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Research Article

Maximizing Banking Profit on a Random Time Interval

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We study the stochastic dynamics of banking items such as assets, capital, liabilities and profit. A consideration of these items leads to the formulation of a maximization problem that involves endogenous variables such as depository consumption, the value of the bank's investment in loans, and provisions for loan losses as control variates. A solution to the aforementioned problem enables us to maximize the expected utility of discounted depository consumption over a random time interval, $[t, \tau]$, and profit at terminal time τ. Here, the term depository consumption refers to the consumption of the bank's profits by the taking and holding of deposits. In particular, we determine an analytic solution for the associated Hamilton-Jacobi-Bellman (HJB) equation in the case where the utility functions are either of power, logarithmic, or exponential type. Furthermore, we analyze certain aspects of the banking model and optimization against the regulatory backdrop offered by the latest banking regulation in the form of the Basel II capital accord. In keeping with the main theme of our contribution, we simulate the financial indices return on equity and return on assets that are two measures of bank profitability.

1. Introduction

Our study of the dynamics and optimization of banking operations involves imperfectly competitive market conditions. In this situation, it is guaranteed that profits are made since the net loan interest margin is greater than the marginal resource cost of deposits and loans. Besides competition policy, the decisions related to how much bank capital to hold play an important role in profit-making with the association between capital, credit issuing, and macroeconomic activity being of considerable significance. In our contribution, we present a banking model that takes cognizance of the balance sheet items such

as assets (loans, treasuries, and reserves), liabilities (deposits), and bank capital (shareholder equity and subordinate debt). As a consequence of this, we are able to formulate a maximization problem that determines the optimal bank depository consumption over a random time interval, $[t, \tau]$, and terminal profit at τ. In this case, the control variates are the depository consumption, value of the bank's investment in loans, and provisions for loan losses. Here the term depository consumption refers to the consumption of the bank's profits by the taking and holding of deposits.

A further factor impacting the procedure for optimizing profit is the prudential aspects of supervision and regulation. Currently, this banking regulation is embodied by the Basel II capital accord (see [1, 2]) that will be implemented globally by the end of year 2007. Basel II adopts a three-pillared approach with the ratio of bank capital to risk-weighted assets (RWAs), also called the *capital adequacy ratio (CAR)*, playing a major role as an index of the adequacy of capital held by banks. Our study expresses the CAR as

$$\mathrm{CAR}(\rho) = \frac{\text{Bank capital}(C)}{\text{Total RWAs}(a)}, \tag{1.1}$$

where the total RWAs, a, are comprised of risk-weighted loans, λ, and treasuries, T. In particular, "procyclicality" has become a buzzword in discussions around the new regulatory framework offered by Basel II. In the sequel, the movement in a financial indicator is said to be "procyclical" if it tends to amplify business cycle fluctuations. In this regard, it is likely that during a recession a decrease in CARs and an increase in regulatory requirements necessitated by the fall in the risk profile of assets may increase the possibility of a credit crunch and result in poor economic growth. Also, since RWAs are sensitive to risk changes, the CAR may increase while the actual levels of bank capital may decrease. This means that a given CAR can only be sustained if banks hold more regulatory capital. The most important role of capital is to mitigate the moral hazard problem that results from asymmetric information between banks, depositors, and debtors. The Modigliani-Miller theorem forms the basis for modern thinking on capital structure (see [3]). In an efficient market, their basic result states that, in the absence of taxes, insolvency costs, and asymmetric information, the bank value is unaffected by how it is financed. In this framework, it does not matter if bank capital is raised by issuing equity or selling debt or what the dividend policy is. By contrast, in our contribution, in the presence of loan market frictions, the value of the bank is dependent on its financial structure (see, e.g., [4–7] for banking). In this case, it is well known that the bank's decisions about lending and other issues may be driven by the CAR (see, e.g., [8–12]). Further evidence of the impact of capital requirements on the lending activities of banks is provided by [13].

In the recent past, research into credit models for monetary policy has considered the relationship between bank capital and loan demand and supply (see, e.g., [14–20]). This credit channel is commonly known as the bank capital channel and propagates that a change in interest rates can affect lending via bank capital. We also discuss the effect of macroeconomic activity on a bank's capital structure and lending activities (see, e.g., [21]). With regard to the latter, for instance, there is considerable evidence to suggest that macroeconomic conditions impact the probability of default and loss given default on loans (see, e.g., [21, 22]).

Empirical evidence suggests that financial indicators like credit prices, asset prices, bond spreads, ratings from credit-rating agencies, provisioning, profitability, capital, leverage and risk-weighted capital adequacy ratios, and other ratios such as write-off/loan ratios and perceived risk, exhibit cyclical tendencies. As was mentioned before, this phenomenon is related to procyclicality and is affected by a risk-sensitive framework such as Basel II. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses, and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. Our interest in cyclicality extends to its relationship with credit prices, risk-weightings, provisioning, profitability, and capital (see, e.g., [15–17, 23–25]). As an example, we incorporate in our models the fact that provisioning behaves procyclically by falling during economic booms and rising during recessions.

Discrete- and continuous-time modeling and optimization problems in banking have been studied in many recent publications (see, e.g., [6, 11, 14, 17, 21, 26–28]). The stochastic model for profit in the present contribution can be considered to be the natural analogue of the corresponding discrete-time model presented in [14] (see, also, [17]). The paper [27] has an especially close connection with the present paper. However, in the current contribution, a major difference is that we focus on profit maximization in the banking sector whereas [28] (see, also, [27]) discusses an optimal control problem for the profit of a more general class of institutions called depository financial institutions (DFIs). Included in this class of institutions are, for instance, insurance companies and pension schemes as well as investment banks. Our paper is also distinct from [28] in that we define the problem on a random time interval while the analysis in the aforementioned contribution is confined to time intervals that are fixed. As a result, our discussion is more general and closer to reality. On the other hand, [26] examines a problem related to the optimal risk management of banks in a continuous-time stochastic dynamic setting. In particular, we minimize market and capital adequacy risk that involves the safety of the assets held and the stability of sources of capital, respectively. In this regard, we suggest an optimal portfolio choice and rate of the bank capital inflow that will keep the loan level as close as possible to an actuarially determined reference process. This setup leads to a nonlinear stochastic optimal control problem whose solution may be determined by means of the dynamic programming algorithm.

The main problems that are solved in this paper can be formulated as follows.

