

Research Article

CP Violating Tri-Bimaximal-Cabibbo Mixing

D. V. Ahluwalia^{1,2}

¹ *Institute of Mathematics, Statistics and Scientific Computation, IMECC-UNICAMP CP, 6065, 13083-859 Campinas, SP, Brazil*

² *Department of Physics and Astronomy, Rutherford Building, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand*

Correspondence should be addressed to D. V. Ahluwalia, dvahluwalia@ime.unicamp.br

Received 8 August 2012; Accepted 3 September 2012

Academic Editors: K. Cho, S. I. Eidelman, H. Hayashii, and W. Li

Copyright © 2012 D. V. Ahluwalia. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In view of the new data from the Daya Bay and RENO collaborations, King has presented a very natural deformation of tri-bimaximal mixing. Here, we show that L/E flatness of the e -like event ratio in the atmospheric neutrino data, when coupled with King's observation that the smallest neutrino mixing angle, θ_{13} , seems to be related to the largest quark mixing angle (the Cabibbo angle θ_C), leading to a CP violating tri-bimaximal-Cabibbo mixing. King's tri-bimaximal-Cabibbo mixing follows as a leading order approximation from our result.

The precise form of the neutrino mixing matrix, U , that defines the relationship between the flavour and mass eigenstates, $|\nu_\ell\rangle$ and $|\nu_j\rangle$, respectively [1, 2], reads

$$|\nu_\ell\rangle = \sum_j U_{\ell j}^* |\nu_j\rangle, \quad \ell = e, \mu, \tau, \quad j = 1, 2, 3, \quad (1)$$

and the knowledge of the masses for the underlying mass eigenstates arises from yet unknown physics. Nevertheless, once the parameters that determine the mixing matrix and the mass-squared differences are deciphered from the data, one can derive their phenomenological consequences on supernova explosions [3–6], on the synthesis of elements [7], on the cosmic microwave background, and, the distribution of large-scale structure [8]. In particular, if the neutrino mixing angle $\theta_{13} \neq 0$, then one can obtain CP violation in the neutrino sector with many interesting physical consequences [9–11].

The T2K, MINOS, and Double Chooz indications that the smallest neutrino mixing angle θ_{13} may be nonzero [12–14] has now been confirmed by the results of the Daya Bay and RENO collaborations [15, 16]. King has made the observation [17] that the smallest neutrino mixing angle θ_{13} seems to be related to the largest quark mixing angle, the Cabibbo

angle θ_C [18], or equivalently to the Wolfenstein parameter, $\lambda = 0.2253 \pm 0.0007$ [2, 19]: (It is worth noting that Mohapatra and Smirnov had earlier conjectured King's observation [20, Section 3.1].)

$$\theta_{13}(\text{or}, \theta_{\text{reac}}) = \arcsin\left(\frac{\sin \theta_C}{\sqrt{2}}\right) = \arcsin\left(\frac{\lambda}{\sqrt{2}}\right). \quad (2)$$

To this observation, we now add that the L/E , where L is the neutrino source-detector distance and E is the neutrino energy, flatness of the e -like event ratio observed for atmospheric neutrinos [21] requires that

$$\theta_{23}(\text{or}, \theta_{\text{atm}}) = \frac{\pi}{4}, \quad \delta = \pm \frac{\pi}{2}. \quad (3)$$

This observation was first made in reference [22]. The value of δ obtained in [22] was also introduced recently as an Ansatz in [23].

Global analysis of neutrino oscillation data by two independent groups shows: (a) δ to be $(0.83_{-0.64}^{+0.54})\pi$ for the normal mass hierarchy while allowing for the full $[0, 2\pi]$ range for the inverted mass hierarchy [24], (b) $\delta \approx \pi$ with no significant difference between the normal and inverted mass hierarchies [25]. A detailed study of these two papers reveals that there is no statistically significant indication which disfavours $\delta = \pm\pi/2$. Regarding θ_{23} : (a) the first of the mentioned groups obtains $\sin^2\theta_{23} = 0.49_{-0.05}^{+0.08}$ for the normal mass hierarchy and $\sin^2\theta_{23} = 0.53_{-0.07}^{+0.05}$ for the inverted mass hierarchy (these values are consistent with $\theta_{23} = \pi/4$), while (b) the second group finds a slight preference for $\theta_{23} < \pi/4$.

