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Research Article

Dark Energy Constraints on Red-Shift-Based $f(R)$ Gravity

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We have studied cosmological dynamics in $f(R)$ gravity theory via cosmographic parameters. We have changed variables of field equations from time to red-shift and solved the achieved differential equation analytically for $f(R)$. Then we have used Taylor expansion to find general form of $f(R)$ function around the present day value of scalar curvature. By introducing $F(z) = df(R)/dR$ we would simplify our calculations; if we consider $F(z)$ as a given function we would restrict our answers of $f(R)$. In this paper we offer a linear form of $F(z) = 1 + \alpha z$ which leads us to a specific $f(R)$ function, where α is a constant which depends on the present day value of deceleration parameter. As an example, using Taylor expansion coefficients, we have compared our analytically calculated function with reconstructed $f(R)$ function for Dark Energy models. To reconstruct $f(R)$ action for Dark Energy models, we have used corresponding $H(z)$ of each Dark Energy model for calculating Taylor expansion coefficients. As our $F(z)$ function is linear, the Taylor expansion coefficients would be a function of present day value of deceleration parameter.

1. Introduction

Recent data indicates that the Universe is in an accelerating expansion phase [1–8]. Being responsible for this expansion, several approaches such as positive cosmological constant, Dark Energy, and modified gravity have been introduced. The positive cosmological constant would be defined either geometrically as modifying the left hand side of Einstein equations or as a kinematic term on the right hand side with the equation of state parameter $\omega = -1$; however, the fine tuning problem causes some difficulties [9–16]. There are various scalar field models of Dark Energy [16–33]. These models indeed are the outcomes of modifying the right hand side of the Einstein equations, $G_{\mu\nu} = \chi T_{\mu\nu}$, by considering a source term with an equation of state parameter $\omega < -1/3$ which is recognized as Dark Energy. In modified gravity, the right hand side is left unchanged and we modify the left hand side of the Einstein field equations, so we have a large class of theories of gravity [34–42]. Particularly, we are interested in fourth order theories [43–55] based on replacing the scalar curvature R in the Hilbert-Einstein action with a generic analytic function $f(R)$ which should be reconstructed starting from data and physically motivated issues. These models fit both the cosmological data and Solar System constraints in several physically interesting cases [56–65]. Vacuum solutions of $f(R)$ gravity theories is one of interesting subjects which are obtained for constant Ricci scalar [66–72], allowing derivation of non constant curvature scalar vacuum solutions. In this paper, following our other work [73], we have considered vacuum solutions of $f(R)$ gravity in a different way which does not assume constant scalar curvature to obtain vacuum solutions.

Vacuum solutions of modified $f(R)$ gravity explain the late time phase transition of cosmological parameters like deceleration parameter just by pure geometry without the need for dark energy companion of the Universe. The pioneering works on reconstruction of modified action through inverse method are done in [66–72]. They developed a general scheme for cosmological reconstruction of modified $f(R)$ gravity in terms of red-shift without using an auxiliary scalar in intermediate calculations. Using this method, it is possible to construct the specific modified gravity which contains any requested FRW cosmology. A number of $f(R)$ gravity examples are constructed where the following background evolutions may be realized: LCDM epoch, deceleration with subsequent transition to effective phantom super-acceleration leading to Big Rip singularity, deceleration with transition to transit phantom phase without future singularity, and oscillating Universe. It is important that all these cosmologies may be realized only by modified gravity without the use of any dark components. Then we have tried to reconstruct an appropriate action for the modified gravity through the semi-inverse solution method. We do not assume any FRW cosmology to reconstruct its related $f(R)$ action [63–81]. Indeed, as curvature scalar itself is a function of red-shift, by introducing a linear correction which is also a function of z, we can reconstruct Taylor expansion of $f(R)$ action around present time value of red-shift ($z = 0$). For restricting Taylor expansion, we would study the variation of its coefficients from GR.

