Research Article

A Note on Some Properties of the Weighted q -Genocchi Numbers and Polynomials

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We consider the weighted q -Genocchi numbers and polynomials. From the construction of the weighted q -Genocchi numbers and polynomials, we investigate many interesting identities and relations satisfied by these new numbers and polynomials.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, $\mathbb{C}$, and $\mathbb{C}_p$, will, respectively, denote the ring of p -adic integers, the field, of p -adic rational numbers, the complex number field and the completion of algebraic closure of $\mathbb{Q}_p$. Let v_p be the normalized exponential valuation of $\mathbb{C}_p$ such that $|p|_p = p^{-v_p(p)} = 1/p$ (see [1–16]).

As well-known definition, the Euler numbers and Genocchi numbers are defined by

$$\frac{2}{e^t + 1} = e^{Et} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \quad (1.1)$$

with the usual convention of replacing E^n by E_n and

$$\frac{2t}{e^t + 1} = e^{Gt} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \quad (1.2)$$

with the usual convention of replacing G^n by G_n . We assume that $q \in \mathbb{C}_p$ with $|1 - q|_p < 1$ and that the q -number of x is defined by

$$[x]_q = \frac{1 - q^x}{1 - q}, \quad [x]_{-q} = \frac{1 - (-q)^x}{1 + q} \quad (1.3)$$

(see [1–19]).

In [9], Kim introduced ordinary fermionic p -adic integral on $\mathbb{Z}_p$, and he studied some interesting relations and identities related to q -extension of Euler numbers and polynomials. In [8], he also introduced the q -extension of the ordinary fermionic p -adic integral on $\mathbb{Z}_p$ and he investigated many physical properties related to q -Euler numbers and polynomials. Recently, Kim firstly introduced the meaning of the weighted q -Euler numbers and polynomials associated with the weighted q -Bernstein polynomials by using the fermionic invariant p -adic integral on $\mathbb{Z}_p$ (see [14, 15]). In [16], Ryoo tried to study the weighted q -Euler number and polynomials by the same method of Kim et al. in [14] and the q -extension of the fermionic p -adic invariant integrals on $\mathbb{Z}_p$. As well-known properties, the Genocchi numbers are integers. The first few Genocchi numbers for $n = 2, 4, \dots$ are $-1, 1, -3, 17, -155, 2073, \dots$. The first few prime Genocchi numbers are -3 and 17 , which occur for $n = 6$ and 8 . There are no others with $n < 10^5$. These properties are very important to study in the area of fermionic distribution and p -adic numbers theory. By this reason, many mathematicians and physicians have studied Genocchi and Euler numbers which are in the different areas. By the same motivation, we consider weighted q -Genocchi polynomials and numbers by using the fermionic p -adic q -integral on $\mathbb{Z}_p$ which are constructed by Kim and Ryoo (cf. [8, 16]).

In this paper, we consider the q -Genocchi numbers and polynomials with weighted α ($\alpha \in \mathbb{Q}$). From the construction of the weighted q -Genocchi numbers and polynomials, we investigate many interesting identities and relations satisfied by these new numbers and polynomials.

2. The Weighted q -Genocchi Numbers and Polynomials

Let $\text{UD}(\mathbb{Z}_p)$ be the space of uniformly differentiable functions and, for $f \in \text{UD}(\mathbb{Z}_p)$, the fermionic p -adic invariant integral of f on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-1}(x) = \lim_{N \rightarrow \infty} \frac{1}{[p^N]_{-1}} \sum_{x=0}^{p^N-1} f(x) (-1)^x \quad (2.1)$$

(see [1–16]). If we take $f(x) = te^{xt}$, then we get

$$t \int_{\mathbb{Z}_p} e^{xt} d\mu_{-1}(x) = \frac{2t}{e^t + 1}. \quad (2.2)$$

By (1.2) and (2.2), we get

$$\begin{aligned}
 \sum_{n=0}^{\infty} G_n \frac{t^n}{n!} &= \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) \frac{t^{n+1}}{n!} \\
 &= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) \frac{t^{n+1}}{(n+1)!} \\
 &= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} x^{n-1} d\mu_{-1}(x) \frac{t^n}{n!}.
 \end{aligned} \tag{2.3}$$

From (2.3),

$$G_0 = 0, \quad \frac{G_n}{n} = \int_{\mathbb{Z}_p} x^{n-1} d\mu_{-1}(x), \quad n \in \mathbb{N}. \tag{2.4}$$

For $f \in \text{UD}(\mathbb{Z}_p)$, the fermionic p -adic q -integral of f on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-q}(x) = \lim_{N \rightarrow \infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x) (-q)^x \tag{2.5}$$

(see [1–16]). From (2.5), we note that

$$q^n I_{-q}(f_n) = (-1)^n I_{-q}(f) + [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l f(l), \tag{2.6}$$

where $n \in \mathbb{N}$ and $f_n(x) = f(x+n)$.

