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Research Article

Metastability of an Extended *Higgs* Model

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We consider a singlet extended supersymmetric *Higgs* model. In the limit of $\tan\beta = 1$ it is possible to unravel the vacuum structure of this model analytically. We span the parameter space of the model. Specially we consider configurations in this space for a Higg's mass of 125 GeV. We provide a detailed discussion of the issue of metastability of this model.

1. Introduction

The Standard Model (SM) of particle physics suffers from a hierarchy problem. Supersymmetric extensions of this model can solve the hierarchy problem [1–3]. But such extensions are encountered with a μ-problem [4, 5]. The bilinear supersymmetric *Higgs* mass term $\mu H_d H_u$ in the superpotential, where H_d and H_u are one pair of *Higgs* doublets, does not violate supersymmetry (susy), and gauge symmetry. Then the natural scale for μ is about Planck scale. However in order to get the weak scale correctly with unnatural cancelation we need μ to be about TeV scale.

One solution for this so-called μ problem is to substitute vacuum expectation value (VEV) of an extra gauge singlet field for the parameter μ. In the past thirty years various singlet extensions of the MSSM has been considered [6–9].

A singlet extension of MSSM with mirror symmetry is defined by the superpotential [10]

$$W = \lambda\left[S\left(H_u \cdot H_d - v^2\right) + \widetilde{S}\left(\widetilde{H}_u \cdot \widetilde{H}_d - v^2\right) + \mu_0 S\widetilde{S}\right]. \tag{1.1}$$

In this model *Higgs* singlet field S is coupled to a mirror world (hidden sector) indicated by tildes. And the dot product of the two *Higgs* doublets is defined by

$$H_u \cdot H_d = H_u^0 \cdot H_d^0 - H_u^+ H_d^-. \tag{1.2}$$

As we do not consider the possibility that the charged *Higgs* fields could acquire vacuum expectation values we suppress occurrence of charged *Higgs* fields. So the product in (1.2) is taken to be equivalent to the product of the neutral fields.

In a recent paper [11] we showed that a true symmetry breaking minimum does not exist. And the model has two critical points, where at these points all first derivatives of the scalar potential with respect to the fields vanish.

Solution 1. Exact *susy* with Electroweak Symmetry Breaking (EWSB).

Solution 2. Exact *susy* with no EWSB.

In our analysis we neglected the phases in the *Higgs* sector. We also did not include the soft *susy* breaking terms.

In this work we assume that the fields and the parameters in the Lagrangian of the *Higgs* sector are real. We attempt a phenomenological treatment of extended *Higgs* model and we give a partial analysis of soft *susy* breaking terms [12]. The motivation for the present work is as follows.

(i) The discovery of a nonzero vacuum energy density $e = (5.9 \pm 0.2)\,\mathrm{meV}^4$ [13] supports this point of view that we live in a metastable universe destined to ultimately undergo a phase transition to a *susy* world. So it is of interest to study field theoretical model that exhibit this property, such as an extended *susy Higgs* model.

(ii) To show that a true *susy* breaking minima is attainable by addition of soft *Higgs* masses.

(iii) Provide a detailed analysis of the parameter space of the model.

(iv) And finally to study the nature of phase transition to a future *susy* universe [13–16].

Here a question may arise on the issue of metastability. In order to tunnel to a stable *susy* vacuum, the *susy* breaking in the metastable vacuum has to be spontaneous. But the *susy* breaking is parameterized in this paper with soft-breaking terms, which are an explicit breaking. How can this theory have a truly *susy* minimum?

The answer is that at present we do not have a good theory of *susy* breaking in our universe. But it is assumed that spontaneous breaking of supersymmetry occurs in a "hidden sector" of particles that have no (or only) very small coupling to the "visible sector" chiral supermultiplets of MSSM or its extensions.

Within this framework spontaneous symmetry breaking is communicated from the hidden sector where it originates to the observable sector by means of a third set of fields, the mediator, or the messenger fields. This mediation may take through gravity [17] or gauge interactions [18]. Supersymmetry breaking may be mediated by anomaly [19, 20] or extra dimension [21] as well. The result is the effective soft supersymmetry breaking in the visible sector.

