

Research Article

A New Efficient Method for Solving Two-Dimensional Burgers' Equation

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We introduce a new hybrid of the Laplace transform method and new homotopy perturbation method (LTNHPM) that efficiently solves nonlinear two-dimensional Burgers' equation. Three examples are given to demonstrate the efficiency of the new method.

1. Introduction

The system of partial differential equation of the following form

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{1}{R} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \\ \frac{\partial v}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} &= \frac{1}{R} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \end{aligned} \quad (1)$$

$$(x, y) \in \Omega, \quad t > 0,$$

subject to the initial conditions:

$$\begin{aligned} u(x, y, 0) &= f(x, y), \quad (x, y) \in \Omega, \\ v(x, y, 0) &= g(x, y), \quad (x, y) \in \Omega, \end{aligned} \quad (2)$$

is called the system of two-dimensional Burgers' equation, where $\Omega = \{(x, y) | a \leq x \leq b, a \leq y \leq b\}$, $u(x, y, t)$ and $v(x, y, t)$ are the velocity components to be determined, f and g are known functions, and R is the Reynolds number. The Burgers model of turbulence is a very important fluid dynamic model, and the study of this model and the theory of shock waves have been considered by many authors, both to obtain a conceptual understanding of a class of physical flows and for testing various numerical methods. The mathematical properties of Burgers' equation have been studied by Burgers [1]. Nonlinear phenomena play a crucial role in applied mathematics and physics. The

importance of obtaining the exact or approximate solutions of PDEs in physics and mathematics is still a hot topic as regards seeking new methods for obtaining new exact or approximate solutions [2–5]. For that purpose, different methods have been put forward for seeking various exact solutions of multifarious physical models described using nonlinear PDEs. A well-known model was first introduced by Bateman [6], who found its steady solutions, descriptive of certain viscous flows. It was later proposed by Burgers [1] as one of a class of equations describing mathematical models of turbulence. In the context of gas dynamics, it was discussed by Hopf [7] and Cole [8]. They also illustrated independently that the Burgers equation can be solved exactly for an arbitrary initial condition. Benton and Platzman [9] have surveyed the analytical solutions of the one-dimensional Burgers equation. It can be considered as a simplified form of the Navier-Stokes equation [10] due to the form of the nonlinear convection term and the occurrence of the viscosity term. The numerical solution of the Burgers equation is of great importance due to the application of the equation in the approximate theory of flow through a shock wave, travelling in a viscous fluid [8] and in the Burgers model of turbulence [11]. It can be solved analytically for arbitrary initial conditions [7]. Numerical methods such as finite difference, finite element, and classical ones like Fourier series, Fourier integral, and Laplace transformation commonly used for solving these methods either need a lot of computations and have less convergence speed and accuracy or solve only certain types of problems. Therefore,

science and engineering researchers attempt to propose new methods for solving functional equations.

In this paper, we propose a new hybrid of Laplace transform method and new homotopy perturbation method [12] to obtain exact and numerical solutions of the system of two-dimensional Burgers' equation. Finally, three examples are given to illustrate the proposed approach.

2. Analysis of the Method

For solving system of two-dimensional Burgers' equation by LTNHPM, we construct the following homotopy:

$$\begin{aligned} U_t - u_0(x, y, t) + p \left(u_0(x, y, t) + UU_x + VU_y \right. \\ \left. - \frac{1}{R} (U_{xx} + U_{yy}) \right) = 0, \\ V_t - v_0(x, y, t) + p \left(v_0(x, y, t) + UV_x + VV_y \right. \\ \left. - \frac{1}{R} (V_{xx} + V_{yy}) \right) = 0, \end{aligned} \quad (3)$$

where $p \in [0, 1]$ is an embedding parameter and $u_0(x, y, t)$ and $v_0(x, y, t)$ are initial approximation of solution of (1).

