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Research Article

Selecting the Best of Portfolio Using OWA Operator Weights in Cross Efficiency-Evaluation

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The present study is an attempt toward evaluating the performance of portfolios and asset selection using cross-efficiency evaluation. Cross-efficiency evaluation is an effective way of ranking decision making units (DMUs) in data envelopment analysis (DEA). The most widely used approach is to evaluate the efficiencies in each row or column in the cross-efficiency matrix with equal weights into an average cross-efficiency score for each DMU and consider it as the overall performance measurement of the DMU. This paper focuses on the evaluation process of the efficiencies in the cross-efficiency matrix and proposes the use of ordered weighted averaging (OWA) operator weights for cross-efficiency evaluation. The OWA operator weights are generated by the minimax disparity approach and allow the decision maker (DM) or investor to select the best assets that are characterized by an orness degree. The problem consists of choosing an optimal set of assets in order to minimize the risk and maximize return. This method is illustrated by application in mutual funds and weights are obtained via OWA operator for making the best portfolio. The finding could be used for constructing the best portfolio in stock companies, in various finance organization, and public and private sector companies.

1. Introduction

In financial literature, a portfolio is an appropriate mix of investments held by an institution or private individuals. Evaluation of portfolio performance has created a large interest among employees also academic researchers because of huge amount of money being invested in financial markets. The theory of mean-variance, Markowitz [1] is considered the basis of many current models and this theory is widely used to select portfolios. This model is due to the nature of the variance in quadratic form. Other problem in Markowitz model is that increasing the number of assets will develop the covariance matrix of asset returns and will be added to the content calculation. Due to these problems sharp one-factor model is proposed by Sharpe [2]. This method reduces the number of calculations requiring information for the decision. Data envelopment analysis (DEA) has proved the efficiency for assessing the relative efficiency of decision making units (DMUs) employing multiple inputs to produce multiple outputs [3]. M. R. Morey and R. C. Morey [4] proposed mean-variance framework based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return is the output. Joro and Na [5] introduced mean-variance-skewness framework and skewness of returns are also considered as an output. The portfolio optimization problem is a well-known problem in financial real world. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. Since there are a large number of assets to invest in, this objective leads to select the best assets via cross-efficiency matrix by using OWA weighted. Cross-efficiency evaluation, proposed by Sexton et al., [6] is the effective way of ranking decision making units (DMUs). It allows the overall efficiencies of the DMUs to be evaluated through self- and peer-evaluations. The self-evaluation allows the efficiencies of the DMUs to be evaluated with the most favorable weights so that each of them can achieve its best possible relative efficiency, whereas the peer-evaluation requires the efficiency of each DMU to be evaluated with the weights determined by the other DMUs. The self-evaluated efficiency and peer-evaluated efficiencies of each DMU are then averaged as the overall efficiency of the DMU. Since, its remarkable discrimination power, the cross-efficiency evaluation has found significant number of applications in a wide variety of areas such as preference voting and project ranking [7, 8], economic-environmental performance assessment [9, 10], and Olympic ranking and benchmarking [11–13]. Besides a large number of applications, theoretical research has also been conducted on the cross-efficiency evaluation. For example, Doyle and Green [14, 15] presented mathematical formulations for possible implementations of aggressive and benevolent cross-efficiencies. Liang et al. [16] suggested the concept of game cross-efficiency and developed a game cross-efficiency model which treats each DMU as a player that seeks to maximize own efficiency under the condition that the cross-efficiency of each of the other DMUs dose not deteriorate. Wu et al. [13] extended the game cross-efficiency model to variable returns to scale later. In our work, the use of equal weights for cross-efficiency model has a significant problem. That is, self-evaluated efficiencies are much less weighted than peer-evaluated efficiencies. This is because each DMU has only one self-evaluated efficiency value, but multiple peer-evaluated efficiency values. When they are simply averaged together, the weight assigned to the self-evaluated efficiency is only $1/n$ if there are n DMUs to be evaluated, whereas the remaining weights $(n-1)/n$ are all given to those peer-evaluated efficiencies. To overcome this problem, the use of ordered weighted averaging (OWA) operator weights is stated for assets cross-efficiencies. The use of OWA operator weights for the assets cross-efficiency allows the weights to be reasonably allocated between self- and peer-evaluated efficiencies by investor's control [17]. The OWA operator weights are generated by the minimax disparity approach and allow the decision maker (DM) or investors to select the best assets that are characterized by an orness degree [18]. The method consists of choosing an optimal set of assets in order to minimize the risk and maximize return in cross-efficiency using OWA operator. Since there are a large number of assets to invest in, the best assets are chosen via cross-efficiency evaluation by using OWA weighted by control investors.

The rest of the paper is organized as follows: Section 2 briefly reviews the portfolio performance literature, OWA operators, and their weight determination methods, thus the cross-efficiency evaluation in DEA. Section 3 develops a proposed method for selecting the best of the portfolio. Section 4 presents computational results using mutual funds data and finally conclusions are given in Section 5.

2. Background

2.1. Portfolio Performance Literature. Portfolio theory to investing is published by Markowitz (1952). This approach starts by assuming that an investor has a given sum of money to invest at the present time. This money will be invested for a time as the investor's holding period. At the end of the holding period, the investor will sell all of the assets that were bought at the beginning of the period and then either consume or reinvest. Since portfolio is a collection of assets, it is better to select an optimal portfolio from a set of possible portfolios. Hence, the investor should recognize the returns (and portfolio returns), expected (mean) return, and standard deviation of return. This means that the investor wants to both maximize expected return and minimize uncertainty (risk). Rate of return (or simply the return) of the investor's wealth from the beginning to the end of the period is calculated as follows:

$$\text{Return} = \frac{(\text{end-of-period wealth}) - (\text{beginning-of-period wealth})}{\text{beginning-of-period wealth}}. \tag{1}$$

Since Portfolio is a collection of assets, its return r_p can be calculated in a similar manner. Thus, according to Markowitz, the investor should view the rate of return associated with any one of these portfolios as what is called in statistics a random variable. These variables can be described as the expected return (min or $\overline{r}_p$) and standard deviation of return. Expected return and deviation standard of return are calculated as follows:

$$\overline{r}_p = \sum_{i=1}^{n} \lambda_i \overline{r}_i, \qquad \sigma_p = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij} \right]^{1/2}, \tag{2}$$

where n is the number of assets in the portfolio, $\overline{r}_p$ is the expected return of the portfolio, λ_i is the proportion of the portfolio's initial value invested in asset i, $\overline{r}_i$ is the expected return of asset i, σ_p is the deviation standard of the portfolio, and σ_{ij} is the covariance of the returns between asset i and asset j.

