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Research Article

Domain Wall Splitting in Extra Dimension

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We study the splitting of domain walls in 5-dimensional flat *Minkowski* spacetime. To construct a domain wall structure we utilize a potential of the form $V(\varphi) = a\varphi^2 - b\varphi^4 + c\varphi^6$ and for simplicity we assume the scalar field to be real. We also assume the coefficient a to be temperature-dependent. We find exact analytical expressions for the energy density of the domain wall and from this expression we find close analytical form for the separation of the divided domain walls. We find that near critical temperature T_C the domain walls split. At the critical temperature, the domain walls rejoin. In the region above this critical temperature, the kink solution is nontopological. We find that the phenomena of splitting of domain walls occurs in this region as well. This effect is especially manifest near the inflection point of the potential.

1. Introduction

The subject of extra dimension has become an active area of research in the past decade and many models have been constructed to address fundamental problems of particle physics and cosmology, for instances the issue of hierarchy and the cosmological constant problem [1–3].

An essential idea in the *Ekpyrotic* model of universe [4] is that the hot big bang is as a result of collisions of two $branes$. An important feature of this model is nucleation and splitting of the $branes$. To realize the phenomena of splitting of $branes$, a complex scalar field is coupled to gravity in a five-dimensional world with warped geometry [5]. In this work, a numerical solution is found and the region close to the critical temperature is studied. In a recent attempt [6], a real scalar field is utilized to describe $brane$ splitting and localization of fermions in five dimensions. They consider the case in the Newtonian limit as well as the case with the inclusion of gravity. Numerical techniques are also employed to obtain their results.

In the present work, we attempt an analytical treatment for the discussion of the $brane$ splitting, and for simplicity we mainly consider the Newtonian limit. In this limit, it is possible to obtain analytical expressions [7, 8] for the equations of motions in the static case. Having these exact solutions, we compute the energy density of the domain wall, and by mapping out the structure of the critical points of this quantity we find analytical expressions for the distance between the separated domain walls.

The plan of this paper is as follows. In Section 2, we describe the model. And we unravel the critical structure of the potential $V(\varphi)$. In Section 3, we discuss the equation of motion in the static case. We find the domain wall solutions in different regions and we compute the energy density for each case. We find that the general form of this energy density has two symmetric maxima with respect to a minima at the origin. We notice that away from a critical value of the mass parameter a the solution is a single kink, but the energy density of this kink is not sharp and has a slight distortion manifested as two maxima of this quantity. But very close to this critical value of the mass parameter these two maxima of energy density comprise two separate kinks. Finally we conclude in Section 4. Technical details are explained in the appendix.

2. The Model

In this work, we assume a flat background metric, $g_{MN} = \mathrm{Diag}[+1,-1,-1,-1,-1]$, where the capital Latin letters $M, N, \ldots$ run over five dimensions and the Greek letters over the four dimensions. The space coordinates $X^M = [x^\mu, w]$ are decomposed into the $4d$ subset x^μ and the extra dimension direction w. The action contains only one real scalar field and it is defined by

$$\mathbf{S} = \int dx_\mu dw \left[\frac{1}{2} g^{MN} \partial_M \varphi \partial_N \varphi - V(\varphi)\right], \tag{2.1}$$

where the potential $V(\varphi)$ is given by

$$V(\varphi) = a\varphi^2 - b\varphi^4 + c\varphi^6. \tag{2.2}$$

a, b and c are real and nonnegative quantities. Now if we assume that all of these parameters be strictly positive, then the potential will have three local minima, one is $\varphi^{(1)} = 0$ corresponding to a disordered bulk phase and the other two are at $\varphi^{(2)} = -\varphi^{(3)} = v$ with

$$v = \sqrt{\frac{\sqrt{b^2 - 3ac}}{3c} + \frac{b}{3c}}. \tag{2.3}$$

They correspond to ordered bulk phases. The potential also contains maxima at $\varphi^{(4)} = -\varphi^{(5)} = v_1$, where

$$v_1 = \sqrt{-\frac{\sqrt{b^2 - 3ac}}{3c} + \frac{b}{3c}}. \tag{2.4}$$

The parameters a and b may not be constant. In this work, we only investigate the variation of the mass parameter a on the structure of the domain wall [6]. We also assume that φ is only a function of the extra dimension w, namely, $\varphi = \varphi(w)$.