Within this picture the appearance of explicit soft supersymmetry breaking terms are as the result of spontaneous supersymmetry breaking in a more fundamental theory.

It is evident when the *susy* is exact these soft mass terms will vanish. Hence during the transition to an exact *susy* phase (Solution 1) these soft mass terms will disappear.

In [16] we studied this model for the case of $\tan\beta \neq 1$. By utilizing numerical method we found configurations pertinent to an exothermic transition to a future *susy* universe.

In this work we consider the case of $\tan\beta = 1$. We find that in this case it is possible to give an analytical treatment of the subject matter.

This paper is organized in the following way. In section two we describe the model. We obtain the critical point condition on the parameters and *vevs* and we obtain some symmetric solutions. In section three we study the *Higgs* mass squared matrix of our symmetric solutions. We find the eigenvalues and we obtain the positivity constraints for these eigenvalues analytically. In section four we discuss the transition to a future *susy* universe. It is found that for the symmetric solution this transition is endothermic. We span over the soft squared masses. And finally in section five we present our conclusions. Technical details and special cases of soft squared masses are treated in the appendices.

2. The Model

The F term in the scalar potential in any supersymmetric model is derivable from the superpotential by

$$V_F = \sum_{\phi} \left| \frac{\partial W}{\partial \phi} \right|^2. \tag{2.1}$$

So for the neutral fields the F term of the scalar potential from the superpotential (1.1) is

$$\begin{aligned} V_F = \lambda^2 \Bigg[& \left| H_u H_d - v^2 + \mu_0 \widetilde{S} \right|^2 + |S|^2 \left(|H_u|^2 + |H_d|^2 \right) \\ & + \left| \widetilde{H}_u \widetilde{H}_d - v^2 + \mu_0 S \right|^2 + \left| \widetilde{S} \right|^2 \left(\left| \widetilde{H}_u \right|^2 + \left| \widetilde{H}_d \right|^2 \right) \Bigg]. \end{aligned} \tag{2.2}$$

The D term in the potential $V_D = V_{D1} + \widetilde{V}_{D1}$, where

$$V_{D1} = \frac{g_1^2 + g_2^2}{8} \left[|H_d|^2 - |H_u|^2 \right]^2 + \frac{g_2^2}{2} \left[|H_d|^2 |H_u|^2 - |H_u H_d|^2 \right]. \tag{2.3}$$

Here g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings. The structure of $\widetilde{V}_{D1}$ is similar to V_{D1} except that the Higgs doublet fields in this case belong to the mirror world.

The general structure of soft *susy* breaking term has a complicated form [13]. Here we only consider the soft mass squared *Higgs* term, namely,

$$V_S = \lambda^2 \left[m_S^2 |S|^2 + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_{\widetilde{S}}^2 \left| \widetilde{S} \right|^2 + m_{\widetilde{H}_u}^2 \left| \widetilde{H}_u \right|^2 + m_{\widetilde{H}_d}^2 \left| \widetilde{H}_d \right|^2 \right]. \tag{2.4}$$

Hence at the tree level the complete scalar potential of our model is

$$V = V_F + V_D + V_S. \tag{2.5}$$

For simplicity we will ignore phases in the *Higgs* sector as well. The vacuum expectation values of the *Higgs* are given by

$$\langle H_u \rangle = \upsilon_1, \qquad \langle H_d \rangle = \upsilon_2. \tag{2.6}$$

Similarly for the *Higgs* in the mirror world we have

$$\begin{aligned} \left\langle \widetilde{H}_u \right\rangle &= \widetilde{\upsilon}_1, \qquad \left\langle \widetilde{H}_d \right\rangle = \widetilde{\upsilon}_2, \\ \langle S \rangle &= S_0, \qquad \left\langle \widetilde{S} \right\rangle = \widetilde{S}_0. \end{aligned} \tag{2.7}$$

