Research Article

Stability and Superstability of Generalized \((\theta, \phi)\)-Derivations in Non-Archimedean Algebras: Fixed Point Theorem via the Additive Cauchy Functional Equation

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Let \(A\) be an algebra, and let \(\theta, \phi\) be ring automorphisms of \(A\). An additive mapping \(H : A \to A\) is called a \((\theta, \phi)\)-derivation if \(H(xy) = H(x)\theta(y) + \phi(x)H(y)\) for all \(x, y \in A\). Moreover, an additive mapping \(F : A \to A\) is said to be a generalized \((\theta, \phi)\)-derivation if there exists a \((\theta, \phi)\)-derivation \(H : A \to A\) such that \(F(xy) = F(x)\theta(y) + \phi(x)H(y)\) for all \(x, y \in A\). In this paper, we investigate the superstability of generalized \((\theta, \phi)\)-derivations in non-Archimedean algebras by using a version of fixed point theorem via Cauchy’s functional equation.

1. Introduction and Preliminaries

In 1897, Hensel [1] has introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications [2, 3].

A non-Archimedean field is a field \(\mathbb{K}\) equipped with a function (valuation) \(|\cdot|\) from \(\mathbb{K}\) into \([0, \infty]\) such that \(|r| = 0\) if and only if \(r = 0\), \(|rs| = |r||s|\), and \(|r + s| \leq \max\{|r|, |s|\}\) for all \(r, s \in \mathbb{K}\). An example of a non-Archimedean valuation is the mapping \(|\cdot|\) taking everything but 0 into 1 and \(|0| = 0\). This valuation is called trivial (see [4]).
Definition 1.1. Let $X$ be a vector space over a scalar field $K$ with a non-Archimedean non-trivial valuation $|·|$. A function $∥·∥ : X → \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

- (NA1) $∥x∥ = 0$ if and only if $x = 0$,
- (NA2) $∥rx∥ = |r||x∥$ for all $r ∈ K$ and $x ∈ X$,
- (NA3) $∥x + y∥ ≤ \max\{∥x∥, ∥y∥\}$ for all $x, y ∈ X$ (the strong triangle inequality).

A sequence $\{x_m\}$ in a non-Archimedean space is Cauchy’s if and only if $\{x_{m+1} - x_m\}$ converges to zero. By a complete non-Archimedean space, we mean one in which every Cauchy’s sequence is convergent. A non-Archimedean-normed algebra is a non-Archimedean-normed space $A$ with a linear associative multiplication, satisfying $∥xy∥ ≤ ∥x∥∥y∥$ for all $x, y ∈ A$. A non-Archimedean complete normed algebra is called a non-Archimedean Banach’s algebra (see [5]).

Definition 1.2. Let $X$ be a nonempty set, and let $d : X × X → [0, ∞]$ satisfy the following properties:

- (D1) $d(x, y) = 0$ if and only if $x = y$,
- (D2) $d(x, y) = d(y, x)$ (symmetry),
- (D3) $d(x, z) ≤ \max\{d(x, y), d(y, z)\}$ (strong triangle inequality),

for all $x, y, z ∈ X$. Then $(X, d)$ is called a non-Archimedean generalized metric space. $(X, d)$ is called complete if every $d$-Cauchy’s sequence in $X$ is $d$-convergent.

Definition 1.3. Let $A$ be a non-Archimedean algebra, and let $θ, φ$ be ring automorphisms of $A$. An additive mapping $H : A → A$ is called a $(θ, φ)$-derivation in case $H(xy) = H(x)θ(y) + φ(x)H(y)$ holds for all $x, y ∈ A$. An additive mapping $F : A → A$ is said to be a generalized $(θ, φ)$-derivation if there exists a $(θ, φ)$-derivation $H : A → A$ such that

$$F(xy) = F(x)θ(y) + φ(x)H(y)$$ (1.1)

for all $x, y ∈ A$.

We need the following fixed point theorem (see [6, 7]).