Capozziello et al. has also studied Taylor expansion of $f(R)$ function in a model independent way where he has shown how it is possible to relate the cosmographic parameters (i.e., the deceleration q_0, the jerk j_0, the snap s_0, and the lerk l_0 parameters) to the present-day values of $f(R)$ and its derivatives $f^{(n)}(R) = d^n f/dR^n$ (with $n = 1, 2, 3$), thus offering a new tool to constrain such higher order models. Their analysis thus offers the possibility to relate the model independent results coming from cosmography to the theoretically motivated assumptions of $f(R)$ cosmology [75]. But here we have added a new condition to the question which has limited our answers to a certain form of the $f(R)$ function.

In Section 2, we have a briefer review of modified field equations and introduce our $F(z)$ model as a linear function of Red-Shift; we have also tried to reconstruct the $f(R)$ function model independently. In Section 3, we calculate the Taylor expansion of $f(R)$ function via $F(z)$. In Section 4, we have discussed influence of Taylor expansion coefficients in compare with the first term of expansion. In Section 5, we have reconstructed Taylor expansion for several Dark Energy models and compared the result with our $f(R)$ function which has been calculated in Section 3.

2. Modified Field Equations

The action of modified theory of gravity is given by

$$S = \int d^4x\sqrt{-g}\,[f(R) + L_m], \tag{1}$$

where L_m is the matter action such as radiation, baryonic matter, dark matter, and so on which we do not consider in field equation. In this essay, we consider the flat Friedmann-Robertson-Walker, (FRW) background as

$$ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + r^2d\theta^2 + r^2\mathrm{Sin}^2\theta d\varphi^2\right], \tag{2}$$

so that the gravitational field equations for $f(R)$ gravity are provided by the following form:

$$-3\frac{\ddot{a}}{a}f' + 3\frac{\dot{a}}{a}\dot{R}f'' + \frac{1}{2}f = 0, \tag{3}$$

$$\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2\right]f' - 2\frac{\dot{a}}{a}\dot{R}f'' - \dot{R}^2f''' - \ddot{R}f'' - \frac{1}{2}f = 0, \tag{4}$$

where the over-dot denotes a derivative with respect to t, and the prime denotes a derivative with respect to R, $a(t)$ is the scale factor, and $H = \dot{a}/a$ is the Hubble parameter. Eliminating f between (3) and (4) results

$$-2\left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right]f' + \frac{\dot{a}}{a}\dot{R}f'' - \ddot{R}f'' - \dot{R}^2f''' = 0. \tag{5}$$

This can be changed in the form of

$$\ddot{F} - H\dot{F} + 2\dot{H}F = 0, \tag{6}$$

where $F = df/dR$. Equation (6) is a second order differential equation of F with respect to time, in which both of F and H are undefined. The usual method to solve (6) is based on definition of $f(R)$. Changing the variable of the above equation from t to a new variable like N, the number of e-folding, was done in [70–72]. The variable N is related to the red-shift z by $e^{-N} = 1+z$. The cosmological dynamic equation is solved by definition of Hubble parameter as a function of N in a general form, then the equation is rewritten by redefinition of the variable from the number of e-folding to the Ricci scalar and solved with respect to the Ricci scalar. Thus they demonstrated that modified $f(R)$ gravity may describe the LCDM epoch without any need for introducing the effective cosmological constant, non-phantom matter, or phantom matter. In this work we would like to replace the variable of (6) by red-shift, z, directly to find the $f(R)$ function around R_0 via Taylor expansion. Each red-shift z has an associated cosmic time t (the time-when objects are observed with red-shift z emitting light), so we can replace all the differentials with respect to t by z via

$$\frac{d}{dt} = \frac{da}{dt}\frac{dz}{da}\frac{d}{dz} = -(1+z)\,H(z)\,\frac{d}{dz}, \tag{7}$$

where we use $1 + z = a_0/a$, and we consider $a_0 = 1$, in the present time. Now, we can replace the variable of (6) from t to z by using (7) and obtain a first order differential equation for H^2 with respect to z as

$$\frac{d}{dz}H(z)^2 = P(z)\,H(z)^2, \tag{8}$$

where $P(z)$ depends on the definition of $F(z)$ as

$$P(z) = \frac{2(1+z)\left(d^2F/dz^2\right) + 4(dF/dz)}{2F - (1+z)(dF/dz)}. \tag{9}$$