For $\alpha \in \mathbb{Q}$, we consider the following fermionic p -adic q -integral on $\mathbb{Z}_p$:

$$t \int_{\mathbb{Z}_p} e^{[x]_{q^\alpha} t} d\mu_{-q}(x) = \sum_{n=0}^{\infty} \tilde{G}_{n,q}^{(\alpha)} \frac{t^n}{n!}, \tag{2.7}$$

where $\tilde{G}_{n,q}^{(\alpha)}$ are called the n th q -Genocchi numbers with weight α . From (2.7), we get

$$\begin{aligned}
 t \int_{\mathbb{Z}_p} e^{[x]_{q^\alpha} t} d\mu_{-q}(x) &= t \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} [x]_{q^\alpha}^n d\mu_{-q}(x) \frac{t^n}{n!} \\
 &= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} [x]_{q^\alpha}^n d\mu_{-q}(x) \frac{t^{n+1}}{(n+1)!} \\
 &= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \frac{t^n}{n!}.
 \end{aligned} \tag{2.8}$$

By comparing the coefficients on the both sides of (2.7) and (2.8), we get

$$n \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) = \tilde{G}_{n,q}^{(\alpha)}, \quad n \in \mathbb{N}, \quad \tilde{G}_{0,q}^{(\alpha)} = 0. \quad (2.9)$$

From (2.9), we obtain the following theorem.

Theorem 2.1. For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has

$$\int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) = \frac{\tilde{G}_{n,q}^{(\alpha)}}{n}, \quad \tilde{G}_{0,q}^{(\alpha)} = 0. \quad (2.10)$$

By the definition of fermionic p -adic q -integrals, we get

$$\begin{aligned} t \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) &= \frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \int_{\mathbb{Z}_p} q^{\alpha l x} d\mu_{-q}(x) \\ &= \frac{[2]_q}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \frac{1}{1+q^{\alpha l+1}}. \end{aligned} \quad (2.11)$$

Therefore, we obtain the following theorem.

Theorem 2.2. For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, we have

$$\frac{\tilde{G}_{n,q}^{(\alpha)}}{n} = \frac{[2]_q}{(1-q)^{n-1} [\alpha]_q^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \frac{1}{1+q^{\alpha l+1}}. \quad (2.12)$$

By Theorem 2.2, we have the generating function of $\tilde{G}_{n,q}^{(\alpha)}$ as follows:

$$\begin{aligned} \sum_{n=0}^{\infty} \tilde{G}_{n,q}^{(\alpha)} \frac{t^n}{n!} &= [2]_q \sum_{n=0}^{\infty} \frac{n}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \sum_{m=0}^{\infty} (-1)^m q^{\alpha l m + m} \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} \frac{n}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha l m} \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} \frac{n}{(1-q^\alpha)^{n-1}} (1-q^{\alpha m})^{n-1} \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} n [m]_{q^\alpha}^{n-1} \frac{t^n}{n!} \end{aligned}$$

$$\begin{aligned}
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=1}^{\infty} [m]_{q^\alpha}^{n-1} \frac{t^n}{(n-1)!} \\
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} [m]_{q^\alpha}^n \frac{t^{n+1}}{n!} \\
&= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[m]_{q^\alpha} t}.
\end{aligned} \tag{2.13}$$

Let $\tilde{F}_q^{(\alpha)}(t)$ be the generating function of $\tilde{G}_{n,q}^{(\alpha)}$. Then, by (2.9) and (2.13), we get

$$\begin{aligned}
\tilde{F}_q^{(\alpha)}(t) &= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[m]_{q^\alpha} t} \\
&= \sum_{n=0}^{\infty} \tilde{G}_{n,q}^{(\alpha)} \frac{t^n}{n!}.
\end{aligned} \tag{2.14}$$

The q -Genocchi polynomials with weight α are defined by

$$\begin{aligned}
\tilde{F}_q^{(\alpha)}(t, x) &= t \int_{\mathbb{Z}_p} e^{[x+y]_{q^\alpha} t} d\mu_{-q}(y) \\
&= \sum_{n=0}^{\infty} \tilde{G}_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}.
\end{aligned} \tag{2.15}$$

From (2.15), we get

$$\begin{aligned}
t \int_{\mathbb{Z}_p} e^{[x+y]_{q^\alpha} t} d\mu_{-q}(y) &= \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^n d\mu_{-q}(y) \frac{t^{n+1}}{n!} \\
&= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^n d\mu_{-q}(y) \frac{t^{n+1}}{(n+1)!} \\
&= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) \frac{t^n}{n!}.
\end{aligned} \tag{2.16}$$

By (2.15) and (2.16), we obtain the following theorem.

