Research Article

Multi-State Dependent Impulsive Control for Pest Management

Huidong Cheng, Fang Wang, and Tongqian Zhang

College of Science, Shandong University of Science and Technology, Qingdao 266510, China

Correspondence should be addressed to Huidong Cheng, chd900517@sdust.edu.cn

Received 17 April 2012; Accepted 6 June 2012

Academic Editor: Zhiwei Gao

According to the integrated pest management strategies, we propose a model for pest control which adopts different control methods at different thresholds. By using differential equation geometry theory and the method of successor functions, we prove the existence of order one periodic solution of such system, and further, the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis. Numerical simulations are carried out to illustrate the feasibility of our main results. Our results show that our method used in this paper is more efficient and easier than the existing ones for proving the existence of order one periodic solution.

1. Introduction

It is of great value to study pest management method applied in agricultural production; entomologists and the whole society have been paying close attention to how to control pests effectively and to save manpower and material resources. In agricultural production, pesticides-spraying (chemical control) and release of natural enemies (biological control) are the ways commonly used for pest control. But if we implement chemical control as soon as pests appear, many problems are caused: the first is environmental pollution; the second is increase of costs including human and material resources and time; the third is killing natural enemies, such as parasitic wasp; the last is pests’ resistance to pesticides, which brings great negative effects instead of working as well as had been expected [1–3]. The second way, which controls pests with the help of the increasing natural enemies, can avoid problems caused by chemical control and gets more and more attention. So many scholars have been studying and discussing it [4–8]. Considering the effectiveness of the chemical control and nonpollution and limitations of the biological one, people have proposed the method of integrated pest management (IPM), which is a pest management system integrating all
appropriate ways and technologies to control economic injury level (EIL) caused by pest populations in view of population dynamics and its relevant environment. In the process of practical application, people usually implement the following two schemes for the integrated pest management: one is to implement control at a fixed time to eradicate pests \[9, 10\]; the other is to implement measures only when the amount of pests reaches a critical level, which is to make the amount less than certain economic impairment level, not to wipe out pests \[11–13\]. Salazar conducted an experiment of broad bean being damaged by bean sprouts worm in 1976 and found “crops’ compensation to damage of pests”, that is, yields of crops which had been damaged a little by pests in the early growth are actually higher than those without damage. In other words, we do not want to wipe out pests but to control them to a certain economic injury level \(\text{EIL}\). So, the second is used most in the process of agricultural industry. Tang and Cheke \[14\] first proposed the “Volterra” model in the from of a state-dependent impulsive model:

\[
\begin{align*}
x'(t) &= x(t)(a - by(t)), \\
y'(t) &= y(t)(-d + cx(t)), \\
\Delta x(t) &= -ax(t), \\
\Delta y(t) &= q,
\end{align*}
\]

and they applied this model to pest management and proved existence and stability of periodic solution of first and second order. Then Tang and Cheke \[14\] also proposed bait-dependent digestive model with state pulse:

\[
\begin{align*}
x'(t) &= x(t)(a - by(t)), \\
y'(t) &= y(t)\left(\frac{lbx(t)}{1 + bx(t)} - d\right), \\
\Delta x(t) &= -ax(t), \\
\Delta y(t) &= q, \\
x &= h_{\text{max}},
\end{align*}
\]

they had the existence of positive periodic solution and stability of orbit. Recently Jiang and Lu et al. \[15–17\] have proposed pest management model with state pulse and phase structure and several predator-prey models with state pulse and had the existence of semi-trivial periodic solution and positive periodic solution and stability of orbit.

It is worth mentioning that the vast majority of research on population dynamics system with state pulse considers single state pulse, which is to say, only when the amount of population reaches the same economic threshold can measures be taken (e.g., chemical control and biological control); but this single state-pulse control does not confirm to reality. In fact, we often need to use different control methods under different states in real life. For example, in the process of pest management, when the amount of pests is small, biological control is implemented; when the amount is large, combination control is applied. Tang et al. \[18\] have investigated and developed a mathematical model with hybrid
impulsive model:

\[
\begin{align*}
x'(t) &= r x(t)(1 - \delta x(t)) - b x(t)y(t), \quad x < ET, \\
y'(t) &= y(t)(c x(t) - a), \quad t = \lambda_m, \\
x(t^+) &= (1 - p_1)x(t), \\
y(t^+) &= (1 - p_2)y(t), \quad x(t) = ET, \\
y(\lambda_m^+) &= (1 + p_3)y(\lambda) + q, \quad t = \lambda_m.
\end{align*}
\] (1.3)