Problem 1.1 (dynamic modeling of banking items). Can we construct stochastic dynamic models to describe banking items such as profit in an economically sound manner? (Section 2).

Problem 1.2 (banking profit maximization problem). Which decisions about the depository consumption, value of the investment in loans, and provisions for loan losses must be made in order to maximize banking profit on a random time interval? (Theorems 3.2 and 3.6).

In the sequel, the modeling and maximization issues raised in Problems 1.1 and 1.2 are discussed in Section 3, respectively. An analysis of the main economic issues is done in Section 4. Finally, Section 5 offers a few concluding remarks and topics for possible future research.

2. Stochastic model for banks

The Basel II capital accord (see, e.g., [1, 2]) encourages banks to view balance sheet items from the viewpoint of the riskiness of assets held and the adequacy of their capital. In this spirit, we consider a balance sheet that consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by bank capital (see, e.g., [5]). This leads to the well-known relation

$$\text{Total Assets}(A) = \text{Total Liabilities}(\Gamma) + \text{Total Bank Capital}(C), \tag{2.1}$$

where

$$A_t = \Lambda_t + \mathrm{T}_t + R_t, \qquad \Gamma_t = \Delta_t, \qquad C_t = n_t E_{t^-} + O_t. \tag{2.2}$$

Here, the symbols Λ, T, R, Δ, n, E, and O denote loans, treasuries, reserves, deposits, number of shares in bank equity, bank equity, and subordinate debt, respectively.

2.1. Assets. In this subsection, the bank assets that we discuss are loans, treasuries, reserves, and risk-weighted assets. In the sequel, we suppose that $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ is a filtered probability space.

2.1.1. Loans. We suppose that, after providing liquidity, the bank grants loans at the *interest rate on loans* or *loan rate*, r_t^Λ. Due to the expenses related to monitoring and screening, we assume that these loans incur a constant *marginal cost*, c^Λ. In addition, we introduce the generic variable, M_t, that represents the level of macroeconomic activity in the bank's loan market. Also, we assume that the *loan supply process*, Λ, follows the geometric Brownian motion process

$$d\Lambda_t = \Lambda_t \{ (r_t^\Lambda - c^\Lambda)\, dt + \sigma_t dZ_t \}, \tag{2.3}$$

where $\sigma_t > 0$ denotes the volatility in the loan supply and Z_t is a standard Brownian motion with respect to a filtration, $(\mathcal{F}_t)_{t\geq 0}$, of the probability space $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$. The *value of the bank's investment in loans*, λ, at t is expressed as

$$\lambda_t = n_t^\Lambda \Lambda_t, \tag{2.4}$$

where n_t^Λ is the number of loans at t.

2.1.2. Provisions for loan losses. The bank's investment in loans may yield substantial returns but may also result in loan losses. In line with reality, our dynamic banking model allows for loan losses for which provision can be made. We also briefly consider the scenario in which the bank makes no provision for such losses. The accompanying default

risk is modeled as a compound Poisson process where N is a Poisson process with a deterministic frequency parameter, $\phi(t)$. Here, N is stochastically independent of the Brownian motion, Z, given in (2.3). Furthermore, we introduce the value of *loan losses* as

$$L(M_t, t) = r^d(M_t)\lambda_t, \tag{2.5}$$

where L is independent of N. The formula for $L(M_t, s)$, presented in (2.5), can also be expressed in terms of profit, Π, as $L(\Pi_t, s)$, by virtue of the evidence from empirical studies that suggest that a strong positive correlation between M_t and Π_t exists (see the discussion on the procyclicality of bank profitability in, e.g., [23, 24]). Also, we assume that the *default* or *loan loss rate*, $r^d \in [0,1]$, increases when macroeconomic conditions deteriorate according to

$$0 \le r^d(M_t) \le 1, \quad \frac{\partial r^d(M_t)}{\partial M_t} < 0. \tag{2.6}$$

As was the case with the relationship between profit and macroeconomic activity, the above description of the loan loss rate is consistent with empirical evidence that suggests that bank losses on loan portfolios are correlated with the business cycle under any capital adequacy regime (see, e.g., [23–25, 29]). Furthermore, we assume that the provision made by the bank for loan losses takes the form of a continuous contribution that can be expressed as

$$[1 + \theta(s)]\phi(s)\mathbf{E}[P_s(L)], \tag{2.7}$$

where θ is a credit risk compensatory term, $\theta(t) \ge 0$, and P_t is the actual provision for loan losses. This means that if the bank suffers a loan loss of $\lambda = l$ at time t, the provision, $P_t(l)$, covers these losses. The actual manner in which banks make provision for loan losses can differ greatly. However, there is invariably some cost incurred by the bank in administering the process. In this regard, we denote the *costs associated with the bank provisioning for loan losses* by c^P (see, e.g., [23, 25, 29]).

2.1.3. Treasury securities. *Treasury securities* are bonds issued by national treasuries. They are the debt financing instruments of the federal government and are often referred to as "treasuries." There are four types of treasuries: treasury bills, treasury notes, treasury bonds, and savings bonds. All of the treasury securities besides savings bonds are very liquid. They are heavily traded on the secondary market. We denote the *interest rate on treasuries* or *treasury rate* by r_t^{T} and assume that for all t, we have

$$r_t^{\Lambda} - c^{\Lambda} > r_t^{\mathrm{T}}. \tag{2.8}$$

2.1.4. Reserves. *Bank reserves* are the deposits held in accounts with a national agency (e.g., the Federal Reserve for banks) plus money that is physically held by banks (vault cash). Such reserves are constituted by money that is not lent out but is earmarked to cater for withdrawals by depositors. Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits will be needed as reserves.

As a result of this description, we may introduce a reserve-deposit ratio, γ, for which

$$R_t = \gamma \Delta_t. \tag{2.9}$$

The bank uses the remaining deposits to earn profit either by issuing loans or by investing in assets such as treasuries and stocks.

2.1.5. Risk-weighted assets. We consider risk-weighted assets (RWAs) that are defined by placing each on- and off-balance sheet item into a risk category. The more risky assets are assigned a larger weight. Table 2.1 below provides a few illustrative risk categories, their risk-weights, and representative items.