Theorem 1.4 (Non-Archimedean Alternative Contraction Principle). Suppose $(X, d)$ is a non-Archimedean generalized complete metric space and $Λ : X → X$ is a strictly contractive mapping; that is,

$$d(Λx, Λy) ≤ Ld(x, y) \quad (x, y ∈ X)$$ (1.2)

for some $L < 1$. If there exists a nonnegative integer $k$ such that $d(Λ^{k+1} x, Λ^k x) < ∞$ for some $x ∈ X$, then the followings are true.

(a) The sequence $\{Λ^n x\}$ converges to a fixed point $x^*$ of $Λ$.
(b) $x^*$ is a unique fixed point of $Λ$ in

$$X^* = \{y ∈ X | d(Λ^k x, y) < ∞\}.$$ (1.3)
(c) If \( y \in X^* \), then

\[
d(y, x^*) \leq d(\Lambda y, y).
\]

(1.4)

A functional equation \((\xi)\) is **superstable** if every approximately solution of \((\xi)\) is an exact solution of it.

The stability of functional equations was first introduced by Ulam [8] during his talk before a mathematical colloquium at the University of Wisconsin in 1940. In 1941, Hyers [9] gave a first affirmative answer to the question of Ulam for Banach spaces. In 1978, Rassias [10] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy’s differences \( \|f(x + y) - f(x) - f(y)\| \leq \epsilon (\|x\|^p + \|y\|^p) \), \((\epsilon > 0, p \in [0, 1])\). Moreover, John Rassias [11–13] investigated the stability of some functional equations when the control function is the product of powers of norms. In 1991, Gajda [14] answered the question for the case \( p > 1 \), which was raised by Rassias. This new concept is known as the Hyers-Ulam-Rassias or the generalized Hyers-Ulam stability of functional equations ([11–13, 15–35]).

In 1992, Gavruta [36] generalized the Rassias theorem as follows.

Suppose \((G, +)\) is an abelian group, \( X \) is a Banach space, \( \varphi : G \times G \rightarrow [0, \infty) \) satisfies

\[
\varphi(x, y) \leq \frac{1}{2} \sum_{n=0}^{\infty} 2^{-n} \varphi(2^n x, 2^n y) < \infty,
\]

for all \( x, y \in G \). If \( f : G \rightarrow X \) is a mapping with

\[
\|f(x + y) - f(x) - f(y)\| \leq \varphi(x, y),
\]

(1.6)

for all \( x, y \in G \), then there exists a unique mapping \( T : G \rightarrow X \) such that \( T(x+y) = T(x) + T(y) \) and \( \|f(x) - T(x)\| \leq \varphi(x, x) \) for all \( x, y \in G \).

In 1949, Bourgin [37] proved the following result, which is sometimes called the superstability of ring homomorphisms: suppose that \( A \) and \( B \) are Banach algebras with unit. If \( f : A \rightarrow B \) is a surjective mapping such that

\[
\|f(x + y) - f(x) - f(y)\| \leq \epsilon,
\]

\[
\|f(xy) - f(x)f(y)\| \leq \delta,
\]

(1.7)

for some \( \epsilon \geq 0, \delta \geq 0 \) and for all \( x, y \in A \), then \( f \) is a ring homomorphism.

The first superstability result concerning derivations between operator algebras was obtained by Šemrl in [38]. Badara [39] proved the superstability of the functional equation \( f(xy) = xf(y) + f(x)y \), where \( f \) is a mapping on normed algebra \( A \) with unit. Ansari-Piri and Anjidani [40] discussed the superstability of generalized derivations on Banach’s algebras. Recently, Eshaghi Gordji et al. [41] investigated the stability and superstability of higher ring derivations on non-Archimedean Banach’s algebras (see also [42]). In this paper, we investigate the superstability of generalized \((\theta, \phi)\)-derivations on non-Archimedean Banach algebras by using the fixed point methods.
2. Non-Archimedean Superstability of Generalized (θ, ϕ)-Derivations

In this paper, we assume that A is a non-Archimedean Banach’s algebra, with unit over a non-Archimedean field K, and θ, ϕ are ring automorphisms of A.