Now we may solve Hubble parameter that depends on the definition of $F(z)$ [63, 64, 73]. In this paper we consider $F(z)$ as a linear function of z:

$$F(z) = 1 + \alpha z. \tag{10}$$

To avoid the Dolgov-Kawasaki instability [82], the second order derivative of $f(R)$ action with respect to Ricci scalar should be positive for different values of α. Solving (8), using linear correction equation (10), the Hubble parameter has been found as

$$H(z)^2 = H_\alpha^2\left[1 - \frac{\alpha}{2}(1+z)\right]^4 \tag{11}$$

in which $H_\alpha^2 = H_0^2/(1-\alpha/2)^4$. Using Ricci scalar definition, $R = 6(\dot{H} + 2H^2)$, the scalar curvature would be

$$R(z) = R_\alpha \left[1 - \frac{\alpha}{2}(1-z)\right]^3 \tag{12}$$

in which

$$R_\alpha = \frac{12(1-\alpha)H_0^2}{(1-\alpha/2)^4}. \tag{13}$$

Eliminating z between (10) and (12) leads us to reach

$$F(R) = 2\left(\frac{R}{R_\alpha}\right)^{1/3} + \alpha - 1. \tag{14}$$

In the case of $\alpha = 0$ or $R_\alpha = R_0$, we would like to recover de'Sitter solution. Using (12) we obtain $R = R_0$ and then $F(R) = 1$ which is equal to the GR or de'Sitter model. Calculating $f(R)$ by integrating of (14) with respect to R, we reach

$$f(R) = R\left(\frac{3}{2}\left(\frac{R}{R_\alpha}\right)^{1/3} + \alpha - 1\right) + C, \tag{15}$$

where C is the constant of integration. Since in the absence of correction term, α, we should recover de'Sitter model ($f_d(R) = R - 2\Lambda$), we compare the value of de'Sitter action with the action of (15) when $\alpha = 0$ or $R = R_0$. Since the scalar curvature of de'Sitter Universe is a constant, then we can determine the value of $f_d(R)$ in the presence of constant curvature. If we solve the standard algebraic equation of $f(R)$ gravity as $2f_d(R) - RF_d(R) = 0$ we can find $R_0 = 4\Lambda$. Then the value of de'Sitter action in the case of $R = R_0$ is $f_d(R_0 = 4\Lambda) = 2\Lambda$ which is equal to $R_0/2$. For the case of our obtained action from (15), we have $f(R = R_0) = R_0/2 + C$ when $\alpha = 0$. Since we would like to have the same solution for de'Sitter model and our obtained action in present of $R = R_0$ we force constant of integration, C, to be equal to zero [73]. One of main late time cosmological constraint which is tested for redshift-based correction of $f(R)$ gravity is SNeIa distance module with respect to redshift constraint. The result of this comparison is shown in Figure 3 [73].

3. Taylor Expansion of $f(R)$ via General Function of $F(z)$

As we are interested in finding appropriate action without solving field equation, here we have used Taylor expansion of $f(R)$ function around $R = R_0 = R(z=0)$, However, as we are using this method, our reconstructed $f(R)$ is acceptable just around present day value of curvature scalar:

$$\begin{aligned} f(R) = {} & f(R_0) + (R - R_0)\left.\frac{df}{dR}\right|_{R_0} + \frac{(R-R_0)^2}{2!}\left.\frac{d^2 f}{dR^2}\right|_{R_0} \\ & + \frac{(R-R_0)^3}{3!}\left.\frac{d^3 f}{dR^3}\right|_{R_0} + \cdots \end{aligned} \tag{16}$$

and as $R = R(z)$, coefficients of Taylor expansion to the third order would be find as

$$\begin{aligned} \frac{d}{dR} f(R) &= F(z), \\ \frac{d^2}{dR^2} f(R) &= \left(\frac{dR}{dz}\right)^{-1} \frac{dF}{dz}, \\ \frac{d^3}{dR^3} f(R) &= \left(\frac{dR}{dz}\right)^{-2} \frac{d^2F}{dz^2} - \frac{d^2R/dz^2}{(dR/dz)^3}\frac{dF}{dz}. \end{aligned} \tag{17}$$

