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Research Article

A New Three-Oscillator Model for the Heart System in the Case of Time Delay and Designing Appropriate Controller for Its Synchronization

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If the *SA* and *AV* oscillators are not synchronized, it may arise some kinds of blocking arrhythmias in the system of heart. In this paper, in order to examine the heart system more precisely, we apply the three-oscillator model of the heart system, and to prevent arrhythmias, perform the following steps. Firstly, we add a voltage with rang a_1 and ω frequency to *SA* node. Then, we use delay time factor in oscillators and finally the appropriate control is designed. In this paper, we have explained how simulating and curing these arrhythmias are possible by designing a three-oscillator system for heart in the state of delay and without delay and by applying an appropriate control. In the end, we present the simulation results.

1. Introduction

As we know, If the *SA* and *AV* oscillators are not synchronized, it may cause some kinds of blocking arrhythmias in the system of the heart. At present, there are devices called pacemaker using electric shocks to synchronize if there is an arrhythmia. [1–3]. The main defect of these devices is the error of diagnosis and not having accurate control. For example, they sometimes apply shocking of 600 to 700 voltages to the heart wrongly even when there is no blocking at all, which is too dangerous. Applied shock to the heart muscle constricts all fibers concurrently and consequently all fibers return to recovery state and hope to return to their normal rhythm after that and it may sometimes occur that the patient involves blocking and the device have not diagnosed it so the patient dies. Two popular types of arrhythmias blocking and ectopic are interesting for researchers [4]. In order to simulate ectopic arrhythmias, a voltage ranging a_1 and ω frequency is added to *SA* node.

2. Three-Oscillator Modelling of the Heart without Time Delay

As we know, cardiac normal rhythm is firstly produced by *SA* node (normal pacemaker) and then causes stimulating of *AV* node. Nevertheless, it has been observed that these two oscillators are not so accurate for producing *ECG* signal. This is because, the signal of first oscillator is related to the activation of *SA* node and the right atrium and signal of second oscillator is only related to the depolarization of the left ventricular. Based on this hypothesis, it is possible to produce *P* curve, but *QRS* complex may not be produced, because this interval is mainly due to ventricular repolarization [5]. These observations and also the existence of blocking arrhythmias force us to put the third oscillator which represents the spread of the pulse through ventricles, which physiologically represents His-Purkinje complex. To create a general model, we assume that there is a mutual coupling asymmetry among all oscillators [6]. In addition, external stimulation is entered into the system, considering periodic stimulus sentence to each oscillator. This developed model can be shown by a system of differential equations as shown below, in which a Vander pol equation has been considered to model *SA* oscillator and another one to model the *AV* oscillator and the third one to model *HP* oscillator [7]:

$$(SA)\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -d_1\left(x_1^2-1\right)x_2 - c_1x_1 + a_1\cos\omega t\\ \qquad + R_{13}\left(x_1-x_3\right) + R_{15}\left(x_1-x_5\right),\end{cases}$$
$$(AV)\begin{cases}\dot{x}_3 = x_4\\ \dot{x}_4 = -d_2\left(x_3^2-1\right)x_4 - c_2x_3 + a_2\cos\omega t\\ \qquad + R_{31}\left(x_3-x_1\right) + R_{35}\left(x_3-x_5\right),\end{cases}$$
$$(HP)\begin{cases}\dot{x}_5 = x_6\\ \dot{x}_6 = -d_3\left(x_5^2-1\right)x_6 - c_3x_5 + a_3\cos\omega t\\ \qquad + R_{51}\left(x_5-x_1\right) + R_{53}\left(x_5-x_3\right),\end{cases} \tag{1}$$

in which pairs (x_1, x_2), (x_3, x_4), and (x_5, x_6) show *SA*, *AV*, and *HP* oscillators, respectively.

To choose appropriate parameters from the Vander-pol system which are close to the system of the heart, we use trying and error test [8]. Regarding this test, appropriate parameters have been mentioned in Table 1. By choosing parameters as shown in Table 1, any oscillator system is synchronized with another system that has different parameters. In the case that two systems are not synchronized, with the help of changing parameters and coupling coefficients, arrhythmias will occur [9, 10].

By changing d_1, d_2, and d_3 coefficients, we can produce different kinds of alternative responses. These coefficients influence the nonlinear sentence of the equation and cause stability of limit cycle in the phase plane (x_1, x_2). Having limit cycle with the heart behavior is adapted physiologically to the behavior of the heart [11].