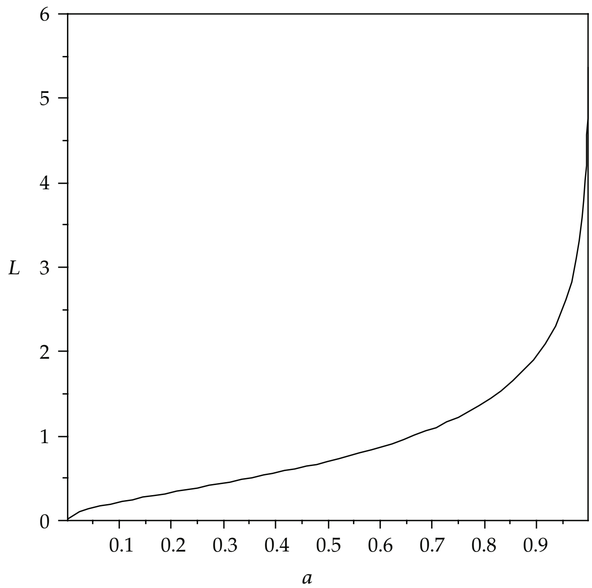

Figure 1: The solid curve represents the distance between the two maxima of the energy density denoted by L (3.6). The value of the mass parameter a is in the range $0.001 \leq a \leq 0.999$.

3. Effects of the Variation of the Mass Parameter a on the Thick *Brane* Solutions

The equation of motion from the action is

$$\frac{d^2\varphi}{dw^2} = \frac{dV(\varphi)}{d\varphi}. \tag{3.1}$$

While resorting to numerical solutions [5, 6], the strategy is to prescribe some boundary conditions. However, in the exact treatment, the boundary conditions are a by-product of the solutions. We distinguish three different regions.

3.1. The Region below Critical Temperature T_c

In this region, $b^2 > 4ac$. We present the exact solution of (3.1) in the following form:

$$\varphi = \frac{A\tanh(Bw)}{\sqrt{1 - D\tanh^2(Bw)}}, \tag{3.2}$$

where the coefficients A, B are

$$B = \sqrt{\frac{2a}{3D-2}}, \qquad A = \sqrt{\frac{(1-D)(3D-1)}{2b}}B, \tag{3.3}$$

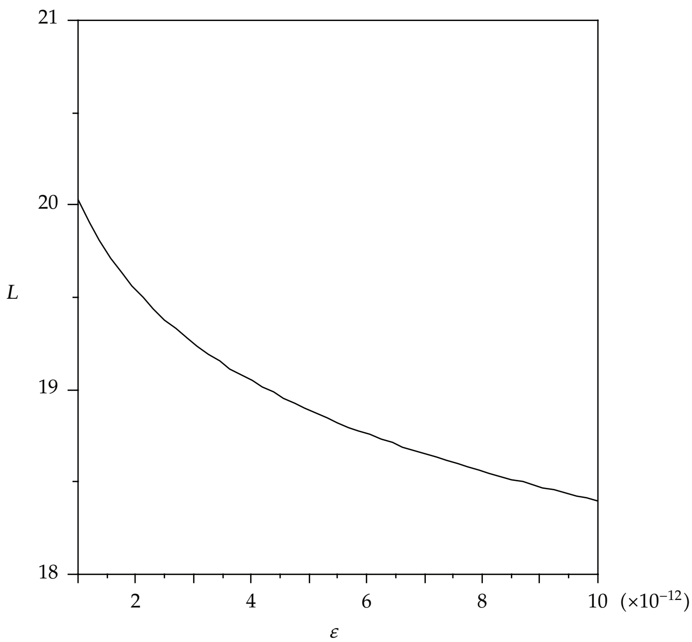

Figure 2: The same as Figure 1 but the mass parameter is parameterized by $a = 1 - \varepsilon$. And we have taken $10^{-12} \leq \varepsilon \leq 10^{-11}$. Therefore, close to the critical value of a_c, the solution is composed of two distinct domain walls.