Via (7), (8), and definition of Ricci scalar, $R = 6(\dot{H} + 2H^2)$, we can rewrite R as a function of z:

$$R(z) = 3H^2\left[4 - (1+z)P(z)\right]. \tag{18}$$

We can normalize $R(z)$ to H_0:

$$\begin{aligned} R(z) &= \frac{3H(z)^2}{H_0^2}\left[4 - (1+z)P(z)\right] \\ &= 3h(z)^2\left[4 - (1+z)P(z)\right]. \end{aligned} \tag{19}$$

Finding the first term of Taylor expansion, $f(R_0)$ we use (3) so it also should be written with respect to z:

$$f(R(z)) = 6h(z)^2\left[q(z)F(z) + (1+z)\frac{dF(z)}{dz}\right], \tag{20}$$

where $q(z) = -1 - (\dot{H}/H^2)$ is the deceleration parameter that after replacing all the differentials with respect to t by z via (7) would be as

$$q(z) = -1 + \frac{1+z}{2H^2}\frac{dH^2}{dz} \tag{21}$$

or

$$q(z) = -1 + \frac{1+z}{2}P(z). \tag{22}$$

Finally the constant term of Taylor expansion would be obtained by substituting $z = 0$ in (20). Using the linear modification $F(z) = 1 + \alpha z$, R derivatives for this function in $z = 0$ would be as