As a result, RWAs are a weighted sum of the various assets of the banks. In the sequel, we denote the risk-weight on treasuries and loans by ω^{T} and ω^{λ}, respectively. With regard to the latter, we can identify a special risk-weight on loans $\omega^{\lambda} = \omega(M_t)$ that is a decreasing function of current macroeconomic conditions, that is,

$$\frac{\partial \omega(M_t)}{\partial M_t} < 0. \tag{2.10}$$

This is in line with the procyclical notion that during booms, when macroeconomic activity increases, the risk-weights will decrease. On the other hand, during recessions, risk-weights may increase because of an elevated probability of default and/or loss given default on loans (see, e.g., [23–25]).

2.2. Capital. In this subsection, we discuss total bank capital, binding capital constraints, and retained earnings for a bank.

2.2.1. Total bank capital. The bank's total capital, C, has the form

$$C_t = C_t^{T1} + C_t^{T2}, \tag{2.11}$$

where C_t^{T1} and C_t^{T2} are Tier 1 and Tier 2 capital, respectively. *Tier 1 (T1) capital* is the book value of bank capital defined as the difference between the accounting value of the assets and liabilities. In our contribution, Tier 1 capital is represented at t^-'s market value of the bank equity, $n_t E_{t^-}$, where n_t is the number of shares and E_t is the market price of the bank's common equity at t. *Tier 2 (T2) capital* consists of preferred stock and subordinate debt. Subordinate debt is subordinate to deposits and hence faces greater default risk. Tier 2 capital, O_t, issued at t^- is represented by bonds that pay an interest rate, r^O (see, e.g., [14]).

2.2.2. Binding capital constraints. To reflect the book value property of regulatory capital and its market valuation sensitivity, we assume that at t^-, the market value of equity and treasuries determines the capital constraint to which the bank is subjected at t. While there are several capital constraints associated with Basel II, it is easy to show that the binding one is the total capital constraint. This constraint requires that

$$\rho_t = \frac{C_t}{a_t} \geq 0.08. \tag{2.12}$$

Table 2.1. Risk categories, risk-weights, and representative Items.

Risk category	Risk-weight	Banking items
1	0%	Cash, reserves, treasuries
2	20%	Shares
3	50%	Home loans
4	100%	Loans to private agents

For the regulatory ratio of total capital to risk-weighted loans plus treasuries, $\overline{\rho}^r$, a capital constraint may be represented by

$$\overline{\rho}^r\left[\omega^\lambda\lambda_t+\omega^{\mathrm{T}}\mathrm{T}_t\right]\leq n_tE_{t^-}+O_t. \tag{2.13}$$

As a result of (2.13), it is not necessary to differentiate between the relative cost of raising debt versus equity. Moreover, when maximizing profits, we consider the regulatory ratio of total capital to risk-weighted loans, ρ^r, as an appropriate capital constraint. This means that we may set $\omega^\lambda=\omega(M_t)$ and $\omega^{\mathrm{T}}=0$ in (2.13), and express the binding capital constraint as

$$\rho^r\omega(M_t)\lambda_t\leq n_tE_{t^-}+O_t. \tag{2.14}$$

The exact value of the regulatory ratio, ρ^r, may differ markedly from institution to institution (see, e.g., [7, 12]). In fact, subject to an appropriate choice for ρ^r, some banks may consider that the equality in (2.14) implies an optimal choice of the investment in loans, λ, so that

$$\lambda_t^*=\frac{n_tE_{t^-}+O_t}{\rho^r\omega(M_t)}. \tag{2.15}$$

2.3. Liabilities. We only consider the banking item deposits in the category of liabilities.

2.3.1. Deposits. The bank takes deposits, Δ_t, at a constant *marginal cost*, c^Δ, that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the *interest rate on deposits* or *deposit rate*, r_t^Δ, is less than the treasuries rate, r_t^{T}. In the sequel, we express the depository consumption, k, as

$$k_t=\left[r_t^\Delta+c^\Delta\right]\Delta_t. \tag{2.16}$$

It is realistic to take cognizance of the possibility that *unanticipated deposit withdrawals*, u, will occur. By way of making provision for these withdrawals, the bank is inclined to hold reserves, R, and treasuries, T, that are very liquid. In our contribution, we assume that u is related to the *probability density function*, $f(u)$, that is independent of time. For the sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[\underline{\Delta},\overline{\Delta}]$ so that the *cost of liquidation*, c^l, or additional external funding is a quadratic function of the sum of reserves and treasuries, $W=R+\mathrm{T}$.

In addition, for any t, if we have that

$$u > W_t, \tag{2.17}$$

then bank assets are liquidated at some *penalty rate*, r_t^p. In this case, the *cost of deposit withdrawals* is

$$c^w(W_t) = r_t^p \int_{W_t}^{\infty} [u - W_t] f(u) du = \frac{r_t^p}{2\Delta} [\overline{\Delta} - W_t]^2. \tag{2.18}$$

2.4. Profit. Suppose that the values of the bank's investment in loans, λ, loan losses, L, depository consumption, k, and cost of withdrawals, $c^w(W_t)$, are given by (2.4), (2.5), (2.16), and (2.18), respectively. Here, the differential equation for the profit dynamics of banks that make provision for loan losses may be represented as

$$\begin{aligned} d\Pi_s = {} & [r_s^{\mathrm{T}} \Pi_s + (r_s^{\Lambda} - c^{\Lambda} - r_s^{\mathrm{T}})\lambda_s + \mu^a(s) - k_s - [1 + \theta(s)]\phi(s)\mathbf{E}[P_s(L)]] ds \\ & - c^w(W_s) + \sigma_s \lambda_s dZ_s - \{L(\Pi_s, s) - P_s(L(\Pi_s, s))\} dN_s, \quad s \geq t,\ \Pi_t = \pi, \end{aligned} \tag{2.19}$$

where $\mu^a(s)$ is the rate term for auxiliary profits that may be generated from activities such as special screening, monitoring, liquidity provision, and access to the payment system. Also, this additional profit may be generated from imperfect competition, barriers to entry, exclusive access to cheap deposits or tax benefits. On the other hand, the expression for the profit dynamics of banks that make no provision for loan losses may be given by

$$\begin{aligned} d\Pi_s = {} & [r_s^{\mathrm{T}} \Pi_s + (r_s^{\Lambda} - c^{\Lambda} - r_s^{\mathrm{T}})\lambda_s + \mu^a(s) - k_s] ds \\ & - c^w(W_s) + \sigma_s \lambda_s dZ_s - L(\Pi_s, s) dN_s, \quad s \geq t,\ \Pi_t = \pi. \end{aligned} \tag{2.20}$$

3. Optimization on random time intervals

In this section, we make use of the models constructed in the preceding discussion to solve an optimization problem for banks. An important feature of our analysis is that this problem is considered on a random time interval, $[t, \tau]$.