Motivated by Tang, on the basis of the above analysis, we set up the following predicator-prey system with different control methods in different thresholds:

\[
\begin{align*}
x'(t) &= x(t)(a - by(t)), \\
y'(t) &= y(t)\left(\frac{\lambda bx(t)}{1 + bhx(t)} - d\right), \quad x \neq h_1, h_2 \text{ or } x = h_1, \ y > y^*, \\
\Delta x(t) &= 0, \quad x = h_1, \ y \leq y^*, \\
\Delta y(t) &= \delta, \\
\Delta x(t) &= -ax(t), \quad x = h_2, \\
\Delta y(t) &= -by(t) + q, \quad x = h_2.
\end{align*}
\] (1.4)

where \(x(t)\) and \(y(t)\) represent, respectively, the prey and the predator population densities at time \(t\); \(a, b, \lambda, h_1, h_2\) and \(d\) are all positive constants and \(h_1 < h_2; y^* = a/b\). \(a, \beta \in (0, 1)\) represent the fraction of pest and predator, respectively, which die due to the pesticide when the amount of prey reaches economic threshold \(h_2\) and \(q\) is the release amount of predator. \(\lambda bx(t)/(1 + bhx(t))\) is the per capita functional response of the predator. When the amount of the prey reaches the threshold \(h_1\) at time \(t_{h_1}\), controlling measures are taken (releasing natural enemies) and the amount of predator abruptly turns to \(y(t_{h_1}) + \delta\). When the amount of the prey reaches the threshold \(h_2\) at time \(t_{h_2}\), spraying pesticide, and releasing natural enemies and the amount of prey and predator abruptly turn to \((1 - a)x(t_{h_1})\) and \((1 - \beta)y(t_{h_2}) + q\), respectively. Refer to [17] Liu et al. for details.

2. Preliminaries

First, we give some basic definitions and lemmas.

**Definition 2.1.** A triple \((X, \Pi, R^+)\) is said to be a semidynamical system if \(X\) is a metric space, \(R^+\) is the set of all nonnegative real, and \(\Pi(P, t) : X \times R^+ \rightarrow X\) is a continuous map such that:

(i) \(\Pi(P, 0) = P\) for all \(P \in X\);

(ii) \(\Pi(P, t)\) is continuous for \(t\) and \(s\);

(iii) \(\Pi(\Pi(P, t) + s) = \Pi(P, t + s)\) for all \(P \in X\) and \(t, s \in R^+\). Sometimes a semi-dynamical system \((X, \Pi, R^+)\) is denoted by \((X, \Pi)\).
**Definition 2.2.** Assuming that

(i) \((X, \Pi)\) is a semi-dynamical system;

(ii) \(M\) is a nonempty subset of \(X\);

(iii) function \(I : M \to X\) is continuous and for any \(P \in M\), there exists a \(\varepsilon > 0\) such that for any \(0 < |t| < \varepsilon\), \(\Pi(P, t) \notin M\).

Then, \((X, \Pi, M, I)\) is called an impulsive semi-dynamical system.

For any \(P\), the function \(\Pi_P : \mathbb{R}^+ \to X\) defined as \(\Pi_P(t) = \Pi(P, t)\) is continuous, and we call \(\Pi_P(t)\) the trajectory passing through point \(P\). The set \(C^+(P) = \{\Pi(P, t) / 0 \leq t < +\infty\}\) is called positive semitrajectory of point \(P\). The set \(C^-(P) = \{\Pi(P, t) / -\infty < t \leq 0\}\) is called the negative semi-trajectory of point \(P\).

**Definition 2.3.** One considers state-dependent impulsive differential equations:

\[
\begin{align*}
    x'(t) &= P(x, y), \\
    y'(t) &= Q(x, y), \\
    \Delta x(t) &= \alpha(x, y), \\
    \Delta y(t) &= \beta(x, y),
\end{align*}
\]

where \(M(x, y)\) and \(N(x, y)\) represent the straight line or curve line on the plane, \(M(x, y)\) is called impulsive set. The function \(I\) is continuous mapping, \(I(M) = N\), \(I\) is called the impulse function. \(N(x, y)\) is called the phase set. We define “dynamic system” constituted by the definition of solution of state impulsive differential equation (2.1) as “semicontinuous dynamic systems”, which is denoted as \((\Omega, f, I, M)\).