In the discussion of synchronization, synchronizing of the slow oscillators with the fast oscillators is done more easily. In the case of heart, slower pacemakers such as *AV* and *HP*, should be synchronized with faster pacemakers such as *SA*. If it is not synchronized by changing the coupling coefficients, we try to synchronize oscillators by applying appropriate input controller to slow oscillators as shown below

$$(SA)\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -d_1\left(x_1^2-1\right)x_2 - c_1x_1 + a_1\cos\omega t\\ \qquad + R_{13}\left(x_1-x_3\right) + R_{15}\left(x_1-x_5\right),\end{cases}$$
$$(AV)\begin{cases}\dot{x}_3 = x_4\\ \dot{x}_4 = -d_2\left(x_3^2-1\right)x_4 - c_2x_3 + a_2\cos\omega t\\ \qquad + R_{31}\left(x_3-x_1\right) + R_{35}\left(x_3-x_5\right) + u_1,\end{cases} \tag{2}$$
$$(HP)\begin{cases}\dot{x}_5 = x_6\\ \dot{x}_6 = -d_3\left(x_5^2-1\right)x_6 - c_3x_5 + a_3\cos\omega t\\ \qquad + R_{51}\left(x_5-x_1\right) + R_{53}\left(x_5-x_3\right) + u_2.\end{cases}$$

Table 1: Appropriate parameters to synchronize three-oscillator system.

Parameter	Definition	Three-oscillators system
c_1	*SA* frequency	1
c_2	*AV* frequency	2
c_3	*HP* frequency	1
a_1	*SA* voltage range	5
a_2	*AV* voltage range	6
a_3	*HP* voltage range	4
d_1	*SA* damping Coefficient	5
d_2	*AV* damping Coefficient	6
d_3	*HP* damping Coefficient	7
ω	Frequency	$2.001 \le \omega < 2.05$
R_{ij}	Coupling coefficients between x_i and x_j	
R_{13}		1
R_{15}		1
R_{35}		1
R_{51}		2
R_{31}		5
R_{53}		3
x_1	*SA* membrance flow	
x_2	*SA* membrance voltage	
x_3	*AV* membrance flow	
x_4	*AV* membrance voltage	
x_5	*HP* membrance flow	
x_6	*HP* membrance voltage	

The state variables of slow systems will be converged to state variables of fast system after a transient time. In fact, the second and third oscillators should follow the first oscillator which contains dominant frequency, so synchronization error is considered as follows [12]:

$$\begin{aligned} e_1 &= x_1 - x_3, \qquad e_2 = x_2 - x_4,\\ e_3 &= x_3 - x_5, \qquad e_4 = x_4 - x_6. \end{aligned} \tag{3}$$

The purpose of the synchronization is to annihilate errors. So, selected control functions to vanish errors as $\dot{e}_i = 0$, $i = 1, 2, 3, 4$, are measured as follows:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_3,\\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_4\\ &= -d_1\left(x_1^2-1\right)x_2 - c_1x_1 + a_1\cos\omega t\\ &\quad + R_{13}\left(x_1-x_3\right) + R_{15}\left(x_1-x_5\right) + d_2\left(x_3^2-1\right)x_4\\ &\quad + c_2x_3 - a_2\cos\omega t - R_{31}\left(x_3-x_1\right) - R_{35}\left(x_3-x_5\right) - u_1, \end{aligned}$$

