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Research Article

Necessary and Sufficient Condition for Mann Iteration Converges to a Fixed Point of Lipschitzian Mappings

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Suppose that E is a real normed linear space, C is a nonempty convex subset of E, $T : C \to C$ is a Lipschitzian mapping, and $x^* \in C$ is a fixed point of T. For given $x_0 \in C$, suppose that the sequence $\{x_n\} \subset C$ is the Mann iterative sequence defined by $x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n$, $n \geq 0$, where $\{\alpha_n\}$ is a sequence in $[0,1]$, $\sum_{n=0}^{\infty} \alpha_n^2 < \infty$, $\sum_{n=0}^{\infty} \alpha_n = \infty$. We prove that the sequence $\{x_n\}$ strongly converges to x^* if and only if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that $\limsup_{n\to\infty} \inf_{j(x_n - x^*) \in J(x_n - x^*)} \{\langle Tx_n - x^*, j(x_n - x^*)\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|)\} \leq 0$.

1. Introduction

Let E be an arbitrary real normed linear space with dual space E^*, and let C be a nonempty subset of E. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \left\{ f \in E^* : \langle x, f \rangle = \|x\|^2 = \|f\|^2 \right\}, \quad \forall\, x \in E, \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing.

A mapping $T : C \to E$ is called *strongly pseudocontractive* if there exists a constant $k \in (0,1)$ such that, for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y) \rangle \leq (1-k)\|x-y\|^2. \tag{1.2}$$

T is called *ϕ-strongly pseudocontractive* if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that, for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \phi(\|x-y\|)\|x-y\|. \tag{1.3}$$

T is called *generalized Φ-pseudocontractive* (see, e.g., [1]) if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \Phi(\|x-y\|) \tag{1.4}$$

holds for all $x, y \in C$ and for some $j(x-y) \in J(x-y)$.

Let $F(T) = \{x \in C : Tx = x\}$ denote the fixed point set of T. If $F(T) \neq \emptyset$, and (1.3) and (1.4) hold for all $x \in C$ and $y \in F(T)$, then the corresponding mapping T is called *ϕ-hemicontractive* and generalized *Φ-hemicontractive*, respectively. It is well known that these kinds of mappings play important roles in nonlinear analysis.

ϕ-hemicontractive (resp., generalized *Φ-hemicontractive*) mapping is also called uniformly pseudocontractive (resp., uniformly hemicontractive) mapping in [2, 3]. It is easy to see that if T is generalized *Φ-hemicontractive* mapping, then $F(T)$ is singleton.

It is known (see, e.g., [4, 5]) that the class of strongly pseudocontractive mappings is a proper subset of the class of ϕ-strongly pseudocontractive mappings. By taking $\Phi(s) = s\phi(s)$, where $\phi : [0,\infty) \to [0,\infty)$ is a strictly increasing function with $\phi(0) = 0$, we know that the class of ϕ-strongly pseudocontractive mappings is a subset of the class of generalized Φ-pseudocontractive mappings. Similarly, the class of ϕ-hemicontractive mappings is a subset of the class of generalized Φ-hemicontractive mappings. The example in [6] demonstrates that the class of Lipschitzian ϕ-hemicontractive mappings is a proper subset of the class of Lipschitzian generalized Φ-hemicontractive mappings.

It is well known (see, e.g., [7]) that if C is a nonempty closed convex subset of a real Banach space E and $T : C \to C$ is a continuous strongly pseudocontractive mapping, then T has a unique fixed point $p \in C$. In 2009, it has been proved in [8] that if C is a nonempty closed convex subset of a real Banach space E and $T : C \to C$ is a continuous generalized Φ-pseudocontractive mappings, then T has a unique fixed point $p \in C$.

Many results have been proved on convergence or stability of Ishikawa iterative sequences (with errors) or Mann iterative sequences (with errors) for Lipschitzian ϕ-hemicontractive mappings or Lipschitzian generalized Φ-hemicontractive mapping (see, e.g., [4–6, 9–12] and the references therein). In 2010, Xiang et al. [6] proved the following result.

Theorem XCZ (see [6, Theorem 3.2]). *Let E be a real normed linear space, let C be a nonempty convex subset of E, and let $T : C \to C$ be a Lipschitzian generalized Φ-hemicontractive mapping. For given $x_0 \in C$, suppose that the sequence $\{x_n\} \subset C$ is the Mann iterative sequence defined by*

$$x_{n+1} = (1-\beta_n)x_n + \beta_n T x_n, \quad n \geq 0, \tag{1.5}$$

where $\{\beta_n\}$ *is a sequence in* $[0,1]$ *satisfying the following conditions:*

(1) $\sum_{n=0}^{\infty} \beta_n = \infty$,

(2) $\sum_{n=0}^{\infty} \beta_n^2 < \infty$.