In the above, optimal portfolio from the set of portfolios will be chosen as maximum expected return for varying levels of risk and minimum risk for varying levels of expected return [19]. Data Envelopment Analysis is a nonparametric method for evaluating the efficiency of systems with multiple inputs and multiple outputs. In this section, we present some basic definitions, models, and concepts that will be used in other sections in DEA. They will not be discussed in details. Consider DMU_j, $(j = 1, \dots, n)$, where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j = (x_{1j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, \dots, y_{sj})$, respectively, and let $X_j \geq 0$ and

$X_j \neq 0$, $Y_j \geq 0$, and $Y_j \neq 0$. A basic DEA formulation in input orientation is as follows:

$$\begin{aligned} \min \quad & \theta - \varepsilon \left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- \right) \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \quad r = 1, \ldots, s \\ & \lambda \in \Lambda, \\ & s^+, s^- \geq 0, \\ & \varepsilon \geq 0, \end{aligned} \tag{3}$$

where λ is an n-vector of λ variables, s^+ as-vector of output slacks, s^- an m-vector of input slacks and set Λ is defined as follows:

$$\Lambda = \begin{cases} \{\lambda \in R_+^n\} & \text{with constant returns to scale,} \\ \{\lambda \in R_+^n, 1\lambda \leq 1\} & \text{with nonincreasing} \\ & \text{returns to scale,} \\ \{\lambda \in R_+^n, 1\lambda = 1\} & \text{with variable returns to scale.} \end{cases} \tag{4}$$

Note that subscript "o" refers to the unit under the evaluation. A DMU is efficient if and only if $\theta = 1$ and all slack variables s^-, s^+ equal zero; otherwise, it is inefficient [20]. In the DEA formulation above, the left-hand sides in the constraints define an efficient portfolio. θ is a multiplier which defines the distance from the efficient frontier. The slack variables are used to ensure that the efficient point is fully efficient. This model is used for asset selection. The portfolio performance evaluation literature is vast. In recent years, these models have been used to evaluate the portfolio efficiency. Also, in the Markowitz theory, it is required to characterize the whole efficient frontier but the proposed models by Joro and Na do not need to characterize the whole efficient frontier but only the projection points. The distance between the asset and its projection which means the ratio between the variance of the projection point and the variance of the asset is considered as an efficiency measure (θ) [5].

2.2. OWA Operators and Their Weight Determination Methods. An OWA operator of dimension n is a mapping $F : \mathfrak{R}^n \to \mathfrak{R}$ with an associated weight vector $W = (w_1, \ldots, w_n)^T$ such that

$$\begin{gathered} w_1 + \cdots + w_n = 1, \quad o \leq w_i \leq 1, \ i = 1, \ldots, n, \\ F(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i, \end{gathered} \tag{5}$$

where b_i is the ith largest of $a_1, \ldots, a_n$.

OWA operators, introduced by Yager [21], provide a unified framework for decision making under uncertainty, where different decision criteria such as maximax (optimistic), maximin (pessimistic), and equally likely (Laplace), and Hurwicz criteria are characterized by different OWA operator weights.

For different weight selections, they are distinguished by the following orness degree [21]:

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i. \tag{6}$$

The orness degree can be regarded as a measure of the optimism level of the DM.

To apply OWA operators for decision making, it is essential to determine the weights of OWA operators. The following models (7) and (8) are two important approaches for determining OWA operator weights under a given orness degree:

$$\begin{aligned} & \text{Maximize Disp}(W) = -\sum_{i=1}^{n} w_i \ln w_i, \\ & \text{Subject to orness}(W) = \alpha = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \\ & \qquad 0 \leq \alpha \leq 1, \\ & \sum_{i=1}^{n} w_i = 1, \\ & w_i \geq 0, \quad i = 1, \ldots, n, \end{aligned} \tag{7}$$

$$\begin{aligned} & \text{Minimize } \delta \\ & \text{Subject to orness}(W) = \alpha = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \\ & \qquad 0 \leq \alpha \leq 1, \\ & \sum_{i=1}^{n} w_i = 1, \\ & w_i - w_{i+1} - \delta \leq 0, \quad i = 1, \ldots, n-1, \\ & w_i - w_{i+1} - \delta \geq 0, \quad i = 1, \ldots, n-1, \\ & w_i \geq 0, \quad i = 1, \ldots, n. \end{aligned} \tag{8}$$

Model (7), suggested by O'Hagan [22], maximizes the entropy of weight distribution and is thus referred to as the maximum entropy method, whereas model (8) that was proposed by Wang and Parkan [18] minimizes the maximum disparity between two adjacent weights and is thus called the minimax disparity approach.

The OWA operator weights determined by the above models have the following characteristics.

The weights are ordered. That is, $w_1 \geq w_2 \geq \cdots \geq w_n \geq 0$ if the orness degree $\alpha > 0.5$ and $0 \leq w_1 \leq w_2 \leq \cdots \leq w_n$ if $\alpha \leq 0.5$.

The weights have nothing to do with the magnitudes of the aggregates $b_1 \sim b_n$ but depend upon their ranking orders and the DM's optimism level (orness degree).

Consider $w_1 = 1$ and $w_j = 0$ $(j = 1)$ if $\alpha = 1$, which means that the DM or investor is purely optimistic and considers only the biggest value $b_1 = \max_i(a_i)$ in decision analysis.

Consider $w_n = 1$ and $w_j = 0$ $(j \neq n)$ if $\alpha = 0$, which represents that the DM or investor is purely pessimistic and is only concerned with the most conservative value $b_n = \min_i(a_i)$ when making decision.

Consider $w_1 = \cdots = w_n = (1/n)$ if $\alpha = 0.5$, which stands for that the DM or investor is neutral and makes use of all the aggregates $b_1 \sim b_n$ equally in decision making.

