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Research Article

Geodesic Lightlike Submanifolds of Indefinite Kenmotsu Manifolds

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The aim of the present paper is to study geodesic contact screen Cauchy Riemannian (SCR-) lightlike submanifolds, geodesic screen transversal lightlike, and geodesic transversal lightlike submanifolds of indefinite Kenmotsu manifolds.

1. Introduction

The study of the geometry of submanifolds of a Riemannian or semi-Riemannian manifold is one of the interesting topics of differential geometry. Despite of some similarities between semi-Riemannian manifolds and Riemannian manifolds, the lightlike submanifolds are different since their normal vector bundle intersect with the tangent bundle making it more interesting and difficult to study. These submanifolds were introduced and studied by Duggal and Bejancu [1]. On the other hand, geodesic CR-lightlike submanifolds in Kähler manifolds were studied by Sahin and Gunes [2], and geodesic lightlike submanifolds of indefinite Sasakian manifolds were investigated by Dong and Liu [3]. In 2006, Sahin [4] initiated the study of transversal lightlike submanifolds of an indefinite Kähler manifold which are different from CR-lightlike [1] and screen CR-lightlike submanifolds [5]. Recently, Sahin [6] introduced the notion of screen transversal lightlike submanifolds of indefinite Kähler manifolds and obtained some useful results. In this paper, we study geometric conditions under which some lightlike submanifolds of an indefinite Kenmotsu manifold are totally geodesic.

2. Preliminaries

We follow [1] for the notation and fundamental equations for lightlike submanifolds used in this paper. A submanifold M^m immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold if it admits a degenerate metric g induced from $\overline{g}$ whose radical distribution $\operatorname{Rad} TM = TM \cap TM^{\perp}$ is of rank r, where $1 \leq r \leq m$ and

$$TM^{\perp} = \cup\left\{U \in T_x\overline{M} : \overline{g}(U, V) = 0,\ \forall V \in T_x\overline{M}\right\}. \tag{2.1}$$

Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $\operatorname{Rad} TM$ in TM, that is,

$$TM = \operatorname{Rad} TM \perp S(TM). \tag{2.2}$$

Consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $\operatorname{Rad} TM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $\operatorname{Rad} TM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $\operatorname{ltr}(TM)$ locally spanned by $\{N_i\}$ [1, page 144]. Let $\operatorname{tr}(TM)$ be the complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$.

Then,

$$\begin{gathered} \operatorname{tr}(TM) = \operatorname{ltr}(TM) \perp S\left(TM^{\perp}\right), \\ T\overline{M} = S(TM) \perp [\operatorname{Rad} TM \oplus \operatorname{ltr}(TM)] \perp S\left(TM^{\perp}\right). \end{gathered} \tag{2.3}$$

Let $\overline{\nabla}$ be the Levi-Civita connection on $\overline{M}$. Then, in view of the decomposition (2.3), the Gauss and Weingarten formulas are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad \forall X, Y \in \Gamma(TM), \tag{2.4}$$

$$\overline{\nabla}_X U = -A_U X + \nabla^t_X U, \quad \forall X \in \Gamma(TM),\ U \in \Gamma(\operatorname{tr}(TM)), \tag{2.5}$$

where $\{\nabla_X Y, A_U X\}$ and $\{h(X, Y), \nabla^t_X U\}$ belong to $\Gamma(TM)$ and $\Gamma(\operatorname{tr}(TM))$, respectively, ∇ and ∇^t are linear connection on M and on the vector bundle $\operatorname{tr}(TM)$, respectively. Moreover, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l(X, Y) + h^s(X, Y), \tag{2.6}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s(X, N), \tag{2.7}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l(X, W) \tag{2.8}$$

for all, $X, Y \in \Gamma(TM), N \in \Gamma(\mathrm{ltr}(TM))$, and $W \in \Gamma(S(TM^{\perp}))$. If we denote the projection of TM on $S(TM)$ by P, then by using (2.6)–(2.8) and the fact that $\overline{\nabla}$ is a metric connection, we obtain

$$\overline{g}(h^s(X,Y),W) + \overline{g}\left(Y, D^l(X,W)\right) = g(A_W X, Y), \tag{2.9}$$

$$\overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X). \tag{2.10}$$

From the decomposition of the tangent bundle of a lightlike submanifold, we have

$$\nabla_X PY = \nabla^*_X PY + h^*(X,PY), \tag{2.11}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi \tag{2.12}$$

for any $X, Y \in \Gamma(TM)$, and $\xi \in \Gamma(\mathrm{Rad}\, TM)$. By using above equation we obtain

$$\begin{aligned} \overline{g}\left(h^l(X,PY),\xi\right) &= g\left(A^*_\xi X, PY\right), \\ \overline{g}(h^*(X,PY),N) &= g(A_N X, PY), \end{aligned} \tag{2.13}$$

$$\overline{g}\left(h^l(X,\xi),\xi\right) = 0, A^*_\xi \xi = 0. \tag{2.14}$$

An odd dimensional semi-Riemannian manifold $(\overline{M}, \overline{g})$ is said to be an indefinite contact metric manifold [7] if there exists a (1,1) tensor field ϕ, a vector field V, called the characteristic vector field, and its 1-form η satisfying

$$\overline{g}(\phi X, \phi Y) = \overline{g}(X,Y) - \epsilon\eta(X)\eta(Y), \quad \overline{g}(V,V) = \epsilon, \tag{2.15}$$

$$\phi^2 X = -X + \eta(X)V, \quad \overline{g}(X,V) = \epsilon\eta(X), \quad \forall X, Y \in \Gamma(TM), \tag{2.16}$$

where $\epsilon = \pm 1$. It is not difficult to show that $\phi V = 0$, $\eta o \phi = 0$, $\eta(V) = \epsilon$.