Then $\{x_n\}$ *converges strongly to the unique fixed point of* T *in* C.

The main purpose of this paper is to give necessary and sufficient condition for the Mann iterative sequence which converges to a fixed point of general Lipschitzian mappings in an arbitrary real normed linear space. As an immediate consequence, we will obtain necessary and sufficient condition for the Mann iterative sequence which converges to a solution of a general Lipschitzian operator equation $Tx = f$.

2. Preliminaries

The following lemmas will be used in the proof of our main results.

Lemma 2.1 (see, e.g., [12]). *Let* E *be a real normed linear space. Then for all* $x, y \in E$, *we have*

$$\|x+y\|^2 \le \|x\|^2 + 2\langle y, j(x+y)\rangle, \quad \forall j(x+y) \in J(x+y). \tag{2.1}$$

Lemma 2.2 (see, e.g., [13]). *Let* $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ *be three nonnegative sequences satisfying the following condition:*

$$a_{n+1} \le (1+b_n)a_n + c_n, \quad \forall\ n \ge n_0, \tag{2.2}$$

where n_0 *is some nonnegative integer,* $\sum_{n=n_0}^{\infty} b_n < \infty$, *and* $\sum_{n=n_0}^{\infty} c_n < \infty$. *Then the limit* $\lim_{n\to\infty} a_n$ *exists.*

Lemma 2.3. *Suppose that* $\varphi : [0,\infty) \to [0,\infty)$ *is a strictly increasing function with* $\varphi(0) = 0$ *and there exists a natural number* n_0 *such that* a_n, b_n, ε_n, *and* α_n *are nonnegative real numbers for all* $n \ge n_0$ *satisfying the following conditions:*

(i) $a_{n+1} \le (1+b_n)a_n - \alpha_n\varphi(a_{n+1}) + \alpha_n\varepsilon_n$, for all $n \ge n_0$,

(ii) $\sum_{n=n_0}^{\infty} b_n < \infty$, $\lim_{n\to\infty}\varepsilon_n = 0$,

(iii) $\sum_{n=n_0}^{\infty} \alpha_n = \infty$.

Then $\lim_{n\to\infty} a_n = 0$.

Proof. Without loss of generality, let $\lim_{n\to\infty}\inf a_n = a$. Now, we will show that $a = 0$. Consider its contrary: $a > 0$ or $a = \infty$. For any given $r \in (0,a)$, there exists a nonnegative integer $n_1 \ge n_0$ such that $a_n \ge r > 0$ and $\varepsilon_n < 1/2\varphi(r) \le 1/2\varphi(a_{n+1})$ for all $n \ge n_1$. By condition (i), we have

$$\begin{aligned} a_{n+1} &\le (1+b_n)a_n - \alpha_n\varphi(a_{n+1}) + \alpha_n \cdot \frac{1}{2}\varphi(a_{n+1}) \\ &= (1+b_n)a_n - \frac{1}{2}\alpha_n\varphi(a_{n+1}) \\ &\le (1+b_n)a_n, \quad \forall n \ge n_1. \end{aligned} \tag{2.3}$$

Using Lemma 2.2 and condition (ii), we obtain that $\lim_{n\to\infty} a_n$ exists and $\{a_n\}$ is bounded. Suppose that $a_n \le M$ (for all $n \ge n_1$), where M is a nonnegative constant. It follows that

$$a_{n+1} \le (1+b_n)a_n - \frac{1}{2}\alpha_n\varphi(a_{n+1}) \le a_n - \frac{1}{2}\alpha_n\varphi(r) + Mb_n (\forall n \ge n_1). \tag{2.4}$$

Thus,

$$\infty = \frac{1}{2}\varphi(r)\sum_{n=n_1}^{\infty}\alpha_n \le a_{n_1} + M\sum_{n=n_1}^{\infty} b_n < \infty, \tag{2.5}$$

which is a contradiction. Therefore,

$$\liminf_{n\to\infty} a_n = 0. \tag{2.6}$$

By condition (ii), for all $\varepsilon > 0$, there exists a nonnegative integer $n_2 \ge n_0$ such that

$$\varepsilon_n < \varphi(\varepsilon) \quad (\forall n \ge n_2), \qquad \sum_{n=n_2}^{\infty} b_n < \ln 2. \tag{2.7}$$