**Definition 2.4.** Suppose that the impulse set \(M\) and the phase set \(N\) are both lines, as shown in Figure 1. Define the coordinate in the phase set \(N\) as follows: denote the point of intersection \(Q\) between \(N\) and \(x\)-axis as \(O\), then the coordinate of any point \(A\) in \(N\) is defined as the distance between \(A\) and \(Q\) and is denoted by \(y_A\). Let \(C\) denote the point of intersection between the trajectory starting from \(A\) and the impulse set \(M\), and \(B\) denote the phase point of \(C\) after impulse with coordinate \(y_B\). Then, we define \(B\) as the successor point of \(A\), and then the successor function of point \(A\) is that \(f(A) = y_B - y_A\).

**Definition 2.5.** A trajectory \(\tilde{\Pi}(P_0, t)\) is called order one periodic solution with period \(T\) if there exists a point \(P_0 \in N\) and \(T > 0\) such that \(P = \Pi(P_0, t) \in M\) and \(P^+ = I(P) = P_0\).

We get these lemmas from the continuity of composite function and the property of continuous function.

**Lemma 2.6.** Successor function defined in Definition 2.1 is continuous.

**Lemma 2.7.** In system (1.4), if there exist \(A \in N\), \(B \in N\) satisfying successor function \(f(A)f(B) < 0\), then there must exist a point \(P\) \((P \in N)\) satisfying \(f(P) = 0\) the function between the point of \(A\) and the point of \(B\), thus there is an order one periodic solution in system (1.4).
Next, we consider the model (1.4) without impulse effects:

\[
\begin{align*}
    x'(t) &= x(t) \left( a - by(t) \right), \\
    y'(t) &= y(t) \left( \frac{\lambda bx(t)}{1 + bhx(t)} - d \right),
\end{align*}
\]  

It is well known that the system (2.2) possesses

(I) two steady states \( O(0,0) \)-saddle point, and \( R(d/b(\lambda - dh), a/b) = R(x^*, y^*)(\lambda > dh) \)-stable centre;

(II) a unique closed trajectory through any point in the first quadrant contained inside the point \( R \).

In this paper, we assume that the condition \( \lambda > dh \) holds. By the biological background of system (1.4), we only consider \( D = \{(x, y) : x \geq 0, y \geq 0\} \). Vector graph of system (2.2) can be seen in Figure 2.

This paper is organized as follows. In the next section, we present some basic definitions and an important lemmas as preliminaries. In Section 3, we prove existence for an order one periodic solution of system (1.4). The sufficient conditions for the attractiveness of order one periodic solutions of system (1.4) are obtained in Section 4. At last, we state conclusion, and the main results are carried out to illustrate the feasibility by numerical simulations.
3. Existence of the Periodic Solution

In this section, we will investigate the existence of an order one periodic solution of system (1.4) by using the successor function defined in this paper and qualitative analysis. For this goal, we denote that \( M_1 = \{ (x, y) / x = h_1, 0 \leq y \leq a/b \} \), and that \( M_2 = \{ (x, y) / x = h_2, y \geq 0 \} \).

Phase set of set \( M \) is that \( N_1 = I(M_1) = \{ (x, y) / x = h_1, a/b < y \leq (a/b) + \delta \} \) and that \( N_2 = I(M_2) = \{ (x, y) / x = (1 - a)h_2, y \geq q \} \). Isoclinic line is denoted, respectively, by lines: \( L_1 = \{ (x, y) / y = a/b, x \geq 0 \} \) and \( L_2 = \{ (x, y) / x = d/b(\lambda - dh), y \geq 0 \} \).