By minimizing the scalar potential we obtain

$$\begin{aligned} &S_0\left(\upsilon_1^2+\upsilon_2^2\right)+\mu_0\left(\widetilde{\upsilon}_1\widetilde{\upsilon}_2-\upsilon^2+\mu_0 S_0\right)+m_S^2 S_0=0, \\ &\widetilde{S}_0\left(\widetilde{\upsilon}_1^2+\widetilde{\upsilon}_2^2\right)+\mu_0\left(\upsilon_1\upsilon_2-\upsilon^2+\mu_0\widetilde{S}_0\right)+m_{\widetilde{S}}^2\widetilde{S}_0=0, \\ &\upsilon_2\left(\upsilon_1\upsilon_2-\upsilon^2+\mu_0\widetilde{S}_0\right)+\upsilon_1 S_0^2+\frac{\upsilon_1}{4\lambda^2}\left(g_1^2+g_2^2\right)\left(\upsilon_1^2-\upsilon_2^2\right)+m_{H_u}^2\upsilon_1=0, \\ &\upsilon_1\left(\upsilon_1\upsilon_2-\upsilon^2+\mu_0\widetilde{S}_0\right)+\upsilon_2 S_0^2+\frac{\upsilon_2}{4\lambda^2}\left(g_1^2+g_2^2\right)\left(\upsilon_2^2-\upsilon_1^2\right)+m_{H_d}^2\upsilon_2=0, \\ &\widetilde{\upsilon}_2\left(\widetilde{\upsilon}_1\widetilde{\upsilon}_2-\upsilon^2+\mu_0 S_0\right)+\widetilde{\upsilon}_1\widetilde{S}_0^2+\frac{\widetilde{\upsilon}_1}{4\lambda^2}\left(g_1^2+g_2^2\right)\left(\widetilde{\upsilon}_1^2-\upsilon_2^2\right)+m_{\widetilde{H}_u}^2\widetilde{\upsilon}_1=0, \\ &\widetilde{\upsilon}_1\left(\widetilde{\upsilon}_1\widetilde{\upsilon}_2-\upsilon^2+\mu_0 S_0\right)+\widetilde{\upsilon}_2\widetilde{S}_0^2+\frac{\widetilde{\upsilon}_2}{4\lambda^2}\left(g_1^2+g_2^2\right)\left(\widetilde{\upsilon}_2^2-\upsilon_1^2\right)+m_{\widetilde{H}_d}^2\widetilde{\upsilon}_2=0, \end{aligned} \tag{2.8}$$

with solutions.

Solution 3. $\upsilon_1 = \upsilon_2 = \widetilde{\upsilon}_1 = \widetilde{\upsilon}_2 = 0$ and

$$S_0 = \frac{\mu_0\upsilon^2}{\mu_0^2+m_S^2}, \qquad \widetilde{S}_0 = \frac{\mu_0\upsilon^2}{\mu_0^2+m_{\widetilde{S}}^2}, \tag{2.9}$$

which denotes a broken *susy* phase with no EWSB.

Solution 4. Here we have $\upsilon_1 = \upsilon_2 = \widetilde{\upsilon}_1 = \widetilde{\upsilon}_2 = \upsilon_0$ moreover $S_0 = \widetilde{S}_0$. This will correspond to a broken *susy* phase with EWSB. But as in this case $\upsilon_1^2 + \upsilon_2^2 = (246)^2$, we have $\upsilon_0 = 174\,\text{GeV}$. Moreover since $\tan\beta = \upsilon_1/\upsilon_2$, this solution then corresponds to the limit of $\tan\beta = 1$.

3. Higgs Mass Squared Matrices

In this section we compute the *Higgs* mass matrices for the solutions.

3.1. Mass Squared Matrix of Solution 3

In the space of H_u, H_d, S, $\widetilde{H}_u$, $\widetilde{H}_d$, and $\widetilde{S}$ this mass squared matrix is obtained from the second derivative of the scalar potential. For simplicity we impose the following conditions on the soft squared masses

$$m_{H_u}^2 = m_{H_d}^2 = m_{\widetilde{H}_u}^2 = m_{\widetilde{H}_d}^2 = m_H^2, \qquad m_S^2 = m_{\widetilde{S}}^2. \tag{3.1}$$