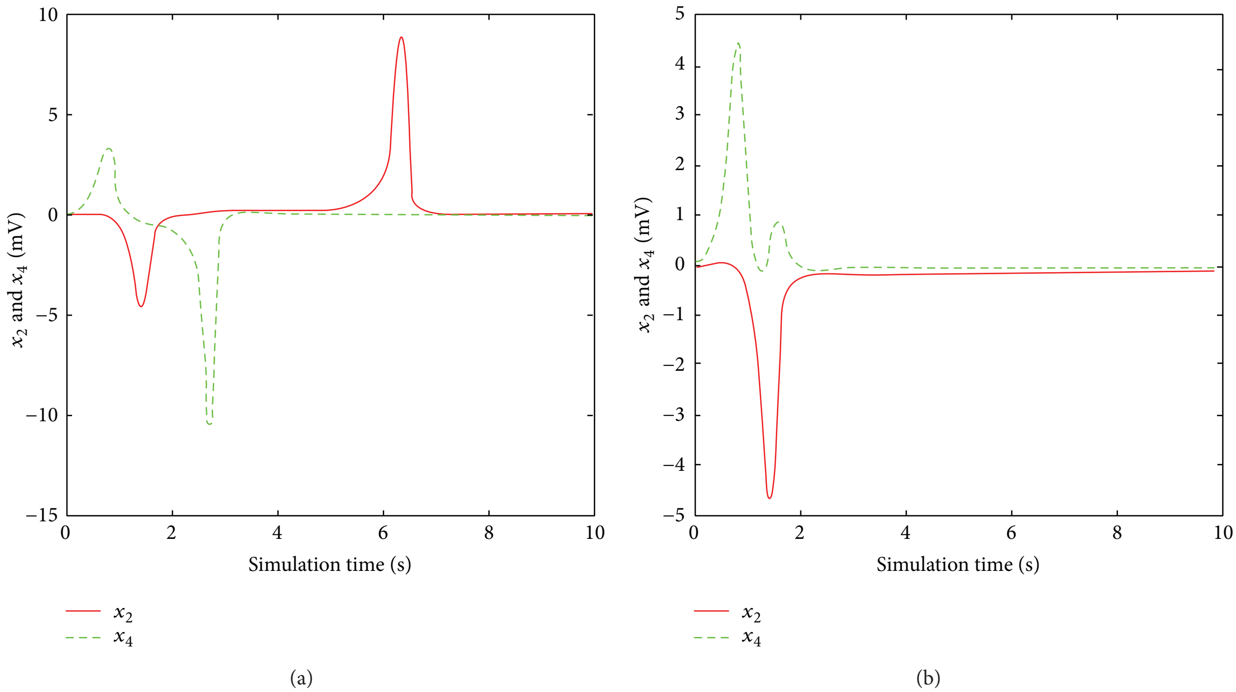


Figure 1: Synchronization of states x_2, x_4 without time delay befor control (a) and after control (b).

$$\begin{aligned}
\dot{e}_3 &= \dot{x}_3 - \dot{x}_5, \\
\dot{e}_4 &= \dot{x}_4 - \dot{x}_6 \\
&= -d_2\left(x_3^2 - 1\right)x_4 - c_2 x_3 + a_2 \cos\omega t \\
&\quad + R_{31}\left(x_3 - x_1\right) + R_{35}\left(x_3 - x_5\right) + u_1 \\
&\quad + d_3\left(x_5^2 - 1\right)x_6 + c_3 x_5 - a_3 \cos\omega t \\
&\quad - R_{51}(x_5 - x_1) - R_{53}(x_5 - x_3) - u_2.
\end{aligned} \tag{4}$$

So, by choosing control rules as follows:

$$\begin{aligned}
u_1 &= -d_1 x_1^2 x_2 + a_1 \cos\omega t + d_2 x_3^2 x_4 - c_1 x_3 \\
&\quad - a_2 \cos\omega t + d_1 x_4 + c_2 x_5 - d_2 x_2 \\
&\quad + k_1 e_1 + k_2 e_2 + k_3 e_3, \\
u_2 &= -d_2 x_3^2 x_4 + a_2 \cos\omega t + d_3 x_5^2 x_6 \\
&\quad + c_3 x_5 - a_3 \cos\omega t + \widehat{d}_2 x_6 - c_2 x_5 \\
&\quad - d_3 x_4 + k_4 e_1 + k_5 e_3 + k_6 e_4 + u_1 + k_7,
\end{aligned} \tag{5}$$

matrix form of error equations is

$$\dot{e} = Ae. \tag{6}$$

By applying controllers u_1 and u_2, the results of simulations show that diagrams related to synchronization are tracking each other and

$$\lim_{t\to\infty} \|e\| = 0. \tag{7}$$

Simulation diagrams related to synchronization in the case of three oscillators without time delay have been shown below. See Figures 1, 2, and 3 (the horizontal axis time (s) and the vertical axis millivolts).

3. Three-Oscillator Modelling of the Heart with Time Delay

Since even small delays may change the dynamics of the system, including of time delay in differential equations can cause drastic changes and chaos in a system that has regular behavior [7, 13]. Thus, system (1) is changed as below in which τ represents time delay and $x_i^\tau = x_i(t-\tau)$:

$$\begin{aligned}
&(SA)\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -d_1\left(x_1^2 - 1\right)x_2 - c_1 x_1 + a_1 \cos\omega t \\
\qquad + R_{13}\left(x_1 - x_3^{\tau_{13}}\right) + R_{15}\left(x_1 - x_5^{\tau_{15}}\right),
\end{cases} \\
&(AV)\begin{cases}
\dot{x}_3 = x_4 \\
\dot{x}_4 = -d_2\left(x_3^2 - 1\right)x_4 - c_2 x_3 + a_2 \cos\omega t \\
\qquad + R_{31}\left(x_3 - x_1^{\tau_{31}}\right) + R_{35}\left(x_3 - x_5^{\tau_{35}}\right),
\end{cases} \\
&(HP)\begin{cases}
\dot{x}_5 = x_6 \\
\dot{x}_6 = -d_3\left(x_5^2 - 1\right)x_6 - c_3 x_5 + a_3 \cos\omega t \\
\qquad + R_{51}\left(x_5 - x_1^{\tau_{51}}\right) + R_{53}\left(x_5 - x_3^{\tau_{53}}\right).
\end{cases}
\end{aligned} \tag{8}$$

By substitution, we will have

$$(SA)\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -d_1\left(x_1^2 - 1\right)x_2 - c_1 x_1 + a_1 \cos\omega t \\
\qquad + R_{13}\left(x_1 - x_3\left(t - \tau_{13}\right)\right) \\
\qquad + R_{15}\left(x_1 - x_5\left(t - \tau_{15}\right)\right),
\end{cases}$$