By applying Laplace transform on both sides of (3), we have

$$L \left\{ U_t - u_0(x, y, t) \right.$$

$$+ p \left(u_0(x, y, t) + UU_x + VU_y - \frac{1}{R} (U_{xx} + U_{yy}) \right) \} = 0,$$

$$L \left\{ V_t - v_0(x, y, t) \right.$$

$$+ p \left(v_0(x, y, t) + UV_x + VV_y - \frac{1}{R} (V_{xx} + V_{yy}) \right) \} = 0. \quad (4)$$

Using the differential property of Laplace transform we have

$$\begin{aligned} sL\{U\} - U(x, y, 0) &= L \left\{ u_0(x, y, t) - p \left(u_0(x, y, t) + UU_x + VU_y - \frac{1}{R} (U_{xx} + U_{yy}) \right) \right\}, \\ sL\{V\} - V(x, y, 0) &= L \left\{ v_0(x, y, t) - p \left(v_0(x, y, t) + UV_x + VV_y - \frac{1}{R} (V_{xx} + V_{yy}) \right) \right\}, \end{aligned} \quad (5)$$

or

$$\begin{aligned} L\{U\} &= \frac{1}{s} \left(U(x, y, 0) + L \left\{ u_0(x, y, t) - p \left(u_0(x, y, t) + UU_x + VU_y - \frac{1}{R} (U_{xx} + U_{yy}) \right) \right\} \right), \\ L\{V\} &= \frac{1}{s} \left(V(x, y, 0) + L \left\{ v_0(x, y, t) - p \left(v_0(x, y, t) + UV_x + VV_y - \frac{1}{R} (V_{xx} + V_{yy}) \right) \right\} \right). \end{aligned} \quad (6)$$

By applying inverse Laplace transform on both sides of (6), we have

$$\begin{aligned} U(x, y, t) &= L^{-1} \left\{ \frac{1}{s} \left(U(x, y, 0) \right. \right. \\ &\quad \left. \left. + L \left\{ u_0(x, y, t) - p \left(u_0(x, y, t) + UU_x + VU_y \right. \right. \right. \right. \\ &\quad \left. \left. \left. \left. - \frac{1}{R} (U_{xx} + U_{yy}) \right) \right\} \right) \right\}, \end{aligned}$$

$$\begin{aligned} V(x, y, t) &= L^{-1} \left\{ \frac{1}{s} \left(V(x, y, 0) \right. \right. \\ &\quad \left. \left. + L \left\{ v_0(x, y, t) - p \left(v_0(x, y, t) + UV_x + VV_y \right. \right. \right. \right. \\ &\quad \left. \left. \left. \left. - \frac{1}{R} (V_{xx} + V_{yy}) \right) \right\} \right) \right\}. \end{aligned} \quad (7)$$

According to the HPM, we use the embedding parameter p as a small parameter and assume that the solutions of (7) can be represented as a power series in p as

$$\begin{aligned} U &= U_0 + pU_1 + p^2U_2 + \dots, \\ V &= V_0 + pV_1 + p^2V_2 + \dots. \end{aligned} \quad (8)$$

TABLE 1: The LTNHMP results for $u(x, y, t)$ for the first five approximations with $R = 0.5$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $
(0.1, 0.1)	0.6249902344	9E - 6	0.6245117213	4E - 4	0.6230474455	1E - 3
(0.5, 0.1)	0.6234278173	9E - 6	0.6229494028	2E - 3	0.6214857057	3E - 3
(0.9, 0.1)	0.6218658937	3E - 3	0.6213877271	3E - 3	0.6199250641	5E - 3
(0.3, 0.3)	0.6249902344	9E - 6	0.6245117213	4E - 4	0.6230474455	1E - 3
(0.7, 0.3)	0.6234278173	1E - 3	0.6229494028	2E - 3	0.6214857057	3E - 3
(0.1, 0.5)	0.6265526528	1E - 3	0.6260741906	1E - 3	0.6246097925	3E - 4
(0.5, 0.5)	0.6249902344	9E - 6	0.6245117213	4E - 4	0.6230474455	3E - 3
(0.9, 0.5)	0.6234278173	1E - 3	0.6229494028	2E - 3	0.6214857057	3E - 3
(0.3, 0.7)	0.6265526528	1E - 3	0.6260741906	1E - 3	0.62460987925	3E - 4
(0.7, 0.7)	0.6249902344	9E - 6	0.6245117213	4E - 4	0.6230474455	1E - 3
(0.1, 0.9)	0.6281145910	3E - 3	0.6276363292	2E - 3	0.6261722650	1E - 3
(0.5, 0.9)	0.6265526528	1E - 3	0.6260741906	1E - 3	0.6246097925	3E - 3
(0.9, 0.9)	0.6249902344	9E - 6	0.6245117213	4E - 4	0.6230474455	1E - 3