The *Higgs* mass matrix squared matrix for Solution 3 is given by

$$M^2 = \begin{pmatrix} a & b & 0 & 0 & 0 & 0 \\ b & a & 0 & 0 & 0 & 0 \\ 0 & 0 & e & 0 & 0 & 0 \\ 0 & 0 & 0 & a & b & 0 \\ 0 & 0 & 0 & b & a & 0 \\ 0 & 0 & 0 & 0 & 0 & e \end{pmatrix}, \tag{3.2}$$

where the elements of the matrix are

$$\begin{aligned} a &= 2\lambda^2\left(S_0^2 + m_H^2\right), \\ b &= -2\lambda^2 S_0^2, \\ e &= 2\lambda^2\left(\mu_0^2 + m_S^2\right). \end{aligned} \tag{3.3}$$

We find that the degenerate physical *Higgs* masses are

$$\xi_1 = \xi_2 = \xi_3 = \xi_4 = 2\lambda^2\left(2S_0^2 + m_H^2\right), \qquad \xi_5 = \xi_6 = 2\lambda^2\left(\mu_0^2 + m_S^2\right). \tag{3.4}$$

The *Higgs* contribution to the vacuum energy for this solution is

$$V_3(0) = 2\lambda^2 m_S^2 \frac{\mu_0^2 v^4 + m_S^2}{\left(\mu_0^2 + m_S^2\right)^2}. \tag{3.5}$$

The conditions for this solution to be a true supersymmetry breaking vacuum is

(i) the physical *Higgs* masses be all positive,

(ii) the vacuum energy be also positive.

These conditions are satisfied if

$$m_S^2 > 0, \qquad m_H^2 > \frac{-\mu_0^2 \upsilon^4}{\left(\mu_0^2 + m_S^2\right)^2}. \tag{3.6}$$

3.2. Mass Squared Matrix of Solution 4

The condition of (3.1) for the soft squared masses is implied by this solution. And the *Higgs* mass squared matrix for this case is

$$M^2 = \begin{pmatrix} a_1 & b_1 & c_1 & 0 & 0 & d_1 \\ b_1 & a_1 & c_1 & 0 & 0 & d_1 \\ c_1 & c_1 & e_1 & d_1 & d_1 & 0 \\ 0 & 0 & d_1 & a_1 & b_1 & c_1 \\ 0 & 0 & d_1 & b_1 & a_1 & c_1 \\ d_1 & d_1 & 0 & c_1 & c_1 & e_1 \end{pmatrix}, \tag{3.7}$$

where the elements of this matrix are given by

$$\begin{gathered} a_1 = 2\lambda^2\left(\upsilon_0^2 + S_0^2 + m_H^2\right) + \upsilon_0^2\left(g_1^2 + g_2^2\right), \\ b_1 = 2\lambda^2\left(\upsilon_0^2 - S_0^2\right) - \upsilon_0^2\left(g_1^2 + g_2^2\right), \\ c_1 = 4\lambda^2 \upsilon_0 S_0, \\ d_1 = 2\lambda^2 \mu_0 \upsilon_0, \\ e_1 = 2\lambda^2\left(2\upsilon_0^2 + \mu_0^2 + m_S^2\right). \end{gathered} \tag{3.8}$$

In a previous work we found the eigenvalues of this matrix [11]. The first two eigenvalues are

$$\eta_1 = \eta_2 = a_1 - b_1 = 2\lambda^2\left(2S_0^2 + m_H^2\right) + 2\upsilon_0^2\left(g_1^2 + g_2^2\right). \tag{3.9}$$

The third and the fourth eigenvalues satisfy

$$\eta^2 - (a_1 + b_1 + e_1)\eta + e_1(a_1 + b_1) - 2(c_1 - d_1)^2 = 0, \tag{3.10}$$

and finally the fifth and the sixth eigenvalues satisfy

$$\eta^2 - (a_1 + b_1 + e_1)\eta + e_1(a_1 + b_1) - 2(c_1 + d_1)^2 = 0. \tag{3.11}$$

If we require that the quantity

$$(a_1 + b_1 + e_1) = 2\lambda^2\left(4\upsilon_0^2 + \mu_0^2 + m_H^2 + m_S^2\right) \tag{3.12}$$

be positive, then the conditions for the last four eigenvalue to be positive are

$$\begin{aligned} e_1(a_1+b_1) &> 2(c_1-d_1)^2, \\ e_1(a_1+b_1) &> 2(c_1+d_1)^2. \end{aligned} \tag{3.13}$$