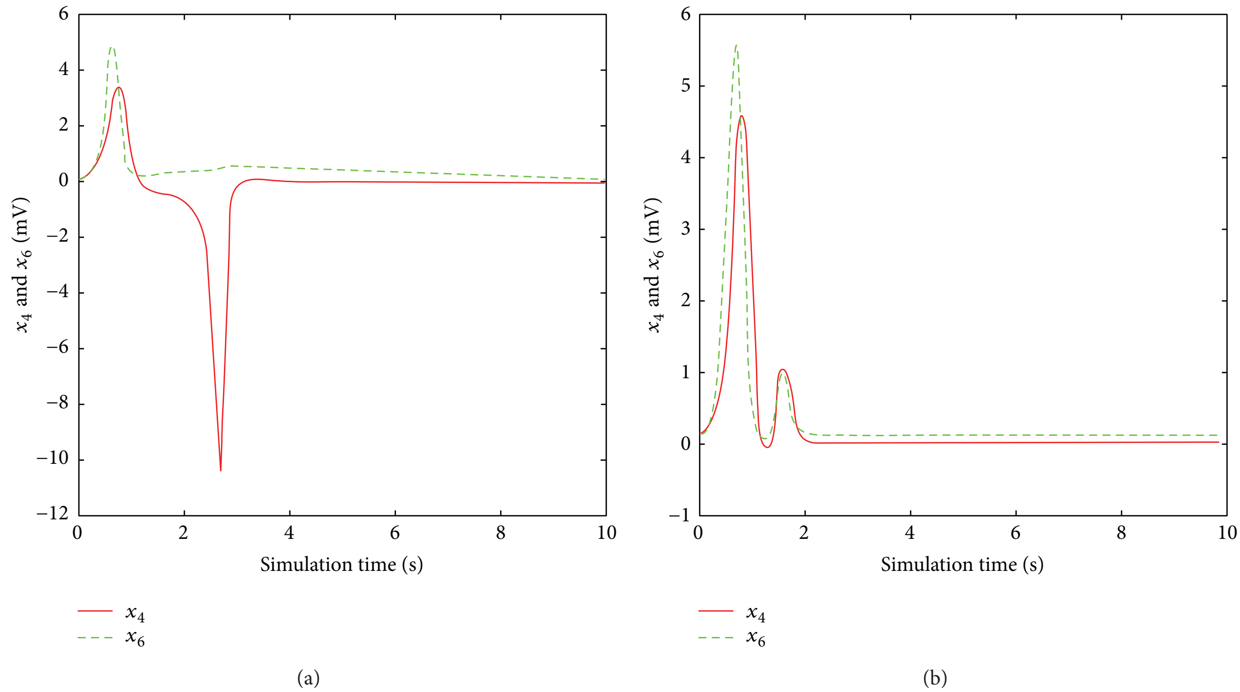


FIGURE 2: Synchronization of states x_4, x_6 without time delay befor control (a) and after control (b).