TABLE 2: The LTNHMP results for $v(x, y, t)$ for the first five approximations with $R = 0.5$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$	$ v^* - v $
(0.1, 0.1)	0.8750097656	9E - 6	0.8754882787	4E - 4	0.8769525545	1E - 3
(0.5, 0.1)	0.8765721828	1E - 3	0.8770505973	2E - 3	0.8785142944	3E - 3
(0.9, 0.1)	0.8781341113	3E - 3	0.8786122779	3E - 3	0.8800749409	5E - 3
(0.3, 0.3)	0.8750097656	9E - 6	0.8754882787	4E - 4	0.8769525545	1E - 3
(0.7, 0.3)	0.8765721828	1E - 3	0.8770505973	2E - 3	0.8785142944	3E - 3
(0.1, 0.5)	0.8734473429	1E - 3	0.8739258051	1E - 3	0.8753902032	3E - 4
(0.5, 0.5)	0.8750097656	9E - 6	0.8754882787	4E - 4	0.8769525545	1E - 3
(0.9, 0.5)	0.8765721828	1E - 3	0.8770505973	2E - 3	0.8785142944	3E - 3
(0.3, 0.7)	0.8734473429	1E - 3	0.8739258051	1E - 3	0.8753902032	3E - 4
(0.7, 0.7)	0.8750097656	9E - 6	0.8754882787	4E - 4	0.8769525545	1E - 3
(0.1, 0.9)	0.8753902032	3E - 3	0.8723636741	2E - 3	0.8738277383	1E - 3
(0.5, 0.9)	0.8734473429	3E - 10	0.8739258051	1E - 3	0.8753902032	3E - 4
(0.9, 0.9)	0.8750097656	1E - 3	0.8754882787	4E - 4	0.8769525545	1E - 3

TABLE 3: The LTNHMP results for $u(x, y, t)$ for the first five approximations with $R = 1$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $
(0.1, 0.1)	0.6249804688	1E - 10	0.6240234576	2E - 10	0.6210993749	4E - 6
(0.5, 0.1)	0.6218561341	2E - 9	0.6208999113	2E - 9	0.6179805324	4E - 6
(0.9, 0.1)	0.6187357207	1E - 10	0.6177814780	2E - 10	0.6148703830	4E - 6
(0.3, 0.3)	0.6249804688	1E - 10	0.6240234576	2E - 10	0.6210993749	4E - 6
(0.7, 0.3)	0.6218561341	2E - 9	0.6208999113	2E - 9	0.6179805324	4E - 6
(0.1, 0.5)	0.6281048314	1E - 9	0.6271482276	1E - 9	0.6242230488	4E - 6
(0.5, 0.5)	0.6249804688	1E - 10	0.6240234576	2E - 10	0.6210993749	4E - 6
(0.9, 0.5)	0.6218561341	2E - 9	0.6208999113	2E - 9	0.6179805324	4E - 6
(0.3, 0.7)	0.6281048314	1E - 9	0.6271482276	1E - 9	0.6242230488	4E - 6
(0.7, 0.7)	0.6249804688	1E - 10	0.6240234576	2E - 10	0.6210993749	4E - 6
(0.1, 0.9)	0.6312253142	0E - 0	0.6302703114	2E - 10	0.6273476542	4E - 6
(0.5, 0.9)	0.6281048314	1E - 9	0.6271482276	1E - 9	0.6242230488	4E - 6
(0.9, 0.9)	0.6249804688	1E - 10	0.6240234576	2E - 10	0.6210993749	4E - 6

