# Mismatch Loss Reduction in Photovoltaic Arrays as a Result of Sorting Photovoltaic Modules by Max-Power Parameters 

J. Webber ${ }^{1}$ and E. Riley ${ }^{2}$<br>${ }^{1}$ European Solar Engineering School, 78170 Borlänge, Sweden<br>${ }^{2}$ Black \& Veatch Energy, San Francisco, CA 94111, USA<br>Correspondence should be addressed to J. Webber; mr.jeffrey.webber@gmail.com

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#### Abstract

Variations in photovoltaic (PV) module current-voltage curves result in a power loss in PV arrays often referred to as mismatch loss (MML). As a means of reducing MML, newly fabricated PV modules are sorted to meet a set tolerance for variation in overall maximum power output with respect to a given module's rated power. Starting with flash test data sets for two different polycrystalline PV modules and a simulated sorting procedure, Monte Carlo techniques were used to generate a large number of artificial PV arrays. The MMLs for each of these arrays were then calculated to assess the sorting procedure's ability to reduce MML. Overall MMLs were quite small ( $0.001-0.01 \%$ ). Sorting by $I_{\mathrm{mp}}$ resulted in the most consistent MML reductions. Sorting by $V_{\mathrm{mp}}$ yielded insignificant results. Sorting by $P_{\mathrm{mp}}$ yielded significant MML reduction in only one of the two PV module data sets. Analysis was conducted to quantify if additional sorting on top of what both manufacturers had already done would make economic sense. Based on high level economic analysis, it appears that additional sorting yields little economic gain; however, this is highly dependent upon manufacturer sorting cost.


## 1. Introduction

Differences in the current-voltage characteristics of photovoltaic (PV) modules give rise to a type of power loss referred to as "electrical mismatch" once modules are connected to networks of series and parallel strings called arrays. The consequence of "mismatch loss" (MML) is that the total power output of a PV array will be less than the sum of the power outputs of the modules as if they were acting independently [1].

This phenomenon has been investigated by a number of authors [1-5] primarily by using one of two methods: (1) the comparison between a PV array's ideal max-power and the actual max-power computed by progressively synthesizing the $I-V$ curves of modules, series strings, and finally the complete PV array; and (2) MML estimates of PV arrays composed of modules with known or statistically generated $I-V$ characteristics. This second method was made possible by an equation developed by Bucciarelli [1] that estimates MML in PV arrays composed of modules with relatively small
variations in their $I-V$ characteristics. Bucciarelli's [1] model was found to adequately estimate MMLs in 100 kWp arrays by Iannone et al. [2], combining and expanding on the work of Bishop [3] who used random numbers to generate PV modules arranged into arrays, and Chamberlin et al. [4], who estimated small MML values in randomly arranged arrays of four 48 Wp PV modules. After a brief outline of some of the causative mechanisms of MML in PV arrays, Kaushika and Rai [5] used Bucciarelli's [1] model to explore the consequences of aging on MML values, showing extremely small MMLs ( $<0.01 \%$ ) and relatively large MMLs ( $>10 \%$ ) in PV arrays composed of newly fabricated cells and aging cells, respectively.

A common practice employed by manufacturers is to sort newly fabricated PV modules prior to installation as a means of reducing MMLs in assembled PV arrays. Previous studies have focused primarily on predicting and estimating MML in existing or simulated PV arrays; however very little research exists on the effectiveness of sorting as a means of MML reduction. The purpose of this paper is to quantify to what
degree manufacturer sorting has upon MML reduction and whether it remains a good economic decision.

This investigation was carried out in three steps. First, a simulated sorting procedure was applied to a population of real PV modules. Sorting methods included sorting modules by reducing variances in a population of modules: maximum power current, maximum power voltage, and maximum power. Second, Monte Carlo techniques were used to randomly select and arrange the sorted PV modules into PV arrays. Bucciarelli's [1] method was then used to calculate the MML in each randomly generated PV array, producing a MML distribution over a series of simulations. This was systematically repeated over ever stricter sorting procedures, the results of which showcase the effect of sorting upon MML reduction. Finally, the impacts of these results were discussed by means of a high level economic analysis.