3.1. Statement of the optimization problem. In the sequel, we study a special case of (2.19) in which

$$c^w(W_t) = 0, \qquad r_s^{\mathrm{T}} = r^{\mathrm{T}}, \qquad r_s^{\Lambda} = r^{\Lambda}, \qquad \sigma_s = \sigma. \tag{3.1}$$

As a consequence of these choices, we have that

$$\begin{aligned} d\Pi_s = {} & [r^{\mathrm{T}} \Pi_s + (r^{\Lambda} - c^{\Lambda} - r^{\mathrm{T}})\lambda_s + \mu^a(s) - k_s - [1 + \theta(s)]\phi(s)\mathbf{E}[P_s(L)]] ds \\ & + \sigma \lambda_s dZ_s - \{L(\Pi_s, s) - P_s(L(\Pi_s, s))\} dN_s, \quad s \geq t,\ \Pi_t = \pi. \end{aligned} \tag{3.2}$$

We suppose that the bank aims to optimize (over allowable $\{k_t, \lambda_t, P_t\}$) its expected utility of the discounted depository consumption during a random time interval, $[t, \tau]$, and

profit at terminal random time, τ. In this case, the set of admissible controls, $\mathscr{A}$, may be given by

$$\mathscr{A} = \{(k_t, \lambda_t, P_t) : \text{measurable with respect to } \mathscr{F}_t, \text{ (3.2) has a unique solution.}\}. \tag{3.3}$$

Also, the value function may be represented by

$$V(\pi, t) = \sup_{\{k_t, \lambda_t, P_t\}} \mathbf{E}\left[\int_t^\tau \exp\{-\delta(s-t)\} U^{(1)}(k_s)\, ds + \exp\{-\delta(\tau - t)\} U^{(2)}(\Pi_\tau) \mid \Pi_t = \pi\right]. \tag{3.4}$$

Here the utilities, $U^{(1)}$ and $U^{(2)}$, are increasing, twice-differentiable, concave functions, and $\delta > 0$ is the rate at which the depository consumption and terminal profit are discounted. The functions $U^{(1)}$ and $U^{(2)}$ measure the utility of depository consumption and terminal profit, respectively. An interesting problem is to determine whether we can obtain a smooth or analytical solution for the Hamilton-Jacobi-Bellman (HJB) equation resulting from the above. In the sequel, we accomplish this for the choices of exponential, power, and logarithmic utility functions. In this regard, we align our optimization procedure with the methodology suggested in such contributions as [30].

Next, the stochastic optimization problem for bank depository consumption and terminal profit on a random time interval, $[t, \tau]$, is formulated.

Problem 3.1 (optimal depository consumption and profit). Suppose that $\mathscr{A} \neq \emptyset$, where the admissible class of control laws, $\mathscr{A}$, is given by (3.3). Also, consider the SDE for the Π-dynamics from (3.2) and the value function, $V : \mathscr{A} \to \mathbb{R}_+$, given by (3.4). In this case, solve

$$\sup_{k_t, \lambda_t, P_t} V(\Pi; k_t, \lambda_t, P_t) \tag{3.5}$$

and, if it exists, the optimal control law $(k_t^*, \lambda_t^*, P_t^*)$ given by

$$(k_t^*, \lambda_t^*, P_t^*) = \arg\sup_{k_t, \lambda_t, P_t} V(\Pi; k_t, \lambda_t, P_t) \in \mathscr{A}. \tag{3.6}$$

3.2. Solution to the optimization problem. In this subsection, we solve Problem 3.1 on the random time interval $[t, \tau]$. In Theorem 3.2, $D_t V(\pi, t)$, $D_\pi V(\pi, t)$, and $D_{\pi\pi} V(\pi, t)$ denote first- and second-order partial derivatives of V with respect to the variables t and π, where appropriate. For example, $D_{\pi\pi} V(\pi, t)$ is the second partial derivative of V with respect to π. A general solution (without utility choices being made) of Problem 3.1 is given in the next result.

Theorem 3.2 (general solution of Problem 3.1). *Suppose that the value function, $V(\pi, t)$, is given by (3.4). In this case, a solution to the loan component of Problem 3.1 is of the form*

$$\lambda_t^* = -\frac{r^\Lambda - c^\Lambda - r^{\mathrm{T}}}{\sigma^2} \frac{D_\pi V(\Pi_t^*, t)}{D_{\pi\pi} V(\Pi_t^*, t)}, \tag{3.7}$$

where Π_t^* is the optimally controlled profit. Also, the optimal depository consumption, $\{k_t^*\}$, solves the equation

$$D_k U^{(1)}(k_t^*) = D_\pi V(\Pi_t^*, t). \tag{3.8}$$

Proof. In our proof, via the dynamic programming approach, V solves the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}\delta V(\pi,t) =& D_t V(\pi,t) + \max_\lambda \left[\left((r^\Lambda - c^\Lambda) - r^{\mathrm{T}} \right) \lambda D_\pi V(\pi,t) + \frac{1}{2}\sigma^2\lambda^2 D_{\pi\pi} V(\pi,t) \right] \\ &+ \max_k \left[U^{(1)}(k) - k D_\pi V(\pi,t) \right] + \left(r^{\mathrm{T}}\pi + \mu^a(t) \right) D_\pi V(\pi,t) \\ &+ \max_P \left[\phi(t) \{ \mathbf{E} V(\pi - (L - P(L)), t) - V(\pi,t) \} - (1+\theta(t))\phi(t)\mathbf{E}[P(L)] D_\pi V(\pi,t) \right] \\ &+ \eta^p(t) \left[U^{(2)}(\pi) - V(\pi,t) \right] \\ &\lim_{s\to\infty} \mathbf{E}\left[\exp\left\{ -\int_t^s (\delta + \eta^p(u))\, du \right\} V(\Pi_s^*, s) \mid \Pi_t^* = \pi \right] = 0,\end{aligned} \tag{3.9}$$

where $\eta^p(t)$ is the rate of inclination towards bankruptcy at time t of a bank which has been in existence for the time $p + t$.