TABLE 4: The LTNHMP results for $v(x, y, t)$ for the first five approximations with $R = 1$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$	$ v^* - v $
(0.1, 0.1)	0.8750195312	1E - 10	0.8759765424	2E - 10	0.8789006251	4E - 6
(0.5, 0.1)	0.8781438713	3E - 9	0.8791000941	3E - 9	0.8820194730	4E - 6
(0.9, 0.1)	0.8812642793	1E - 10	0.8822185220	2E - 10	0.8851296170	4E - 6
(0.3, 0.3)	0.8750195312	1E - 10	0.8759765424	2E - 10	0.8789006251	4E - 6
(0.7, 0.3)	0.8781438713	3E - 9	0.8791000941	3E - 9	0.8820194730	4E - 6
(0.1, 0.5)	0.8718951722	2E - 9	0.8728517760	2E - 9	0.8757769548	4E - 6
(0.5, 0.5)	0.8750195312	1E - 10	0.8759765424	2E - 10	0.8789006251	4E - 6
(0.9, 0.5)	0.8781438713	3E - 9	0.8791000941	3E - 9	0.8820194730	4E - 6
(0.3, 0.7)	0.8718951722	2E - 10	0.8728517760	2E - 9	0.8757769548	4E - 6
(0.7, 0.7)	0.8750195312	1E - 10	0.8759765424	2E - 10	0.8789006251	4E - 6
(0.1, 0.9)	0.8687746864	6E - 10	0.8697296892	4E - 10	0.8726523464	4E - 6
(0.5, 0.9)	0.8718951722	1E - 9	0.8728517760	2E - 9	0.8757769548	4E - 6
(0.9, 0.9)	0.8750195312	1E - 10	0.8759765424	2E - 10	0.8789006251	4E - 6

TABLE 5: The LTNHMP results for $u(x, y, t)$ for the first five approximations with $R = 100$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$	$ u^* - u $
(0.1, 0.1)	-0.01433515980	4E - 10	-0.01146399722	3E - 6	-0.04985414081	4E - 2
(0.5, 0.1)	0.01932053175	0E - 0	0.01517417083	3E - 6	-0.04626741859	5E - 2
(0.9, 0.1)	-0.01718352263	4E - 19	-0.01319841673	1E - 5	0.06125231399	6E - 2
(0.3, 0.3)	0.01130347869	1E - 11	0.009435085671	1E - 6	-0.01389095422	1E - 2
(0.7, 0.3)	0.02532577158	9E - 9	0.01777109572	2E - 4	0.1210568093	1E - 1
(0.1, 0.5)	-0.03913650004	8E - 11	-0.03227952068	2E - 5	0.2751624426	2E - 1
(0.5, 0.5)	0.06252255411	1E - 11	0.04901387388	5E - 5	-0.8092220313	1E - 1
(0.3, 0.7)	0.01130347871	3E - 11	0.009435085675	1E - 6	-0.01389095417	1E - 2
(0.7, 0.7)	0.02532586111	9E - 8	0.01777138161	2E - 4	0.1210567369	1E - 1
(0.1, 0.9)	-0.01433515851	8E - 10	-0.01146399707	3E - 6	-0.04985413728	4E - 2
(0.5, 0.9)	0.01932053170	0E - 0	0.01517417079	3E - 6	-0.04626741849	5E - 2
(0.9, 0.9)	-0.01718352694	2E - 10	-0.01319842063	1E - 5	0.06125228267	6E - 2

TABLE 6: The LTNHMP results for $v(x, y, t)$ for the first five approximations with $R = 100$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$			
	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$	$ v^* - v $		
(0.1, 0.1)	-0.01602719682	1E - 9	-0.01281338076	5E - 7	-0.01158528430	5E - 3
(0.5, 0.1)	-5.521281362E - 16	5E - 16	-7.159309431E - 6	7E - 6	-0.01817103931	1E - 2
(0.9, 0.1)	0.01921176621	7E - 10	0.01476978811	2E - 7	-0.06185015166	6E - 2
(0.3, 0.3)	-0.01263767338	1E - 11	-0.01054980461	7E - 7	2.61506638E - 3	8E - 3
(0.7, 0.3)	0.2831506768	5E - 9	0.01969169193	7E - 5	0.6129399566	6E - 1
(0.1, 0.5)	-0.00000000000	0E - 0	-0.00000000000	0E - 0	0.00000000000	0E - 0
(0.5, 0.5)	0.00000000000	0E - 0	0.00000000000	0E - 0	0.00000000000	0E - 0
(0.9, 0.5)	-0.00000000000	0E - 0	-0.00000000000	0E - 0	0.00000000000	0E - 0
(0.3, 0.7)	0.01263767338	0E - 0	0.01054980461	7E - 7	-2.61506637E - 3	8E - 3
(0.7, 0.7)	-0.02831513946	7E - 8	-0.019969205679	7E - 5	-0.6129371973	6E - 1
(0.1, 0.9)	0.01602719556	1E - 10	0.01281338024	5E - 7	0.01158528474	5E - 3
(0.5, 0.9)	-5.521281351E - 16	5E - 16	7.159309412E - 6	7E - 6	0.01817103920	1E - 2
(0.9, 0.9)	-0.01146399722	3E - 6	-0.01476978755	2E - 7	0.06185014690	6E - 2