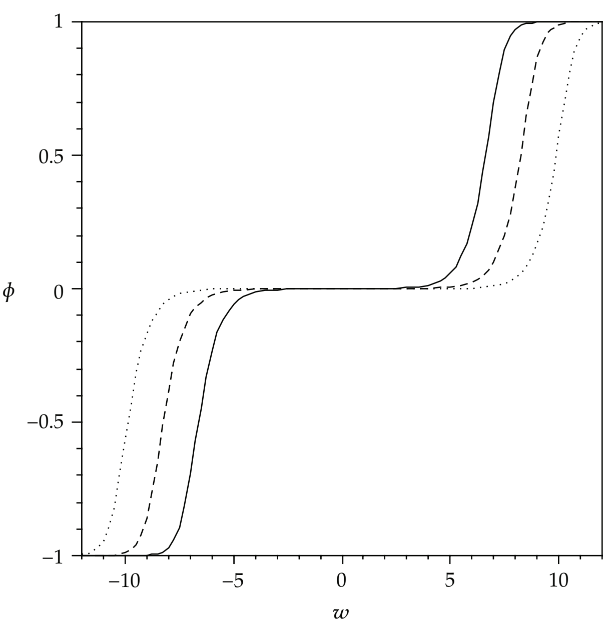

Figure 3: The scalar field (3.2) versus the extra dimensional coordinate which we designate by w. The mass parameter is parameterized by $a = 1-\varepsilon$. The solid curve corresponds to the case where $\varepsilon = 10^{-8}$, the dashed curve is for $\varepsilon = 10^{-10}$, and finally the dotted curve is for $\varepsilon = 10^{-12}$.

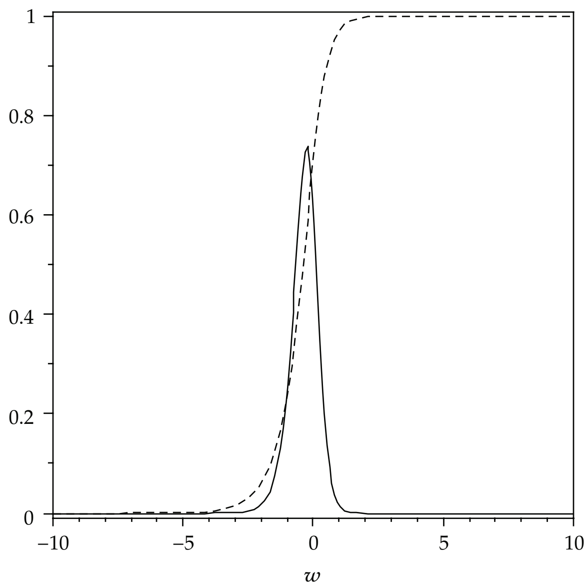

Figure 4: The dashed curve represents the scalar field (3.7) when $a = a_c$. The solid curves shows the associated energy density (3.5).

and finally the coefficient D is given by

$$D = \frac{1}{3}\left[1 + \frac{b}{\sqrt{b^2 - 3ac}}\right]. \tag{3.4}$$

The energy density of the scalar field is given by

$$\rho(w) = \frac{1}{2}\left(\frac{d\varphi}{dw}\right)^2 + V(\phi(w)). \tag{3.5}$$