An indefinite almost contact metric manifold $\overline{M}$ is said to be an indefinite Kenmotsu manifold [8] if

$$\overline{\nabla}_X V = -X + \eta(X)V, \tag{2.17}$$

$$\left(\overline{\nabla}_X \phi\right) Y = -\overline{g}(\phi X, Y)V + \epsilon(Y)\phi X \tag{2.18}$$

for any $X, Y \in \Gamma(T\overline{M})$.

Without loss of generality, we take $\epsilon = 1$. For any vector field X tangent to M, we put

$$\phi X = TX + \omega X, \tag{2.19}$$

where TX and ωX are the tangential and transversal parts of ϕX, respectively.

For any $U \in \Gamma(\mathrm{tr}(TM))$, we have

$$\phi U = BU + CU, \tag{2.20}$$

where BU and CU are the tangential and transversal parts of ϕU, respectively.

From now on, we denote $(M, g, S(TM), S(TM^{\perp}))$ by M in this paper.

3. Geodesic Contact *SCR*-Lightlike Submanifolds

In this section, we study the geometric conditions under which the distributions involved in the definition of a contact SCR-lightlike submanifold M and the submanifold itself are totally geodesic. We recall the following definition of contact SCR-lightlike submanifold of an indefinite Kenmotsu manifold given by Duggal and Sahin [5].

Definition 3.1. A lightlike submanifold M, tangent to structure vector field V, immersed in an indefinite Kenmotsu manifold $(\overline{M}, \overline{g})$ is said to be contact SCR-lightlike submanifold of $\overline{M}$ if the following conditions are satisfied.

(a) There exists real nonnull distribution D and $D^{\perp}$ such that

$$S(TM) = D \perp D^{\perp} \perp \{V\}, \quad \phi\left(D^{\perp}\right) \subset S\left(TM^{\perp}\right), \tag{3.1}$$

$D \cap D^{\perp} = 0$, where $D^{\perp}$ is the orthogonal complementary to $D \perp \{V\}$ in $S(TM)$.

(b) $\phi D = D, \phi \,\mathrm{Rad}\, TM = \mathrm{Rad}\, TM$, $\phi \mathrm{ltr}(TM) = \mathrm{ltr}(TM)$.

The tangent bundle of a contact SCR-lightlike submanifold is decomposed as

$$TM = \overline{D} \perp D^{\perp} \perp \{V\}, \quad \overline{D} = D \perp \mathrm{Rad}\, TM. \tag{3.2}$$

We will use the symbol μ to denote the orthogonal complement of $\phi D^{\perp}$ in $S(TM^{\perp})$.

Definition 3.2. A contact SCR-lightlike submanifold M of an indefinite Kenmostsu manifold $\overline{M}$ is said to be

(i) $D^{\perp}$-totally geodesic contact SCR-lightlike submanifold if $h(X, Y) = 0$ for any $X, Y \in \Gamma(D^{\perp})$,

(ii) mixed totally geodesic contact SCR-lightlike submanifold if $h(X, Y) = 0$ for any $X \in \Gamma((\overline{D}) \perp \{V\})$ and $Y \in \Gamma(D^{\perp})$.

Let M be a contact SCR-lightlike submanifold of indefinite Kenmotsu manifold $\overline{M}$ and let P and Q be the projection morphisms on $\overline{D}$ and $D^{\perp}$, respectively. Then for any vector field X tangent to M, we can write

$$X = PX + QX + \eta(X)V. \tag{3.3}$$

Applying ϕ to (3.3) and using (2.19), we obtain

$$\phi X = TPX + \omega QX. \tag{3.4}$$

If we denote TPX by TX and ωQX by ωX, then (3.4) can be rewritten as

$$\phi X = TX + \omega X, \tag{3.5}$$

where $TX \in \Gamma(\overline{D})$ and $\omega X \in \Gamma(\phi(D^{\perp})) \subset S(TM^{\perp})$.

For any $W \in \Gamma(S(TM^{\perp}))$, we have

$$\phi W = BW + CW, \tag{3.6}$$

where $BW \in \Gamma(D^{\perp})$ and $CW \in \Gamma(\mu) \subset S(TM^{\perp})$.

In view of the above arguments, we are in a position to prove the following characterization theorem for the existence of a $D^{\perp}$-totally geodesic contact SCR-lightlike submanifold immersed in indefinite Kenmotsu manifolds.

Theorem 3.3. *Let M be a contact SCR–lightlike submanifold of an indefinite Kenmotsu manifolds $\overline{M}$. Then M is $D^{\perp}$-totally geodesic if and only if*

(i) $h^s(X, \phi\xi) \in \mu$.

(ii) $A_{\omega Y}X \notin D^{\perp}$ *and* $\nabla^s_X \omega Y \notin \mu$ *for any* $X, Y \in \Gamma(D^{\perp})$.

Proof. Suppose that the contact SCR-lightlike submanifold M is totally geodesic. Then we see that $\overline{g}(h^l(X,Y),\xi) = 0$ and $\overline{g}(h^s(X,Y),W) = 0$ for all, $X, Y \in \Gamma(D^{\perp})$.