TABLE 7: The LTNHMP results for $u(x, y, t)$ for the first five approximations with $R = 500$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$
	$u^*(x, y, t)$	$ u^* - u $	$u^*(x, y, t)$
(0.1, 0.1)	-0.002877429822	3E - 10	-0.002752396466
(0.5, 0.1)	0.003879391382	0E - 0	0.003696240199
(0.9, 0.1)	-0.003451655816	3E - 10	-0.003273279771
(0.3, 0.3)	0.002267155545	2E - 12	0.002188809320
(0.7, 0.3)	0.005097690552	5E - 9	0.004717986784
(0.1, 0.5)	-0.007851234710	2E - 12	-0.007561582238
(0.5, 0.5)	0.01255397423	1E - 11	0.01196126856
(0.9, 0.5)	-0.01437772613	1E - 8	-0.01343545467
(0.3, 0.7)	0.002267155545	3E - 12	0.002188809324
(0.7, 0.7)	0.005097745644	4E - 8	0.004718074710
(0.1, 0.9)	-0.002877429571	9E - 11	-0.002752396375
(0.5, 0.9)	0.003879391372	0E - 0	0.0036962440189
(0.9, 0.9)	-0.003451655844	3E - 9	-0.003273279804

TABLE 8: The LTNHMP results for $v(x, y, t)$ for the first five approximations with $R = 500$.

(x, y)	$t = 0.01$	$t = 0.5$	$t = 2$
	$v^*(x, y, t)$	$ v^* - v $	$v^*(x, y, t)$
(0.1, 0.1)	-0.003217063991	2E - 11	-0.003077270449
(0.5, 0.1)	-4.350477708E - 20	4E - 20	-6.738702226E - 10
(0.9, 0.1)	-0.003859067764	1E - 9	0.003659637690
(0.3, 0.3)	-0.002534756956	0E - 0	-0.002447164709
(0.7, 0.3)	0.005699386966	1E - 8	0.005274890097
(0.1, 0.5)	-0.00000000000	0E - 0	-0.00000000000
(0.5, 0.5)	0.00000000000	0E - 0	0.00000000000
(0.9, 0.5)	-0.00000000000	0E - 0	-0.00000000000
(0.3, 0.7)	0.002534756959	0E - 0	0.002447164718
(0.7, 0.7)	-0.005699453379	5E - 8	-0.005275019330
(0.1, 0.9)	0.003217064054	9E - 11	0.003077270418
(0.5, 0.9)	4.350477662E - 20	4E - 20	6.738702149E - 10
(0.9, 0.9)	-0.003859067867	1E - 9	-0.003659637235

Substituting (8) into (7) and equating the terms with the identical powers of p lead to

$$\begin{aligned}
 p^0 : & \begin{cases} U_0 = L^{-1} \left\{ \frac{1}{s} (U(x, y, 0) + L\{u_0(x, y, t)\}) \right\}, \\ V_0 = L^{-1} \left\{ \frac{1}{s} (V(x, y, 0) + L\{v_0(x, y, t)\}) \right\}, \end{cases} \\
 p^1 : & \begin{cases} U_1 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ u_0(x, y, t) + U_0(U_0)_x + V_0(U_0)_y - \frac{1}{R} ((U_0)_{xx} + (U_0)_{yy}) \right\} \right) \right\}, \\ V_1 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ v_0(x, y, t) + U_0(V_0)_x + V_0(V_0)_y - \frac{1}{R} ((V_0)_{xx} + (V_0)_{yy}) \right\} \right) \right\}, \end{cases} \\
 p^2 : & \begin{cases} U_2 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(U_1)_x + U_1(U_0)_x + V_0(U_1)_y + V_1(U_0)_y - \frac{1}{R} ((U_1)_{xx} + (U_1)_{yy}) \right\} \right) \right\}, \\ V_2 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(V_1)_x + U_1(V_0)_x + V_0(V_1)_y + V_1(V_0)_y - \frac{1}{R} ((V_1)_{xx} + (V_1)_{yy}) \right\} \right) \right\}, \end{cases}
 \end{aligned}$$