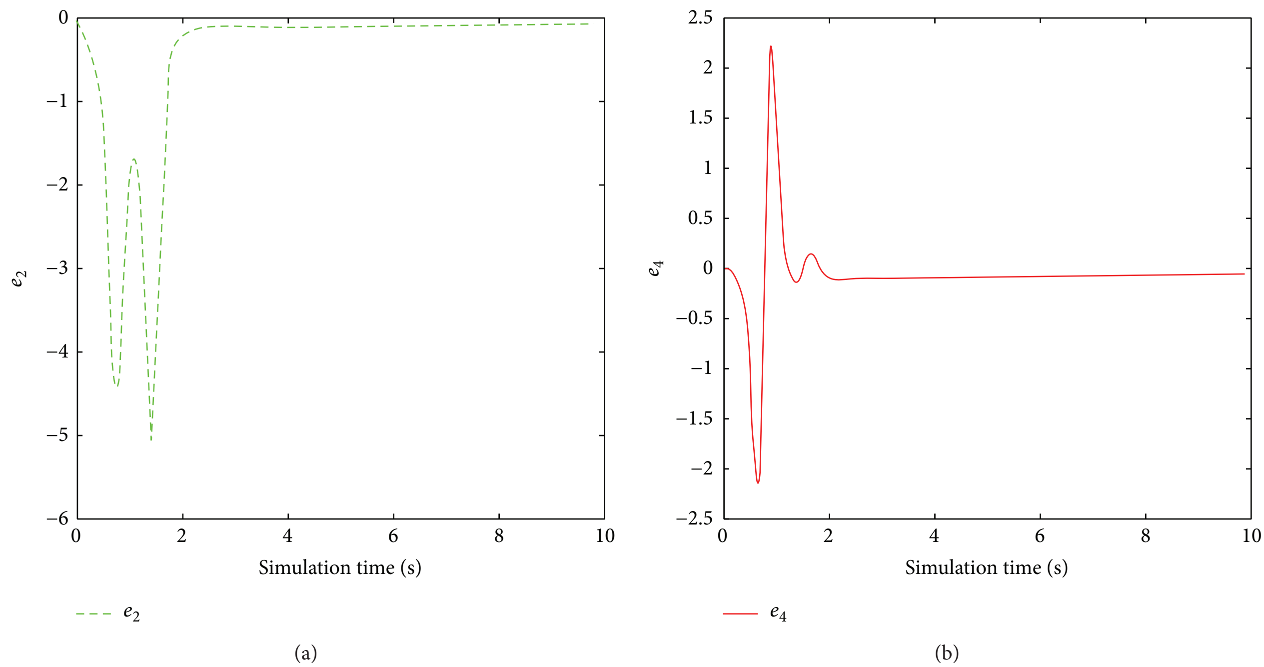


FIGURE 3: Error states x_2, x_4 and x_4, x_6 without time delay after control ((a), (b)).

$$(AV)\begin{cases}\dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2\left(x_3^2 - 1\right)x_4 - c_2 x_3 + a_2\cos\omega t \\ \qquad + R_{31}\left(x_3 - x_1\left(t - \tau_{31}\right)\right) \\ \qquad + R_{35}\left(x_3 - x_5\left(t - \tau_{35}\right)\right),\end{cases} \qquad (HP)\begin{cases}\dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3\left(x_5^2 - 1\right)x_6 - c_3 x_5 + a_3\cos\omega t \\ \qquad + R_{51}\left(x_5 - x_1\left(t - \tau_{51}\right)\right) \\ \qquad + R_{53}\left(x_5 - x_3\left(t - \tau_{53}\right)\right).\end{cases} \tag{9}$$

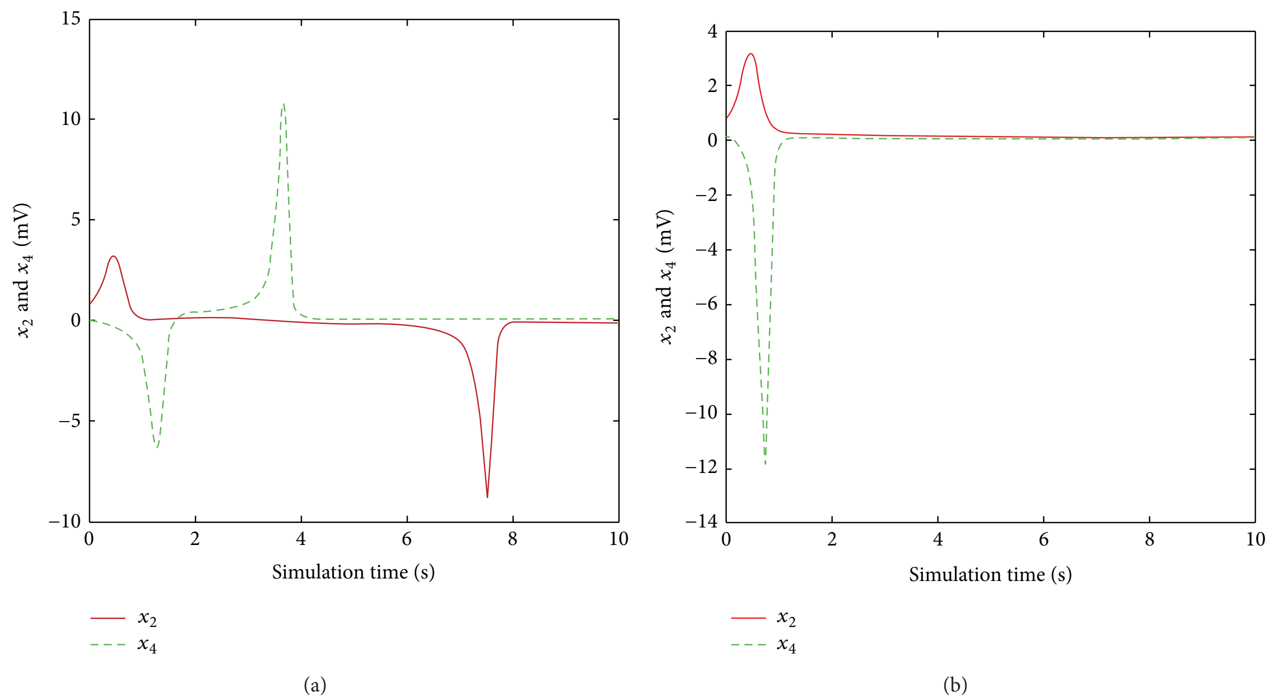


Figure 4: Synchronization of states x_2, x_4 with time delay befor control (a) and after control (b).