Also, from (2.6) and (2.15), we obtain

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}\left(\overline{\nabla}_X \phi Y - \left(\overline{\nabla}_X \phi\right)Y, \phi\xi\right), \tag{3.7}$$

from which we derive

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}\left(D^l(X,\omega Y),\phi\xi\right), \tag{3.8}$$

where we have used (2.8), (2.18), and (3.5). Using (2.9) in the above equation, we get

$$\overline{g}\left(h^l(X,Y),\xi\right) = -\overline{g}(h^s(X,\phi\xi),\omega Y). \tag{3.9}$$

On the other hand, making use of (2.6), (2.8), (2.15), (2.18), (3.5), and (3.6), we arrive at

$$\overline{g}(h^s(X,Y),W) = -g(A_{\omega Y}X, BW) + \overline{g}(\nabla^s_X \omega Y, CW). \tag{3.10}$$

Hence, (i) and (ii) follows from (3.9) and (3.10) together with the fact that $\overline{g}(h^l(X,Y),\xi) = 0$ and $\overline{g}(h^s(X,Y),W) = 0$.

Converse part directly follow from (3.9) and (3.10). □

The necessary and sufficient conditions for contact *SCR*-lightlike submanifolds to be mixed totally geodesic is given by the following theorem.

Theorem 3.4. *Let M be a contact SCR-lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is mixed totally geodesic if and only if*

(i) $h^s(X,\phi\xi) \in \mu$,

(ii) $A_{\omega Y}X \notin D^{\perp}$ *and* $\nabla^s_X \omega Y \notin \mu$ *for any* $X \in \Gamma(\overline{D} \perp \{V\})$ *and* $Y \in \Gamma(D^{\perp})$.

Proof. Assume that M is mixed totally geodesic. Then $\overline{g}(h^l(X,Y),\xi) = 0$, $\overline{g}(h^l(X,Y),W) = 0$, for any $X \in \Gamma(\overline{D} \perp \{V\})$ and $Y \in \Gamma(D^{\perp})$.

Moreover, using (2.6), (2.8), (2.15), (2.18), and (3.5) a direct calculation shows that

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = \overline{g}\Big(D^l(X,\omega Y),\phi\xi\Big). \tag{3.11}$$

From (2.9) and (3.11), we have

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = -\overline{g}(h^s(X,\phi\xi),\omega Y). \tag{3.12}$$

On the other hand, using (2.6), (2.8), (2.15), (2.18), (3.5), and (3.6), we derive

$$\overline{g}(h^s(X,Y),W) = -g(A_{\omega Y}X,BW) + g(\nabla^s_X \omega Y, CW). \tag{3.13}$$

Thus, (i) and (ii) follow from (3.12) and (3.13) along with $\overline{g}(h^l(X,Y),\xi) = 0$, $\overline{g}(h^l(X,Y),W) = 0$. □

Converse part directly follows from (3.12) and (3.13).

4. Screen Transversal Lightlike Submanifolds

We begin this section by recalling the following definitions from [6].

Definition 4.1. An r-lightlike submanifold M of an indefinite Kenmotsu manifold $\overline{M}$ is said to be screen transversal (ST) lightlike submanifold of $\overline{M}$ if there exists a screen transversal bundle $S(TM^{\perp})$ such that

$$\phi(\operatorname{Rad} TM) \subset S\Big(TM^{\perp}\Big). \tag{4.1}$$

Definition 4.2. An ST-lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$ is said to be

(i) radical ST-lightlike submanifold if $S(TM)$ is invariant with respect to ϕ.

(ii) ST-anti-invariant lightlike submanifold if $S(TM)$ is screen transversal with respect to ϕ, that is,

$$\phi((S(TM))) \subset S\left(TM^{\perp}\right). \tag{4.2}$$

For a radical screen transversal lightlike submanifolds M immersed in indefinite Kenmotsu manifold $\overline{M}$, we will denote the projection morphisms of $S(TM)$ and $\operatorname{Rad} TM$ by P and Q, respectively. Then for $X \in \Gamma(TM)$, we can write

$$X = PX + QX. \tag{4.3}$$

We apply ϕ to (4.3) and then using (2.19), we obtain

$$\phi X = TPX + \omega QX. \tag{4.4}$$

Denoting TPX by TX and ωQX by ωX, (4.4) can be we rewritten as

$$\phi X = TX + \omega X, \tag{4.5}$$

where $TX \in \Gamma(S(TM))$ and $\omega X \in \Gamma(\phi(\operatorname{Rad} TM)) \subset S(TM^{\perp})$.

For $W \in \Gamma(S(TM^{\perp}))$, we write

$$\phi W = BW + C_1 W + C_2 W, \tag{4.6}$$

where $BW \in \Gamma(\operatorname{Rad} TM)$, $C_1 W \in \Gamma(\operatorname{ltr}(TM))$, and $C_2 W \in \Gamma(\mu)$ (μ is the orthogonal complement of $\phi(\operatorname{Rad} TM) \oplus \phi(\operatorname{ltr}(TM))$ in $S(TM^{\perp})$).

The geometric conditions under which the distribution $\operatorname{Rad} TM$ is totally geodesic is given by the following theorem.

Theorem 4.3. *Let M be a radical screen transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then* Rad TM*is totally geodesic if and only if $h^s(\xi_1, BW) + D^s(\xi_1, C_1 W) + \nabla^s_{\xi_1} C_2 W$ has no component in $\phi(\operatorname{ltr}(\mathrm{TM})) \perp {}^{-}$.*

Proof. If the distribution $\operatorname{Rad} TM$ is totally geodesic, then

$$\overline{g}\left(h^l(\xi_1, \xi_2), \xi_3\right) = 0, \qquad \overline{g}(h^s(\xi_1, \xi_2), W) = 0 \tag{4.7}$$

for any $\xi_1, \xi_2, \xi_3 \in \Gamma(\mathrm{Rad}\,TM)$, $W \in \Gamma(S(TM^{\perp}))$ and $h^l = 0$ on $\mathrm{Rad}\,TM$ [9]. On the other hand, using (2.6), (2.15), (2.18), and (4.6), we arrive at

$$\overline{g}(h^s(\xi_1,\xi_2),W) = -\overline{g}\Big(\phi\xi_2, h^s(\xi_1,BW) + D^s(\xi_1,C_1W) + \nabla^s_{\xi_1}C_2W\Big). \tag{4.8}$$

Thus, our assertion follows from (4.8) and (4.7).