**Theorem 2.1.** Let φ, ψ : A × A → [0, ∞) be functions. Suppose that f : A → A is a mapping such that

\[
\|f(x + y) - f(x) - f(y)\| \leq \varphi(x, y), \tag{2.1}
\]

\[
\|f(xy) - f(x)\theta(y) - \phi(x)g(y)\| \leq \psi(x, y), \tag{2.2}
\]

for all x, y ∈ A. If there exist constants K, L < 1 and a natural number k ∈ ℤ,

\[
|k|^{-1}\varphi(kx, ky) \leq L\varphi(x, y), \quad |k|^{-1}\varphi(kx, y), \quad |k|^{-1}\varphi(x, ky) \leq K\psi(x, y), \tag{2.3}
\]

for all x, y ∈ A, then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

**Proof.** By induction on i, we prove that for each n ∈ ℤ₀, for all x ∈ A and i ≥ 2,

\[
\|f(ix) - if(x)\| \leq \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \ldots, \varphi((i - 1)x, x)\}. \tag{2.4}
\]

Let x = y in (2.1), then

\[
\|f(2x) - 2f(x)\| \leq \max\{\varphi(0, 0), \varphi(x, x)\}, \quad n ∈ ℤ₀, \ x ∈ A. \tag{2.5}
\]

This proves (2.4) for i = 2. Let (2.4) hold for i = 1, 2, ..., j. Replacing x by jx and y by x in (2.1) for each n ∈ ℤ₀, and for all x ∈ A, we get

\[
\|f((j + 1)x) - f(jx) - f(x)\| \leq \max\{\varphi(0, 0), \varphi(jx, x)\}. \tag{2.6}
\]

Since

\[
f((j + 1)x) - f(jx) - f(x) = f((j + 1)x) - (j + 1)f(x) + (j + 1)f(x) - f(jx) - f(x)
\]

\[= f((j + 1)x) - (j + 1)f(x) + jf(x) - f(jx), \tag{2.7}
\]

for all x ∈ A, it follows from induction hypothesis and (2.6) that, for all x ∈ A,

\[
\|f((j + 1)x) - (j + 1)f(x)\| \leq \max\{\|f((j + 1)x) - f(jx) - f(x)\|, \|jf(x) - f(jx)\|\}
\]

\[\leq \max\{\varphi(0, 0), \varphi(x, x), \varphi(2x, x), \ldots, \varphi((j)x, x)\}. \tag{2.8}
\]
This proves (2.4) for all \( i \geq 2 \). In particular, for all \( x \in A \),
\[
\| f(kx) - kf(x) \| \leq \Phi(x),
\]  
(2.9)
where
\[
\Phi(x) = \max \{ \varphi(0,0), \varphi(x,x), \varphi(2x,x), \ldots, \varphi((k-1)x,x) \} \quad (x \in A).
\]  
(2.10)
Let us define a set \( X \) of all functions \( r : A \to A \) by
\[
X = \{ r : A \to A \}
\]  
(2.11)
and introduce \( d \) on \( X \) as follows:
\[
d(r,s) = \inf \{ \alpha > 0 : \| r(x) - s(x) \| \leq \alpha \Phi(x) \forall x \in A \}.
\]  
(2.12)
It is easy to see that \( d \) defines a generalized complete metric on \( X \). Define \( J : X \to X \) by
\[
J(r)(x) = k^{-1}r(kx).
\]  
Then \( J \) is strictly contractive on \( X \), in fact if
\[
\| r(x) - s(x) \| \leq \alpha \Phi(x) \quad (x \in A),
\]  
(2.13)
then, by (2.3),
\[
\| J(r)(x) - J(s)(x) \| = |k|^{-1}\| r(kx) - s(kx) \| \leq \alpha |k|^{-1} \Phi(kx) \leq L \Phi(x) \quad (x \in A).
\]  
(2.14)
It follows that
\[
d(J(r), J(s)) \leq Ld(r,s) \quad (g, h \in X).
\]  
(2.15)
Hence, \( J \) is strictly contractive mapping with the Lipschitz constant \( L \). By (2.9),
\[
\| (Jf)(x) - f(x) \| = \| k^{-1}f(kx) - f(x) \|,
\]  
(2.16)
\[
|k|^{-1}\| f(kx) - kf(x) \| \leq |k|^{-1} \Phi(x) \quad (x \in A).
\]
This means that \( d(J(f), f) \leq 1/|k| \). By Theorem 1.4, \( J \) has a unique fixed point \( h : A \to A \) in the set
\[
U = \{ r \in X : d(r, J(f)) < \infty \},
\]  
(2.17)
and, for each \( x \in A \),
\[
h(x) = \lim_{m \to \infty} f^m(kx) = \lim_{m \to \infty} k^{-m}f(k^m x).
\]  
(2.18)
Therefore, each \( x, y \in A \),
\[
\| h(x + y) - h(x) - h(y) \| = \lim_{m \to \infty} |k|^m \| f(k^m(x + y)) - f(k^m x) - f(k^m y) \|
\]
\[
\leq \lim_{m \to \infty} |k|^m \max \{ \varphi(0,0), \varphi(k^n x, k^n y) \}
\]
\[
\leq \lim_{m \to \infty} L^n \varphi(x, y) = 0.
\]