$$R_0 = \frac{24(1-\alpha)H_0^2}{(2-\alpha)}, \tag{23}$$

$$R'(z=0) = \frac{72(1-\alpha)\alpha H_0^2}{(2-\alpha)^2}, \tag{24}$$

$$R''(z=0) = \frac{144(1-\alpha)\alpha^2 H_0^2}{(2-\alpha)^3}. \tag{25}$$

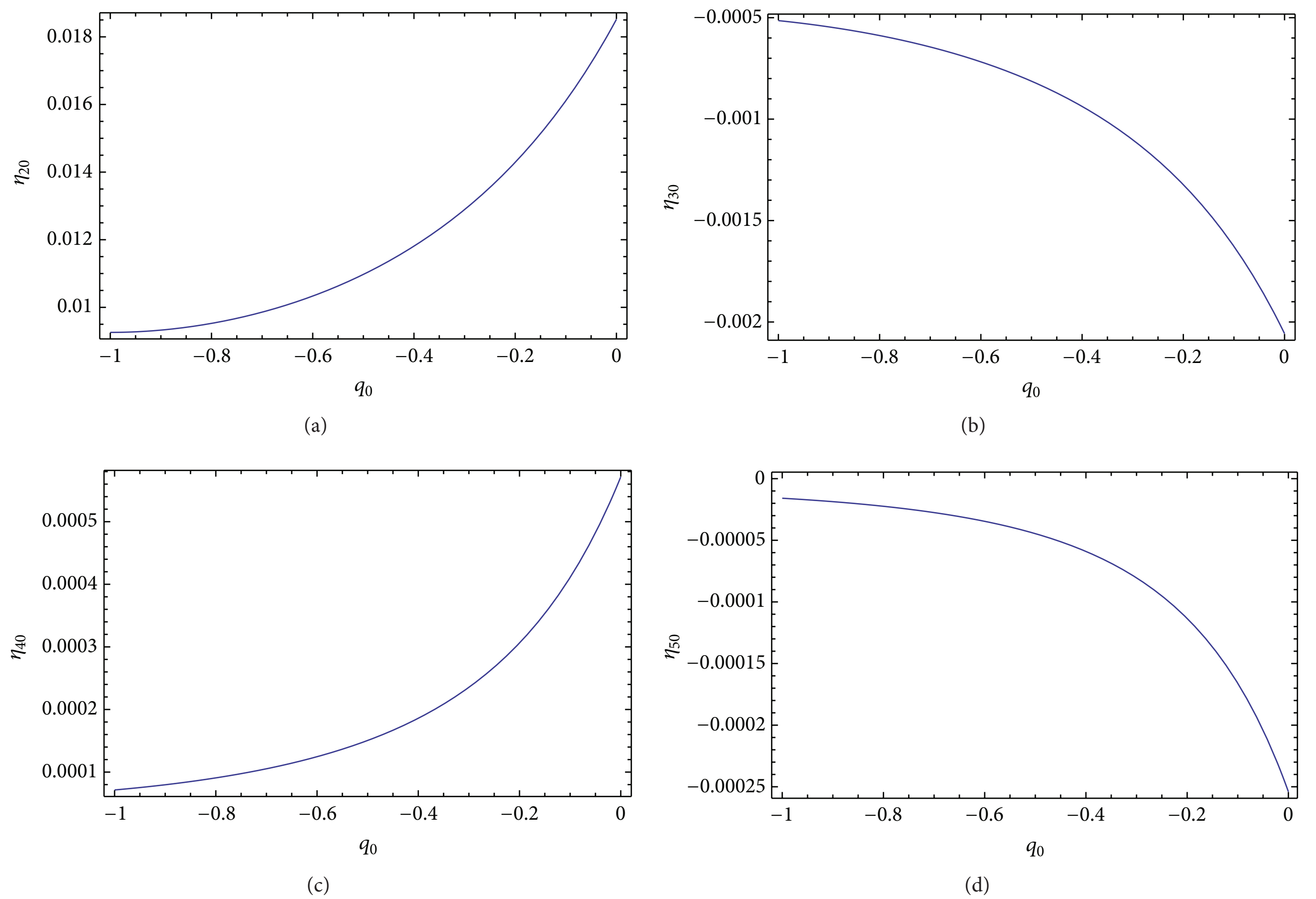

Figure 1: The dimensionless ratio between the present-day values of $f^{(i)}(R)$ and $f(R)$ as a function of q_0.

Hence, using (16) Taylor expansion coefficients would be

$$\begin{gathered} f\left(R_0\right)=-\frac{6\left(\alpha^2+\alpha-2\right)H_0^2}{2-\alpha}, \\ \frac{d}{dR}f(R)\bigg|_{R_0}=1, \\ \frac{d^2}{dR^2}f(R)\bigg|_{R_0}=\frac{(2-\alpha)^2}{72\left(1-\alpha\right)H_0^2}, \\ \frac{d^3}{dR^3}f(R)\bigg|_{R_0}=-\frac{(2-\alpha)^3}{2592(1-\alpha)^2H_0^4}. \end{gathered} \tag{26}$$

4. Order of Taylor Expansion

In this section we keep Taylor expansion of $f(R)$ action up to the third order term and we ignore higher order modifications. Therefore, $f(R)$ Taylor expansion coefficients have been studied as a function with respect to present day value of deceleration parameter. For linear correction, $F(z)=1+\alpha z$, $q(z)$ would be obtained by (22) as

$$q\left(z=0\right)=-1+\frac{2\alpha}{2-\alpha}. \tag{27}$$

Therefore

$$\alpha=\frac{2\left(1+q_0\right)}{3+q_0}. \tag{28}$$

Rewriting $f(R)$ Taylor expansion coefficients with respect to q_0, via (26) and then introducing function $\eta_{i0}(q_0)=h_0^{2i}f^{(i)}(q_0)/f(q_0)$ [75], in which $i=2,3,4,5,\ldots$ is the order of differentiation, we have:

$$\begin{gathered} \eta_{20}\left(q_0\right)=\frac{1}{27\left(2-3q_0+q_0^3\right)}, \\ \eta_{30}\left(q_0\right)=\frac{1}{243(-1+q_0)^3\left(2+q_0\right)}, \\ \eta_{40}\left(q_0\right)=\frac{5}{4374(-1+q_0)^4\left(2+q_0\right)}, \\ \eta_{50}\left(q_0\right)=\frac{10}{19683(-1+q_0)^5\left(2+q_0\right)}. \end{gathered} \tag{29}$$