Converse part directly follows from (4.8) and (4.7). □

For the screen distribution $S(TM)$ to be totally geodesic in M, we have the following.

Theorem 4.4. *Let M be a radical screen transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then, $S(TM)$ is totally geodesic if and only if $A_{\omega\xi}X, A^*_{BW}X + A_{C_1W}X + A_{C_2W}X \notin S(TM)$ for all $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$ and $W \in \Gamma(S(TM^{\perp}))$.*

Proof. Suppose that the distribution $S(TM)$ is totally geodesic. Then

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = 0, \qquad \overline{g}(h^s(X,Y),W) = 0 \tag{4.9}$$

for any $X, Y \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^{\perp}))$. Using (2.6), (2.8), (2.15), (2.18), and (4.5), a direct calculation shows that

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = \overline{g}(TY, A_{\omega\xi}X). \tag{4.10}$$

On the other hand, from (2.6), (2.7), (2.12), (2.15), (2.18), (4.5), and (4.6), we obtain

$$\overline{g}(h^s(X,Y),W) = \overline{g}(TY, A^*_{BW}X + A_{C_1W}X + A_{C_2W}X). \tag{4.11}$$

Thus, our assertion follows from (4.9),(4.10), and (4.11).

Converse part directly follows from (4.10) and (4.11). □

In respect of a radical screen transversal lightlike submanifold to be mixed totally geodesic and totally geodesic, we have the following two theorems.

Theorem 4.5. *Let M be a radical screen transversal lightlike submanifold of $\overline{M}$. Then M is mixed totally geodesic if and only if $A_{\omega\xi}X \notin (\mathrm{Rad}\,TM), \nabla^s_X\omega\xi \notin \mu$ and $D^l(X,\omega\xi) = 0$ for all, $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$.*

Proof. Assume that the submanifold M is mixed geodesic. Then

$$\overline{g}\Big(h^l(X,\xi),\xi\Big) = 0, \qquad \overline{g}(h^s(X,\xi),W) = 0 \tag{4.12}$$

for any $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^{\perp}))$. Also, from (2.14) we have

$$\overline{g}\Big(h^l(X,\xi),\xi\Big) = 0. \tag{4.13}$$

On the other hand, by the use of (2.6), (2.8), (2.15), (2.18), and (4.6), we obtain

$$\overline{g}(h^s(X,\xi),W) = -\overline{g}(A_{\omega\xi}X, C_1W) + \overline{g}(\nabla^s_X\omega\xi, C_2W) + \overline{g}\left(D^l(X,\omega\xi), BW\right). \tag{4.14}$$

Thus, our assertion follows from (4.12) and (4.14).

Converse part directly follows from (4.14) and the fact that $\overline{g}(h^l(X,\xi),\xi) = 0$. □

Theorem 4.6. *Let M be a radical screen transversal lightlike submanifold of $\overline{M}$. Then M is totally geodesic if and only if*

(i) $A_{\phi\xi}X \notin \Gamma(S(TM))$ *and* $\nabla^s_X\phi\xi \notin \phi(\mathrm{ltr}(TM)) \perp \mu$,

(ii) $h^l(X,TY) + D^l(X,\omega Y) \notin (\mathrm{ltr}\,(TM)), h^*(X,TY) - A_{\omega Y}X \notin \mathrm{ltr}\,TM$ *and* $h^s(X,TY) + \nabla^s_X\omega Y \notin \mu$

for any $X,Y \in \Gamma(TM)$ *and* $\xi \in \Gamma(\mathrm{Rad}\,TM)$.

Proof. If the submanifold M is totally geodesic, then

$$\overline{g}\left(h^l(X,Y),\xi\right) = 0, \qquad \overline{g}(h^s(X,Y),W) = 0 \tag{4.15}$$

for any $X,Y \in \Gamma(TM)$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^\perp))$. Also, making use of (2.6), (2.8), (2.15), (2.17), (2.18), and (4.5), a direct calculation shows that

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}(TY, A_{\phi\xi}X) - g(\nabla^s_X\phi\xi, \omega Y). \tag{4.16}$$

On the other hand, from (2.6), (2.8), (2.11), (2.15), (2.18), (4.5), and (4.6), we obtain

$$\begin{aligned}\overline{g}(h^s(X,Y),W) &= \overline{g}\left(h^l(X,TY) + D^l(X,\omega Y), BW\right)\\ &\quad + g(h^*(X,TY) - A_{\omega Y}X, C_1W) + \overline{g}(h^s(X,TY) + \nabla^s_X\omega Y, C_2W).\end{aligned} \tag{4.17}$$

Thus, (i) and (ii) follow from (4.16), (4.17), and (4.15).

Converse part directly follows from (4.16) and (4.17).

For a ST-anti-invariant lightlike submanifold M immersed in $\overline{M}$, if we denote the projection morphism of $S(TM)$ and $\mathrm{Rad}\,TM$ by P and Q, respectively, then for any vector field tangent to M we can write

$$X = PX + QX. \tag{4.18}$$

By applying ϕ to (4.18) and then using (2.19), we obtain

$$\phi X = \omega PX + \omega QX. \tag{4.19}$$

Denoting ωPX by ω_2 and ωQX by ω_1. Then (4.19) can be rewritten as

$$\phi X = \omega_1 X + \omega_2 X, \tag{4.20}$$

where $\omega_1 X \in \Gamma(\mathrm{ltr}(TM))$ and $\omega_2 X \in \Gamma(S(TM^{\perp}))$.