Both groups agree with the tri-bimaximal mixing value for the remaining angle [24, 25]

$$\theta_{12}(\text{or}, \theta_{\odot}) = \arcsin\left(\frac{1}{\sqrt{3}}\right). \quad (4)$$

With all the angles and phases thus fixed, the neutrino mixing matrix for the choice $\delta = \pi/2$ in (3) takes the form

$$U^+ = \begin{pmatrix} \sqrt{\frac{2}{3}}\left(1 - \frac{\lambda^2}{2}\right)^{1/2} & \sqrt{\frac{1}{3}}\left(1 - \frac{\lambda^2}{2}\right)^{1/2} & i\frac{1}{\sqrt{2}}\lambda \\ -\frac{1}{\sqrt{6}}(1 - i\lambda) & \frac{1}{\sqrt{3}}\left(1 + i\frac{1}{2}\lambda\right) & \frac{1}{\sqrt{2}}\left(1 - \frac{\lambda^2}{2}\right)^{1/2} \\ \frac{1}{\sqrt{6}}(1 + i\lambda) & -\frac{1}{\sqrt{3}}\left(1 - i\frac{1}{2}\lambda\right) & \frac{1}{\sqrt{2}}\left(1 - \frac{\lambda^2}{2}\right)^{1/2} \end{pmatrix}. \quad (5)$$

Its counterpart, U^- , for $\delta = -\pi/2$ is obtained by letting $i \rightarrow -i$ in U^+ . As a measure of CP violation, following [2], we define the asymmetries

$$A_{\text{CP}}^{(\ell'\ell)} := P(\nu_{\ell} \rightarrow \nu_{\ell'}) - P(\bar{\nu}_{\ell} \rightarrow \bar{\nu}_{\ell'}) \quad (6)$$

and find

$$\begin{aligned}
A_{\text{CP}}^{(\mu e)} &= -A_{\text{CP}}^{(\tau e)} = A_{\text{CP}}^{(\tau \mu)} \\
&= \mp \frac{1}{3} \lambda (2 - \lambda^2) \left(\sin \frac{\Delta m_{32}^2}{2p} L + \sin \frac{\Delta m_{21}^2}{2p} L + \sin \frac{\Delta m_{13}^2}{2p} L \right) \\
&\approx \mp 0.146 \left(\sin \frac{\Delta m_{32}^2}{2p} L + \sin \frac{\Delta m_{21}^2}{2p} L + \sin \frac{\Delta m_{13}^2}{2p} L \right),
\end{aligned} \tag{7}$$

where all symbols have their usual meaning. The \mp sign holds for $\delta = \pm \pi/2$. For $\lambda = 0$, or equivalently $\theta_{13} = 0$, the U^\pm matrix reduces to the standard tri-bimaximal mixing matrix [26]. (This may be compared with [27, equation (26)] that gives an interpolating matrix with θ_\odot as a variable. In one limit the interpolating matrix gives the bimaximal mixing [28–30] and in another it yields tri-bimaximal mixing [26].)

The result (7) is modified by matter effects [31, 32]. Its general features are studied in detail by various authors [11, 33–35]. In gravitational environment, the following argument suggests that one must expect a significant modification to the result (7). Neutrino oscillations provide us with a set of flavour oscillation clocks. These clocks must redshift according to the general expectations of the theory of general relativity. In gravitational environments of neutron stars, the dimensionless gravitational potential is $\Phi_{\text{grav}}^{\text{NS}} \approx 0.2$ (cf. for Earth, $\Phi_{\text{grav}}^\oplus \approx 6.95 \times 10^{-10}$). For a given source-detector distance and a given energy, the asymmetries A_{CP} for supernovae modeling must be accordingly modified [36–41] at the 20% level, or thereabouts.

An examination of the U^\pm matrix immediately shows that the expectation values of the ν_μ and ν_τ masses are identical. To $\mathcal{O}(\lambda^2)$ the U^- obtained above reproduces King's result [17, equation (8)] for $\delta = \pi/2$. The presented U^\pm matrix not only accommodates the implications of the Daya Bay and RENO collaborations, but also the L/E flatness of the e -like event ratio seen in the atmospheric neutrino data while respecting all other known data on neutrino oscillations.

Acknowledgments

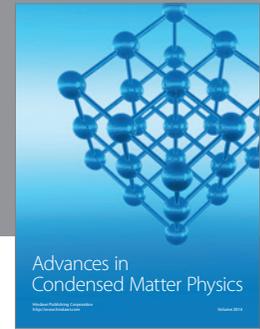
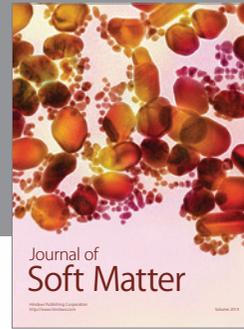
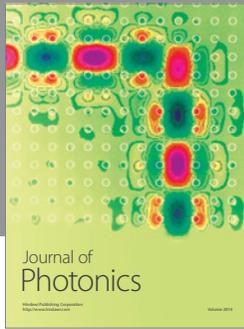
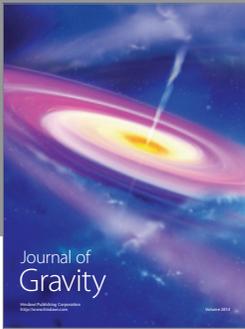
The result presented here was obtained on 10 May, 2012 and was presented the next day at a MatScience Seminar. The author would like to thank the Institute of Mathematical Sciences (MatScience, Chennai, India) for its hospitality and for its vibrant scholarly environment. The author is equally thankful to the Institute of Mathematics, Statistics and Scientific Computation (IMECC, Unicamp, Brazil) where the work was written during a sabbatical year.