4. Metastable Aspects of the Model

To discuss transition from Solution 4 to Solution 1, we consider the equations of motions, for the symmetric solution they are

$$2S_0\upsilon_0^2+\mu_0\left(\upsilon_0^2-\upsilon^2+\mu_0 S_0\right)+m_S^2 S_0=0, \tag{4.1}$$

$$\upsilon_0^2-\upsilon^2+\mu_0 S_0+S_0^2+m_H^2=0. \tag{4.2}$$

The *Higgs* contribution to the vacuum energy for a symmetric solution is

$$V(0)=2\lambda^2\left[\left(\upsilon_0^2-\upsilon^2+\mu_0 S_0\right)^2+2\upsilon_0^2S_0^2+2m_H^2\upsilon_0^2+m_S^2S_0^2\right]. \tag{4.3}$$

From (4.1) the value of S_0 is

$$S_0=-\frac{\mu_0(\upsilon_0^2-\upsilon^2)}{A}, \quad \text{where } A=\mu_0^2+2\upsilon_0^2+m_S^2. \tag{4.4}$$

From (4.2) the value of m_H^2 is

$$m_H^2=-\left(\upsilon_0^2-\upsilon^2\right)+\frac{\mu_0^2(\upsilon_0^2-\upsilon^2)}{A}-\frac{\mu_0^2(\upsilon_0^2-\upsilon^2)^2}{A^2}. \tag{4.5}$$

Upon substituting the values of S_0 and m_H^2 the vacuum energy of Solution 4 is

$$V_4(0)=-\frac{2\lambda^2(\upsilon_0^2-\upsilon^2)}{A^2}\left[Bm_S^4+Cm_S^2+D\right], \tag{4.6}$$

where

$$B=\upsilon_0^2+\upsilon^2, \qquad C=\left(4\upsilon_0^2+\upsilon^2\right)B, \qquad D=4\upsilon_0^2\left(B+\mu_0^2\right). \tag{4.7}$$

But due to the positivity constraints of the *Higgs* mass matrix an exothermic phase transition ($\upsilon<\upsilon_0$) does not occur (see Appendix A for the proof).

For the case of an endothermic transition ($\upsilon>\upsilon_0$), the vacuum energy is positive if the values of the m_S^2 satisfy

$$m_S^2>\varsigma_1 \quad \text{or} \quad m_S^2<\varsigma_2, \tag{4.8}$$

where ς_1 and ς_2 are the roots of

$$B\varsigma^2 + C\varsigma + D = 0. \tag{4.9}$$

We note that $\varsigma_1 < 0$ and $\varsigma_2 < 0$. In Appendix B we show that the region $m_S^2 < \varsigma_2$ is ruled out by the positivity constraint of the *Higgs* mass squared matrix, there we also show that $m_H^2 > 0$.

As noted earlier the experimental value of EWSB requires $\upsilon_0 = 174\,\mathrm{GeV}$.

For the Higgs mass we consider $m_H = 125\,\mathrm{GeV}$ [22].

In units of υ_0 a typical solution with positive soft squared masses is

$$\upsilon_0^2 = 1, \qquad \upsilon^2 = 2, \qquad \mu_0 = 2, \qquad m_S^2 = 3.06, \qquad m_H^2 = 0.51. \tag{4.10}$$

Choosing a larger value for m_S^2 will result in a smaller value for m_H^2.

Another type of solution when $m_S^2 < 0$ is

$$\upsilon_0^2 = 1, \qquad \upsilon^2 = 2, \qquad \mu_0 = 1, \qquad m_S^2 = -0.22, \qquad m_H^2 = 0.51. \tag{4.11}$$

The special cases where one of the soft squared masses is zero is treated in Appendix C.

5. Conclusions

For simplicity we did not included phases in the *Higgs* sector. However by the inclusion of soft squared masses we showed that the model has a rich vacuum structure. For the symmetric solution we discussed the phase transition and we showed it was an endothermic transition. We also provided bounds on the soft squared masses. It will be interesting to consider other solutions of the model. Or consider the case where all the fields in the *Higgs* sector are complex. We plan to report on these issues in the future.