In the appendix, we show that this function has two maxima which are symmetric with respect to $w = 0$. We also compute the distance between these two maxima, which is given by

$$L = \sqrt{\frac{3c}{2\xi(b+\xi)}}\operatorname{Ln}\left[\frac{\sqrt{ac} + b - \xi}{\sqrt{ac} - b - \xi}\right], \tag{3.6}$$

where $\xi = \sqrt{b^2 - 3ac}$. If these maxima are well separated then they designate the case of two separate domain walls. In this work, all of the figures are drawn for the case $b = 2$, $c = 1$, and we only consider the effect of variation of the mass parameter a on physical quantities. Figure 1 shows the separation between two maxima L versus a for $0.001 < a < 0.999$. We see that close to the critical point $a_c = 1$, the separation between domain walls is increased. To investigate this region, we assume $a = 1 - \varepsilon$ with small positive ε. Figure 2 shows this phenomena of domain wall splitting when $10^{-12} < \varepsilon < 10^{-11}$.

Figure 3 shows the scalar field for $\varepsilon = 10^{-8}$ (solid curve), $\varepsilon = 10^{-10}$ (dashed curve), and $\varepsilon = 10^{-12}$ (dotted curve). Our analytical results confirm the previous result that near

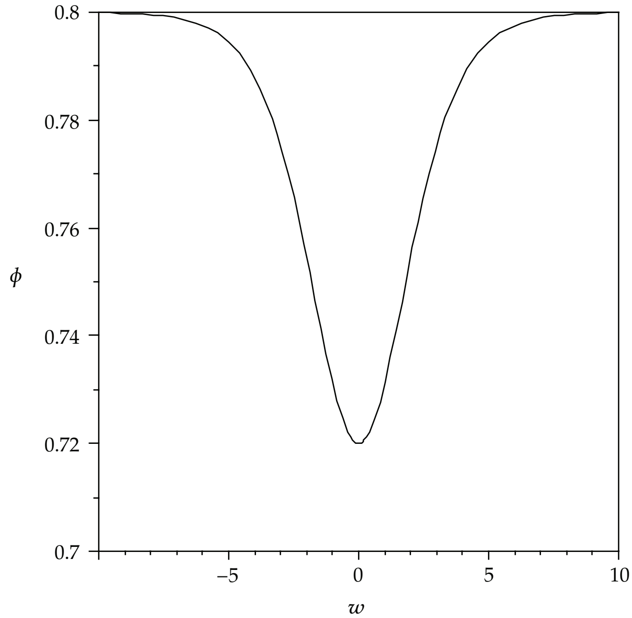

Figure 5: The profile of a nontopological solution (3.8). The mass parameter is $a = 1.2$.

the phase transition the domain wall splits to two domain walls, and each of these domain walls connects an ordered phase to a common disordered phase [6].

3.2. At Critical Temperature T_c

The case of critical temperature corresponds to $a_c = (b^2/(4c))$. From (3.4), the value of D becomes unity, and hence from (3.3) the coefficient A is identically zero. Therefore, the solution as described by (3.2) does not exist at critical temperature. The treatment of [6] is numerical and its discussion in the case of $a = a_c$ is incorrect. Namely its discussion is valid only for $a < a_c$. The scalar field in this case is given by

$$\varphi = \sqrt{\frac{a}{b}\left[1 + \tanh\left(\sqrt{2a}w\right)\right]}. \tag{3.7}$$

Figure 4 shows the shape of the scalar field (dashed curve), and the associated energy density (solid curve) is shown. The splitting of the domain walls disappears at this temperature and we have a scalar field which connects an ordered phase to a disordered phase.