References

- [1] L. L. Chau and W. Y. Keung, "Comments on the parametrization of the Kobayashi-Maskawa matrix," *Physical Review Letters*, vol. 53, no. 19, pp. 1802–1805, 1984.
- [2] J. Beringer, J.-F. Arguin, R. M. Barnett et al., "Review of particle physics," *Physical Review D*, vol. 86, no. 1, Article ID 010001, 2012.
- [3] D. V. Ahluwalia-Khalilova, "Erratum: first prize essay for 1996: Neutrino oscillations and supernovae (General Relativity and Gravitation (1996) 28 (1161))," *General Relativity and Gravitation*, vol. 36, no. 9, pp. 2183–2187, 2004.

- [4] C. Lunardini, B. Müller, and H.-Th. Janka, "Neutrino oscillation signatures of oxygen-neon-magnesium supernovae," *Physical Review D*, vol. 78, no. 2, Article ID 023016, 2008.
- [5] H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian, "Simulation of coherent nonlinear neutrino flavor transformation in the supernova environment: correlated neutrino trajectories," *Physical Review D*, vol. 74, no. 10, Article ID 105014, 2006.
- [6] H. Duan, G. M. Fuller, J. Carlson, and Y. Z. Qian, "Flavor evolution of the neutronization neutrino burst from an O-Ne-Mg core-collapse supernova," *Physical Review Letters*, vol. 100, no. 2, Article ID 021101, 2008.
- [7] T. Yoshida, T. Kajino, H. Yokomakura, K. Kimura, A. Takamura, and D. H. Hartmann, "Neutrino oscillation effects on supernova light-element synthesis," *Astrophysical Journal*, vol. 649, no. 1 I, pp. 319–331, 2006.
- [8] J. Lesgourgues and S. Pastor, "Massive neutrinos and cosmology," *Physics Reports*, vol. 429, no. 6, pp. 307–379, 2006.
- [9] M. Y. Khlopov and S. T. Petcov, "Possible cosmological effect of CP-violation in neutrino oscillations," *Physics Letters B*, vol. 99, no. 2, pp. 117–121, 1981.
- [10] P. H. Frampton, S. L. Glashow, and T. Yanagida, "Cosmological sign of neutrino CP violation," *Physics Letters*, vol. 548, no. 3-4, pp. 119–121, 2002.
- [11] A. B. Balantekin, J. Gava, and C. Volpe, "Possible CP-violation effects in core-collapse supernovae," *Physics Letters B*, vol. 662, no. 5, pp. 396–404, 2008.
- [12] K. Abe, Y. Hayato, T. Iida et al., "Search for differences in oscillation parameters for atmospheric neutrinos and antineutrinos at Super-Kamiokande," *Physical Review Letters*, vol. 107, no. 24, Article ID 241801, 2011.
- [13] P. Adamson, D. J. Auty, D. S. Ayres et al., "Improved search for muon-neutrino to electron-neutrino oscillations in MINOS," *Physical Review Letters*, vol. 107, no. 18, Article ID 181802, 2011.
- [14] Y. Abe, C. Aberle, T. Akiri et al., "Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment," *Physical Review Letters*, vol. 108, no. 13, Article ID 131801, 2012.
- [15] F. P. An, J. Z. Bai, A. B. Balantekin et al., "Observation of electron-antineutrino disappearance at Daya Bay," *Physical Review Letters*, vol. 108, no. 17, Article ID 171803, 2012.
- [16] J. K. Ahn, S. Chebotaryov, J. H. Choi et al., "Observation of reactor electron antineutrinos disappearance in the RENO experiment," *Physical Review Letters*, vol. 108, no. 19, Article ID 191802, 2012.
- [17] S. King, "Tri-bimaximal-cabibbo mixing," <http://arxiv.org/abs/1205.0506>.
- [18] N. Cabibbo, "Unitary symmetry and leptonic decays," *Physical Review Letters*, vol. 10, no. 12, pp. 531–533, 1963.
- [19] L. Wolfenstein, "Parametrization of the Kobayashi-Maskawa matrix," *Physical Review Letters*, vol. 51, no. 21, pp. 1945–1947, 1983.
- [20] R. N. Mohapatra and A. Y. Smirnov, "Neutrino mass and new physics," *Annual Review of Nuclear and Particle Science*, vol. 56, pp. 569–628, 2006.
- [21] Y. Fukuda, T. Hayakawa, E. Ichihara et al., "Evidence for oscillation of atmospheric neutrinos," *Physical Review Letters*, vol. 81, no. 8, pp. 1562–1567, 1998.
- [22] D. V. Ahluwalia, Y. Liu, and I. Stancu, "CP-violation in neutrino oscillations and L/E flatness of the E-like event ratio at Super-Kamiokande," *Modern Physics Letters A*, vol. 17, no. 1, pp. 13–21, 2002.
- [23] X. Zhang and B.-Q. Ma, "A prediction of neutrino mixing matrix with CP violating phase," *Physics Letters B*, vol. 713, no. 3, pp. 202–205, 2012.
- [24] M. Tortola, J. Valle, and D. Vanegas, "Global status of neutrino oscillation parameters after recent reactor measurements," <http://arxiv.org/abs/1205.4018>.
- [25] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. M. Rotunno, "Global analysis of neutrino masses, mixings, and phases: entering the era of leptonic CP violation searches," *Physical Review D*, vol. 86, no. 1, Article ID 013012, 2012.
- [26] P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Physics Letters B*, vol. 530, no. 1-4, pp. 167–173, 2002.
- [27] I. Stancu and D. V. Ahluwalia, "L/E-flatness of the electron-like event ratio, in Super-Kamiokande and a degeneracy, in neutrino masses," *Physics Letters B*, vol. 460, no. 3-4, pp. 431–436, 1999.
- [28] F. Vissani, "A Study of the scenario with nearly degenerate Majorana neutrinos," <http://arxiv.org/abs/hep-ph/9708483>.
- [29] D. V. Ahluwalia, "On reconciling atmospheric, LSND, and solar neutrino-oscillation data," *Modern Physics Letters A*, vol. 13, no. 28, pp. 2249–2264, 1998.
- [30] V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, "Bi-maximal mixing of three neutrinos," *Physics Letters B*, vol. 437, no. 1-2, pp. 107–116, 1998.