## 2. Mismatch Loss Estimation Model

2.1. Current-Voltage Relationship in a PV Cell. In terms of light generated current, the $I-V$ curve of a PV cell can be expressed as [5]

$$
\begin{equation*}
I^{\prime}=I_{\mathrm{ph}}-I_{o}\left(e^{\left[E\left(V^{\prime}+I^{\prime} R_{s}\right) / k A T_{c}\right]}-1\right)-\frac{V^{\prime}+I^{\prime} R_{s}}{R_{\mathrm{sh}}} \tag{1}
\end{equation*}
$$

Dividing (1) by the average max-power current $\bar{I}_{\mathrm{mp}}$, introducing the average max-power voltage $\bar{V}_{\mathrm{mp}}$ as a unity fraction, and setting

$$
\begin{gather*}
V=V^{\prime}+I^{\prime} R_{s}  \tag{2}\\
I=I^{\prime}+\frac{V}{R_{\mathrm{sh}}}  \tag{3}\\
\frac{I}{\bar{I}_{\mathrm{mp}}}=\frac{\left(I_{\mathrm{ph}}+I_{o}\right)}{\bar{I}_{\mathrm{mp}}}-\frac{I_{o}}{\bar{I}_{\mathrm{mp}}} e^{\left[\left(E V / k A T_{c}\right) *\left(\bar{V}_{\mathrm{mp}} / \bar{V}_{\mathrm{mp}}\right)\right]} . \tag{4}
\end{gather*}
$$

Substituting (4a), (4b), and (4c) into (4) results in (5):

$$
\begin{gather*}
\alpha=I_{\mathrm{ph}}+\frac{I_{\mathrm{ph}}}{\bar{I}_{\mathrm{mp}}}  \tag{4a}\\
\beta=\frac{I_{o}}{\bar{I}_{\mathrm{mp}}}  \tag{4b}\\
C=\frac{e \bar{V}_{\mathrm{mp}}}{k A T_{c}}  \tag{4c}\\
\frac{I}{\bar{I}_{\mathrm{mp}}}=\alpha-\beta e^{C\left(V / \bar{V}_{\mathrm{mp}}\right)} \tag{5}
\end{gather*}
$$

This derivation is similar in method to Kaushika and Rai [5], resulting in an expression identical to that of Bucciarelli [1]. Equation (5) is assumed to adequately represent the $I-V$ curve of a photovoltaic cell (or network of cells) near their maxpower point $[1,2,5]$. $C$ is commonly referred to as the cell


Figure 1: Cell characteristic factor versus fill factor.
characteristic factor and can be expressed in terms of the fill factor (FF) defined as [1]

$$
\begin{equation*}
\mathrm{FF}=\frac{I_{\mathrm{mp}} V_{\mathrm{mp}}}{I_{\mathrm{sc}} V_{\mathrm{oc}}}=\frac{C^{2}}{(1+C)[C+\ln (1+C)]} \tag{6}
\end{equation*}
$$

The relationship between the fill factor and $C$ can be seen in Figure 1.

This study takes into consideration the placement of individual modules within a PV array. To account for this, we introduce the modified cell characteristic factor $C^{\prime}$, which is based upon the average fill factor of only the PV modules used within the assembled PV array (as opposed to the entire available population):

$$
\begin{equation*}
\overline{\mathrm{FF}}=\frac{\left(C^{\prime}\right)^{2}}{\left(1+C^{\prime}\right)\left[C^{\prime}+\ln \left(1+C^{\prime}\right)\right]} \tag{7}
\end{equation*}
$$