Theorem 2.3. For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has

$$n \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) = \tilde{G}_{n,q}^{(\alpha)}(x), \quad \tilde{G}_{0,q}^{(\alpha)}(x) = 0. \tag{2.17}$$

We note that

$$\begin{aligned} \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) &= \sum_{l=0}^{n-1} \binom{n-1}{l} [x]_{q^\alpha}^{n-1-l} q^{alx} \int_{\mathbb{Z}_p} [y]_{q^\alpha}^l d\mu_{-q}(y) \\ &= \sum_{l=0}^{n-1} \binom{n-1}{l} [x]_{q^\alpha}^{n-1-l} q^{alx} \frac{\tilde{G}_{l+1,q}^{(\alpha)}}{l+1}. \end{aligned} \quad (2.18)$$

From (2.17) and (2.18), we obtain the following theorem.

Theorem 2.4. For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has

$$\frac{\tilde{G}_{n,q}^{(\alpha)}(x)}{n} = \sum_{l=0}^{n-1} \binom{n-1}{l} [x]_{q^\alpha}^{n-1-l} q^{alx} \frac{\tilde{G}_{l+1,q}^{(\alpha)}}{l+1}. \quad (2.19)$$

From (2.15), we note that

$$\begin{aligned} \tilde{F}_q^{(\alpha)}(t, x) &= \sum_{n=0}^{\infty} \tilde{G}_{n,q}^{(\alpha)}(x) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} n \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} n \left(\frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} q^{alx} (-1)^l \int_{\mathbb{Z}_p} q^{aly} d\mu_{-q}(y) \right) \frac{t^n}{n!} \\ &= [2]_q \sum_{n=0}^{\infty} n \left(\frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} q^{alx} \frac{(-1)^l}{1+q^{al+1}} \right) \frac{t^n}{n!} \\ &= [2]_q \sum_{n=0}^{\infty} n \left(\frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} q^{alx} (-1)^l \sum_{m=0}^{\infty} (-1)^m q^{alm+m} \right) \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} n \left(\frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha(x+m)l} \right) \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} n \left(\frac{1}{(1-q^\alpha)^{n-1}} (1-q^{\alpha(x+m)})^{n-1} \right) \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} n [x+m]_{q^\alpha}^{n-1} \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=1}^{\infty} [x+m]_{q^\alpha}^{n-1} \frac{t^n}{(n-1)!} \end{aligned}$$

$$\begin{aligned}
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} [x+m]_{q^\alpha}^n \frac{t^{n+1}}{n!} \\
&= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[x+m]_{q^\alpha} t}.
\end{aligned} \tag{2.20}$$

Therefore, we obtain the following theorem.

Theorem 2.5. For $\alpha \in \mathbb{Q}$, one has

$$\tilde{F}_q^{(\alpha)}(t, x) = [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[x+m]_{q^\alpha} t}. \tag{2.21}$$

From (2.15) and (2.21), we obtain that

$$\begin{aligned}
\tilde{G}_{n,q}^{(\alpha)}(x) &= \left. \frac{d^n}{dt^n} \tilde{F}_q^{(\alpha)}(t, x) \right|_{t=0} \\
&= n[2]_q \sum_{m=0}^{\infty} (-1)^m q^m [x+m]_{q^\alpha}^{m-1} \\
&= n[2]_q \frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \frac{\binom{n-1}{l} q^{\alpha l x} (-1)^l}{1+q^{\alpha l+1}} \\
&= \frac{n[2]_q}{(1-q)^{n-1} [\alpha]_q^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha l x} \frac{1}{1+q^{\alpha l+1}}.
\end{aligned} \tag{2.22}$$

Therefore, we obtain the following theorem.

Theorem 2.6. For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has

$$\tilde{G}_{n,q}^{(\alpha)}(x) = \frac{n[2]_q}{(1-q)^{n-1} [\alpha]_q^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} \frac{(-1)^l q^{\alpha l x}}{1+q^{\alpha l+1}}. \tag{2.23}$$

From (2.6), if we take $f(x) = [x]_{q^\alpha}^m = ((1-q^{\alpha x})/(1-q^\alpha))^m$, then we get

$$q^n \int_{\mathbb{Z}_p} [x+n]_{q^\alpha}^m d\mu_{-q}(x) = (-1)^n \int_{\mathbb{Z}_p} [x]_{q^\alpha}^m d\mu_{-q}(x) + [2]_q \sum_{l=0}^{n-1} (-1)^l q^l [l]_{q^\alpha}^m. \tag{2.24}$$

By (2.17) and (2.24), we obtain the following theorem.