For $W \in \Gamma(S(TM^{\perp}))$, writing

$$\phi W = B_1 W + B_2 W + C_1 W + C_2 W, \tag{4.21}$$

where $B_1 W \in \Gamma(\mathrm{Rad}\, TM)$, $B_2 W \in \Gamma(S(TM))$, $C_1 W \in \Gamma(\mathrm{ltr}(TM))$, and $C_2 W \in \Gamma(\mu)$ (μ is the orthogonal complement of $\{\phi(\mathrm{Rad}\, TM) \oplus \phi(\mathrm{ltr}(TM))\} \oplus_{\mathrm{ortho}} \phi(S(TM)$ in $S(TM^{\perp})$). □

In view of the above discussions, the conditions under which the distribution Rad TM of a *ST*-anti-invariant lightlike submanifold immersed in indefinite Kenmotsu manifolds to be totally geodesic is given by the following result.

Theorem 4.7. *Let M be a ST-anti-invariant lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the distribution* Rad TM *is totally geodesic if and only if $A_{\omega\xi_2}\xi_1 = 0, \nabla^s_{\xi_1}\omega\xi_2 \notin \mu$ and $D^l(\xi_1, \phi\xi_2) = 0$, for any $\xi_1, \xi_2 \in \Gamma(\mathrm{Rad}\, TM)$.*

Proof. Suppose that the distribution Rad TM is totally geodesic. Then we see that

$$\overline{g}\left(h^l(\xi_1, \xi_2), \xi\right) = 0, \qquad \overline{g}(h^s(\xi_1, \xi_2), W) = 0 \tag{4.22}$$

for any $\xi_1, \xi_2 \in \Gamma(\mathrm{Rad}\, TM)$ and $W \in \Gamma(S(TM^{\perp}))$. Recall that $h^l = 0$ on Rad TM [9]. On the other hand, by the use of (2.6), (2.8), (2.15), (4.20), and (4.21), we obtain

$$\begin{aligned} &\overline{g}(h^s(\xi_1, \xi_2), W) \\ &\quad = -\overline{g}(A_{\omega\xi_2}\xi_1, B_2 W + C_1 W) + \overline{g}\left(\nabla^s_{\xi_2}\omega\xi_2, C_2 W\right) + \overline{g}\left(D^l(\xi_1, \phi\xi_2), B_1 W\right). \end{aligned} \tag{4.23}$$

Thus, our assertion follows from (4.22) and (4.23).

Converse part directly follows from (4.22) and (4.23). □

For the screen distribution $S(TM)$ of a *ST*-anti-invariant lightlike submanifold to be totally geodesic, we have the following.

Theorem 4.8. *Let M be a ST-anti-invariant lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the distribution $S(TM)$ is totally geodesic if and only if $\nabla^s_X \omega Y \notin \phi(\mathrm{ltr}(TM)) \perp \mu$, $A_{\omega Y} X = 0$ and $D^l(X, \omega Y) = 0$, for any $X, Y \in \Gamma(S(TM))$.*

Proof. If $S(TM)$ is totally geodesic, then

$$\overline{g}\left(h^l(X, Y), \xi\right) = 0, \qquad \overline{g}(h^s(X, Y), W) = 0, \tag{4.24}$$

for any $X, Y \in \Gamma(S(TM)), \xi \in \Gamma(\mathrm{Rad}\, TM)$ and $W \in \Gamma(S(TM^{\perp}))$.

On the other hand, using (2.6), (2.15), (2.18), and (4.20), we obtain

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}(\nabla^s_X \omega Y, \omega\xi). \tag{4.25}$$

Also,

$$\begin{aligned}
&\overline{g}(h^s(X,Y),W) \\
&\quad = \overline{g}(-A_{\omega Y}X, B_2W + C_1W) + g(\nabla^s_X \omega Y, C_2W) + \overline{g}\left(D^l(X,\omega Y), B_1W\right),
\end{aligned} \tag{4.26}$$

where we have used (2.6), (2.8), (2.15), (2.18), (4.20), and (4.21). Thus, our assertion follows from (4.25), (4.26), and (4.24).

Converse part directly follows from (4.25) and (4.26). □

The necessary and sufficient conditions for a ST-anti-invariant lightlike submanifold to be mixed totally geodesic, we have the following.

Theorem 4.9. *Let M be a ST-anti-invariant lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is mixed totally geodesic if and only if $A_{\omega\xi}X = 0$, $D^l(X,\omega\xi) = 0$ and $\nabla^s_X \omega\xi \notin \mu$. for any $X \in \Gamma(S(TM))$ and $\xi \in \Gamma(\mathrm{Rad}\ TM)$.*

Proof. Assume that the submanifold M is mixed gedesic. Then

$$\overline{g}\left(h^l(X,\xi),\xi\right) = 0, \qquad \overline{g}\left(h^l(X,\xi),W\right) = 0 \tag{4.27}$$

for any $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^{\perp}))$. By virtue of (2.14), we have

$$\overline{g}\left(h^l(X,\xi),\xi\right) = 0. \tag{4.28}$$

On the other hand, using (2.6), (2.8), (2.15), (2.18), (4.20), and (4.21), we get

$$\begin{aligned}
&\overline{g}(h^s(X,\xi),W) \\
&\quad = \overline{g}(-A_{\omega\xi}X, B_2W + C_1W) + \overline{g}(\nabla^s_X \omega\xi, C_2W) + \overline{g}\left(D^l(X,\omega\xi), B_1W\right).
\end{aligned} \tag{4.29}$$

Thus, our assertion follows from (4.27) and (4.29).

Converse part directly follows from (4.29). □

Now, we prove the following.