Consider $w_1, \ldots, w_n$ determined by model (7) vary in the form of geometric progression, that is $w_{i+1}/w_i \equiv q$ for $i = 1, \ldots, n-1$, where $q > 0$, while $w_1, \ldots, w_n$ determined by model (8) vary in the form of arithmetical progression; namely, $w_i - w_{i+1} = d$ for $i = 1, \ldots, K$ $(K \leq n)$ or $w_{i+1} - w_i = d$ for $i = K, \ldots, n$ $(K \geq 1)$, where $d > 0$.

2.3. The Cross-Efficiency Evaluation. Consider n DMUs that are to be evaluated with m inputs and s output. Denote by x_{ij} $(1, \ldots, m)$ and y_{rj} $(r = 1, \ldots, s)$ the input and output values of DMU_j $(j = 1, \ldots, n)$. The efficiencies of the n DMUs can then be computed by solving the following CCR model for each of the n DMUs, respectively [3]:

$$\begin{aligned} \max \quad & \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk}, \\ \text{subject to} \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{r=1}^{s} v_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\ & u_{rk}, v_{ik} \geq 0, \quad r = 1, \ldots, s, \ i = 1, \ldots, m, \end{aligned} \tag{9}$$

where DMU_k is the DMU under evaluation and v_{ik} $(i = 1, \ldots, m)$ and u_{rk} $(r = 1, \ldots, s)$ are input and output weights. Let u_{rk}^* $(r = 1, \ldots, s)$ and v_{ik}^* $(i = 1, \ldots, m)$ be the optimal solution to the above CCR model. Then, $\theta_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ is referred to as the CCR-efficiency of DMU_k, which is the best relative efficiency of DMU_k by self-evaluation. If $\theta_{kk}^* = 1$, DMU_k is said to be CCR-efficient; otherwise, it is said to be non-CCR-efficient. $\theta_{jk} = \sum_{r=1}^{s} u_{rk}^* y_{rj} / \sum_{i=1}^{m} v_{ik}^* x_{ij}$ is referred to as the cross-efficiency of DMU_k to DMU_j by peer-evaluation; where $j = 1, \ldots, n$; $j \neq k$.

Model (9) is solved n times, each time for one particular DMU. As a result, we can get one CCR-efficiency value and $(n-1)$ cross-efficiency values for each DMU. The n efficiency values constitute a cross-efficiency matrix, as shown in Table 1, where θ_{kk} $(k = 1, \ldots, n)$ are the CCR-efficiency values of the n DMUs; that is, $\theta_{kk} = \theta_{kk}^*$. The n efficiency values of each DMU are then simply averaged as its overall performance, which is called average cross-efficiency value. Based on these overall performance values, the n DMUs can be compared or fully ranked.

Table 1: Cross-efficiency matrix for n DMUs.

DMU	Target DMU				Average crosses efficiency
	1	2	$\cdots$	n	
1	θ_{11}	θ_{12}	$\cdots$	θ_{1n}	$\left(\frac{1}{n}\right)\sum_{k=1}^{n}\theta_{1k}$
2	θ_{21}	θ_{22}	$\cdots$	θ_{2n}	$\left(\frac{1}{n}\right)\sum_{k=1}^{n}\theta_{2k}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\cdots$	θ_{nn}	$\left(\frac{1}{n}\right)\sum_{k=1}^{n}\theta_{nk}$

Table 2: Re-ordered cross-efficiency matrix of the n DMUs.

DMU	Re-ordered efficiencies in descending order				Weighted average cross efficiency
	1st	2nd	$\cdots$	nth	
	w_1	w_2	$\cdots$	w_n	
1	ϑ_{11}	ϑ_{12}	$\cdots$	ϑ_{1n}	$\sum_{k=1}^{n} w_k \vartheta_{1k}$
2	ϑ_{21}	ϑ_{22}	$\cdots$	ϑ_{2n}	$\sum_{k=1}^{n} w_k \vartheta_{2k}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
n	ϑ_{n1}	ϑ_{n2}	$\cdots$	ϑ_{nn}	$\sum_{k=1}^{n} w_k \vartheta_{nk}$

The above approach about cross-efficiency value in CCR efficiencies or constant returns to scale (CRS) DEA model was extended to the variable returns to scale (VRS) DEA model [13]. The VRS DEA model can generate negative cross-efficiency scores.

The VRS DEA model is as follows [23]:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_{rk} y_{rk} - u_0 \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} - u_0 \leq 0, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & u_{rk} \geq 0, \quad r = 1, \ldots, s \\ & v_{ik} \geq 0, \quad i = 1, \ldots, m \\ & u_0 \geq 0. \end{aligned} \tag{10}$$

For each DMU_k $(k = 1, \ldots, n)$ under evaluation in model (10), we obtain a set of optimal weights (u_{rk}^*, v_{rk}^*). Using this set of weights, the DMU_k-based cross-efficiency for any DMU_j $(j = 1, \ldots, n)$ is calculated as

$$E_{kj} = \frac{\sum_{r=1}^{s} u_{rk}^* y_{rj} - u_0}{\sum_{i=1}^{m} v_{ik}^* x_{ij}} \qquad k, j = 1, 2, \ldots, n. \tag{11}$$

Table 3: Descriptive statistics of the mutual funds.