We note that the value function, V, is increasing and concave with respect to profit π, because the utility functions $U^{(1)}$ and $U^{(2)}$ are increasing and concave and because the differential equation for profit is linear with respect to the controls. These observations culminate in the fact that the optimal investment strategy in (3.7) holds. □

3.2.1. Optimal provisioning process. Suppose that the profit dynamics of banks that make provision for loan losses are represented by (2.19) and the dynamics of banks that make no provision are given by (2.20). With regard to the optimal provisioning process, $P^* = \{P_t^*\}_{0\le t\le\tau}$, we have the following result from [28].

Proposition 3.3 (optimal provisioning process). *Suppose that the value function, $V(\pi,t)$, is described by (3.4).*

(1) *The optimal provisioning process, P^*, is either no provisioning or per-loan loss provisioning, in which the provisioning costs may vary with respect to time. Specifically, at a given time, the optimal provisioning costs $c^{P*} = \{c_t^{P*}\}_{0\le t\le\tau}$ solve*

$$[1 - \theta(t)] D_\pi V(\Pi_t^*, t) = D_\pi V(\Pi_t^* - c_t^P, t). \tag{3.10}$$

No provisioning is optimal at time t if and only if

$$[1 - \theta(t)] D_\pi V(\Pi_t^*, t) \ge D_\pi V(\Pi_t^* - ess\sup \lambda(\Pi_t^*, t), t). \tag{3.11}$$

(2) *An increase in the instantaneous price of provisioning reduces the instantaneous inclination towards provisioning.*

(3) *Suppose that c_t^{p*} exists. In this case, if V in (3.4) exhibits decreasing absolute risk aversion with respect to profit, then the inclination towards provisioning decreases with increasing profit.*

We are now in a position to make our utility choices mentioned earlier, namely, exponential, power, and logarithmic utility functions.

3.2.2. Optimization with exponential utility. Suppose

$$U^{(1)}(k) = 0, \qquad U^{(2)}(\pi) = -\frac{1}{\gamma}\exp\{-\gamma\pi\}, \quad \gamma > 0. \tag{3.12}$$

We can verify the following result.

Theorem 3.4 (optimization with exponential utility). *Suppose that the exponential utilities are given as in (3.12) and assume that the loan loss L is independent of profit, with the probability distribution of L being a deterministic function of time. In this case,*

$$V(\pi,t) = f(t)\exp\{-\gamma\pi\} \tag{3.13}$$

in which $f(t)$ solves

$$f' + G(t)f = \frac{\eta^p(t)}{\gamma},$$
$$G(t) = \phi(t)\left[\exp\{\gamma c^{P^*}\} - 1 + (1+\theta(t))\left(L(t) - c_t^{P^*}\right)\gamma\right] - \frac{\left(r^\Lambda - c^\Lambda\right)^2}{2\sigma^2} + \pi(t)\gamma + \eta^p(t) + \delta. \tag{3.14}$$

Assume that the parameter values are such that $f(t) < 0$. Moreover, optimal provisioning cost is given by

$$c_t^{p^*} = \min\left[\frac{1}{\gamma}\ln(1+\theta(t)), ess\sup L(t)\right]. \tag{3.15}$$

The optimal bank investment in loans is

$$\lambda_t^* = \frac{r^\Lambda - c^\Lambda}{\sigma^2\gamma}, \tag{3.16}$$

and the optimal consumption is given by

$$k_t^* = 0. \tag{3.17}$$

Proof. The proof proceeds via standard arguments from stochastic optimization theory in continuous time. For instance, the formula for the optimal provisioning cost, c_t^{p*}, in (3.15) is a direct consequence of Proposition 3.3. Also, the optimal bank investment in loans, λ_t^*, given by (3.16) can be derived from an application of (3.7) in Theorem 3.2 to the expression for $V(\pi,t)$ in (3.13). From $U^{(1)}(k) = 0$, it follows that the optimal depository consumption is identically 0 as in (3.17). □

3.2.3. Optimization with exponential utility and a once-off provisioning payment. Suppose that, as in Section 3.2.2, we consider the utility functions

$$U^{(1)}(k) = 0, \qquad U^{(2)}(\pi) = -\frac{1}{\gamma}\exp\{-\gamma\pi\}, \quad \gamma > 0. \tag{3.18}$$

Furthermore, we assume that the model is stationary so that

$$\mu^a(t) \equiv \mu^a, \qquad \phi(t) = \phi, \qquad \theta(t) \equiv \theta, \qquad \eta^p(t) \equiv \eta^p. \tag{3.19}$$

Also, we suppose that the loan loss, L, does not depend on time and profit, and the loan loss provisions are made as a single payment at time t and are equal in value to the product of the expected present value of the loan loss and the credit risk compensatory term, $1+\theta$. As a consequence, the single provisioning payment made at time t is given by

$$\frac{1}{\eta^p}(1+\theta)\phi\mathbf{E}(L-c^p)_+. \tag{3.20}$$

Since the model is stationary, the provisioning installment payable at time t is independent of t. In this situation, for a fixed cost of provisioning, c^p, the value function $V(\pi)$ (cf. (3.9)) solves

$$\begin{gathered}\delta V = \mu^a D_t V - \frac{(r^\Lambda - c^\Lambda)^2 (D_t V)^2}{2\sigma^2 D_{tt} V} + \phi\{\mathbf{E}V(\pi - L\wedge c^p) - V(\pi)\} + \eta^p[U^{(2)}(\pi) - V], \\ \lim_{s\to\infty}\mathbf{E}[\exp\{-(\delta+\eta^p)(s-t)\}V(\Pi^*) \mid \Pi_t^* = \pi] = 0.\end{gathered} \tag{3.21}$$

This means that

$$V(\pi) = -\frac{1}{\gamma}\exp\{-\gamma\pi\}\frac{\eta^p}{\eta^p + \delta + \varrho + \gamma\mu^a - \theta[M_{L\wedge c^p}(\gamma) - 1]}, \tag{3.22}$$

where

$$\varrho = \frac{(r^\Lambda - c^\Lambda)^2}{2\sigma^2}. \tag{3.23}$$

The next result follows from the preceding discussion.