By (2.6), there exists a natural number $N \ge n_2$ such that $a_N < \varepsilon$. Now, we prove the following inequality (2.8) holds for all $k \ge N$:

$$a_k \le \varepsilon \cdot \exp\left(\sum_{n=N}^{k-1} b_n\right). \tag{2.8}$$

It is obvious that (2.8) holds for $k = N$. Assuming (2.8) holds for some $k \ge N$, we prove that (2.8) holds for $k+1$. Suppose this is not true, that is, $a_{k+1} > \varepsilon \cdot \exp(\sum_{n=N}^{k} b_n)$. Then $a_{k+1} \ge \varepsilon$ and so $\varphi(a_{k+1}) \ge \varphi(\varepsilon)$. Noting that $1 + b_k \le \exp(b_k)$, it follows from condition (i), (2.7), and (2.8) that

$$\begin{aligned} a_{k+1} &\le (1+b_k)a_k - \alpha_k\varphi(a_{k+1}) + \alpha_k\varepsilon_k \\ &\le (1+b_k)a_k - \alpha_k\varphi(\varepsilon) + \alpha_k\varphi(\varepsilon) \\ &\le \varepsilon\cdot(1+b_k)\exp\left(\sum_{n=N}^{k-1} b_n\right) \\ &\le \varepsilon\cdot\exp\left(\sum_{n=N}^{k} b_n\right), \end{aligned} \tag{2.9}$$

which is a contradiction. This implies that (2.8) holds for $k+1$. By induction, (2.8) holds for all $k \geq N$. From (2.7), and (2.8), we have

$$\limsup_{k\to\infty} a_k \leq \varepsilon \cdot \exp\left(\sum_{n=N}^{\infty} b_n\right) < 2\varepsilon. \tag{2.10}$$

Taking $\varepsilon \to 0$, we obtain $\lim_{n\to\infty} \sup a_k = 0$. By (2.6), we have $\lim_{n\to\infty} a_n = 0$. This completes the proof. □

Remark 2.4. Lemma 2.3 is different from Lemma 3 in [14], which requires that $b_n = 0$ for all $n \geq 0$. It is also different from Lemma 2.3 in [6], which requires that $\sum_{n=n_0}^{\infty} \alpha_n \varepsilon_n < \infty$.

3. Main Results

Theorem 3.1. *Let E be a real normed linear space, C be a nonempty convex subset of E, let $T : C \to C$ be a Lipschitzian mapping, and let $x^* \in C$ be a fixed point of T. For given $x_0 \in C$, suppose that the sequence $\{x_n\} \subset C$ is the Mann iterative sequence defined by*

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad n \geq 0, \tag{3.1}$$

where $\{\alpha_n\}$ is a sequence in $[0,1]$ satisfying the following conditions:

(i) $\sum_{n=0}^{\infty} \alpha_n^2 < \infty$,

(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Then $\{x_n\}$ converges strongly to x^* if and only if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that

$$\limsup_{n\to\infty} \inf_{j(x_n-x^*)\in J(x_n-x^*)} \left\{ \langle Tx_n - x^*, j(x_n - x^*)\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|) \right\} \leq 0. \tag{3.2}$$

Proof. First, we prove the sufficiency of Theorem 3.1.

Suppose there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that (3.2) holds. Let

$$\gamma_n = \inf_{j(x_n-x^*)\in J(x_n-x^*)} \left\{ \langle Tx_n - x^*, j(x_n - x^*)\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|) \right\}. \tag{3.3}$$

Then there exists $j(x_n - x^*) \in J(x_n - x^*)$ such that

$$\langle Tx_n - x^*, j(x_n - x^*)\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|) < \gamma_n + \frac{1}{n}, \quad \forall n \geq 1. \tag{3.4}$$

By (3.2), we obtain $\lim_{n\to\infty} \sup \gamma_n \leq 0$. Taking $\varepsilon_n = 1/(n+1) + \max\{\gamma_{n+1}, 0\}$ (for all $n \geq 0$), then