Mutual fund	Expected return	Variance
ACEFX	2.671	64.173
ACEGX	2.734	64.254
ACEMX	2.668	64.098
AELAX	1.241	22.976
AELGX	1.287	22.970
AGTHX	1.902	23.445
ARCGX	2.017	30.697
AVLFX	1.615	19.817
BJBIX	1.506	33.203
CUCAX	1.990	24.858
FAGIX	0.533	5.471
FAIAX	1.404	39.099
FAICX	1.420	39.095
FDEGX	2.445	53.059
FEURX	1.696	30.655
FIUIX	1.398	13.813
FSUTX	1.697	18.829
GEGTX	1.727	19.483
HRCPX	2.376	34.856
IDETX	2.356	35.331
IDEUX	1.874	24.913
IGLBX	1.858	25.178
IGLCX	2.488	39.982
JAMRX	1.991	35.740
JAOSX	1.870	23.264
JAVLX	2.237	30.771
JAWWX	1.946	20.705
LMVTX	1.735	19.463
MAFGX	1.849	20.490
MBFGX	1.923	20.626
MCFGX	2.463	60.247
MCOBX	1.959	19.729
MCOFX	1.875	18.821
MDFGX	1.942	18.833
MGCAX	1.789	31.720
MSEQX	1.842	31.348
NAWCX	1.318	24.631
NAWGX	1.384	24.676
POVBX	2.579	149.234
POVSX	1.619	21.647
RYOSX	2.690	42.951
SCGEX	1.656	18.530
SRGFX	2.224	40.412
SSGWX	2.044	26.771
TALGX	1.368	30.664
TRGEX	1.786	25.739
TWIEX	1.773	23.208
UMBIX	1.823	12.379

Table 3: Continued.

Mutual fund	Expected return	Variance
USBOX	2.093	24.542
VGHCX	1.334	26.919
VPMCX	1.513	20.243
WAGEX	1.625	17.619
WBIGX	0.979	25.015
FMAGX	1.681	14.225
JANSX	2.643	35.453
VFINX	1.690	27.731
VWNDX	1.889	21.968

The average of all E_{kj} $(k = 1, \ldots, n)$

$$\overline{E}_j = \frac{1}{n}\sum_{k=1}^{n} E_{kj} \tag{12}$$

is used as the cross-efficiency score for DMU_j $(j = 1, \ldots, n)$.

Note that the cross-efficiency score obtained in the above manner can be negative. This subject is presented by a simple numerical example involving five DMUs, with two input and single output [13].

The negative VRS cross-efficiency score is due to the fact that $\sum_{r=1}^{s} u_{rk} y_{rj} - u_0 < 0$ for some DMU_j; that is, some DMU_j will have negative efficiency ratios when they use a set of optimal weights obtained when DMU_k is under evaluation. Naturally, we want every output-input efficiency ratio to be positive regardless of the chosen weights. Therefore, adding $\sum_{r=1}^{s} u_{rk} y_{rj} - u_0 \geq 0$ into the VRS model is proposed when calculating the cross-efficiency scores [13]. This will also guarantee nonnegativity of both VRS cross-efficiency scores and VRS efficiency ratios.

Therefore, the following modified VRS DEA model is used for model (10) development and application:

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_{rk} y_{rk} - u_0 \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} - u_0 \leq 0, \quad j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - u_0 \geq 0, \quad j = 1, 2, \ldots, n \\
& u_{rk} \geq 0, \quad r = 1, \ldots, s \\
& v_{ik} \geq 0, \quad i = 1, \ldots, m \\
& u_0 \geq 0.
\end{aligned} \tag{13}$$

3. Methodology

Return of assets consists of money which we receive among period plus difference of buying and selling. Return is

Table 4: OWA operator weights for cross efficiency evaluation.

$\alpha = 1$	$\alpha = 0.9613$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$	$\alpha = 0.6$	$\alpha = 0.5$	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$
1	0.2	0.106	0.056	0.038	0.028	0.018	0.007	0	0	0
0	0.178	0.1	0.055	0.037	0.027	0.018	0.008	0	0	0
0	0.156	0.094	0.053	0.037	0.027	0.018	0.008	0	0	0
0	0.134	0.088	0.051	0.036	0.027	0.018	0.008	0	0	0
0	0.112	0.082	0.05	0.035	0.026	0.018	0.009	0	0	0
0	0.09	0.077	0.048	0.034	0.026	0.018	0.009	$3.00E-04$	0	0
0	0.068	0.071	0.046	0.034	0.026	0.018	0.01	0.001	0	0
0	0.046	0.065	0.045	0.033	0.025	0.018	0.01	0.002	0	0
0	0.024	0.059	0.043	0.032	0.025	0.018	0.01	0.003	0	0
0	0.002	0.053	0.042	0.031	0.024	0.018	0.011	0.003	0	0
0	0	0.047	0.04	0.031	0.024	0.018	0.011	0.004	0	0
0	0	0.041	0.038	0.03	0.024	0.018	0.011	0.005	0	0
0	0	0.035	0.037	0.029	0.023	0.018	0.012	0.005	0	0
0	0	0.029	0.035	0.029	0.023	0.018	0.012	0.006	0	0
0	0	0.023	0.033	0.028	0.023	0.018	0.012	0.007	0	0
0	0	0.017	0.032	0.027	0.022	0.018	0.013	0.008	0	0
0	0	0.011	0.03	0.026	0.022	0.018	0.013	0.008	0	0
0	0	0.005	0.029	0.026	0.022	0.018	0.014	0.009	0	0
0	0	0	0.027	0.025	0.021	0.018	0.014	0.01	0	0
0	0	0	0.025	0.024	0.021	0.018	0.014	0.011	0	0
0	0	0	0.024	0.023	0.02	0.018	0.015	0.011	0	0
0	0	0	0.022	0.023	0.02	0.018	0.015	0.012	0	0
0	0	0	0.02	0.022	0.02	0.018	0.015	0.013	0	0
0	0	0	0.019	0.021	0.019	0.018	0.016	0.014	0.003	0
0	0	0	0.017	0.02	0.019	0.018	0.016	0.014	0.004	0
0	0	0	0.016	0.02	0.019	0.018	0.016	0.015	0.006	0
0	0	0	0.014	0.019	0.018	0.018	0.017	0.016	0.007	0
0	0	0	0.012	0.018	0.018	0.018	0.017	0.017	0.009	0
0	0	0	0.011	0.017	0.018	0.018	0.018	0.017	0.011	0
0	0	0	0.009	0.017	0.017	0.018	0.018	0.018	0.012	0
0	0	0	0.007	0.016	0.017	0.018	0.018	0.019	0.014	0
0	0	0	0.006	0.015	0.016	0.018	0.019	0.02	0.016	0
0	0	0	0.004	0.014	0.016	0.018	0.019	0.02	0.017	0
0	0	0	0.003	0.014	0.016	0.018	0.019	0.021	0.019	0
0	0	0	0	0.013	0.015	0.018	0.02	0.022	0.02	0
0	0	0	0	0.012	0.015	0.018	0.02	0.023	0.022	0
0	0	0	0	0.011	0.015	0.018	0.02	0.023	0.024	0
0	0	0	0	0.011	0.014	0.018	0.021	0.024	0.025	0
0	0	0	0	0.01	0.014	0.018	0.021	0.025	0.027	0
0	0	0	0	0.009	0.014	0.018	0.022	0.026	0.029	0.005
0	0	0	0	0.008	0.013	0.018	0.022	0.026	0.03	0.011
0	0	0	0	0.008	0.013	0.018	0.022	0.027	0.032	0.017
0	0	0	0	0.007	0.012	0.018	0.023	0.028	0.033	0.023
0	0	0	0	0.006	0.012	0.018	0.023	0.029	0.035	0.029
0	0	0	0	0.005	0.012	0.018	0.023	0.029	0.037	0.035
0	0	0	0	0.005	0.011	0.018	0.024	0.03	0.038	0.041
0	0	0	0	0.004	0.011	0.018	0.024	0.031	0.04	0.047
0	0	0	0	0.003	0.011	0.018	0.024	0.031	0.042	0.053