Theorem 3.5 (optimization with exponential utility and a once-off provisioning installment). *Assume that the conditions are as given in Theorem 3.4 and that (3.19) and (3.20) hold. Furthermore, suppose that the exponential utilities are defined by (3.18) and that*

$$\eta^p + \delta + \varrho + \gamma\mu^a - \theta[M_{L\wedge c^p}(\gamma) - 1] > 0. \tag{3.24}$$

The optimal provisioning cost solves

$$\eta^p \exp\{\gamma c^p\} = (1+\theta)(\eta^p + \delta + \varrho + \gamma\mu^a - \theta[M_{L\wedge c^p}(\gamma) - 1]). \tag{3.25}$$

Furthermore, the optimal bank investment in loans is

$$\lambda_t^* = \frac{r^\Lambda - c^\Lambda}{\sigma^2\gamma}, \tag{3.26}$$

and the optimal consumption is given by

$$k_t^* = 0. \tag{3.27}$$

Proof. We note that we maximize

$$V\left(\pi - \frac{1}{\eta^p}(1+\theta)\phi\mathbf{E}(L-c^p)_+\right), \tag{3.28}$$

with respect to c^p, in order to conclude that (3.25) should hold. □

3.2.4. Optimization with power utility. In this regard, for a choice of power utility, we have that

$$\overline{U}^{(1)}(k) = \frac{k^\alpha}{\alpha}, \qquad \overline{U}^{(2)}(\pi) = b\frac{\pi^\alpha}{\alpha} \tag{3.29}$$

for some $0 \neq \alpha < 1$ and $b \geq 0$. The parameter b represents the weight that the bank gives to terminal profit versus depository consumption and can be viewed as a measure of the bank's inclination towards deposit taking. This leads to the following important result.

Theorem 3.6 (optimization with power utility). *Suppose that the additional profit rate is zero, the power utilities are given as in (3.29), and assume that the loan loss is proportional to profit via*

$$L(\Pi_t, t) = \beta(t)\Pi_t \tag{3.30}$$

for some deterministic loan loss severity function, β, where $0 \leq \beta(t) \leq 1$. In this case,

$$\overline{V}(\pi, t) = \frac{\pi^\alpha}{\alpha}\overline{\xi}(t). \tag{3.31}$$

Here, $\overline{\xi}$ solves the nonlinear, nonhomogeneous, differential equation

$$0 = \overline{\xi}' - \overline{H}\xi + (1-\alpha)\overline{\xi}^{\alpha/\alpha-1} + b\eta^p(t) \tag{3.32}$$

with

$$\overline{H} = H(t) + \eta^p(t), \tag{3.33}$$

in which H is given by

$$\begin{aligned} H(t) &= \delta + \phi(t) - \kappa\alpha + (1+\theta(t))\phi(t)\alpha\max\left(0, (1+\theta(t))^{1/1-\alpha} - (1-\beta(t))\right) \\ &\quad - \phi(t)\max\left((1+\theta(t))^{1/1-\alpha}, 1-\beta(t)\right), \\ \kappa &= r^{\mathrm{T}} + \frac{(r^\Lambda - c^\Lambda - r^{\mathrm{T}})^2}{2\sigma^2(1-\alpha)}. \end{aligned} \tag{3.34}$$

Assume the parameters are such that $\overline{\xi}(t) > 0$. This implies that the optimal provisioning cost is

$$c_t^{p^*} = \min\left[1 - (1+\theta(t))^{1/\alpha-1}, \beta(t)\right]\Pi_t^*, \tag{3.35}$$

the optimal consumption is given by

$$k_t^* = D_\pi \overline{V}^{1/1-\alpha} = \overline{\xi}(t)^{1/1-\alpha}\Pi_t^*, \tag{3.36}$$

and the optimal bank investment in loans is

$$\lambda_t^* = \frac{r^\Lambda - c^\Lambda - r^{\mathrm{T}}}{\sigma^2(1-\alpha)}\Pi_t^*. \tag{3.37}$$

Proof. The proof relies on standard arguments from stochastic optimization theory. Also, (3.36) is determined by using (3.8) with power utility. □

The following corollary to Theorem 3.6 comments on the relationship between formulas for the optimal bank investment in loans, λ^*, obtained in (2.15) and (3.37).

Corollary 3.7 (optimization with power utility). *Suppose that the optimal bank investment in loans, λ^*, given by (2.15) and (3.37) are equal. Then, a formula for the optimal profit is given by*

$$\Pi_t^* = \frac{\sigma^2(1-\alpha)(n_t E_{t^-} + O_t)}{\rho^r \omega(M_t)(r^\Lambda - c^\Lambda - r^{\mathrm{T}})}. \tag{3.38}$$

3.2.5. Optimization with logarithmic utility. Suppose that the value function, $V(\pi,t)$, is described by

$$\widetilde{U}^{(1)}(k) = \ln k, \qquad \widetilde{U}^{(2)}(\pi) = b\ln\pi. \tag{3.39}$$

Corollary 3.8. *Assume that the value function is given by (3.39). Then, the optimal provisioning cost and bank investment in loans are consistent with the results for $\alpha \neq 0$. The optimal consumption is given by*

$$k_t^* = D_\pi \widetilde{V}^{-1}(\pi,t) = \widetilde{\xi}(t)^{-1}\Pi_t^*, \tag{3.40}$$

where $\widetilde{\xi}(t)$ is given by

$$\widetilde{\xi}(t) = b\overline{A}_{x+t}^{\delta}. \tag{3.41}$$

4. Analysis of the main economic issues

In accordance with the dictates of Basel II, the models of banking items constructed in this paper are related to the methods currently being used to assess the riskiness of bank portfolios and their minimum capital requirement (see [1, 2]).

4.1. Stochastic banking model. In this subsection, we analyze aspects of banking items as introduced in Section 2. As far as the bank behavior is concerned, we make a specific choice of the "bank capital channel" approach as alluded to in such contributions as [14–20].

4.1.1. Assets. Section 2.1.1 suggests that the dynamics of the macroeconomic process, $M = \{M_t\}_{t\geq 0}$, may follow the geometric Brownian motion process

$$dM_t = M_t\left[\mu_t^M dt + \sigma_t^M dZ_t^M\right], \tag{4.1}$$

where σ_t^M and Z_t^M denote volatility in macroeconomic activity and the Brownian motion driving the macroeconomic activity, respectively.

Our model also allows us to comment on loan demand. In this regard, the bank may face a *Hicksian demand for loans* given by

$$\Lambda_t = l_o - l_1 \int_0^t r_s^\Lambda ds + \int_0^t \sigma_s^\Lambda dZ_s^d + l_2 M_t, \tag{4.2}$$

where σ_s^Λ and Z_s^d denote volatility in the loan demand and the Brownian motion driving the demand for loans (which may be correlated with the macroeconomic activity), respectively. We note that the loan demand in (4.2) is an increasing function of M and a decreasing (increasing) function of $\int_0^t r_s^\Lambda ds > 0 (< 0)$.