This shows that \( h \) is additive.
Replacing \( y \) by \( k^n y \) in (2.2), we get
\[
\| f(k^n xy) - f(x) \theta(k^n y) - \phi(x) g(k^n y) \| \leq \varphi(x, k^n y),
\]
(2.20)
and so
\[
\left\| f\left(\frac{k^n xy}{k^n}\right) - f(x) \theta(y) - \phi(x) \frac{g(k^n y)}{k^n} \right\| \leq \frac{1}{|k|^n} \varphi(x, k^n y) \leq K^n \varphi(x, y),
\]
(2.21)
for all \( x, y \in A \) and each \( n \in \mathbb{N} \). By taking \( n \to \infty \), we have
\[
h(xy) = f(x) \theta(y) + \lim_{n \to \infty} \phi(x) \frac{g(k^n y)}{k^n}.
\]
(2.22)
for all \( x, y \in A \).
Fix \( m \in \mathbb{N} \). By (2.22), we have
\[
f(k^m x) \theta(y) = h(k^m xy) - \lim_{n \to \infty} \phi(k^m x) \left( \frac{g(k^n y)}{k^n} \right)
\]
\[
= f(x) \theta(k^m y) + \lim_{n \to \infty} \phi(x) \left( \frac{g(k^n k^m x)}{k^n} \right) - k^m \lim_{n \to \infty} \phi(x) \left( \frac{g(k^n x)}{k^n} \right)
\]
\[
= k^m f(x) \theta(y) + k^m \lim_{n \to \infty} \phi(x) \left( \frac{g(k^n x)}{k^n} \right) - k^m \lim_{n \to \infty} \phi(x) \left( \frac{g(k^n x)}{k^n} \right)
\]
\[
= k^m f(x) \theta(y),
\]
(2.23)
for all \( x, y \in A \). Then \( f(x) \theta(y) = (f(k^m x)/k^m) \theta(y) \) for all \( x, y \in A \) and each \( m \in \mathbb{N} \), and so, by taking \( m \to \infty \), we have \( f(x) \theta(y) = h(x) \theta(x) \). Now we obtain \( h = f \), since \( A \) is with unit. Replacing \( x \) by \( k^n x \) in (2.2), we obtain
\[
\| f(k^n xy) - f(k^n x) \theta(y) - \phi(k^n x) g(y) \| \leq \varphi(k^n x, y),
\]
(2.24)
and; hence,

\[
\left\| \frac{f(k^nxy)}{k^n} - \frac{f(k^n)}{k^n} \theta(y) - \phi(x) g(y) \right\| \leq \frac{1}{|k|} \psi(k^n, y) \leq K^n \psi(x, y),
\]