$$\lim_{n\to\infty} \varepsilon_n = 0. \tag{3.5}$$

From (3.1) and (3.4), by using Lemma 2.1, we obtain

$$\begin{aligned}
&\|x_{n+1}-x^*\|^2 \\
&\quad = \|(1-\alpha_n)(x_n-x^*)+\alpha_n(Tx_n-x^*)\|^2 \\
&\quad \le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\langle Tx_n-x^*, j(x_{n+1}-x^*)\rangle \\
&\quad \le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\langle Tx_{n+1}-x^*, j(x_{n+1}-x^*)\rangle \\
&\qquad + 2\alpha_n\langle Tx_n-Tx_{n+1}, j(x_{n+1}-x^*)\rangle \\
&\quad \le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\left[\|x_{n+1}-x^*\|^2-\Phi(\|x_{n+1}-x^*\|)+\gamma_{n+1}+\frac{1}{n+1}\right] \\
&\qquad + 2L\alpha_n\|x_n-x_{n+1}\|\cdot\|x_{n+1}-x^*\| \\
&\quad \le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\left[\|x_{n+1}-x^*\|^2-\Phi(\|x_{n+1}-x^*\|)+\varepsilon_n\right] \\
&\qquad + 2L\alpha_n\|x_n-x_{n+1}\|\cdot\|x_{n+1}-x^*\|,
\end{aligned} \tag{3.6}$$

where L is the Lipschitzian constant of T. It follows from (3.1) that

$$\begin{aligned}
\|x_n-x_{n+1}\| &= \|\alpha_n(x_n-Tx_n)\| \\
&\le \alpha_n(\|x_n-x^*\|+\|Tx^*-Tx_n\|) \\
&\le \alpha_n(1+L)\|x_n-x^*\|.
\end{aligned} \tag{3.7}$$

Substituting (3.7) into (3.6), we have

$$\begin{aligned}
\|x_{n+1}-x^*\|^2 &\le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\|x_{n+1}-x^*\|^2-2\alpha_n\Phi(\|x_{n+1}-x^*\|) \\
&\quad + 2\alpha_n\varepsilon_n+2L(1+L)\alpha_n^2\|x_n-x^*\|\cdot\|x_{n+1}-x^*\| \\
&\le (1-\alpha_n)^2\|x_n-x^*\|^2+2\alpha_n\|x_{n+1}-x^*\|^2-2\alpha_n\Phi(\|x_{n+1}-x^*\|) \\
&\quad + 2\alpha_n\varepsilon_n+L(1+L)\alpha_n^2\left(\|x_n-x^*\|^2+\|x_{n+1}-x^*\|^2\right).
\end{aligned} \tag{3.8}$$

Setting $a_n=\|x_n-x^*\|^2$(for all $n\ge 0$), $\varphi(s)=2\Phi(\sqrt{s})$, it follows from (3.8) that

$$\begin{aligned}
a_{n+1} &\le (1-\alpha_n)^2a_n+2\alpha_na_{n+1}-\alpha_n\varphi(a_{n+1})+2\alpha_n\varepsilon_n \\
&\quad + L(1+L)\alpha_n^2(a_n+a_{n+1}) \\
&= \left[1-2\alpha_n+\alpha_n^2+L(1+L)\alpha_n^2\right]a_n+\left[2\alpha_n+L(1+L)\alpha_n^2\right]a_{n+1} \\
&\quad - \alpha_n\varphi(a_{n+1})+2\alpha_n\varepsilon_n.
\end{aligned} \tag{3.9}$$

It follows from condition (i) that $\lim_{n\to\infty}[2\alpha_n + L(1+L)\alpha_n^2] = 0$. Thus, there exists a natural number n_0 such that $2\alpha_n + L(1+L)\alpha_n^2 \le 1/2$ for all $n \ge n_0$. Let

$$b_n = \frac{1 - 2\alpha_n + \alpha_n^2 + L(1+L)\alpha_n^2}{1 - 2\alpha_n - L(1+L)\alpha_n^2} - 1 = \frac{\alpha_n^2 + 2L(1+L)\alpha_n^2}{1 - 2\alpha_n - L(1+L)\alpha_n^2}, \quad \forall n \ge n_0. \tag{3.10}$$

Since $1/2 \le 1 - 2\alpha_n - L(1+L)\alpha_n^2 \le 1$ for all $n \ge n_0$, by (3.9) and (3.10), we have

$$\begin{gathered} a_{n+1} \le (1+b_n)a_n - \alpha_n\varphi(a_{n+1}) + 4\alpha_n\varepsilon_n, \quad \forall n \ge n_0, \\ 0 \le b_n \le 2[1 + 2L(1+L)]\alpha_n^2, \quad \forall n \ge n_0. \end{gathered} \tag{3.11}$$

It follows from condition (i) that $\sum_{n=n_0}^{\infty} b_n < \infty$. Therefore, by (3.5), condition (ii), and Lemma 2.3, we obtain that $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\|x_n - x^*\|^2 = 0$. That is, $\{x_n\}$ converges strongly to x^*.