4.1.2. Capital. Despite the effort made in Section 2.2, bank capital has proven to be difficult to define, monitor, and measure. For instance, the valuing of all the bank's financial instruments and other assets is key in measuring equity, E. In our case, the modeling of E is largely motivated by the following two observations. Firstly, it is meant to reflect the nature of the book value of equity and the second observation being recognized is that the book and market value of equity is highly correlated. Under Basel II, the bank capital requirements have replaced reserve requirements (see Section 2.1.4) as the main constraint on the behaviour of banks. A first motivation for this is that bank capital has a major role to play in overcoming the moral hazard problem arising from asymmetric information between banks, creditors, and debtors. Also, bank regulators require capital to be held to protect themselves against the costs of financial distress, agency problems, and the reduction of market discipline caused by the safety net. And again, it is only proper to provide for deposit withdrawals.

Section 2.2.2 suggests that a close relationship exists between bank capital holding and macroeconomic activity in the loan market. As was mentioned before, Basel II dictates that a macroeconomic shock will affect the loan risk-weights in the CAR. In general, a negative (positive) shock results in the tightening (loosening) of the capital constraint given by (2.14). As a consequence, in terms of a possible binding capital constraint, banks are free to increase (decrease) the loan supply when macroeconomic conditions improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply, but rather results in a change in the loan rate when the capital constraint binds. It is not always true that the Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that macroeconomic conditions not only necessarily affect loan demand but also influence the total capital constraint from (2.14). Furthermore, banks do not necessarily need to raise new capital to expand their loan supply since a positive macroeconomic shock may result in a decrease in the RWAs with a corresponding increase in CARs (cf. (1.1)). Similiarly, banks are not compelled to

decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a negative macroeconomic shock. A further complication is that an improvement in economic conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint will be binding. Banks may react to this situation by increasing capital to maximize profits (cf. the definition of the return on equity (ROE) presented subsequently in Section 4.1.4). Our main conclusion is that bank capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macroeconomic activity.

4.1.3. Liabilities. In some quarters, the deposit rate, r^{Δ}, described in Section 2.3.1 from Section 2.3, is considered to be a strong approximation of bank monetary policy. Since such policy is usually affected by macroeconomic activity, M, we expect the aforementioned items to share a close connection.

4.1.4. Profit. We are able to establish a connection between our research and the two main measures of bank profitability. The first measure is the return on assets (ROA) which may be given by

$$\text{ROA} = \frac{\text{Net Profit After Taxes}}{\text{Assets}}. \tag{4.3}$$

The ROA provides information about how much profit is generated per average by each unit of assets. Therefore, the ROA is an indicator of how efficiently a bank is being managed. The second measure is the return on equity (ROE) that is represented by

$$\text{ROE} = \frac{\text{Net Profit After Taxes}}{\text{Equity Capital}}. \tag{4.4}$$

The ROE provides information about how much shareholders are earning on their investment in the bank equity.

4.1.5. Numerical results. We would like to simulate the two measures of profitability, namely, return on assets (ROA) and return on equity (ROE) mentioned in Section 4.1.4. In order to accomplish this, we use appropriate data provided by the US Federal Deposit Insurance Corporation (FDIC) on their website [31]. The relevant data about ROA and ROE are provided in Tables 4.1 and 4.2 with the corresponding simulations being presented subsequently in Figures 4.1 and 4.2.

Table 4.1 provides data of both ROA and ROE from the year 1999 to the year 2005. Also, Table 4.2 contains all the parameters used in the calculations which lead to the simulations given in Figure 4.1. The latter figure provides information about how much profit is generated per average by each unit asset of the FDIC-insured institution. Furthermore, the simulation presented in Figure 4.2 gives an indication of how much shareholders are earning on their investment in FDIC-insured institutional equity.

4.2. Optimization on random time intervals. In this subsection, we discuss some of the issues related to the optimal control problem presented in Section 3. We note from Theorem 3.2, that [30] can be used to derive the associated HJB equation (cf. (3.9)).

Table 4.1. Source FDIC-insured institutions.

	ROA	ROE
2005	1.31	12.91
2004	1.3	13.74
2003	1.4	15.31
2002	1.33	14.46
2001	1.15	13.08
2000	1.18	13.99
1999	1.31	15.3

Table 4.2. Parameter choices for the ROE and ROA simulation.

Parameter	Symbol	Value
Bank equity	E	1164
Volatility of E	σ_e	0.286
Total expected returns on E	μ_e	0.06
Value of net profit after tax	Π_t^n	16878
Dividend payments on E	δ_e	0.05
Interest and principal payments on O	δ_s	1.06
Interest rate	r	0.06
Subordinate debt	O	135
Volatility of assets	σ_A	0.3
Net expected returns on assets	μ^A	0.003

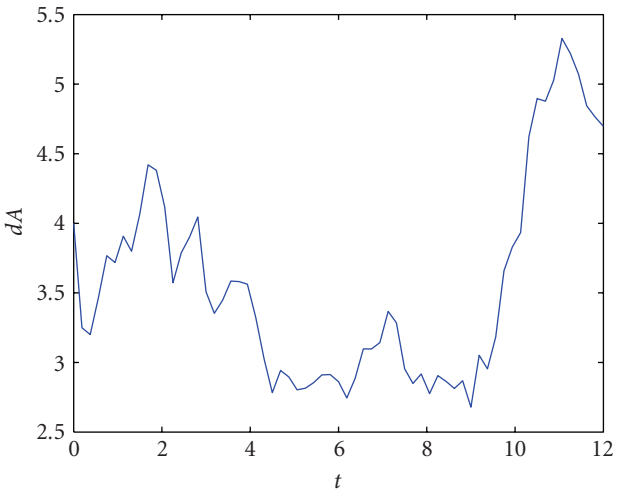

Figure 4.1. Trajectories of simulated ROA.