⋮

$$p^j : \begin{cases} U_j = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ \sum_{k=0}^{j-1} \left(U_k (U_{j-k-1})_x + V_k (U_{j-k-1})_y \right) - \frac{1}{R} \left((U_{j-1})_{xx} + (U_{j-1})_{yy} \right) \right\} \right) \right\}, \\ V_j = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ \sum_{k=0}^{j-1} \left(U_k (V_{j-k-1})_x + V_k (V_{j-k-1})_y \right) - \frac{1}{R} \left((V_{j-1})_{xx} + (V_{j-1})_{yy} \right) \right\} \right) \right\}, \\ \vdots \end{cases} \quad (9)$$

Suppose that the initial approximation has the form $U(x, y, 0) = u_0(x, y, t) = f(x, y)$ and $V(x, y, 0) = v_0(x, y, t) = g(x, y)$; therefore the exact solution may be obtained as follows

$$u(x, y, t) = \lim_{p \rightarrow 1} U(x, y, t) = U_0(x, y, t) + U_1(x, y, t) + \dots,$$

$$v(x, y, t) = \lim_{p \rightarrow 1} V(x, y, t) = V_0(x, y, t) + V_1(x, y, t) + \dots.$$

(10)

subject to the initial condition

$$u(x, y, 0) = x + y,$$

(12)

$$v(x, y, 0) = x - y.$$

3. Examples

Example 1. Consider the following homogeneous form of a coupled Burgers equation [13]:

$$\begin{aligned} u_t + uu_x + vu_y &= \frac{1}{R} (u_{xx} + u_{yy}), \\ v_t + uv_x + vv_y &= \frac{1}{R} (v_{xx} + v_{yy}), \end{aligned} \quad (11)$$

The exact solution of this equation is $u(x, y, t) = (x + y - 2xt)/(1 - 2t^2)$ and $v(x, y, t) = (x - y - 2yt)/(1 - 2t^2)$.

Starting with $U(x, y, 0) = u_0 = x + y$, $V(x, y, 0) = v_0 = x - y$ and using (9), we obtain

$$U_0 = L^{-1} \left\{ \frac{1}{s} (x + y + L\{x + y\}) \right\} = (x + y)(1 + t),$$

$$V_0 = L^{-1} \left\{ \frac{1}{s} (x - y + L\{x - y\}) \right\} = (x - y)(1 + t),$$

$$U_1 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ x + y + U_0(U_0)_x + V_0(U_0)_y - \frac{1}{R} ((U_0)_{xx} + (U_0)_{yy}) \right\} \right) \right\}$$

$$= -(3x + y)t - 2xt^2 - \frac{2}{3}xt^3,$$

$$V_1 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ x - y + U_0(V_0)_x + V_0(V_0)_y - \frac{1}{R} ((V_0)_{xx} + (V_0)_{yy}) \right\} \right) \right\}$$

$$= -(x + y)t - 2yt^2 - \frac{2}{3}yt^3,$$

$$U_2 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(U_1)_x + U_1(U_0)_x + V_0(U_1)_y + V_1(U_0)_y - \frac{1}{R} ((U_1)_{xx} + (U_1)_{yy}) \right\} \right) \right\}$$

$$= (4x + 2y)t^2 + \left(4x + \frac{8}{3}y \right) t^3 + \left(\frac{4}{3}x + \frac{4}{3}y \right) t^4 + \left(\frac{4}{15}x + \frac{4}{15}y \right) t^5,$$

$$V_2 = L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(V_1)_x + U_1(V_0)_x + V_0(V_1)_y + V_1(V_0)_y - \frac{1}{R} ((V_1)_{xx} + (V_1)_{yy}) \right\} \right) \right\}$$

$$= 2xt^2 + \left(\frac{8}{3}x - \frac{4}{3}y \right) t^3 + \left(\frac{4}{3}x - \frac{4}{3}y \right) t^4 + \left(\frac{4}{15}x - \frac{4}{15}y \right) t^5,$$