TABLE 4: Continued.

$\alpha = 1$	$\alpha = 0.9613$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$	$\alpha = 0.6$	$\alpha = 0.5$	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$
0	0	0	0	0.003	0.01	0.018	0.025	0.032	0.043	0.059
0	0	0	0	0.002	0.01	0.018	0.025	0.033	0.045	0.065
0	0	0	0	0.001	0.01	0.018	0.026	0.034	0.046	0.071
0	0	0	0	$3.00E-04$	0.009	0.018	0.026	0.034	0.048	0.077
0	0	0	0	0	0.009	0.018	0.026	0.035	0.05	0.082
0	0	0	0	0	0.008	0.018	0.027	0.036	0.051	0.088
0	0	0	0	0	0.008	0.018	0.027	0.037	0.053	0.094
0	0	0	0	0	0.008	0.018	0.027	0.037	0.055	0.1
0	0	0	0	0	0.007	0.018	0.028	0.038	0.056	0.106

not definitely usually obvious. This is uncertain in rate of expected return defined as deviation of return. Deviation of return is called risk. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. In this regard, mean-variance model Markowitz expected return is treated as output and deviation as input. The methodology in this paper starts with asset selection via cross-efficiency evaluation using OWA operator weights. The data used for this methodology is from 57 mutual funds [5]. In many cases similar to this example, there are a lot of assets. It is better that starts with asset selection. The choice of the asset can be random or discrete. The random choice of assets is usually biased and does not promise an optimum portfolio; hence, it is more rational to have an objective choice while selecting the assets to be included in the portfolio. Among many evaluation methods, Data Envelopment Analysis (DEA) is one of the best ways for assessing the relative efficiency a group of homogenous decision making units (DMUs) that use multiple inputs to produce multiple outputs, originated from the work by Charnes et al. [3]. Selection of assets to be included in portfolio is followed by using cross-efficiency in DEA. The variable returns to scale (VRS) DEA model is used for efficiency evaluation. In the analysis, the variance of the assets is used as an input to the DEA and expected return is used as an output. Because the VRS DEA model can generate negative cross-efficiency score, thus model (13) is proposed so that the cross-efficiency scores are nonnegative. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A significant problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the DM's or investor's subjective preferences in to the evaluation. For example, the investors may wish self-evaluated efficiencies to account for 20% or play a leading role in the final overall efficiency assessment. Obviously, equal evaluation has no method to obtain this purpose. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation (see Table 4). This requires the reordering of the efficiencies, both self-evaluated and peer-evaluated, of each DMU, as shown in Table 2, where $w_1, \dots, w_n$ are OWA operator weights and ϑ_{ij} $(i, j = 1, \dots, n)$ are reordered efficiencies of each DMU from the biggest to the smallest. Obviously, self-evaluated efficiencies are always ranked in the first place; that is, $\vartheta_{i1} = \theta_{ii}^*, i = 1, \dots, n$. In order to determine the weights of OWA operator, it is necessary for the investor to provide his/her preferences on different efficiencies or optimism level towards the best relative efficiencies. For example, if the investor wants the self-evaluated efficiencies to account for 20% in the final overall efficiency assessment, then w_1 should take 0.2, whereas the other weights can be designated minimax disparity approach. With regard to the minimax disparity OWA operator weights, the following theorems are existed [17].

Theorem 1. *For a given w_1, there exists an integer $k \le n$ such that $w_i = w_1 - (i-1)d \ge 0$ for $i = 1, \dots, k$ and $w_i = 0$ for $i = k+1, \dots, n$, where k and d are determined by*

$$k = \min\left(n, \mathrm{INT}\left[\frac{2}{w_1}\right]\right), \qquad d = \frac{2(kw_1 - 1)}{k(k-1)}. \tag{14}$$

Theorem 2. *For a given orness degree $\alpha \in (0.5, 1)$, there exists an integer $k \le n$ such that $w_i = w_1 - (i-1)d \ge 0$ for $i = 1, \dots, k$ and $w_i = 0$ for $i = k+1, \dots, n$, where k, w_1, and d are determined by*

$$w_1 = \frac{4(k+1) - 6n + 6\alpha(n-1)}{k(k+1)}, \qquad d = \frac{2(kw_1 - 1)}{k(k-1)}. \tag{15}$$

In this paper, the OWA operator weights can be determined by using the minimax disparity approach. In the following, some very special OWA operator weights for the cross-efficiency evaluation are given.

Consider $w_1 = 1$ and $w_j = 0$ $(j \ne 1)$. In this case, $\mathrm{orness}(W) = 1$ and $\overline{\theta}_i = \sum_{k=1}^{n} w_k \vartheta_{ik} = \vartheta_{i1} = \theta_{ii}^*$ for $i = 1, \dots, n$. The investor considers only self-evaluated efficiencies in the final overall efficiency assessment and is purely optimistic.

Consider $w_n = 1$ and $w_j = 0$ $(j \ne 1)$. In this case, $\mathrm{orness}(W) = 0$ and $\overline{\theta}_i = \sum_{k=1}^{n} w_k \vartheta_{ik} = \vartheta_{in} = \min_k(\theta_{ik})$ for $i = 1, \dots, n$. The investor chooses the least efficiency value of each DMU as its overall efficiency and is extremely pessimistic.