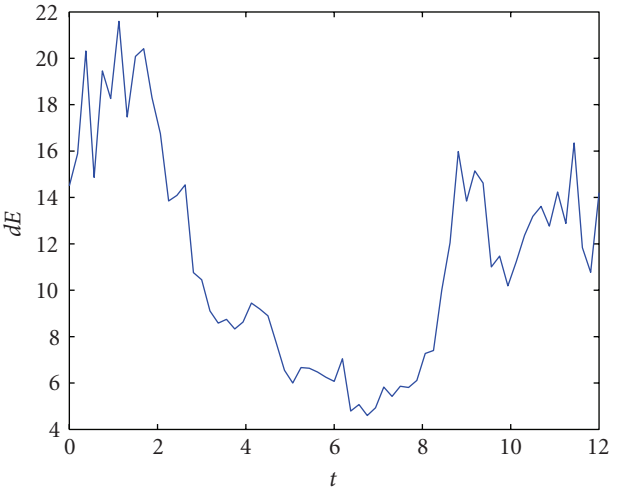

Figure 4.2. Trajectories of simulated ROE.

4.2.1. Statement of the optimization problem. Problem 3.1 (see, also, Problem 1.2) addresses issues in bank operations that are related to the optimal implementation of financial economic principles. The parameter δ is an idiosyncratic discount rate that is not a market parameter, but rather a part of the utility functional. Note that an assumption is that $U^{(1)}$ and $U^{(2)}$ are additively separable which is not necessarily true for all banks.

4.2.2. Solution of the optimization problem. The boundary condition

$$\lim_{s\to\infty} \mathbf{E}\left[\exp\left\{-\int_t^s (\delta+\eta^p(u))\,du\right\} V(\Pi_s^*, s) \mid \Pi_t^* = \pi\right] = 0 \tag{4.5}$$

emanates from the work of Merton in [32]. The contributions [33, 34] provide a proof to Theorem 3.2 based on a martingale approach. In order to determine an exact solution for our optimization problem in Theorem 3.2, we are required to make a specific choice for the utilities $U^{(1)}$ and $U^{(2)}$. Essentially, these utilities can be almost any function involving k and π, respectively. However, in order to obtain smooth analytic solutions to the maximization problem in the ensuing discussion, we choose power, logarithmic, and exponential utility functions and analyze the effect of the different choices.

From the discussion in Section 3.2.2, we may conclude that the optimal loan investment strategy does not depend on the parameters of the loan loss. Also, the optimal cost of provisioning is independent of the price process for the loan. The strategies related to the optimal cost of provisioning and loan allocation are not reliant on the inclination towards bankruptcy, η^p, and thus also independent of the bank's horizon. However, the value function, V, is driven by the horizon through the reliance of f and G on η^p. At this stage, it is not absolutely clear how this impacts the banking dynamics.

Section 3.2.3 suggests that the optimal cost of provisioning depends on the parameters of the loan price process and on the Poisson frequency parameter, ϕ. In addition,c^{p*}

depends on the bank's horizon via its reliance on the inclination towards bankruptcy, η^p. A number of interesting properties of the optimal cost of provisioning can be discerned. For instance, we have that

$$\frac{\partial c^{p*}}{\partial \theta} > 0, \qquad \frac{\partial c^{p*}}{\partial \varrho} > 0, \qquad \frac{\partial c^{p*}}{\partial \delta} > 0, \qquad \frac{\partial c^{p*}}{\partial \phi} < 0, \qquad \frac{\partial c^{p*}}{\partial \mu^a} > 0. \tag{4.6}$$

The first inequality implies that as the expense associated with provisioning increases (θ increases), the demand for provisioning decreases (c^{p*} increases). Also, the second inequality intimates that as the credit risk increases (ϱ decreases), the demand for provisioning increases (c^{p*} decreases). Moreover, the third inequality suggests that as the value of future profit decreases (δ increases), the demand for provisioning decreases (c^{p*} increases). The penultimate inequality implies that as the frequency, ϕ, of the loan losses increases, the demand for provisioning increases (c^{p*} decreases). The final inequality implies that as the auxiliary profits increase (μ^a increases), the demand for provisioning decreases (c^{p*} increases).

The optimal provisioning cost, depository consumption, and investment in loans, obtained in Theorem 3.6 of Section 3.2.4, can each be represented as a linear function of optimal profit. Another observation is that $\overline{\xi}$, and hence $\overline{V}$ from (3.31), and k^* in (3.36) are affected explicitly by the inclination towards bankruptcy η^p. Also, c^{P*} and λ^* given by (3.35) and (3.36), respectively, are influenced by the inclination towards bankruptcy via the impact of k^*s on profit. Note also that the optimal provisioning costs, c^{P*}, are not reliant on the loan price parameters, and the optimal loan investment, λ^*, does not depend on loan losses.

For (3.41) in Corollary 3.8, we define the provisions which the bank is compensated with in case it faces bankruptcy at a random time τ with the rate of discount equal to δ. In a special case of constant rate of inclination towards bankruptcy, η^p, we have that

$$\widetilde{\xi}(t)^{-1} = \frac{\eta^p + \delta}{1 + b\eta^p}. \tag{4.7}$$

Thus if $\delta b < 1$, we have that $\widetilde{\xi}^{-1}$ increases with η^p. As a consequence, it appears that a bank with a longer horizon consumes a smaller proportion of profit.

5. Concluding remarks

In our contribution, we solve a stochastic maximization problem that is related to depository consumption and banking profit on a random time interval. In particular, we demonstrate that a bank is able to maximize its expected utility of discounted depository consumption on a random time interval, $[t, \tau]$, and its final profit at time τ. Here, the associated Hamilton-Jacobi-Bellman (HJB) equation has a smooth solution whenthe

optimal controls are computed by means of power, logarithmic, and exponential utility functions. This enables us to make a direct comparison between the economic properties of the solutions for different choices of utility function. By way of conclusion, we provide an analysis of the economic aspects of the banking modeling and optimization discussed in the main body of the paper. A feature of our approach throughout is that we can incorporate inherent cyclical effects in credit prices, risk-weightings, provisioning, profitability, and capital in the modeling of the aforementioned items.

Current research entails constructing dynamic models of bank items driven by general processes that deviate from the classical Black-Scholes models. An example of such processes is the Lévy process (example of a semimartingale) that facilitates the characterization of the dynamics of noncontinuous economic and financial indices more accurately. In keeping with this new thrust of research, recent investigations have substituted the Brownian motion-based bank models (see, e.g., [6, 9, 12, 35]) by systems driven by such processes. Also, further studies on risk management within a Basel II regulatory framework (see, e.g., [26, 36]) are sorely needed. These effects are not fully recognized in our contribution and require further attention.

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