2.2. Power Loss due to Mismatch. From Bucciarelli [1],T total PV modules arranged into a network of $M$ parallel series strings of $L$ modules; each has a mismatch loss $\Delta P$ estimated by

$$
\begin{equation*}
\Delta P=\frac{\left(C^{\prime}+2\right)}{2}\left[\sigma_{\eta}^{2}\left(1-\frac{1}{L}\right)+\frac{\sigma_{\xi}^{2}}{L}\left(1-\frac{1}{M}\right)\right] \tag{8}
\end{equation*}
$$

where $\sigma_{\eta}^{2}$ and $\sigma_{\xi}^{2}$ represent the variance of max-power current and voltage, respectively, in our network of PV cells. Figure 2 shows a typical $L \times M$ network of modules:

$$
\begin{gather*}
\sigma_{\eta}^{2}=\left(\frac{\sigma_{I_{\mathrm{mp}}}}{\bar{I}_{\mathrm{mp}}}\right)^{2}, \\
\sigma_{\xi}^{2}=\left(\frac{\sigma_{V_{\mathrm{mp}}}}{\bar{V}_{\mathrm{mp}}}\right)^{2} . \tag{9}
\end{gather*}
$$

Expressions (9) are derived on the assumption that variations in max-power current and max-power voltage in the $s$ th PV cell in our network take the nondimensional form [1]:

$$
\begin{gather*}
\frac{I_{\mathrm{mp}, s}}{\bar{I}_{\mathrm{mp}}}=1+\epsilon_{\eta} \eta_{s} \\
\epsilon_{\eta}=I_{\mathrm{mp}, \mathrm{MAX}}-I_{\mathrm{mp}, \mathrm{MIN}}  \tag{10}\\
\frac{V_{\mathrm{mp}, \mathrm{~s}}}{\bar{V}_{\mathrm{mp}}}=1+\epsilon_{\xi} \xi_{s} \\
\epsilon_{\xi}=V_{\mathrm{mp}, \mathrm{MAX}}-V_{\mathrm{mp}, \mathrm{MIN}} .
\end{gather*}
$$

$\eta_{s}$ and $\xi_{s}$ are respective measurements of variations in maxpower current and max-power voltage and take on values between -1 and 1. $\epsilon_{\eta}$ and $\epsilon_{\xi}$ are small numbers, respectively, measuring the percentage range of variation in $I_{\mathrm{mp}}$ and $V_{\mathrm{mp}}$. In order for (8) to yield acceptably accurate results, the product of $\epsilon$ and $C^{\prime}$, known as the weighted variation of max-power $\epsilon C^{\prime}$, must be less than 1 [1]. In practice, this limits the standard deviations of max-power parameters in a population of PV modules to within a few percentage points of their mean values [5]. It is also assumed that max-power current and voltage in each PV module are uncorrelated; specifically that the Pearson's Correlation between $I_{\mathrm{mp}}$ and $V_{\mathrm{mp}}$ is small with respect to $\pm 1$ [1].

In prior MML studies, the term $\sigma_{\eta}$ is calculated based upon the entire sample population of modules used in a given array without taking into consideration module placement. This neglects the effect that module placement has on series MML in that different configurations of the same modules within the same array will yield significantly different series MML values. In order to account for module placement, a unique $\sigma_{\eta}, C^{\prime}$, and series MML value are calculated for each string based solely upon the modules within that string. These MML values are then given a weight $\omega$ based upon the ideal power output $P_{\text {ideal }}$ of that string relative to the average ideal power output of every string in the array. The final series MML is taken as the average of the weighted series MML of each string in the array. Thus, for an array of $M$ strings, the series MML term in (8) becomes

$$
\begin{gather*}
\Delta P_{\text {series }}=\frac{1}{M} \sum_{q=1}^{M} \omega_{q} \Delta P_{q} \\
\Delta P_{q}=\frac{\left(C_{q}^{\prime}+2\right)}{2} \sigma_{\eta, q}^{2}\left(1-\frac{1}{L}\right),  \tag{11}\\
\omega_{q}=\frac{P_{\text {ideal }, q}}{\bar{P}_{\text {ideal }}}
\end{gather*}
$$