- [31] L. Wolfenstein, "Neutrino oscillations in matter," *Physical Review D*, vol. 17, no. 9, pp. 2369–2374, 1978.
- [32] S. Mikheev and A. Y. Smirnov, "Resonance amplification of oscillations in matter and spectroscopy of solar neutrinos," *Soviet Journal of Nuclear Physics*, vol. 42, pp. 913–917, 1985.
- [33] J. Gava and C. Volpe, "Collective neutrino oscillations in matter and CP violation," *Physical Review D*, vol. 78, no. 8, Article ID 083007, 2008.
- [34] J. P. Kneller and G. C. McLaughlin, "Three flavor neutrino oscillations in matter: flavor diagonal potentials, the adiabatic basis, and the CP phase," *Physical Review D*, vol. 80, no. 5, Article ID 053002, 2009.
- [35] L. S. Kisslinger, E. M. Henley, and M. B. Johnson, "Neutrino oscillation in matter and parameters s_{13} , δ_{cp} ," *International Journal of Modern Physics E*, vol. 21, no. 7, Article ID 1250065, 2012.
- [36] D. V. Ahluwalia and C. Burgard, "Gravitationally induced neutrino-oscillation phases," *General Relativity and Gravitation*, vol. 28, no. 10, pp. 1161–1170, 1996.
- [37] D. V. Ahluwalia and C. Burgard, "Interplay of gravitation and linear superposition of different mass eigenstates," *Physical Review D*, vol. 57, no. 8, pp. 4724–4727, 1998.
- [38] K. Konno and M. Kasai, "General relativistic effects of gravity in quantum mechanics: a case of ultra-relativistic, spin 1/2 particles," *Progress of Theoretical Physics*, vol. 100, no. 6, pp. 1145–1157, 1998.
- [39] J. Wudka, "Mass dependence of the gravitationally induced wave-function phase," *Physical Review D*, vol. 64, no. 6, Article ID 065009, 2001.
- [40] B. Mukhopadhyay, "Neutrino asymmetry around black holes: neutrinos interact with gravity," *Modern Physics Letters A*, vol. 20, no. 28, pp. 2145–2155, 2005.
- [41] P. Singh and B. Mukhopadhyay, "Gravitationally induced neutrino asymmetry," *Modern Physics Letters A*, vol. 18, no. 11, pp. 779–785, 2003.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