Theorem 4.10. *Let M be a ST-anti-invariant lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is totally geodesic if and only if $\nabla^s_X \omega Y \notin \mu$, $A_{\omega Y}X = 0$ and $D^l(X,\omega Y) = 0$, for any $X, Y \in \Gamma(TM)$.*

Proof. The submanifold M is totally geodesic if and only if

$$\overline{g}\left(h^l(X,Y),\xi\right) = 0, \qquad \overline{g}(h^s(X,Y),W) = 0 \tag{4.30}$$

for any $X, Y \in \Gamma(TM)$, $\xi \in \Gamma(\operatorname{Rad} TM)$ and $W \in \Gamma(S(TM^{\perp}))$. By the use of (2.6), (2.8), (2.15), (2.18), and (4.20), we obtain

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}(\nabla^s_X \omega Y, \omega\xi). \tag{4.31}$$

On the other hand, from (2.6), (2.8), (2.15), (4.20), and (4.21), we have

$$\begin{aligned} &\overline{g}(h^s(X,Y),W) \\ &\quad = -\overline{g}(A_{\omega Y}X, B_2W + C_1W) + \overline{g}(\nabla^s_X\omega Y, C_2W) + \overline{g}\left(D^l(X,\omega Y), B_1W\right). \end{aligned} \tag{4.32}$$

Thus, our assertion follows from (4.31), (4.32), and (4.30).

Converse part directly follows from (4.31) and (4.32). □

5. Transversal Lightlike Submanifolds

The purpose of this section is to study transversal and radical transversal lightlike submanifolds in an indefinite Kenmotsu manifold. We recall here the definitions of these submanifolds given by Yıldırım and Sahin [10].

Definition 5.1. A lightlike submanifold M tangent to structure vector field V immersed in an indefinite Kenmotsu manifold $\overline{M}$ is said to be

(i) radical transversal lightlike submanifold of $\overline{M}$ if

$$\phi(\operatorname{Rad} TM) = \operatorname{ltr}(TM), \qquad \phi(S(TM)) = S(TM), \tag{5.1}$$

(ii) transversal lightlike submanifold of $\overline{M}$ if

$$\phi(\operatorname{Rad} TM) = \operatorname{ltr}(TM), \qquad \phi(S(TM)) \subseteq S\left(TM^{\perp}\right). \tag{5.2}$$

For a radical transversal lightlike submanifold M of an indefinite Kenmotsu manifold $\overline{M}$, if P and Q are the projection morphism on $S(TM)$ and $\operatorname{Rad} TM$, respectively, then any vector field X tangent to M can be written as

$$X = PX + QX. \tag{5.3}$$

We apply ϕ to (5.3) and then using (2.19), we get

$$\phi X = TPX + \omega QX. \tag{5.4}$$

If we denote TPX by TX and ωQX by ωX, then (5.4) can be rewritten as

$$\phi X = TX + \omega X, \tag{5.5}$$

where $TX \in \Gamma(S(TM))$ and $\omega X \in \Gamma(\text{ltr}(TM))$.

Moreover, if $W \in \Gamma(S(TM^{\perp}))$, then

$$\phi W = CW, \tag{5.6}$$

from which we observe that $\phi W \in \Gamma(S(TM^{\perp}))$.

Using the above notations, one can prove the following.

Theorem 5.2. *Let M be a radical transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the distribution $\text{Rad}\, TM$ is totally geodesic if and only if $A_{CW}\xi_1 \notin \Gamma(\text{Rad}\, TM)$ for any $\xi_1 \in \Gamma(\text{Rad}\, TM)$ and $W \in \Gamma(S(TM^{\perp}))$.*

Proof. Since $h^l = 0$ on $\text{Rad}\, TM$ [9], we observe that the distribution $\text{Rad}\, TM$ is totally geodesic if and only if

$$\overline{g}\Big(h^l(\xi_1, \xi_2), \xi\Big) = 0, \qquad \overline{g}(h^s(\xi_1, \xi_2), W) = 0 \tag{5.7}$$

for any $\xi_1 \in \Gamma(\text{Rad}\, TM)$ and $W \in \Gamma(S(TM^{\perp}))$. On the other hand, using (2.6), (2.8), (2.15), (2.18), (5.5), and (5.6), we arrive at

$$\overline{g}(h^s(\xi_1, \xi_2), W) = \overline{g}(\omega\xi_2, A_{CW}\xi_1). \tag{5.8}$$

Thus, our assertion follows from (5.7) and (5.8).

Converse part directly follows from (5.8). □

A screen distribution $S(TM)$ of a radical transversal lightlike submanifold in indefinite Kenmotsu manifolds to be totally geodesic, we have the following.

Theorem 5.3. *Let M be a radical transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the distribution $S(TM)$ is totally geodesic if and only if $h^*(X, TY) = 0$ and $A_{CW}X \notin \Gamma(S(TM))$ for any $X, Y \in \Gamma(S(TM))$ and $W \in \Gamma(S(TM^{\perp}))$.*

Proof. We note that the distribution $S(TM)$ is totally geodesic if and only if

$$\overline{g}\Big(h^l(X, Y), \xi\Big) = 0, \qquad \overline{g}(h^s(X, Y), W) = 0 \tag{5.9}$$

for any $X, Y \in \Gamma(S(TM))$, $\xi \in \Gamma(\text{Rad}\, TM)$, and $W \in \Gamma(S(TM^{\perp}))$. Making use of (2.6), (2.11), (2.15), (2.18), and (5.5), we get

$$\overline{g}\Big(h^l(X, Y), \xi\Big) = \overline{g}(h^*(X, TY), \omega\xi). \tag{5.10}$$

On the other hand, from (2.6), (2.8), (2.15), (2.18), (5.5), and (5.6), we have

$$\overline{g}(h^s(X,Y),W) = \overline{g}(TY, A_{CW}X). \tag{5.11}$$

Thus, our assertion follows from (5.10), (5.11), and (5.9).