Appendices

A. The Case of an Exothermic Phase Transition

Here we show that an exothermic transition from the symmetric solution does not exist.

For the case of an exothermic transition ($\upsilon < \upsilon_0$), the vacuum energy is positive if the values of the m_S^2 satisfy

$$\varsigma_2 < m_S^2 < \varsigma_1, \tag{A.1}$$

and from (4.9)

$$\varsigma_1 = -2\upsilon_0^2 - \frac{\mu_0}{2}\left(1 - \sqrt{E}\right), \qquad \varsigma_2 = -2\upsilon_0^2 - \frac{\mu_0}{2}\left(1 + \sqrt{E}\right), \tag{A.2}$$

where

$$E = 1 - \frac{8\upsilon_0^2(\upsilon_0^2 - \upsilon^2)}{\mu_0^2(\upsilon_0^2 + \upsilon^2)}. \tag{A.3}$$

The physical *Higgs* masses are positive if

$$2\upsilon_0^2\left(2\upsilon_0^2 + m_S^2\right) + m_H^2\left(2\upsilon_0^2 + m_S^2 + \mu_0^2\right) > 8\upsilon_0^2\left(S_0^2 \pm \mu_0 S_0\right). \tag{A.4}$$

But utilizing (4.5)

$$m_H^2\left(2\upsilon_0^2 + m_S^2 + \mu_0^2\right) = -\left(\upsilon_0^2 - \upsilon^2\right)\left(2\upsilon_0^2 + m_S^2\right) + \mu_0 S_0 \upsilon_0^2\left(1 - \frac{\upsilon^2}{\upsilon_0^2}\right). \tag{A.5}$$

So the positivity requirement of the physical *Higgs* masses becomes

$$\left(\upsilon_0^2 + \upsilon^2\right)\left(2\upsilon_0^2 + m_S^2\right) + \mu_0 S_0 \upsilon_0^2\left(1 - \frac{\upsilon^2}{\upsilon_0^2}\right) > 8\upsilon_0^2\left(S_0^2 \pm \mu_0 S_0\right). \tag{A.6}$$

From the values of ς_1 and ς_2 we find that $2\upsilon_0^2 + m_S^2 < 0$. Therefore the condition stated in (A.4) cannot be satisfied for an exothermic transition.

B. The Case of an Endothermic Phase Transition

In this appendix first we show that for an endothermic transition ($\upsilon > \upsilon_0$) the region $m_S^2 < \varsigma_2$ is ruled out by the positivity constraints of the *Higgs* squared mass matrix. The form of the positivity condition is identical to that of (A.4). But in this region again $2\upsilon_0^2 + m_S^2 < 0$. Therefore the first term in the left hand side of this equation is negative. By using (4.5) it is easy to see that for an endothermic transition the second term in the left hand side is negative as well. But depending on the choice of the parameters of the model the right hand is always positive (either for plus sign or minus sign). Hence the values $m_S^2 < \varsigma_2$ are not acceptable.

Here we prove that the negative values of m_H^2 are not permitted in an endothermic transition. First we assume $m_H^2 < 0$ and then from the equations of motion we calculate m_S^2 and S_0 and finally we show that the positivity constraints for the *Higgs* mass squared matrix is violated. From (4.5) we obtain

$$m_H^2 = \frac{(\upsilon^2 - \upsilon_0^2)}{A^2}\left[\left(2\upsilon_0^2 + m_S^2\right)^2 + \mu_0^2\left(2\upsilon_0^2 + m_S^2\right) - \mu_0^2\left(\upsilon^2 - \upsilon_0^2\right)\right]. \tag{B.1}$$

So for an endothermic transition and assuming a negative value for m_H^2 we have

$$\left(2\upsilon_0^2 + m_S^2\right)^2 + \mu_0^2\left(2\upsilon_0^2 + m_S^2\right) - \mu_0^2\left(\upsilon^2 - \upsilon_0^2\right) < 0. \tag{B.2}$$