Theorem 2.7. For $n \in \mathbb{N}$, $m \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}$, and $\alpha \in \mathbb{Q}$, one has

$$q^n \frac{\tilde{G}_{m+1,q}^{(\alpha)}(n)}{m+1} = (-1)^n \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1} + [2]_q \sum_{l=0}^{n-1} (-1)^l q^l [l]_{q^\alpha}^m. \quad (2.25)$$

We remark that if we take $n = 2s$ ($s \in \mathbb{Z}_+$) in Theorem 2.7, then we have

$$q^{2s} \frac{\tilde{G}_{m+1,q}^{(\alpha)}(2s)}{m+1} = \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1} + [2]_q \sum_{l=0}^{2s-1} (-1)^l q^l [l]_{q^\alpha}^m \quad (2.26)$$

and if we take $n = 2s + 1$ ($s \in \mathbb{Z}_+$) in Theorem 2.7, then we have

$$q^{2s+1} \frac{\tilde{G}_{m+1,q}^{(\alpha)}(2s+1)}{m+1} + \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1} = [2]_q \sum_{l=0}^{2s} (-1)^l q^l [l]_{q^\alpha}^m. \quad (2.27)$$

From (2.27) with $s = 0$, we obtain the following corollary.

Corollary 2.8. For $\alpha \in \mathbb{Q}$ and $m \in \mathbb{Z}_+$, one has

$$q \frac{\tilde{G}_{m+1,q}^{(\alpha)}(1)}{m+1} + \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1} = \begin{cases} [2]_q & \text{if } m = 0, \\ 0 & \text{if } m > 0. \end{cases} \quad (2.28)$$

From (2.19), we note that

$$\begin{aligned} \frac{\tilde{G}_{m+1,q}^{(\alpha)}(n)}{m+1} &= \sum_{l=0}^m \binom{m}{l} [x]_{q^\alpha}^{m-l} \frac{\tilde{G}_{l+1,q}^{(\alpha)}}{l+1} q^{alx} \\ &= \frac{1}{m+1} \sum_{l=0}^m \binom{m+1}{l+1} [x]_{q^\alpha}^{m-l} \tilde{G}_{l+1,q}^{(\alpha)} q^{alx} \\ &= \frac{1}{m+1} \sum_{l=1}^m \binom{m+1}{l} [x]_{q^\alpha}^{m+1-l} \tilde{G}_{l,q}^{(\alpha)} q^{a(l-1)x} \\ &= \frac{1}{q^\alpha} \frac{1}{m+1} \sum_{l=0}^m \binom{m+1}{l} [x]_{q^\alpha}^{m+1-l} \tilde{G}_{l,q}^{(\alpha)} q^{alx}. \end{aligned} \quad (2.29)$$

From (2.29), we get

$$\begin{aligned} q^\alpha \tilde{G}_{m+1,q}^{(\alpha)}(x) &= \sum_{l=0}^{m+1} \binom{m+1}{l} [x]_{q^\alpha}^{m+1-l} \tilde{G}_{l,q}^{(\alpha)} q^{alx} \\ &= \left([x]_{q^\alpha} + q^{\alpha x} \tilde{G}_q^{(\alpha)} \right)^{m+1}, \end{aligned} \quad (2.30)$$

with the usual convention about replacing $(\tilde{G}_q^{(\alpha)})^n$ by $\tilde{G}_{n,q}^{(\alpha)}$. By (2.28) and (2.30), we get

$$\frac{q^{1-\alpha} q^\alpha \tilde{G}_{m+1,q}^{(\alpha)}(1)}{m} + \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1} = \frac{q^{1-\alpha} (1 + q^\alpha \tilde{G}_q^{(\alpha)})^{m+1}}{m+1} + \frac{\tilde{G}_{m+1,q}^{(\alpha)}}{m+1}. \quad (2.31)$$

From (2.28) and (2.31), we obtain the following theorem.

Theorem 2.9. For $\alpha \in \mathbb{Q}$ and $m \in \mathbb{Z}_+$, one has

$$q^{1-\alpha} (1 + q^\alpha \tilde{G}_q^{(\alpha)})^{m+1} + \tilde{G}_{m+1,q}^{(\alpha)} = \begin{cases} [2]_q & \text{if } m = 0, \\ 0 & \text{if } m > 0. \end{cases} \quad (2.32)$$

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