Converse part directly follows from (5.10) and (5.11). □

The conditions under which a radical transversal lightlike submanifold immersed in indefinite Kenmotsu manifolds to be mixed totally geodesic is given by the following theorem.

Theorem 5.4. *Let M be a radical transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is mixed totally geodesic if and only if $A_{CW}X \notin \Gamma(\mathrm{Rad}\,TM)$ for any $X \in \Gamma(S(TM))$ and $W \in \Gamma(S(TM^{\perp}))$.*

Proof. The submanifold M is mixed totally geodesic if and only if

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = 0, \qquad \overline{g}(h^s(X,\xi),W) = 0 \tag{5.12}$$

for any $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^{\perp}))$. From (2.14), we have

$$\overline{g}\Big(h^l(X,\xi),W\Big) = 0. \tag{5.13}$$

On the other hand, using (2.6), (2.8), (2.15), (2.18), (5.5), and (5.6), we obtain

$$\overline{g}(h^s(X,Y),W) = \overline{g}(\omega\xi, A_{CW}X). \tag{5.14}$$

Thus, our assertion follows from (5.12) and (5.14).

Converse part directly follows from (5.14). □

Theorem 5.5. *Let M be a radical transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the submanifold M is totally geodesic if and only if*

(i) $h^*(X,TY) - A_{\omega Y}X \notin \Gamma(\mathrm{Rad}\,M)$.

(ii) $A_{CW}X = 0$

for any $X,Y \in \Gamma(TM)$.

Proof. We observe that the submanifold M is totally geodesic if and only if

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = 0, \qquad \overline{g}(h^s(X,Y),W) = 0 \tag{5.15}$$

for any $X,Y \in \Gamma(TM)$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^{\perp}))$.

By the use of (2.6), (2.8), (2.11), (2.15), (2.18), (5.5), and (5.6), we arrive at

$$\overline{g}\Big(h^l(X,Y),\xi\Big) = \overline{g}(h^*(X,TY),\omega\xi) - g(A_{\omega Y}X,\omega\xi). \tag{5.16}$$

On the other hand, from (2.6), (2.8), (2.15), (2.18), (5.5), and (5.6), we have

$$\overline{g}(h^s(X,Y),W) = g(TY, A_{CW}X) + \overline{g}(\omega Y, A_{CW}X). \tag{5.17}$$

Thus, our assertion follows from (5.15), (5.16), and (5.17).

Converse part directly follows from (5.16), and (5.17). □

If we denote the projections on the distributions $S(TM)$ and $\mathrm{Rad}\,TM$ involved with the definition of a transversal lightlike submanifold M immersed in indefinite Kenmotsu manifold $\overline{M}$ by P and Q, respectively, then any vector field X tangent to M can be written as

$$X = PX + QX. \tag{5.18}$$

Applying ϕ to (5.18) and then using (2.19), we get

$$\phi X = \omega PX + \omega QX. \tag{5.19}$$

If we denote ωQX by ω_1 and ωPX by ω_2, then (5.19) can be written as

$$\phi X = \omega_2 X + \omega_1 X, \tag{5.20}$$

where $\omega_1 \in \Gamma(\mathrm{ltr}(TM))$ and $\omega_2 X \in \Gamma(S(TM^{\perp}))$.

For $W \in \Gamma(S(TM^{\perp}))$, we have

$$\phi W = BW + CW, \tag{5.21}$$

where $BW \in \Gamma(S(TM))$ and $CW \in \Gamma(\mu)$ (μ is the orthogonal complement of $\phi(S(TM))$ in $S(TM^{\perp})$).

Theorem 5.6. *Let M be a transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then the distribution $\mathrm{Rad}\,TM$ is totally geodesic if and only if $A_{\omega\xi_2}\xi_1 \notin S(TM)$ and $D^s(\xi_1, \phi\xi_2) \in (\mu)$ for any $\xi_1, \xi_2 \in \Gamma(\mathrm{Rad}\,TM)$.*

Proof. The distribution $\mathrm{Rad}\,TM$ is totally geodesic if and only if

$$\overline{g}\left(h^l(\xi_1,\xi_2),\xi\right) = 0, \qquad \overline{g}(h^s(\xi_1,\xi_2),\xi) = 0 \tag{5.22}$$

for any $\xi_1, \xi_2, \xi \in \Gamma(\mathrm{Rad}\,TM)$. In view of $h^l = 0$ on $\mathrm{Rad}\,TM$ [9], we have

$$\overline{g}\left(h^l(\xi_1,\xi_2),\xi\right) = 0. \tag{5.23}$$

On the other hand, making use of (2.6), (2.7), (2.15), (2.18), (5.20), and (5.21), we get

$$\overline{g}(h^s(\xi_1,\xi_2),W) = -g\left(A_{\omega\xi_2}\xi_1, BW\right) + \overline{g}\left(D^s(\xi_1,\phi\xi_2), CW\right). \tag{5.24}$$

Thus, our assertion follows from (5.22), (5.23), and (5.24).

Converse part directly follows from (5.23) and (5.24). □

Theorem 5.7. *Let M be a transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then $S(TM)$ is totally geodesic if and only if*

$$D^s(X,\omega_1\xi), \quad \nabla^s_X\omega_2Y \notin \mu, \quad A_{\omega_2Y}X \notin \Gamma(S(TM)) \tag{5.25}$$

for any $X,Y \in \Gamma(S(TM))$.