$$\begin{aligned}
U_3 &= L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(U_2)_x + U_1(U_1)_x + U_2(U_0)_x + V_0(U_2)_y + V_1(U_1)_y + V_2(U_0)_y - \frac{1}{R} ((U_2)_{xx} + (U_2)_{yy}) \right\} \right) \right\} \\
&= - \left(\frac{22}{3}x + \frac{8}{3}y \right) t^3 - \left(\frac{28}{3}x + \frac{8}{3}y \right) t^4 - \left(\frac{16}{3}x + \frac{4}{5}y \right) t^5 - \frac{68}{45}xt^6 - \frac{68}{315}xt^7, \\
V_3 &= L^{-1} \left\{ -\frac{1}{s} \left(L \left\{ U_0(V_2)_x + U_1(V_1)_x + U_2(V_0)_x + V_0(V_2)_y + V_1(V_1)_y + V_2(V_0)_y - \frac{1}{R} ((V_2)_{xx} + (V_2)_{yy}) \right\} \right) \right\} \\
&= - \left(\frac{8}{3}x + 2y \right) t^3 - \left(\frac{8}{3}x + 4y \right) t^4 - \left(\frac{4}{5}x + \frac{56}{15}y \right) t^5 - \frac{68}{45}yt^6 - \frac{68}{315}yt^7 \\
&\vdots
\end{aligned} \tag{13}$$

Therefore we gain the solution of (11) as

$$\begin{aligned}
u(x, y, t) &= U_0(x, y, t) + U_1(x, y, t) + U_3(x, y, t) + \dots \\
&= x + y - 2xt + 2xt^2 + 2yt^2 - 4xt^3 \\
&\quad + 4xt^4 + 4yt^4 - 8xt^5 + \dots \\
&= x(1 + 2t^2 + 4t^4 + \dots) + y(1 + 2t^2 + 4t^4 + \dots) \\
&\quad - 2xt(1 + 2t^2 + 4t^4 + \dots) \\
&= \frac{x + y - 2xt}{1 - 2t^2},
\end{aligned}$$

$$\begin{aligned}
v(x, y, t) &= V_0(x, y, t) + V_1(x, y, t) + V_3(x, y, t) + \dots \\
&= x - y - 2yt + 2xt^2 - 2yt^2 - 4yt^3 \\
&\quad + 4xt^4 - 4yt^4 - 8yt^5 + \dots \\
&= x(1 + 2t^2 + 4t^4 + \dots) \\
&\quad - y(1 + 2t^2 + 4t^4 + \dots) \\
&\quad - 2yt(1 + 2t^2 + 4t^4 + \dots) \\
&= \frac{x - y - 2yt}{1 - 2t^2}
\end{aligned} \tag{14}$$

which is exact solution.

Example 2. Let us consider system of Burgers' equations (1), with the following initial conditions [14]:

$$\begin{aligned}
u(x, y, 0) &= \frac{3}{4} - \frac{1}{4[1 + \exp(y - x)]R/8}, \\
v(x, y, 0) &= \frac{3}{4} + \frac{1}{4[1 + \exp(y - x)]R/8},
\end{aligned} \tag{15}$$

for which exact solutions are

$$\begin{aligned}
u(x, y, t) &= \frac{3}{4} - \frac{1}{4[1 + \exp(4y - 4x - t)]R/32}, \\
v(x, y, t) &= \frac{3}{4} + \frac{1}{4[1 + \exp(4y - 4x - t)]R/32}.
\end{aligned} \tag{16}$$

To solve system (1) by LTNHMP, following the same procedure discussed in Section 2 and Example 1, we obtain the iterative relations (9); in this example we take initial approximations (15). The accuracy of LTNHMP for the system of two-dimensional Burgers' equation agrees good with the exact solution, and absolute errors are very small for the present choice of x , y , and t . These results are listed in Tables 1, 2, 3, and 4 for $R = 0.5$ and $R = 1$.

Example 3. Let us consider system of Burgers' equations (8), with the following initial conditions [14]:

$$\begin{aligned}
u(x, y, 0) &= -\frac{4\pi \cos(2\pi x) \sin(\pi y)}{R(2 + \sin(2\pi x) \sin(\pi y))}, \\
v(x, y, 0) &= -\frac{2\pi \sin(2\pi x) \cos(\pi y)}{R(2 + \sin(2\pi x) \sin(\pi y))},
\end{aligned} \tag{17}$$

for which exact solutions are

$$\begin{aligned}
u(x, y, t) &= -\frac{4\pi \exp(-5\pi^2 t/R) \cos(2\pi x) \sin(\pi y)}{R(2 + \exp(-5\pi^2 t/R) \sin(2\pi x) \sin(\pi y))}, \\
v(x, y, t) &= -\frac{2\pi \exp(-5\pi^2 t/R) \sin(2\pi x) \cos(\pi y)}{R(2 + \exp(-5\pi^2 t/R) \sin(2\pi x) \sin(\pi y))}.
\end{aligned} \tag{18}$$