Finally, we prove the necessity of Theorem 3.1.

Assume that $\{x_n\}$ converges strongly to x^*. Let L be the Lipschitzian constant of T. For all $j(x_n - x^*) \in J(x_n - x^*)$, we have

$$\left|\langle Tx_n - x^*, j(x_n - x^*)\rangle\right| \le L\|x_n - x^*\|^2. \tag{3.12}$$

Taking $\Phi(s) = \sqrt{s}$, then $\Phi : [0,\infty) \to [0,\infty)$ is a strictly increasing function with $\Phi(0) = 0$, and $\lim_{n\to\infty}\Phi(\|x_n - x^*\|) = 0$. From (3.12), we obtain

$$\lim_{n\to\infty} \inf_{j(x_n - x^*)\in J(x_n - x^*)} \left\{\langle Tx_n - x^*, j(x_n - x^*)\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|)\right\} = 0, \tag{3.13}$$

which implies (3.2) holds. This completes the proof of Theorem 3.1. □

Remark 3.2. If $T : C \to C$ is a generalized Φ-hemicontractive mapping, then (3.2) holds. By Theorem 3.1, we obtain Theorem XCZ.

Theorem 3.3. *Let E be a real Banach space, let C be a nonempty closed convex subset of E, and let $T : C \to C$ be a Lipschitzian generalized Φ-pseudocontractive mapping. For given $x_0 \in C$, suppose that the sequence $\{x_n\} \subset C$ is the Mann iterative sequence defined by*

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad n \ge 0, \tag{3.14}$$

where $\{\alpha_n\}$ is a sequence in $[0,1]$ satisfying the following conditions:

(1) $\sum_{n=0}^{\infty} \alpha_n = \infty$,

(2) $\sum_{n=0}^{\infty} \alpha_n^2 < \infty$.

Then $\{x_n\}$ converges strongly to the unique fixed point of T in C.

Proof. By Theorem 2.1 in [8], T has a unique fixed point x^* in C. By Theorem 3.1, $\{x_n\}$ converges strongly to x^*. This completes the proof of Theorem 3.3. □

Theorem 3.4. *Let E be a real normed linear space, let $S : E \to E$ be a Lipschitzian operator, and let $f \in E$ and x^* be a solution of the equation $Sx = f$. For given $x_0 \in E$, suppose that the sequence $\{x_n\}$ is the Mann iterative sequence defined by*

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n\left(f + x_n - Sx_n\right), \quad n \geq 0, \tag{3.15}$$

where $\{\alpha_n\}$ is a sequences in $[0,1]$ satisfying the following conditions:

(i) $\sum_{n=0}^{\infty} \alpha_n^2 < \infty$,

(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Then $\{x_n\}$ converges strongly to x^* if and only if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that

$$\liminf_{n\to\infty} \sup_{j(x_n-x^*)\in J(x_n-x^*)} \left\{\left\langle Sx_n - Sx^*, j(x_n - x^*)\right\rangle - \Phi(\|x_n - x^*\|)\right\} \geq 0. \tag{3.16}$$

Proof. Define $T : E \to E$ by $Tx = f + x - Sx$. Since $Sx^* = f$, we have $Tx^* = x^*$. From (3.15), we obtain $x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n,\ n \geq 0$. Since

$$\begin{aligned}&\left\langle Sx_n - Sx^*, j(x_n - x^*)\right\rangle - \Phi(\|x_n - x^*\|)\\ &\quad = -\left\{\left\langle Tx_n - x^*, j(x_n - x^*)\right\rangle - \|x_n - x^*\|^2 + \Phi(\|x_n - x^*\|)\right\}.\end{aligned} \tag{3.17}$$

Therefore,

$$\begin{aligned}&\liminf_{n\to\infty} \sup_{j(x_n-x^*)\in J(x_n-x^*)} \left\{\left\langle Sx_n - Sx^*, j(x_n - x^*)\right\rangle - \Phi(\|x_n - x^*\|)\right\}\\ &\quad = -\limsup_{n\to\infty} \inf_{j(x_n-x^*)\in J(x_n-x^*)} \left\{\left\langle Tx_n - x^*, j(x_n - x^*)\right\rangle - \|x_n - x^*\| + \Phi(\|x_n - x^*\|)\right\}.\end{aligned} \tag{3.18}$$

The condition (3.16) is equivalent to condition (3.2). Since S is a Lipschitzian operator, T is a Lipschitzian mapping. By Theorem 3.1, Theorem 3.4 holds. This completes the proof of Theorem 3.4. □

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