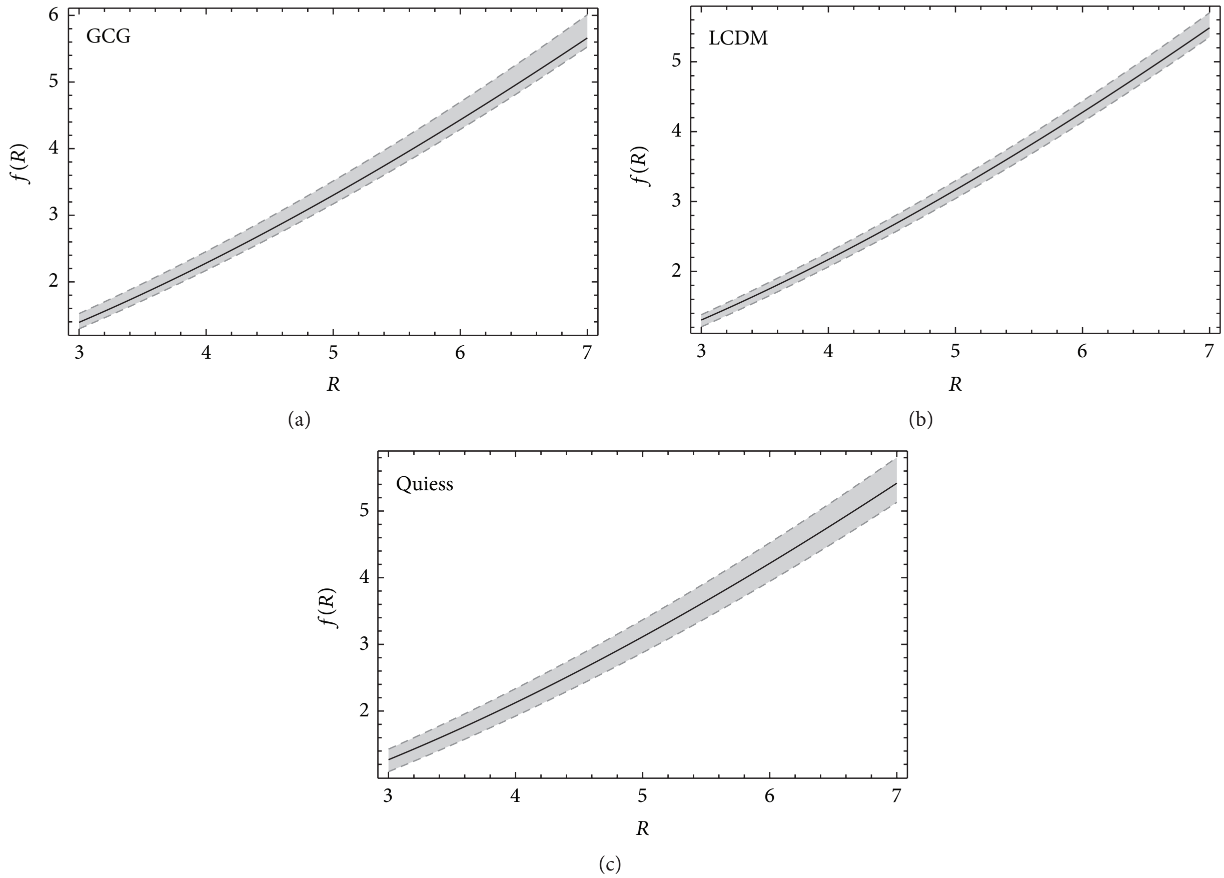

Figure 2: Gray band demonstrates permitted $f(R)$ for each model. The black line is analytically calculated $f(R)$, (15), in which we replaced α by that of Dark Energy.

Table 1: $H(z)^2$ for Dark Energy models [81].