where $q$ refers to the string in question. The final, modified MML equation is found to be

$$
\begin{align*}
\Delta P= & \left(1-\frac{1}{L}\right) \frac{1}{M}\left[\sum_{q=1}^{M} \omega_{q} \frac{\left(C_{q}^{\prime}+2\right)}{2} \sigma_{\eta, q}^{2}\right]  \tag{12}\\
& +\frac{\left(C^{\prime}+2\right)}{2} \frac{\sigma_{\xi}^{2}}{L}\left[1-\frac{1}{M}\right] .
\end{align*}
$$



Figure 2: Network of $L$ modules in series and $M$ modules in parallel.

## 3. Sorting Procedure and PV Array Simulation

Most PV manufacturers sort their modules by power output at $1,000 \mathrm{~W} / \mathrm{m}^{2}$ and 25 degree Celcius cell temperature and Atmospheric Mass of 1.5, guaranteeing some small amount of variation from their rated power. Current industry tolerances between the modules nameplate rating and measured power rating are $\pm 3-5 \%$.

Sorting for the purpose of this paper was conducted by imposing a maximum allowed deviation from the mean for each max-power parameter: $P_{\mathrm{mp}}, I_{\mathrm{mp}}$, and $V_{\mathrm{mp}}$ on the entire population of modules. This maximum deviation was imposed by removing outlier modules and thus reducing the variance of max-power current $\sigma_{\eta}^{2}$ and maximum power voltage $\sigma_{\xi}^{2}$ within the remaining population of PV modules. This maximum allowed deviation from the mean was then reduced to examine the sensitivity of MML specific to each sorting parameter and maximum allowed deviation.

Monte Carlo techniques for randomly selecting and arranging available remaining PV modules into arrays were carried out using Microsoft Excel's random number generator, which has been shown to be suitable for generating small batches of random numbers [6, 7]. After specifying PV array dimensions ( $L$ in series, $M$ in parallel), the required number of modules was selected from sorted groups and placed at random into the array. Finally, a MML value was calculated using (12).
3.1. Model Verification. This model was validated by reevaluating a problem treated by Bucciarelli [1] for estimating MML in a single series string of PV cells with no $V_{\mathrm{mp}}$ variation and an $I_{\mathrm{mp}}$ variation defined by a Gaussian probability density function. Bucciarelli [1] assigns the following values:

$$
\begin{aligned}
& \bar{I}_{\mathrm{mp}}=39.2 \mathrm{~mA}, \\
& \sigma_{I_{\mathrm{mp}}}=2.86 \mathrm{~mA},
\end{aligned}
$$

$$
\mathrm{FF}=0.67
$$

The $1 / L$ term is dropped with respect to 1 , and (8) results in $\Delta P=2.35 \%$.

Starting with the same parameters, a Gaussian probability density function was used to generate $I_{\mathrm{mp}}$ 's for a distribution of 204 PV cells with $\bar{I}_{\mathrm{mp}}$ and $\sigma_{I_{\mathrm{mp}}}$ approximately equal to those listed above. The PV module $I_{\mathrm{mp}}$ distribution can be

Table 1: Statistical measures for the Helios Solar Works 250 Wp module data set.

| Helios Solar Works mono-Cx 250 Wp |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $I_{\text {sc }}(\mathrm{A})$ | $V_{\text {oc }}(\mathrm{V})$ | $V_{\text {mp }}(\mathrm{V})$ | $I_{\text {mp }}$ (I) | $P_{\text {mp }}$ (W) | FF |
| Number of modules | 2,132 | Avg | 8.85 | 37.6 | 30.5 | 8.27 | 252.3 | 0.758 |
| $\varrho\left(I_{\mathrm{mp}}, V_{\mathrm{mp}}\right)$ | -0.35 | Max | 9.01 | 38.5 | 31.1 | 8.46 | 260.4 | 0.772 |
| $\epsilon C^{\prime}\left(V_{\mathrm{mp}}\right)$ | 0.51 | Min | 8.59 | 37.0 | 29.8 | 8.05 | 247.0 | 0.741 |
| $\epsilon C^{\prime}\left(I_{\mathrm{mp}}\right)$ | 0.58 | $\sigma$ | 0.083 | 0.21 | 0.24 | 0.076 | 2.47 | 0.0055 |

