mathbb{W}^\epsilon) \\
& = -\epsilon \int ([\partial_x, \mathbf{u} \cdot \nabla] d \partial_x^\alpha d + [\partial_x, \mathbf{u} \cdot \nabla] \mathbf{c} \partial_x^\alpha \mathbf{c} + [\partial_x, \mathbf{u} \cdot \nabla] n \partial_x^\alpha n) dx \\
& \leq C\epsilon(\|\mathbf{u}\|_s \|\nabla d\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty} \|\nabla d\|_{s-1}) \|d\|_s \\
& \quad + C\epsilon(\|\mathbf{u}\|_s \|\nabla \mathbf{c}\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty} \|\nabla \mathbf{c}\|_{s-1}) \|\mathbf{c}\|_s \\
& \quad + C\epsilon(\|\mathbf{u}\|_s \|\nabla n\|_{0,\infty} + \|\mathbf{u}\|_{0,\infty} \|\nabla n\|_{s-1}) \|n\|_s \\
& \leq C\epsilon(\mathcal{E}_s^\epsilon(t))^{3/2}.
\end{aligned} \tag{3.19}$$

Here, we have used the inequality

$$\|\mathbf{u}\|_s \leq C\|\nabla \mathbf{u}\|_{s-1} \leq C(\|d\|_{s-1} + \|\mathbf{c}\|_{s-1}). \tag{3.20}$$

Finally, the Young inequality gives

$$(\partial_x^\alpha R, \partial_x^\alpha \mathbb{W}^\epsilon) \leq \left\| \partial_t \Delta \mathcal{V}^{\text{NS}} + \mathbf{u}^{\text{NS}} \cdot \nabla \Delta \mathcal{V}^{\text{NS}} \right\|_s \|n\|_s \leq C(1 + \mathcal{E}_s^\epsilon(t)). \tag{3.21}$$

Notice that, to get the last line, we have used (3.2).

Now, we collect all the previous estimates (3.10)–(3.21) and we sum over α to find

$$\frac{d}{dt} \mathcal{E}_s^\epsilon(t) \leq C\left(1 + \mathcal{E}_s^\epsilon(t) + \epsilon(E_s^\epsilon)^{3/2}(t)\right). \tag{3.22}$$

We can conclude using a standard Gronwall's lemma, that if the solution $(\mathbf{u}^{\text{NS}}, \mathcal{V}^{\text{NS}})$ of Navier-Stokes equations (1.2) is smooth on the time interval $[0, T]$, for any $T_1 < T$ there exists ϵ_0 such that the sequence $(\mathbb{W}^\epsilon)_{\epsilon < \epsilon_0}$ is bounded in $C([0, T_1], H^s(\mathbf{T}^3))$. Then we have

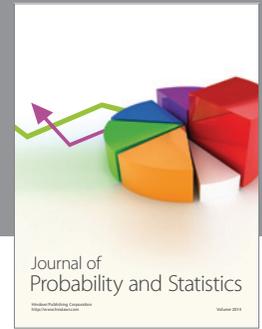
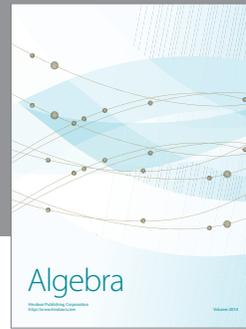
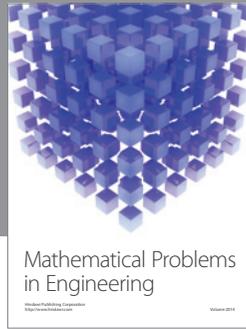
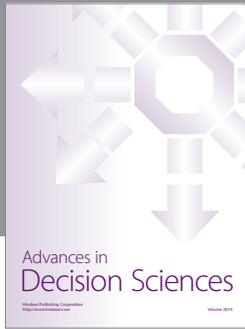
$$\begin{aligned}
\mathbb{W}^\epsilon &= (\operatorname{div} \mathbf{u}, \operatorname{curl} \mathbf{u}, n), \\
\mathbf{u}^\epsilon &= \mathbf{u}^{\text{NS}} + \epsilon \mathbf{u}, \\
n^\epsilon &= 1 + \epsilon^2 (n + \Delta \mathcal{V}^{\text{NS}}).
\end{aligned} \tag{3.23}$$

The assumptions that we have made on the initial data imply that $(1/\epsilon)(\mathbf{u}^\epsilon - \mathbf{u}^{\text{NS}})$, $(1/\epsilon^2)(n - 1)$ is bounded. This proves Theorem 2.1.

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