Proof. We note that the distribution $S(TM)$ is totally geodesic if and only if $\overline{g}(h^l(X,Y),\xi) = 0$ and $\overline{g}(h^s(X,Y),W) = 0$ for any $X,Y \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$, and $W \in \Gamma(S(TM^\perp))$.

Combining (2.6), (2.7), (2.15), (2.18), (5.20), and (5.21), we obtain

$$\overline{g}\left(h^l(X,Y),\xi\right) = -\overline{g}(\omega_2Y, D^s(X,\omega_1\xi)). \tag{5.26}$$

On the other hand, from (2.6), (2.8), (2.15), (2.18), (5.20), and (5.21), we have

$$\overline{g}(h^s(X,Y),W) = -\overline{g}(-A_{\omega_2Y}X, BW) + \overline{g}\left(\nabla^s_X\omega_2Y, CW\right). \tag{5.27}$$

Thus, our assertion follows from (5.25), (5.26), and (5.27).

Converse part directly follows from (5.26) and (5.27). □

Theorem 5.8. *Let M be a transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is mixed totally geodesic if and only if $A_{\omega_1\xi}X \notin \Gamma(S(TM))$ and $D^s(X,\omega_1\xi) \notin (\mu)$ for any $X \in \Gamma(S(TM))$ and $\xi \in \Gamma(\mathrm{Rad}\,TM)$.*

Proof. We observe that the submanifold M is mixed totally geodesic if and only if

$$\overline{g}\left(h^l(X,\xi),\xi\right) = 0, \qquad \overline{g}(h^s(X,\xi),W) = 0 \tag{5.28}$$

for all $X \in \Gamma(S(TM))$, $\xi \in \Gamma(\mathrm{Rad}\,TM)$ and $W \in \Gamma(S(TM^\perp))$.

From (2.14), we infer that

$$\overline{g}\left(h^l(X,\xi),\xi\right) = 0. \tag{5.29}$$

On the other hand, by the use of (2.6), (2.7), (2.15), (2.18), (5.20), and (5.21), we arrive at

$$\overline{g}(h^s(X,\xi),W) = -g\left(A_{\omega_1\xi}X, BW\right) + \overline{g}(D^s(X,\omega_1\xi), CW). \tag{5.30}$$

Thus, our assertion follows from (5.28), (5.29), and (5.30).

Converse part directly follows from (5.29) and (5.30). □

Theorem 5.9. *Let M be a transversal lightlike submanifold of an indefinite Kenmotsu manifold $\overline{M}$. Then M is totally geodesic if and only $A_{\omega Y}X = 0$ and $D^s(X,\omega_1 Y) + \nabla^s_X \omega_2 Y \notin \Gamma(\mu)$ for all $X, Y \in \Gamma(TM)$.*

Proof. The submanifold M is totally geodesic if and only if

$$\overline{g}\left(h^l(X,Y),\xi\right) = 0, \qquad \overline{g}(h^s(X,Y),W) = 0 \tag{5.31}$$

for any $X, Y \in \Gamma(TM)$, $\xi \in \Gamma(\operatorname{Rad} TM)$ and $W \in \Gamma(S(TM^{\perp}))$.

By virtue of (2.6), (2.7), (2.8), (2.15), (2.18), (5.20) and (5.21), we have

$$\overline{g}\left(h^l(X,Y),\xi\right) = \overline{g}(-A_{\omega Y}X, \phi\xi). \tag{5.32}$$

On the other hand, by the use of (2.6), (2.7), (2.8), (2.15), (2.18), (5.20), and (5.21), we get

$$\overline{g}(h^s(X,Y),\xi) = g(-A_{\omega Y}X, BW) + \overline{g}\left(D^s(X,\omega_1 Y) + \nabla^s_X \omega_2 Y, CW\right). \tag{5.33}$$

Thus, our assertion follows from (5.31), (5.32), and (5.33).

Converse part directly follows from (5.32), and (5.33). □

References

[1] K. L. Duggal and A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, vol. 364, Kluwer Academic, Dordrecht, The Netherlands, 1996.

[2] B. Sahin and R. Gunes, "Geodesic CR-lightlike submanifolds," *Contributions to Algebra and Geometry*, vol. 42, no. 2, pp. 583–594, 2001.

[3] J. Dong and X. Liu, "Geodesic lightlike submanifolds of indefinite Sasakian manifolds," *Advances in Pure Mathematics*, vol. 1, pp. 378–383, 2011.

[4] B. Sahin, "Transversal lightlike submanifolds of indefinite Kaehler manifolds," *Analele Universităţii de Vest din Timişoara*, vol. 44, no. 1, pp. 119–145, 2006.

[5] K. L. Duggal and B. Sahin, "Screen Cauchy Riemann lightlike submanifolds," *Acta Mathematica Hungarica*, vol. 106, no. 1-2, pp. 137–165, 2005.

[6] B. Sahin, "Screen transversal lightlike submanifolds of indefinite Kaehler manifolds," *Chaos, Solitons and Fractals*, vol. 38, no. 5, pp. 1439–1448, 2008.

[7] K. L. Duggal, "Space time manifolds and contact structures," *International Journal of Mathematics and Mathematical Sciences*, vol. 13, no. 3, pp. 545–553, 1990.

[8] N. Aktan, "On non-existence of lightlike hypersurfaces of indefinite Kenmotsu space form," *Turkish Journal of Mathematics*, vol. 32, no. 2, pp. 1–13, 2008.

[9] C. L. Bejan and K. L. Duggal, "Global lightlike manifolds and harmonicity," *Kodai Mathematical Journal*, vol. 28, no. 1, pp. 131–145, 2005.

[10] C. Yıldırım and B. Şahin, "Transversal lightlike submanifolds of indefinite Sasakian manifolds," *Turkish Journal of Mathematics*, vol. 33, pp. 1–21, 2009.

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