Now, by applying the appropriate controller on it which is as follows [14]:

$$(SA)\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -d_1\left(x_1^2-1\right)x_2 - c_1x_1 + a_1\cos\omega t\\ \qquad + R_{13}\left(x_1 - x_3\left(t-\tau_{13}\right)\right)\\ \qquad + R_{15}\left(x_1 - x_5\left(t-\tau_{15}\right)\right),\end{cases}$$

$$(AV)\begin{cases}\dot{x}_3 = x_4\\ \dot{x}_4 = -d_2\left(x_3^2-1\right)x_4 - c_2x_3 + a_2\cos\omega t\\ \qquad + R_{31}\left(x_3 - x_1\left(t-\tau_{31}\right)\right)\\ \qquad + R_{35}\left(x_3 - x_5\left(t-\tau_{35}\right)\right) + u_1,\end{cases} \tag{10}$$

$$(HP)\begin{cases}\dot{x}_5 = x_6\\ \dot{x}_6 = -d_3\left(x_5^2-1\right)x_6 - c_3x_5 + a_3\cos\omega t\\ \qquad + R_{51}\left(x_5 - x_1\left(t-\tau_{51}\right)\right)\\ \qquad + R_{53}\left(x_5 - x_3\left(t-\tau_{53}\right)\right) + u_2,\end{cases}$$

the state variables of slow systems will be converged to state variables of fast system, after a transient time. In fact, the second and third oscillators should follow the first oscillator containing dominant frequency, so synchronization error is considered as follows [11]:

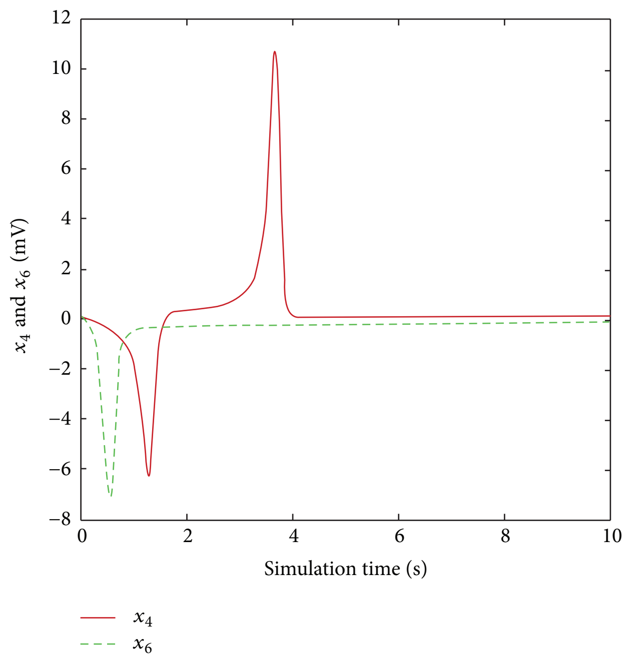
$$\begin{aligned} e_1(t) &= x_1(t) - x_3\left(t-\tau_{13}\right),\\ e_2(t) &= x_2(t) - x_4\left(t-\tau_{24}\right),\\ e_3(t) &= x_3(t) - x_5\left(t-\tau_{35}\right),\\ e_4(t) &= x_4(t) - x_6\left(t-\tau_{46}\right).\end{aligned} \tag{11}$$

The purpose of the synchronization is to annihilate errors. So, chose control functions to vanish errors as $\dot{e}_i = 0, i = 1, 2, 3, 4$, that are measured as follows:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_3\left(t-\tau_{13}\right),\\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_4\left(t-\tau_{24}\right)\\ &= -d_1\left(x_1^2-1\right)x_2 - c_1x_1 + a_1\cos\omega t\\ &\quad + R_{13}\left(x_1 - x_3\left(t-\tau_{13}\right)\right) + R_{15}\left(x_1 - x_5\left(t-\tau_{15}\right)\right)\\ &\quad + d_2\left(x_3^2-1\right)x_4 + c_2x_3 - a_2\cos\omega t\\ &\quad - R_{31}\left(x_3 - x_1\left(t-\tau_{31}\right)\right) - R_{35}\left(x_3 - x_5\left(t-\tau_{35}\right)\right) - u_1,\\ \dot{e}_3 &= \dot{x}_3 - \dot{x}_5\left(t-\tau_{35}\right),\\ \dot{e}_4 &= \dot{x}_4 - \dot{x}_6\\ &= -d_2\left(x_3^2-1\right)x_4 - c_2x_3 + a_2\cos\omega t\\ &\quad + R_{31}\left(x_3 - x_1\left(t-\tau_{31}\right)\right) + R_{35}\left(x_3 - x_5\left(t-\tau_{35}\right)\right)\\ &\quad + u_1 + d_3\left(x_5^2-1\right)x_6 + c_3x_5 - a_3\cos\omega t - R_{51}\\ &\quad \times\left(x_5 - x_1\left(t-\tau_{51}\right)\right) - R_{53}\left(x_5 - x_3\left(t-\tau_{53}\right)\right) - u_2.\end{aligned} \tag{12}$$