To solve system (1) by LTNHMP, following the same procedure discussed in Section 2 and Example 1, we obtain the iterative relations (9); in this example we take initial approximations (17). The accuracy of LTNHMP for the system of two-dimensional Burgers' equation agrees good with the exact solution, and absolute errors are very small for the present choice of x , y , and t . These results are listed in Tables 5, 6, 7, and 8 for $R = 100$ and $R = 500$.

4. Conclusions

In this work, we considered a new hybrid of Laplace transform method and homotopy perturbation method (LTNHPM) for solving system of two-dimensional Burgers' equation. Using this method we obtained new efficient relations to solve these systems. New method is a powerful straightforward method. The LTNHPM is apt to be utilized as an alternative approach to current techniques being employed to a wide variety of mathematical problems.

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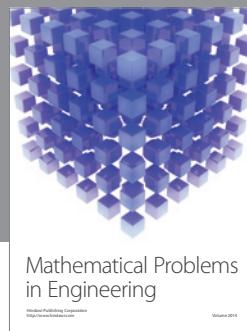
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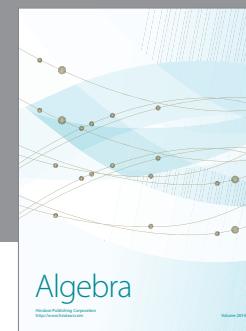
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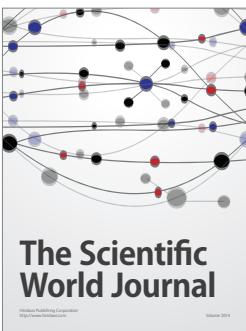
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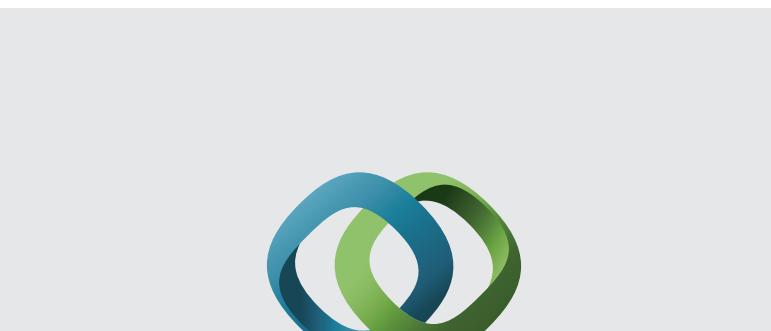
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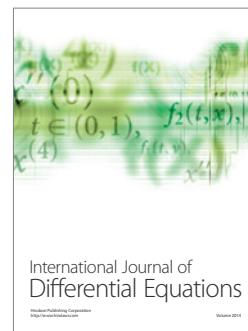


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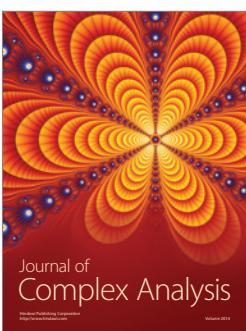
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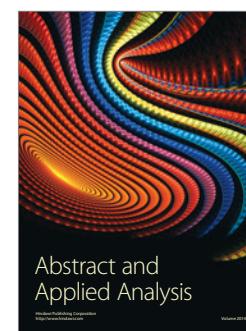
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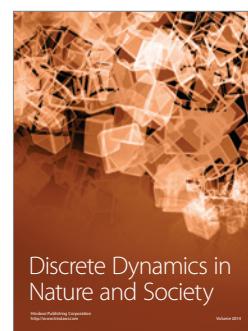
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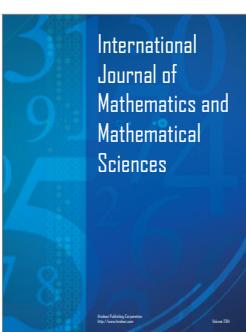
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