(2.25)

for all \( x, y \in A \) and each \( n \in \mathbb{N} \). Sending \( n \) to infinite, we have

\[
f(xy) = f(x)\theta(y) + \phi(x)g(y).
\]

(2.26)

By (2.26), we get

\[
\phi(z)g(xy) = f(zxy) - f(z)\theta(xy)
\]

\[
= f(zx)\theta(y) + \phi(zx)g(y) - f(z)\theta(xy)
\]

\[
= [f(z)\theta(x) + \phi(z)g(x)]\theta(y) + \phi(zx)g(y) - f(z)\theta(xy)
\]

\[
= \phi(z)[g(x)\theta(y) + \phi(x)g(y)],
\]

(2.27)

for all \( x, y, z \in A \). Therefore, we have \( g(xy) = g(x)\theta(y) + \phi(x)g(y) \).

Since \( f(xy) = f(x)\theta(y) + \phi(x)g(y) \), \( f \) is additive, and \( A \) is with unit, \( g \) is additive. \( \Box \)

The proof of the following theorem is similar to that in Theorem 2.1; hence, it is omitted.

**Theorem 2.2.** Let \( \varphi, \psi : A \times A \rightarrow [0, \infty) \) be functions. Suppose that \( f : A \rightarrow A \) and \( g : A \rightarrow A \) are mappings such that

\[
\left\| f(x + y) - f(x) - f(y) \right\| \leq \varphi(x, y),
\]

\[
\left\| f(xy) - xf(y) - g(x)y \right\| \leq \psi(x, y),
\]

(2.28)

for all \( x, y \in A \). If there exists constants \( K, L < 1 \) and a natural number \( k \in \mathbb{K} \),

\[
|k|\varphi(k^{-1}x, k^{-1}y) \leq L\varphi(x, y), \ |k|\varphi(k^{-1}x, y), \ |k|\varphi(x, k^{-1}y) \leq K\psi(x, y),
\]

(2.29)

for all \( x, y \in A \), then \( f \) is a generalized \((\theta, \phi)\)-derivation and \( g \) is a \((\theta, \phi)\)-derivation.

In the following corollaries \( \mathbb{Q}_p \) is the field of \( p \)-adic numbers.

**Corollary 2.3.** Let \( A \) be a non-Archimedean Banach algebra over \( \mathbb{Q}_p \), \( \varepsilon > 0 \), and let \( p_1, p_2 \in (1, \infty) \). Suppose that

\[
\left\| f(x + y) - f(x) - f(y) \right\| \leq \varepsilon(p1 \left\| x \right\|^{p_1} \left\| y \right\|^{p_2}),
\]

\[
\left\| f(xy) - xf(y) - g(x)y \right\| \leq \varepsilon(p2 \left\| x \right\|^{p_1} \left\| y \right\|^{p_2}),
\]

(2.30)

for all \( x, y \in A \). Then \( f \) is a generalized \((\theta, \phi)\)-derivation and \( g \) is a \((\theta, \phi)\)-derivation.
Proof. Let \( \varphi(x, y) = \psi(x, y) = \epsilon(\|x\|^{p_1} \|y\|^{p_2}) \) for all \( x, y \in A \); then

\[
|p|^{-1} \varphi(px, py) = |p|^{p_1+p_2-1} \epsilon(\|x\|^{p_1} \|y\|^{p_2}), \\
|p|^{-1} \varphi(px, y) = |p|^{p_2-1} \epsilon(\|x\|^{p_1} \|y\|^{p_2}), \\
|p|^{-1} \varphi(x, py) = |p|^{p_1-1} \epsilon(\|x\|^{p_1} \|y\|^{p_2}).
\] (2.31)

Put

\[
L = K = \max\left\{ |p|^{p_1-1}, |p|^{p_2-1}, |p|^{p_1+p_2-1} \right\} = \max\left\{ p^{1-p_1}, p^{1-p_2}, p^{1-p_1-p_2} \right\}.
\] (2.32)