3.3. The Region above Critical Temperature T_c

Here we will discuss the region $T > T_c$. The scalar field in this case is given by

$$\varphi = \frac{A_1}{\sqrt{1 - D_1 \tanh^2(B_1 w)}}, \tag{3.8}$$

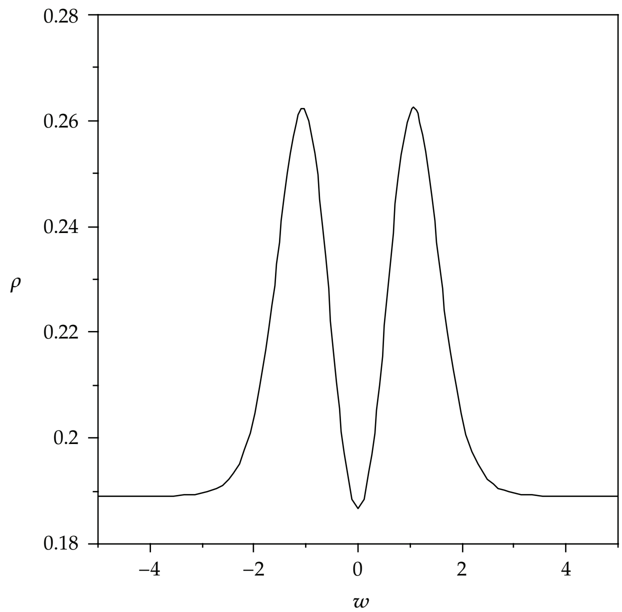

Figure 6: The profile of energy density of the nontopological solution of previous figure as a function of extra dimension coordinate w. This profile also has two maxima.

where

$$D_1 = \frac{\xi}{ac}(b - \xi), \qquad B_1 = \sqrt{\frac{2aD_1}{3 - 2D_1}}, \qquad A_1 = \sqrt{\frac{a(1 - D_1)(3 - D_1)}{b(3 - 2D_1)}} \tag{3.9}$$

and again $\xi = \sqrt{b^2 - 3ac}$.

Figure 5 shows the scalar field for the case $a = 1.2$. This kink does not connect two different minima and hence is a nontopological solution. Figure 6 shows the energy density for this case.

The distance between the two maxima of the energy density in this region is

$$L_1 = \frac{1}{B_1}\mathrm{Ln}\left[\frac{b - \xi + \sqrt{ac}}{b - \xi - \sqrt{ac}}\right]. \tag{3.10}$$

The structure of critical points of the potential as stated in Section 2 holds if $3ac < b^2$. Therefore, the mass parameter is bounded from above. Hence in this situation, $3ac < b^2 < 4ac$. Figure 7 shows the the variation of L_1 versus a. We see that away from the boundaries we have two maxima but the distance between these maxima is such that we do not have two separate kinks. However, again we see that close to the boundaries we have an enhancement in the distance between the two kinks. To study this effect, we assume $a = 1 + \varepsilon$. Figure 8 represents this effect.

With $b = 2$ and $c = 1$ we will have $a < (4/3)$. By choosing $a = (4/3) - \varepsilon$, we can have an estimate of splitting of domain wall near the inflection point ($b^2 = 3ac$). Figure 9 shows this result.

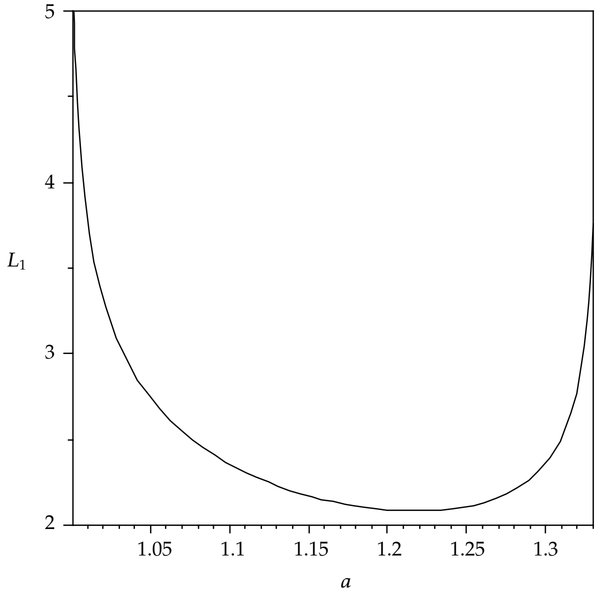

Figure 7: The distance between two maxima of the energy density of the nontopological solution (3.10) versus a. We see that the phenomena of splitting of domain walls occurs near the boundaries.