From this expression the allowed range of m_S^2 is

$$-\frac{\mu_0^2}{2}\left(1+\sqrt{\acute{E}}\right) < m_S^2 + 2v_0^2 < -\frac{\mu_0^2}{2}\left(1-\sqrt{\acute{E}}\right), \tag{B.3}$$

where

$$\acute{E} = 1 + \frac{4\left(v^2 - v_0^2\right)}{\mu_0^2}. \tag{B.4}$$

Combining this result with similar constraint from the positivity of *Higgs* vacuum energy we obtain

$$-\frac{\mu_0^2}{2}\left(1-\sqrt{E}\right) < m_S^2 + 2v_0^2 < -\frac{\mu_0^2}{2}\left(1-\sqrt{\acute{E}}\right), \tag{B.5}$$

as $\acute{E} > E$. For an endothermic transition the values of E and $\acute{E}$ are greater than unity and in the domain of (B.5) the quantity $m_S^2 + 2v_0^2$ has positive value.

Next we consider the case of $\mu_0 > 0$. From (4.4) we see that the value of S_0 is positive. So for negative value of m_H^2 and from the equation of motion we find

$$S_0 > \frac{\mu_0}{2}\left(\sqrt{\acute{E}} - 1\right). \tag{B.6}$$

Now if we substitute the upper bound of the quantity $m_S^2 + 2v_0^2$ in the physical Higgs mass constraint we get

$$\mu_0^2 v_0^2\left(\sqrt{\acute{E}} - 1\right) + \frac{\mu_0^2}{2} m_H^2\left(\sqrt{\acute{E}} + 1\right) > 8v_0^2\left(S_0^2 + \mu_0 S_0\right), \tag{B.7}$$

but the second term in the left hand side of (B.7) is negative, and upon substituting the value of S_0 in the right hand side of this expression we find that the physical *Higgs* mass constraint is violated. Similar result holds for $\mu_0 < 0$ case. So negative values of m_H^2 are not allowed.

C. Special Cases of Soft Squared Masses

In this appendix we study the model with vanishing $m_H^2 = 0$ or vanishing $m_S^2 = 0$.

When $m_H^2 = 0$ from equations of motions we find

$$2S_0 v_0^2 - \mu_0 S_0^2 + m_S^2 S_0 = 0, \tag{C.1}$$

with solutions

$$S_0 = 0, \qquad S_0 = \frac{2v_0^2 + m_S^2}{\mu_0}. \tag{C.2}$$

The *Higgs* vacuum energy in this situation is

$$V_4(0) = 2\lambda^2 S_0^2 \left(S_0^2 + 2\upsilon_0^2 + m_S^2\right). \tag{C.3}$$

We note that when $S_0 = 0$ we have $V_4(0) = 0$. The condition for the positivity of *Higgs* mass squared matrix is

$$2\upsilon_0^2 + m_S^2 > 0, \tag{C.4}$$

in previous we have shown that for an endothermic transition this condition is satisfied. Hence this solution corresponds to an exact *susy* with broken EWSB.

However we see that the solution $S_0 = (2\upsilon_0^2 + m_S^2)/\mu_0$ cannot satisfy the positivity of *Higgs* mass squared matrix for this case which is

$$2\upsilon_0^2 + m_S^2 > 4\left(S_0^2 \pm S_0\mu_0\right), \tag{C.5}$$

therefore this value of S_0 does not correspond to a true minimum.

When $m_S = 0$, the *Higgs* vacuum energy will be

$$V_4(0) = -\frac{8\lambda^2 (\upsilon_0^2 - \upsilon^2)\upsilon_0^4}{(2\upsilon_0^2 + \mu_0^2)^2}\left(\upsilon_0^2 + \upsilon^2 + \mu_0^2\right). \tag{C.6}$$

This energy is positive if $\upsilon_0 < \upsilon$. Therefore this case corresponds to an endothermic transition.

Again we consider $m_H = 125\,\text{GeV}$. An acceptable configuration for in this case is

$$\upsilon_0^2 = 1, \qquad \upsilon^2 = 2, \qquad \mu_0 = 1.13, \qquad m_S^2 = 0.0, \qquad m_H^2 = 0.51. \tag{C.7}$$

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