In this case,

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{x}_4\left(t-\tau_{24}\right)\\ &= -d_1x_1^2x_2 + d_1x_2 + d_1x_4 - d_1x_4 - c_1x_1 + c_1x_3\\ &\quad - c_1x_3 + a_1\cos\omega t + R_{13}\left(x_1 - x_3\left(t-\tau_{13}\right)\right)\end{aligned}$$



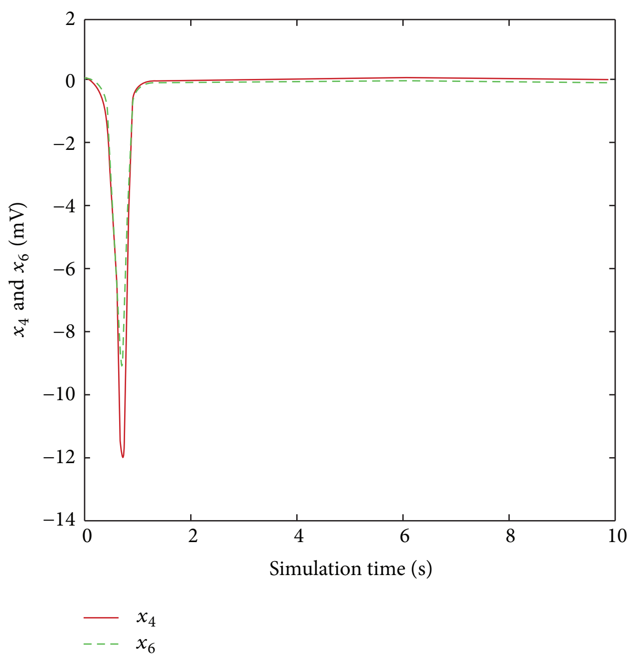


FIGURE 5: Synchronization of states x_4, x_6 with time delay befor control (a) and after control (b).

$$
\begin{aligned}
&+ R_{15}\left(x_1 - x_5\left(t - \tau_{15}\right)\right) + d_2 x_3^2 x_4 - d_2 x_4 \\
&+ d_2 x_2 - d_2 x_2 + c_2 x_3 + c_2 x_5 - c_2 x_5 - a_2 \cos \omega t \\
&- R_{31}\left(x_3 - x_1\left(t - \tau_{31}\right)\right) - R_{35}\left(x_3 - x_5\left(t - \tau_{35}\right)\right) - u_1.
\end{aligned} \tag{13}
$$

By substitution, we will have

$$
\begin{aligned}
\dot{e}_2 = {} & -d_1 x_1^2 x_2 + d_2 x_3^2 x_4 + a_1 \cos \omega t - a_2 \cos \omega t \\
& + R_{13} e_1 + R_{31} e_1 - R_{35} e_3 + R_{15} e_1 + R_{15} e_3 \\
& + d_1 e_2 + d_1 x_4 - c_1 e_1 - c_1 x_3 + d_2 e_2 \\
& - d_2 x_2 + c_2 e_3 + c_2 x_5 - u_1.
\end{aligned} \tag{14}
$$

Also,

$$
\begin{aligned}
\dot{e}_2 = {} & -d_1 x_1^2 x_2 + d_2 x_3^2 x_4 + a_1 \cos \omega t - a_2 \cos \omega t \\
& + d_1 x_4 - c_1 x_3 - d_2 x_2 + c_2 x_5 \\
& + \left(R_{13} + R_{31} + R_{15} - c_1\right) e_1 \\
& + \left(\hat{d}_2 + d_1\right) e_2 + \left(-R_{35} + R_{15} + c_2\right) e_3 - u_1.
\end{aligned} \tag{15}
$$

And control functions are as follows:

$$
\begin{aligned}
u_1 = {} & -d_1 x_1^2 x_2 + d_2 x_3^2 x_4 + a_1 \cos \omega t - a_2 \cos \omega t \\
& + d_1 x_4 - c_1 x_3 - d_2 x_2 + c_2 x_5 + k_1 e_2 \\
& + k_2 e_2 + k_3 e_3.
\end{aligned} \tag{16}
$$

Also,

$$
\begin{aligned}
\dot{e}_4 &= \dot{x}_4 - \dot{x}_6\left(t - \tau_{46}\right) \\
&= -d_2 x_3^2 x_4 + d_2 x_4 + d_2 x_6 - d_2 x_6 - c_2 x_3 \\
&\quad + c_2 x_5 - c_2 x_5 + a_2 \cos \omega t - R_{31} e_1 \\
&\quad + R_{35} e_3 + u_1 + d_3 x_5^2 x_6 - d_3 x_6 \\
&\quad + d_3 x_4 - d_3 x_4 + c_3 x_5 - a_3 \cos \omega t - u_2.
\end{aligned} \tag{17}
$$