Consider $w_1, \dots, w_n = 1/n$. In this case, $\mathrm{orness}(W) = 0.5$ and $\overline{\theta}_i = \sum_{k=1}^{n} w_k \vartheta_{ik} = (1/n) \sum_{k=1}^{n} \theta_{ik}$ for $i = 1, \dots, n$, which is the average cross-efficiency value in the traditional cross-efficiency evaluation.

Table 5: Cross efficiency by optimism level of the investor for $\alpha \geq 0.5$.

DMU	Mutual funds	VRS efficiency	$\alpha = 1$	$\alpha = 0.9613$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$	$\alpha = 0.6$	$\alpha = 0.5$	Ranking $\alpha = 0.8$
1	ACEFX	0.32	0.324	0.323	0.323	0.319	0.312	0.306	0.308	50
2	ACEGX	0.33	0.334	0.333	0.333	0.328	0.322	0.315	0.317	48
3	ACEMX	0.32	0.324	0.287	0.323	0.319	0.312	0.306	0.308	49
4	AELAX	0.37	0.370	0.373	0.370	0.363	0.350	0.3392	0.336	44
5	AELGX	0.38	0.377	0.371	0.383	0.377	0.365	0.3550	0.353	43
6	AGTHX	0.56	0.569	0.572	0.566	0.560	0.558	0.556	0.568	21
7	ARCGX	0.46	0.472	0.454	0.468	0.463	0.459	0.456	0.465	34
8	AVLFX	0.55	0.560	0.563	0.558	0.552	0.546	0.541	0.549	22
9	BJBIX	0.31	0.311	0.313	0.310	0.307	0.301	0.297	0.300	51
10	CUCAX	0.56	0.572	0.564	0.568	0.561	0.558	0.554	0.565	20
11	FAGIX	**0.66**	**0.669**	**0.667**	**0.670**	**0.627**	**0.509**	**0.410**	**0.320**	**9**
12	FAIAX	0.24	0.246	0.248	0.246	0.242	0.236	0.232	0.233	55
13	FAICX	0.25	0.249	0.251	0.249	0.245	0.239	0.235	0.236	54
14	FDEGX	0.35	0.351	0.352	0.349	0.345	0.339	0.334	0.337	46
15	FEURX	0.38	0.379	0.382	0.378	0.3751	0.372	0.370	0.377	42
16	FIUIX	**0.69**	**0.694**	**0.693**	**0.693**	**0.683**	**0.667**	**0.653**	**0.655**	**6**
17	FSUTX	0.61	0.618	0.615	0.620	0.611	0.607	0.603	0.615	13
18	GEGTX	0.60	0.608	0.605	0.607	0.601	0.598	0.595	0.607	16
19	HRCPX	0.51	0.516	0.518	0.512	0.506	0.498	0.491	0.496	27
20	IDETX	0.50	0.503	0.505	0.500	0.494	0.486	0.480	0.485	31
21	IDEUX	0.52	0.525	0.527	0.521	0.517	0.515	0.513	0.525	26
22	IGLBX	0.50	0.513	0.515	0.510	0.506	0.504	0.503	0.514	28
23	IGLCX	0.47	0.477	0.478	0.474	0.468	0.459	0.452	0.456	34
24	JAMRX	0.39	0.398	0.399	0.395	0.390	0.388	0.386	0.393	39
25	JAOSX	0.55	0.561	0.562	0.557	0.552	0.550	0.548	0.560	19
26	JAVLX	0.53	0.540	0.542	0.536	0.530	0.523	0.517	0.524	24
27	JAWWX	**0.65**	**0.666**	**0.668**	**0.661**	**0.654**	**0.650**	**0.647**	**0.661**	**7**
28	LMVTX	0.61	0.611	0.604	0.610	0.605	0.602	0.599	0.611	15
29	MAFGX	**0.62**	**0.627**	**0.628**	**0.622**	**0.618**	**0.616**	**0.614**	**0.628**	**10**
30	MBFGX	**0.65**	**0.657**	**0.659**	**0.653**	**0.646**	**0.643**	**0.640**	**0.654**	**8**
31	MCFGX	0.31	0.312	0.314	0.310	0.306	0.301	0.299	0.282	52
32	MCOBX	**0.69**	**0.705**	**0.707**	**0.700**	**0.693**	**0.688**	**0.685**	**0.699**	**4**
33	MCOFX	**0.68**	**0.696**	**0.698**	**0.691**	**0.685**	**0.682**	**0.680**	**0.695**	**5**
34	MDFGX	**0.72**	**0.730**	**0.732**	**0.725**	**0.717**	**0.713**	**0.710**	**0.725**	**3**
35	MGCAX	0. 38	0.387	0.387	0.386	0.383	0.382	0.381	0.389	40
36	MSEQX	0.40	0.407	0.409	0.405	0.402	0.400	0.399	0.408	36
37	NAWCX	0.36	0.367	0.312	0.366	0.360	0.350	0.340	0.340	45
38	NAWGX	0.38	0.385	0.337	0.384	0.378	0.369	0.361	0.362	41
39	POVBX	0.13	0.134	0.134	0.133	0.131	0.128	0.126	0.127	57
40	POVSX	0.51	0.513	0.511	0.512	0.506	0.501	0.497	0.504	30
41	RYOSX	0.48	0.490	0.492	0.486	0.480	0.470	0.462	0.465	32
42	SCGEX	0.61	0.613	0.593	0.612	0.606	0.600	0.596	0.606	14
43	SRGFX	0.40	0.408	0.410	0.405	0.401	0.395	0.391	0.396	38
44	SSGWX	0.54	0.551	0.552	0.547	0.541	0.536	0.532	0.542	23
45	TALGX	0.30	0.306	0.266	0.305	0.301	0.293	0.286	0.287	53
46	TRGEX	0.47	0.476	0.476	0.475	0.472	0.470	0.469	0.479	33
47	TWIEX	0.52	0.524	0.523	0.523	0.519	0.517	0.515	0.526	25

Table 5: Continued.