Model	$H(z)^2$	Best fit parameter
GCG	$H^2(z) = H_0^2\left[\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})(a_1 + (1-a_1)(1+z)^3)^{\omega}\right]$	$a_1 = 0.9992^{+0.0008}_{-0.0060}$ $\omega = 18.13 \pm 4.95$
LCDM	$H^2(z) = H_0^2[\Omega_{0m}(1+z)^3 + 1 - \Omega_{0m}]$	$\Omega_{0m} = 0.31 \pm 0.04$
Quiess	$H^2(z) = H_0^2[\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})(1+z)^{3(1+\omega)}]$	$\omega = -1.02 \pm 0.11$

From Figure 1 it is obvious that for $i > 3$, the Taylor expansion coefficients have only slight effects.

5. Reconstruction of $f(R)$ for Dark Energy Model

In this section we have related Dark Energy models to cosmographic parameters by using Taylor expansion and then compared the result with the $f(R)$ function calculated in Section 3. For reconstructing $f(R)$ for Dark Energy models we need to use corresponding $H(z)^2$ s from Table 1, as follows:

(1) Using $H(z)^2$ we have calculated $q(z)$ for each Dark Energy model.

(2) Red shift must be fixed to zero.

(3) Using (28) to calculate α for each Dark Energy model, and replacing it in (23) and (26), we complete the Taylor expansion.

In this paper we have used GCG, LCDM, and Quiess models, since their present value of Ricci scalar is approximately the same ($R_0 \approx 4.43$). As errors in Dark Energy model parameters cannot be ignored, we would reach to a group of possible $f(R)$ function for each model, which has been demonstrated as a gray band in Figure 2. For comparing results, we have plotted analytically calculated action from (15), with these diagrams, where we have replaced α by that of Dark Energy according to the values of Table 2. Seeing that, for a linear correction equation (10), both analytically calculated $f(R)$ (15) and reconstructed $f(R)$ for Dark Energy models are in good agreement in neighboring with the present day value of curvature scalar.

Table 2: Parameters for Dark Energy models with $H_0 = 0.697$.

	q_0	R_0	α	$f''(R_0)$
GCG	$-0.48528^{+0.100067}_{-0.06472}$	$4.32025^{+0.19777}_{-0.28257}$	$0.40817^{+0.06208}_{-0.04082}$	$0.12294^{+0.00163}_{-0.00249}$
LCDM	-0.55000 ± 0.06000	4.51802 ± 0.17489	0.36636 ± 0.04000	0.12059 ± 0.00171
Quiess	-0.57100 ± 0.11550	4.57924 ± 0.33666	0.34950 ± 0.07848	0.120375 ± 0.00313

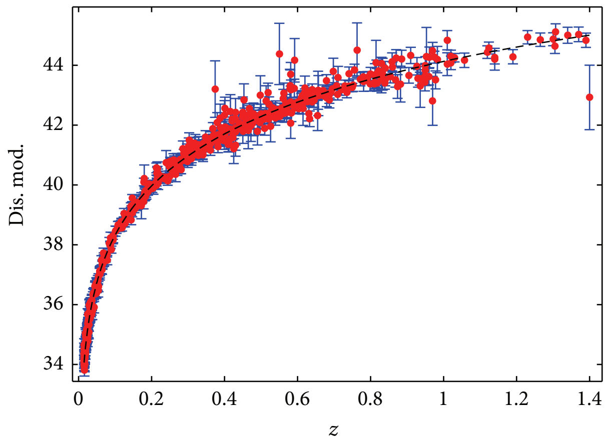

Figure 3: Accordance between Union2 SNeIa data and late time linear correction of our model for $\alpha \approx 0.418$ and $H_0 \approx 0.697$ [62].

6. Conclusions

Here we presented a model independent approach to calculate $f(R)$ by relating it to the present day value of cosmographic parameters. Since we had a linear form of $F(z)$, the η_{i0}s values made us using Taylor expansion up to the third order. In higher order $F(z)$ corrections it might be possible to keep higher terms of Taylor expansion which depend on value of η_{i0}s for these corrections and then we would discuss about validity of these terms in compare with desirable conditions. The Taylor expansion based $f(R)$ functions are viable in a small range about a certain value of Ricci scalar which itself is related to cosmographic parameters; since only present day value of these parameters are derived from data, we cannot discuss behavior of Universe at high red shifts.

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