Table 2: Statistical measures for the Anonymous Module Manufacturer 285 Wp module data set.

|  |  | ymous | dule M | acturer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $I_{\text {sc }}(\mathrm{A})$ | $V_{\text {oc }}(\mathrm{V})$ | $V_{\text {mp }}(\mathrm{V})$ | $I_{\text {mp }}$ (I) | $P_{\text {mp }}$ (W) | FF |
| Number of modules | 3,850 | Avg | 8.54 | 44.8 | 35.7 | 8.03 | 286.9 | 0.750 |
| $\varrho\left(I_{\text {mp }}, V_{\text {mp }}\right)$ | -0.14 | Max | 8.75 | 45.3 | 36.2 | 8.14 | 290.1 | 0.759 |
| $\epsilon C^{\prime}\left(V_{\mathrm{mp}}\right)$ | 0.30 | Min | 8.44 | 44.5 | 35.2 | 7.93 | 284.8 | 0.732 |
| $\epsilon C^{\prime}\left(I_{\text {mp }}\right)$ | 0.29 | $\sigma$ | 0.041 | 0.13 | 0.15 | 0.030 | 1.51 | 0.0037 |



Figure 3: $I_{\mathrm{mp}}$ frequency distribution for generated data set used to verify the MML model. A Gaussian pdf was used to create the distribution based upon the example carried out originally in Bucciarelli [1].
seen in Figure 3. 10,000 simulations were run calculating the MML in a series string composed of 100 randomly selected modules (so as to neglect the $1 / L$ term with respect to 1 ). As seen in Figure 4, the average value of the resulting series MML distribution is nearly identical to that found by Bucciarelli [1], only differing by $0.01 \%$.

## 4. Data

Flash test data was provided by an anonymous module manufacturer and Helios Solar Works. The anonymous module manufacturer provided flash test data for $3,850285 \mathrm{Wp}$ modules, the details of which are shown in Figures 5(a), 5(b), and 5(c). Helios Solar Works is a Milwaukee, WI-based manufacturer of mono-Cx PV modules and provided flash test data for $2,132250 \mathrm{Wp}$ PV modules upon request, the details of which are shown in Figures 6(a), 6(b), and 6(c).

As stated in Section 2, it is a requirement that the weighted variation of max-power $\epsilon C^{\prime}$ is less than 1 and the crosscorrelation $\varrho$ for $I_{\mathrm{mp}}$ and $V_{\mathrm{mp}}$ is small with respect to $\pm 1$ [1]. Statistical analysis shows that both measures are within


Figure 4: Series MML frequency distribution using Monte Carlo techniques for module selection and placement. Results are nearly identical to those of Bucciarelli [1].
acceptable ranges for each module type, the values of which are shown in Tables 1 and 2.

## 5. Results and Analysis

Repetition of the process outlined in Section 3 yields a distribution of MMLs specific to the dimensions of that particular PV array and module sorting criteria. Keeping PV array dimensions constant while systematically restricting the sorting criteria upon the population of available PV modules yields different results dependent upon which particular parameter one sorts by. A realistically dimensioned PV array ( $400 \mathrm{kWp}, 700 \mathrm{~V}$ ) [8] was considered. For this array, MMLs were calculated by sorting based on each of the three maxpower parameters: $P_{\mathrm{mp}}, I_{\mathrm{mp}}$, and $V_{\mathrm{mp}}$. The effect of sorting upon average MML, represented in terms of a reduction in the standard deviations of each respective max-power


Figure 5: $V_{\mathrm{mp}}, I_{\mathrm{mp}}$, and $P_{\mathrm{mp}}$ distributions of the anonymous module manufacturer's 285 Wp module.
parameter distribution, can be seen in Figures 7, 8(a), and 8(b).