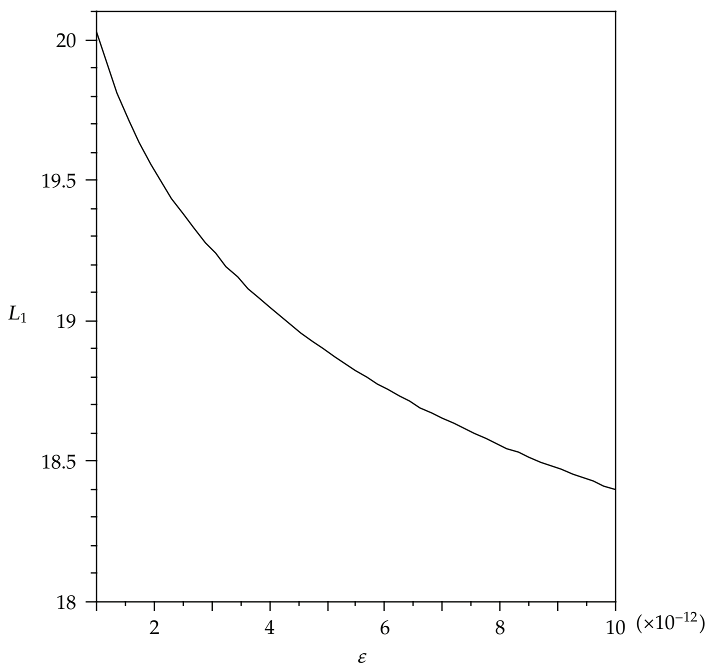

Figure 8: The distance between the two maxima of the energy density of the nontopological solution (3.10) versus ε. Here the mass parameter is parameterized by $a = 1 + \varepsilon$. Large values of L_1 corresponds to the existence of two separate domain walls.

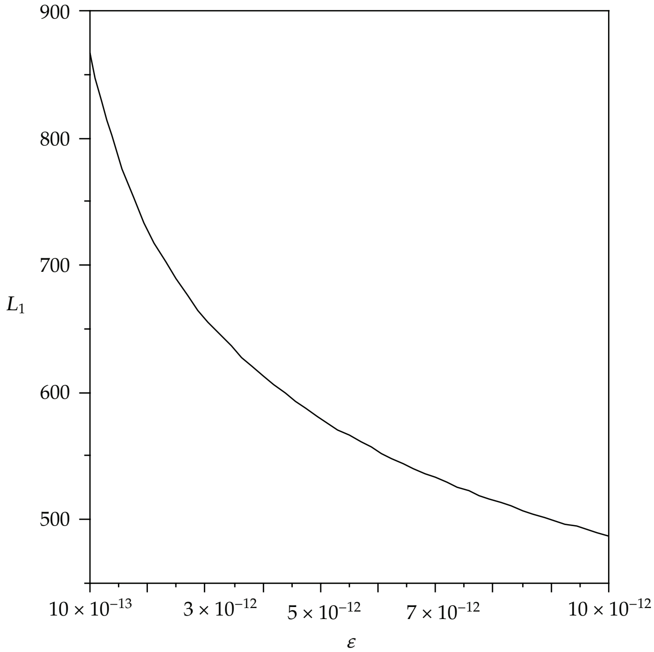

Figure 9: The distance between the two maxima of the energy density of the nontopological solution (3.10) versus ε, where the mass parameter is parameterized by $a = (4/3) - \varepsilon$. We see that the phenomena of splitting of domain walls occurs near this boundary as well.