DMU	Mutual funds	VRS efficiency	$\alpha = 1$	$\alpha = 0.9613$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$	$\alpha = 0.6$	$\alpha = 0.5$	Ranking $\alpha = 0.8$
48	UMBIX	**1.00**	**1.000**	**1.020**	**1.012**	**1.005**	**1.002**	**1.021**	**0.988**	**1**
49	USBOX	0.61	0.620	0.622	0.616	0.609	0.603	0.598	0.608	12
50	VGHCX	0.34	0.342	0.339	0.342	0.336	0.327	0.3191	0.319	47
51	VPMCX	0.51	0.513	0.473	0.512	0.505	0.497	0.490	0.495	29
52	WAGEX	0.63	0.633	0.627	0.631	0.624	0.618	0.613	0.623	10
53	WBIGX	0.27	0.174	0.174	0.172	0.172	0.187	0.201	0.220	56
54	FMAGX	**0.80**	**0.810**	**0.790**	**0.808**	**0.801**	**0.795**	**0.790**	**0.804**	**2**
55	JANSX	0.57	0.580	0.583	0.576	0.569	0.558	0.548	0.552	18
56	VFINX	0.41	0.418	0.408	0.417	0.413	0.410	0.408	0.415	36
57	VWNDX	0.59	0.602	0.583	0.597	0.592	0.590	0.588	0.601	17

The orness degree can be regarded as a measure of the optimism level of the investor. If the investor wants self-evaluated to be more influenced, it should be used of orness > 0.5. And if investor wants peer-evaluated to be more influenced, it should be used of orness < 0.5. Obviously, the best selection of mutual funds is not fixed. It varies with the investor's optimism level or subjective performance.

4. Application in Mutual Funds

We illustrate our approach using OWA operator weights in cross-efficiency evaluation for a dataset of 57 mutual funds. A list of funds used is provided in Table 3. In this report, there is expected return and variance of mutual funds which expected return is considered as output and variance is as input. The example is received from Joro and Na [5] and is about portfolio performance evaluation in a mean-variance framework. Four mutual funds are evaluated as efficient in model [1] which portfolio can be composed with them. But it is better to use cross-efficiency to choose the best portfolio. Because the model (10) can generate negative cross-efficiency score, thus model (13) is used so that the cross-efficiency scores are nonnegative. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A main problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the investor's subjective preferences into the evaluation. Obviously, equal evaluation has no way to obtain this goal. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation. This requires the reordering of the efficiencies. The orness degree can be regarded as a measure of the optimism level of the investor. If the investor wants self-evaluated to be more influenced, it should be used of orness > 0.5. And if investor wants peer-evaluated to be more influenced, it should be used of orness < 0.5. In the traditional equal of cross-efficiencies, the weight assigned to the self-evaluated efficiencies is only $0.017\% = (1/57)$. For an optimistic investor, he/she may wish the self-evaluated efficiencies to play a more role in the final overall efficiency assessment. For example, the investor may wish the weight for the self-evaluated efficiencies to account for 20% rather than 0.017% in the final overall efficiency assessment. By Theorem 1, $k = \min(57, \mathrm{INT}[2/0.2]) = 10$ and $d = 2(kw_1 - 1)/k(k-1) = 2(10 \times 0.2 - 1)/(10 \times 9) = 0.022$ are obtained. As a result, the weights for cross-efficiency are computed as $w_1 = 0.2$, $w_2 = w_1 - d = 0.178$, $w_3 = w_1 - 2d = 0.156$, $w_4 = w_1 - 3d = 0.134$, $w_5 = w_1 - 4d = 0.112$, $w_6 = w_1 - 5d = 0.09$, $w_7 = w_1 - 6d = 0.068$, $w_8 = w_1 - 7d = 0.046$, $w_9 = w_1 - 8d = 0.024$, $w_{10} = w_1 - 9d = 0.002$, and $w_{11} = \cdots = w_{57} = 0$. The investor's optimism level is measured as $\mathrm{orness}(W) = (1/(n-1)) \sum_{i=1}^{n} (n-i) w_i = 0.9613$. The weighted average cross-efficiency values of the 57 mutual funds are computed by $\overline{\theta}_i = \sum_{k=1}^{n} w_k \vartheta_{ik}$ for $i = 1, \ldots, n$ and the results are presented in the fifth column of the Table 5. As is seen in Tables 5 and 6, ranks are not the same. We calculated these ranks for $\alpha = 0.8$ and $\alpha = 0.1$. Some of the best ranks are designated according to investor. We consider ten of the best ranks. Five of the best ranks become the same, in this example,incidentally. Selecting of mutual funds to be included in portfolio is followed by ten of the best ranks in Tables 7 and 8 for $\alpha \geq 0.5$, $\alpha \leq 0.4$, respectively.

5. Conclusion

In this paper, a new method is suggested for selecting the best of portfolio with one input (variance) and one output (expected return) in the DEA context. As an advanced management decision tool, DEA has been widely used for performance evaluation [24, 25], productivity analysis [26–28], resource allocation [29], and so on. The cross-efficiency evaluation is an important method for ranking DMUs in DEA. Traditional approaches for the cross-efficiency evaluation do not differentiate between self-evaluated and peer-evaluated efficiencies. A main problem with these approaches is that the weight assigned to the self-evaluated efficiency of each DMU is fixed and has no way of incorporating the investor's subjective preferences in to the evaluation. To show the investor's subjective preferences on different efficiencies, the use of OWA operator weights is stated for cross-efficiency evaluation. In this case, if the investor wants self-evaluated

Table 6: Cross efficiency by OWA operator weights for $\alpha \leq 0.4$.