So, by Theorem 2.1, \( f \) is a generalized \((\theta, \phi)\)-derivation and \( g \) is a \((\theta, \phi)\)-derivation.

\[\Box\]

**Corollary 2.4.** Let \( A \) be a non-Archimedean Banach algebra over \( \mathbb{Q}_p \), \( \epsilon > 0 \), and let \( p_1, p_2, p_1 + p_2 \in (-\infty, 1) \). Suppose that

\[
\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^{p_1} \|y\|^{p_2}), \\
\|f(xy) - xf(y) - g(x)y\| \leq \epsilon(\|x\|^{p_1} \|y\|^{p_2}).
\] (2.33)

for all \( x, y \in A \). Then \( f \) is a generalized \((\theta, \phi)\)-derivation and \( g \) is a \((\theta, \phi)\)-derivation.

**Proof.** Let \( \varphi(x, y) = \psi(x, y) = \epsilon(\|x\|^{p_1} \|y\|^{p_2}) \) for all \( x, y \in A \), then

\[
|p|\varphi(p^{-1}x, p^{-1}y) = |p|^{1-p_1-p_2} \epsilon(\|x\|^{p_1} \|y\|^{p_2}), \\
|p|\varphi(p^{-1}x, y) = |p|^{1-p_2} \epsilon(\|x\|^{p_1} \|y\|^{p_2}), \\
|p|\varphi(x, p^{-1}y) = |p|^{1-p_1} \epsilon(\|x\|^{p_1} \|y\|^{p_2}).
\] (2.34)

Put

\[
L = K = \max\left\{ |p|^{1-p_1}, |p|^{1-p_2}, |p|^{1-p_1-p_2} \right\} = \max\left\{ p^{1-p_1}, p^{1-p_2}, p^{1-p_1-p_2} \right\}.
\] (2.35)

So, by Theorem 2.2, \( f \) is a generalized \((\theta, \phi)\)-derivation and \( g \) is a \((\theta, \phi)\)-derivation.

\[\Box\]
Similarly, we can obtain the following results.

**Corollary 2.5.** Let $A$ be a non-Archimedean Banach’s algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2 \in (1, \infty)$. Suppose that

\[
\|f(x + y) - f(x) - f(y)\| \leq \varepsilon (\|x\|^{p_1} + \|y\|^{p_2}),
\]

\[
\|f(xy) - xf(y) - g(x)y\| \leq \delta (\|x\|^{p_1} \|y\|^{p_2}),
\]

(2.36)

for all $x, y \in A$. Then $f$ is a generalized $(\theta, \phi)$-derivation and $g$ is a $(\theta, \phi)$-derivation.

**Corollary 2.6.** Let $A$ be a non-Archimedean Banach’s algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2 \in (1, \infty)$. Suppose that

\[
\max \{\|f(x + y) - f(x) - f(y)\|, \|f(xy) - xf(y) - g(x)y\|\}
\]

\[
\leq \varepsilon \min \{ (\|x\|^{p_1} + \|y\|^{p_2}), \|x\|^{p_1} \|y\|^{p_2}\},
\]

(2.37)

for all $x, y \in A$. Then $f$ is a generalized $(\theta, \phi)$-derivation and $g$ is a $(\theta, \phi)$-derivation.

**Corollary 2.7.** Let $A$ be a non-Archimedean Banach’s algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2, p_1 + p_2 \in (-\infty, 1)$. Suppose that

\[
\|f(x + y) - f(x) - f(y)\| \leq \varepsilon (\|x\|^{p_1} + \|y\|^{p_2}),
\]

\[
\|f(xy) - xf(y) - g(x)y\| \leq \delta (\|x\|^{p_1} \|y\|^{p_2}),
\]

(2.38)

for all $x, y \in A$. Then $f$ is a generalized $(\theta, \phi)$-derivation and $g$ is a $(\theta, \phi)$-derivation.

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