It is immediately clear that sorting by $I_{\mathrm{mp}}$ has the most consistent, immediate, and greatest reduction on MML. Each module works at its own $I_{\mathrm{mp}}$ under max-power conditions, and in a series string the worst performing module will lower the current output of every module in that string. As a result, individual module placement heavily influences series MML in a PV array. As outlier modules are removed with respect to the imposed $I_{\mathrm{mp}}$ restriction, a greater net reduction in MML is observed.

Compared to $I_{\mathrm{mp}}$, sorting modules by $V_{\mathrm{mp}}$ results in negligible or even counterproductive MML reduction (as in the case of the 250 Wp module). Parallel MML represents the overall voltage drop in a PV array as a result of the series string with the lowest voltage. In a realistically dimensioned array such as the one considered in this study, each string in the array is composed of twenty or more PV modules. This effectively reduces the impact that individual module placement has upon string voltage variance, thus reducing the importance of $V_{\mathrm{mp}}$ sorting. This phenomenon can be inferred by the MML estimate (8) in that parallel MML is inversely proportional to string length $(L)$. The greater the string length, the less string voltage variation one will find and the less parallel MML one will measure. Evidence of the heavy
dependence of overall MML upon $I_{\mathrm{mp}}$ over $V_{\mathrm{mp}}$ variance has been demonstrated by Kaushika and Rai [5], in which parallel MMLs are found on average to be a full order of magnitude less than series MMLs even in arrays with a string length of one.

Sorting by $P_{\mathrm{mp}}$ yielded the most unpredictable impact on MML reduction. This is not surprising given that a PV module's $P_{\mathrm{mp}}$ is the product of both $V_{\mathrm{mp}}$ and $I_{\mathrm{mp}}$. With overall MML being shown to heavily favour series MML over parallel MML, sorting by any parameter that does not directly reduce the variation of $I_{\mathrm{mp}}$ (such as $P_{\mathrm{mp}}$ ) is not going to result in consistent, reliable MML reduction.

Both module populations were subjected to some degree of sorting with respect to $P_{\mathrm{mp}}$ prior to use by this study and reflect each manufacturer's $P_{\mathrm{mp}}$ variation tolerance. There is a significant difference between the two module sets, with the Helios module showing a modest $\pm 3 \%$ variation from the mean, whereas the modules from the anonymous manufacturer show $\pm 1 \%$. The consequences of this can be seen in average MMLs found before we applied the simulated sorting procedure, with the modules from the anonymous manufacturer showing a MML of $0.009 \%$ compared to the Helios module MML of $0.055 \%$-roughly 6 times more but still orders of magnitude smaller than MML estimates reported in prior studies $[1,2,5]$.


Figure 6: $V_{\mathrm{mp}}, I_{\mathrm{mp}}$, and $P_{\mathrm{mp}}$ distributions of the Helios Solar Works 250 Wp module.


Figure 7: MML frequency distributions for a $700 \mathrm{~V}, 400 \mathrm{kWp}$ PV array populated with the Helios 250 Wp module. Each distribution is the result of ever stricter sorting with respect to $I_{\mathrm{mp}}$, as represented by the reduction in $I_{\mathrm{mp}}$ standard deviations displayed above.

Any reduction in MML resulting from sorting modules similar in fashion to this study would only be reliably seen in the short term. The work by Kaushika and Rai [5] shows dramatic increases in MMLs as PV cells age. Outside the
scope of this study, but clearly the next step, would be to investigate the long-term reduction in PV array MML as a function of sorting modules prior to installation.