By substitution, we will have

$$
\begin{aligned}
\dot{e}_4 &= \dot{x}_4 - \dot{x}_6\left(t - \tau_{46}\right) \\
&= -d_2 x_3^2 x_4 + d_3 x_5^2 x_6 + a_2 \cos \omega t - a_3 \cos \omega t \\
&\quad - R_{31} e_1 + R_{35} e_3 + u_1 + c_3 x_5 \\
&\quad + R_{51} e_1 + R_{51} e_3 + R_{53} e_3 + d_2 e_4 + d_2 x_6 \\
&\quad - c_2 e_3 - c_2 x_5 + d_3 e_4 - d_3 x_4 - u_2.
\end{aligned} \tag{18}
$$

And control functions is as follows:

$$
\begin{aligned}
u_2 = {} & -d_2 x_3^2 x_4 + d_3 x_5^2 x_6 + a_2 \cos \omega t \\
& - a_3 \cos \omega t + u_1 + c_3 x_5 + d_2 x_6 \\
& - c_2 x_5 - d_3 x_4 + k_4 e_1 + k_5 e_3 + k_6 e_4 + k_7
\end{aligned} \tag{19}
$$

and matrix form of error equations is

$$
\dot{e} = A e. \tag{20}
$$

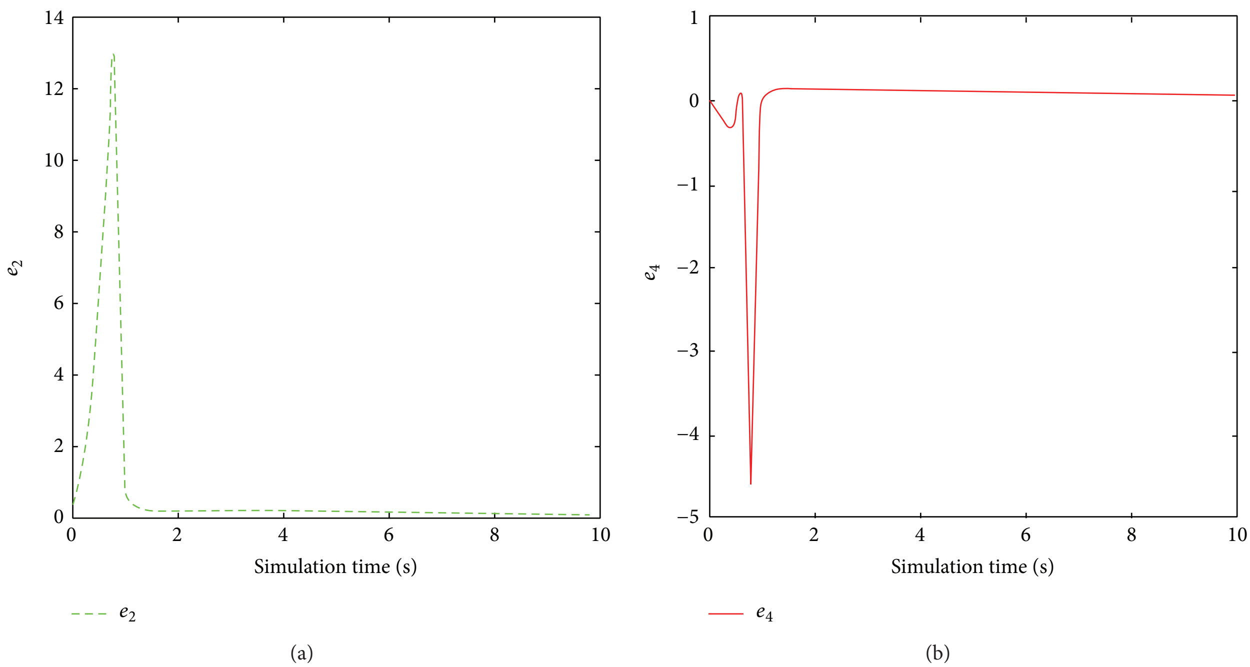


FIGURE 6: Error states x_2, x_4 and x_4, x_6 with time delay after control ((a), (b)).

In this case, the third oscillator acts as a pacemaker and its advantage is that instead of a sudden shock and a voltage of *DC*, it applies sinusoidal voltage for treatment.

Simulation diagrams related to synchronization in the case of three oscillators with time delay have been shown below. See Figures 4, 5, and 6 (the horizontal axis time (s) and the vertical axis millivolts).

4. Conclusion

In this paper, it has been explained that how simulating and curing some arrhythmias are possible, that one example of curing method, is modelling three-oscillator heart system in the state of delay and without delay and by applying an appropriate control.

In this paper after applying control u to the model of three-oscillator heart system in case of time delay and without time delay and also observing of simulated diagrams corresponding to synchronization of this model, we observe that, the time of synchronization x_2, x_4 and x_4, x_6 and also the time when the error converges to zero in x_2, x_4 and x_4, x_6 after applying designed controller; in case of without time delay is less than 2(s), and in case of time delay less than 1.5(s).

These results indicate that the model of three-oscillator of the heart system in case of time delay and designed controller corresponding to synchronization of this model, is more appropriate and accurate than the state of without time delay.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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