4. Conclusions

In this paper, we have provided an analytical treatment of the phenomena of domain wall splitting. For simplicity, we have neglected gravity in this paper. In the case of gravity, the expression for the matter energy density is

$$T_{00} = \left(\frac{1}{2}\left(\frac{d\varphi}{dw}\right)^2 + V\right)e^{2A}, \tag{4.1}$$

where $A(w)$ is the warp factor and it is found from the solution of the Einstein equations. In general, it is not easy to obtain an analytical expression for the warp factor for the φ^6 potential. For $A(w)$, we suppose that an analytical treatment presented here, with inclusion of gravity, is not possible due to the complicated nature of T_{00}. However the numerical work on this subject [5, 6] demonstrates that this phenomena occurs with inclusion of gravity as well. But these studies do not address the case of critical temperature or beyond the critical temperature.

We only considered the static case in this work. It will be of interest to consider the time-dependent solutions and its implications. We hope to report on these issues in the future.

Appendix

Profile of the Energy Density of the Scalar Field

From (3.5), we can obtain

$$\frac{d\rho}{dw} = 2\frac{dV}{d\varphi}\frac{d\varphi}{dw}. \tag{A.1}$$

At points $w_1 = -\infty$, $w_2 = \infty$, we have $d\varphi/dw = 0$. Therefore, they are the extremal points of the function $\rho(w)$. To see the nature of these extremal points we take derivative of (A.1) with respect to w, and we have

$$\frac{d^2\rho}{dw^2} = 2\left[\frac{d^2V}{d\varphi^2}\left(\frac{d\varphi}{dw}\right)^2 + \left(\frac{dV}{d\varphi}\right)^2\right]. \tag{A.2}$$

But for w_1 and w_2, the first term in the right-hand side of (A.2) vanishes. So for these points $d^2\rho/dw^2 > 0$, and they are the two minima for the function $\rho(w)$. A well-known result that a topological kink connects two distinct minima of the potential [9]. We can repeat the argument for $w_3 = 0$ and show that this point is also a minima of the energy density profile. The energy density of the scalar field versus w has three minima: $w_1 = -\infty$, $w_2 = \infty$, and $w_3 = 0$. These are associated to the three minima of the $V(\varphi)$. So from (3.2), we have

$$v = \frac{A}{\sqrt{1-D}}. \tag{A.3}$$

The energy density has two maxima, w_{m+} and w_{m-}, for the case of $a < a_c$, their values are obtained from

$$\pm v_1 = \frac{A\tanh(Bw)}{\sqrt{1 - D\tanh^2(Bw)}}. \tag{A.4}$$

We can see that at these points $d\rho/dw = 0$ as $dV/d\varphi = 0$ at $\varphi = v_1$. And $d^2\rho/dw^2 < 0$ as $d^2V/d\varphi^2$ is negative for $\varphi = \pm v_1$. Therefore w_{m+} and w_{m-} are indeed the two maxima for the energy density. So we have managed to map out the structure of the critical point of the function $\rho(w)$. Now utilizing (A.3) and (A.4), we find the separation of the two maxima L as $L = w_{m+} - w_{m-}$ and the result is

$$L = \sqrt{\frac{3c}{2\xi(b+\xi)}}\,\mathrm{Ln}\left[\frac{\sqrt{ac} + b - \xi}{\sqrt{ac} - b - \xi}\right]. \tag{A.5}$$

But when $a > a_c$, the values of w_{m+} and w_{m-} are determined from

$$v_1 = \frac{A_1}{\sqrt{1 - D_1\tanh^2(B_1 w)}}. \tag{A.6}$$

And by repeating the same procedure, we find the distance between the two maxima of the energy density for the nontopological solution is

$$L_1 = \sqrt{\frac{3-2D_1}{2aD_1}}\,\mathrm{Ln}\left[\frac{\sqrt{ac}+b-\xi}{b-\sqrt{ac}-\xi}\right]. \tag{A.7}$$

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