## 6. Economic Benefit of Sorting PV Modules

Reducing max-power parameter standard deviations of a given set of PV modules results in a $\triangle M M L$ reduction corresponding to a net economic gain over the lifetime of the PV array. For a set Power Purchase Agreement (PPA), a given PV array will produce on average $\nu \$ /$ year. A reduction in MML will result in a yearly net cash flow increase given by

$$
\begin{gather*}
B_{j}=v \Delta \mathrm{MML}(1+\mathrm{EER})^{j},  \tag{13}\\
v=x y z,
\end{gather*}
$$

where
$\Delta \mathrm{MML}=$ reduction in MML due to sorting (\%),
$x=$ array power output per $\mathrm{kWp}(\mathrm{kWh} / \mathrm{kWp})$,
$y=$ array capacity (kWp),
$z=$ energy cost set by PPA ( $\$ / \mathrm{kWh}$ ),
EER = Energy Escalation Rate set by PPA (\%),
$j=$ year.


Figure 8: Average MML as a function of sorting by each max-power parameter for each module. Sorting is represented by a percentage reduction in the standard deviation of each max-power parameter's distribution with respect to their unsorted standard deviations.


FIGURE 9: Allowed manufacturer sorting cost for a given MML reduction resulting from module sorting to be considered a good economic decision. Labelled data points correspond to the strictest sorting criteria for each parameter.

The net present value NPV of this series of yearly cash flows $B_{j}$ with a discount rate $d$ over an array lifetime of $n$ years is given by

$$
\begin{align*}
\mathrm{NPV} & =\sum_{j=1}^{n} \frac{B_{j}}{(1+d)^{j}},  \tag{14}\\
d & =\frac{r-i}{1+i}
\end{align*}
$$

$r=$ opportunity cost of capital (\%),
$i=$ inflation rate (\%),
resulting in the expression

$$
\begin{equation*}
\mathrm{NPV}=v \Delta \mathrm{MML} \sum_{j=1}^{n}\left(\frac{1+\mathrm{EER}}{1+d}\right)^{j} \tag{15}
\end{equation*}
$$

The cost $C_{o}$ to the owner of the PV array to presort the modules is given by

$$
\begin{equation*}
C_{o}=C_{s}(1+\delta), \tag{16}
\end{equation*}
$$

where $C_{s}$ is the manufacturer's cost of sorting and $\delta$ is their desired profit margin. In order to yield a net economic gain,

NPV $>C_{o}$, resulting in an expression for the minimum required decrease in MML from sorting:

$$
\begin{equation*}
\Delta \mathrm{MML}>\frac{C_{s}(1+\delta)}{\nu}\left[\sum_{j=1}^{n}\left(\frac{1+\mathrm{EER}}{1+d}\right)^{j}\right]^{-1} . \tag{17}
\end{equation*}
$$

6.1. Example. We take as an example the Helios 250 Wp module assembled into the 400 kWp PV array simulated previously place it in a region of high irradiance such as the American South-West, and assume a specific yield of $2,000 \mathrm{kWh} / \mathrm{kW}$. For simplicity we assume an array lifetime and PPA of 20 years each. Using recent data for $\$ / \mathrm{kWh}$, EER, and discount rates [9-12] and assuming a modest profit margin of $20 \%$, the following parameters take the values

$$
\begin{aligned}
& x=2,000 \mathrm{kWh} / \mathrm{kWp} \\
& y=400 \mathrm{kWp} \\
& z=\$ 0.16 / \mathrm{kWh} \\
& n=20 \text { years } \\
& \mathrm{EER}=2.4 \% \\
& r=2.72 \% \\
& i=2.42 \% \\
& \delta=20 \%
\end{aligned}
$$

Substituting into (17) yields

$$
\begin{equation*}
\frac{C_{s}}{\$ 2,673,000}<\Delta \mathrm{MML} \tag{18}
\end{equation*}
$$

The net economic benefit of sorting modules can be seen in Figures 9(a) and 9(b). Even in the most significant case, that of sorting by $I_{\mathrm{mp}}$ and reducing the $I_{\mathrm{mp}}$ standard deviation to $40 \%$ in the Helios 250 Wp module, the net economic gains over the lifetime of the PV array are minimal. This is, however, highly dependent upon manufacturer sorting cost. Despite this, all indications suggest that sorting modules by $I_{\mathrm{mp}}$ will yield the most consistent and reliable reduction in MML.