DMU	Mutual funds	VRS efficiency	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$	Ranking $\alpha = 0.1$
1	ACEFX	0.32	0.295	0.288	0.282	0.282	48
2	ACEGX	0.33	0.302	0.296	0.289	0.287	46
3	ACEMX	0.32	0.295	0.288	0.282	0.281	49
4	AELAX	0.37	0.316	0.305	0.293	0.291	45
5	AELGX	0.38	0.334	0.323	0.312	0.310	43
6	AGTHX	0.56	0.551	0.548	0.545	0.545	17
7	ARCGX	0.46	0.450	0.446	0.443	0.443	32
8	AVLFX	0.55	0.530	0.524	0.518	0.515	20
9	BJBIX	0.31	0.288	0.283	0.278	0.276	49
10	CUCAX	0.56	0.547	0.543	0.540	0.539	18
11	FAGIX	0.66	0.213	0.115	0.011	0	57
12	FAIAX	0.24	0.222	0.217	0.211	0.210	55
13	FAICX	0.25	0.225	0.220	0.215	0.214	54
14	FDEGX	0.35	0.323	0.318	0.312	0.311	42
15	FEURX	0.38	0.365	0.362	0.359	0.358	40
16	FIUIX	**0.69**	**0.625**	**0.610**	**0.595**	**0.591**	**9**
17	FSUTX	0.61	0.595	0.590	0.586	0.584	11
18	GEGTX	0.60	0.588	0.585	0.581	0.578	13
19	HRCPX	0.51	0.477	0.469	0.461	0.460	28
20	IDETX	0.50	0.466	0.459	0.451	0.450	31
21	IDEUX	0.52	0.509	0.507	0.505	0.504	23
22	IGLBX	0.50	0.499	0.497	0.494	0.494	25
23	IGLCX	0.47	0.437	0.430	0.421	0.420	34
24	JAMRX	0.39	0.381	0.378	0.376	0.375	35
25	JAOSX	0.55	0.544	0.542	0.539	0.539	19
26	JAVLX	0.53	0.505	0.498	0.492	0.490	26
27	JAWWX	**0.65**	**0.640**	**0.637**	**0.633**	**0.633**	**6**
28	LMVTX	0.61	0.592	0.589	0.585	0.583	12
29	MAFGX	**0.62**	**0.609**	**0.607**	**0.604**	**0.603**	**8**
30	MBFGX	**0.65**	**0.634**	**0.631**	**0.627**	**0.627**	**7**
31	MCFGX	0.31	0.287	0.282	0.277	0.276	51
32	MCOBX	**0.69**	**0.677**	**0.673**	**0.669**	**0.669**	**4**
33	MCOFX	**0.68**	**0.675**	**0.672**	**0.669**	**0.668**	**5**
34	MDFGX	**0.72**	**0.702**	**0.698**	**0.695**	**0.694**	**3**
35	MGCAX	0. 38	0.378	0.376	0.374	0.373	38
36	MSEQX	0.40	0.396	0.395	0.393	0.392	36
37	NAWCX	0.36	0.322	0.313	0.303	0.300	44
38	NAWGX	0.38	0.345	0.336	0.327	0.325	41
39	POVBX	0.13	0.122	0.119	0.117	0.116	56
40	POVSX	0.51	0.486	0.481	0.475	0.473	27
41	RYOSX	0.48	0.444	0.435	0.425	0.422	33
42	SCGEX	0.61	0.586	0.580	0.574	0.572	16
43	SRGFX	0.40	0.382	0.377	0.372	0.371	39
44	SSGWX	0.54	0.524	0.519	0.515	0.515	21
45	TALGX	0.30	0.273	0.266	0.258	0.257	53
46	TRGEX	0.47	0.465	0.463	0.460	0.459	29

Table 6: Continued.

DMU	Mutual funds	VRS efficiency	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$	Ranking $\alpha = 0.1$
47	TWIEX	0.52	0.511	0.508	0.505	0.504	22
48	UMBIX	**1.00**	**0.992**	**0.988**	**0.983**	**0.981**	**1**
49	USBOX	0.61	0.587	0.581	0.576	0.575	15
50	VGHCX	0.34	0.303	0.295	0.286	0.284	47
51	VPMCX	0.51	0.475	0.467	0.459	0.457	30
52	WAGEX	**0.63**	**0.601**	**0.594**	**0.588**	**0.585**	**10**
53	WBIGX	0.27	0.228	0.242	0.259	0.264	52
54	FMAGX	**0.80**	**0.778**	**0.772**	**0.765**	**0.761**	**2**
55	JANSX	0.57	0.527	0.517	0.505	0.503	24
56	VFINX	0.41	0.402	0.398	0.395	0.394	35
57	VWNDX	0.59	0.583	0.580	0.578	0.577	14

Table 7: Selecting the best assets for making portfolio for $\alpha \geq 0.5$.

		Expected return	Variance	Ranking
48	UMBIX	1.833	12.379	1
54	FMAGX	1.681	14.225	2
34	MDFGX	1.942	18.833	3
32	MCOBX	1.959	19.729	4
33	MCOFX	1.875	18.821	5
16	FIUIX	**1.398**	**13.813**	**6**
27	JAWWX	**1.946**	**20.705**	**7**
30	MBFGX	**1.923**	**20.626**	**8**
11	FAGIX	**0.533**	**5.471**	**9**
29	MAFGX	**1.849**	**20.490**	**10**

Table 8: Selecting the best assets for making portfolio for $\alpha \leq 0.4$.

		Expected return	Variance	Ranking
48	UMBIX	1.833	12.379	1
54	FMAGX	1.681	14.225	2
34	MDFGX	1.942	18.833	3
32	MCOBX	1.959	19.729	4
33	MCOFX	1.875	18.821	5
27	JAWWX	**1.946**	**20.705**	**6**
30	MBFGX	**1.923**	**20.626**	**7**
29	MAFGX	**1.849**	**20.490**	**8**
16	FIUIX	**1.398**	**13.813**	**9**
52	WAGEX	**1.625**	**17.619**	**10**

to be more influenced, it should be used of orness > 0.5. Thus, if investor wants peer-evaluated to be more influenced, it should be used of orness < 0.5. In Tables 7 and 8, rankings have been designated for ten of the best mutual funds via OWA operator weights in cross-efficiency evaluation. Since there are a large number of assets to invest in, this objective leads to two investment problems. First, the assets are selected for making portfolio and, second, the proportion or weights are determined to be allocated to the selected assets. Selection of assets to be included in portfolio is followed by using cross-efficiency evaluation. Model (13) is used for this purpose. In this regard, this model is used to analyze the given 57 mutual funds and ten of the best mutual funds are obtained. The other methods of ranking can be used for ten of the best mutual funds for asset allocation in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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