## 7. Summary and Conclusions

A procedure for estimating average MMLs of PV arrays built from a given set of real PV modules was developed by simulating preinstallation sorting, followed by generating random artificial PV arrays using Monte Carlo techniques. Bucciarelli's [1] equation for estimating MML in a PV networks was modified to take into account individual module placement within the network. Two different PV modules ( 250 Wp and 285 Wp ) were used to populate a realistically dimensioned, large-scale PV array ( $700 \mathrm{~V} ., 400 \mathrm{kWp}$ ) [8].

The analysis indicated that sorting PV modules by maxpower current $I_{\mathrm{mp}}$ yielded the most consistent and greatest overall reduction in MML when compared to sorting by $P_{\mathrm{mp}}$, which yielded inconsistent MML reduction, and $V_{\mathrm{mp}}$, which yielded insignificant MML reduction. These results were expressed in terms of a reduction in the standard deviations of each max-power parameter distribution.

A brief economic analysis was carried out describing the net economic benefit over the lifetime of the PV array of MML reduction by means of sorting. Even in the best case scenario, it appears that sorting yields little or no economic gain; however this is highly dependent upon manufacturer sorting cost.

This study was limited by the lack of data pertaining to the time sensitivity of PV module current-voltage variation. As modules degrade over time, their electrical characteristics change which can impact MML.

## Nomenclature

$I_{\text {ph }}$ : Light generated current
$I_{0}$ : Diode saturation current
E: Electron charge
$V^{\prime}$ : Cell voltage
$I^{\prime}: \quad$ Cell current
$R_{s}$ : Cell series resistance
$R_{\text {sh }}$ : Cell shunt resistance
A: Diode ideality factor
$k$ : Boltzmann's constant
$T_{c}$ : Cell temperature
$\Delta P$ : Fractional power loss due to electrical mismatch
$\Delta$ MML: Reduction in mismatch loss
C: Cell characteristic factor
$C^{\prime}$ : Modified cell characteristic factor
FF: Fill factor
$P_{\mathrm{mp}}$ : Module max-power output
$V_{\mathrm{mp}}$ : Module max-power voltage
$\bar{V}_{\mathrm{mp}}$ : Average module max-power voltage
$I_{\mathrm{mp}}$ : Module max-power current
$\bar{I}_{\mathrm{mp}}$ : Average module max-power current
$V_{\text {oc }}$ : Module open circuit voltage
$I_{\mathrm{sc}}$ : Module short circuit voltage
$\sigma_{V_{\mathrm{mp}}}$ : Standard deviation of max-power voltage
$\sigma_{I_{\mathrm{mp}}}: \quad$ Standard deviation of max-power current
$\sigma_{\xi}: \quad$ Coefficient of variation of max-power voltage
$\sigma_{\eta}$ : Coefficient of variation of max-power current
$\sigma_{\xi}^{2}: \quad$ Variance of max-power voltage
$\sigma_{\eta}^{2}$ : Variance of max-power current
L: $\quad$ Number of modules connected in series
M: Number of series strings connected in parallel
$T$ : $\quad$ Total number of modules in the PV network
$\epsilon C^{\prime}$ : Weighted variation of max-power current/voltage
$\varrho: \quad$ Pearson's correlation
NPV: Net present value
PPA: Power purchase agreement
EER: Energy escalation rate
$d$ : Discount rate
$C_{s}: \quad$ Cost to manufacturer to sort modules
$\delta: \quad$ Manufacturer profit margin.

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