For the convenience, if \( P \in \Omega - M, F(P) \) is defined as the first point of intersection of \( C^+(P) \) and \( M \), that is, there exists a \( t_1 \in R \) such that \( F(P) = \Pi(P, t_1) \in M \), and for \( 0 < t < t_1, \Pi(P, t) \notin M \); if \( B \in N, R(B) \) is defined as the first point of intersection of \( C^-(P) \) and \( N \), that is, there exists a \( t_2 \in R \) such that \( R(B) = \Pi(B, -t_2) \in N \), and for \( -t < t < 0, \Pi(B, t) \notin N \). For any point \( P \), we denote \( y_P \) as its ordinate. If the point \( P(h, y_P) \in M \), then pulse occurs at the point \( P \), the impulsive function transfers the point \( P \) into \( P^+ \in N \). Without loss of generality, unless otherwise specified we assume the initial point of the trajectory lies in phase set \( N \).

Due to the practical significance, in this paper we assume the set always lies in the left side of stable centre \( R \), that is, \( h_1 < d/b(\lambda - dh) \) and \( (1 - a)h_2 < d/b(\lambda - dh) \).

In the light of the different position of the set \( N_1 \) and the set \( N_2 \), we consider the following three cases.

Case 1 \( (0 < h_1 < d/b(\lambda - dh)) \). In this case, set \( M_1 \) and \( N_1 \) are both in the left side of stable center \( R \) (as shown in Figure 3). Take a point \( B_1(h_1, (a/b) + \varepsilon) \in N_1 \) above \( A \), where \( \varepsilon > 0 \) is small enough, then there must exist a trajectory passing through \( B_1 \) which intersects with \( M_1 \) at point \( P_1(h_1, y_{p_1}) \), we have \( y_{p_1} < a/b \). Since \( p_1 \in M_1 \), pulse occurs at the point \( P_1 \), the impulsive function transfers the point \( P_1 \) into \( P^+_1(h_1, y_{p_1} + \delta) \) and \( P^+_1 \) must lie above \( B_1 \), therefore inequation \( (a/b) + \varepsilon < y_{p_1} + \delta \) holds, thus the successor function of \( B_1 \) is \( f(B_1) = y_{p_1} + \delta - ((a/b) + \varepsilon) > 0 \).
Figure 3: \(0 < h_1 < \frac{d/b}{1 - dh}(a/b) < y_{p_2} + \delta + y_{p_1} + \delta\).

On the other hand, the trajectory with the initial point \(P_1^+\) intersects \(M_1\) at point \(P_2(h_1, y_{p_2})\), in view of vector field and disjointness of any two trajectories, we know \(y_{p_2} < y_{p_1} < a/b\). Supposing the point \(P_2\) is subject to impulsive effects to point \(P_2^+(h_1, y_{p_2}^+),\) where \(y_{p_2}^+ = y_{p_2} + \delta\), the position of \(P_2^+\) has the following two cases.

**Subcase 1.1** \((a/b < y_{p_2} + \delta < y_{p_1} + \delta)\). In this case, the point \(P_2^+\) lies above the point \(A\) and below \(P_1^+\), then we have \(f(P_2^+) = y_{p_2} + \delta - (y_{p_1} + \delta) < 0\).

By Lemma 2.7, there exists an order one periodic solution of system (1.4), whose initial point is between \(B_1\) and \(P_1^+\) in set \(N_1\).

**Subcase 1.2** \((a/b \geq y_{p_2} + \delta)\) (as shown in Figure 4). The point \(P_2^+\) lies below the point \(A\), that is, \(P_2^+ \in M_1\), then pulse occurs at the point \(P_2^+\), the impulsive function transfers the point \(P_2^+\) into \(P_2^{++}(h_1, y_{p_2} + 2\delta)\).

If \(a/b < y_{p_2} + 2\delta < y_{p_1} + \delta\), like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.4).

If \(a/b > y_{p_2} + 2\delta\), that is, \(P_2^{++} \in M_1\), then we repent the above process until there exists \(k \in Z\), such that \(P_2^{++}\) jumps to \(P_2^{++}(h_1, y_{p_2} + (k + 2)\delta)\) after \(k\) times impulsive effects which satisfies \(a/b < y_{p_2} + (k + 2)\delta < y_{p_1} + \delta\). Like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.4).

Now, we can summarize the above results as the following theorem.

**Theorem 3.1.** If \(\lambda > dh, 0 < h_1 < \frac{d/b}{(\lambda - dh)}\), then there exists an order one periodic solution of the system (1.4).
Remark 3.2. It shows from the proved process of Theorem 3.1 that the number of natural enemies should be selected appropriately, which aims to reduce releasing impulsive times to save manpower and resources.

Case 2 ($h_2 < d/b(\lambda - dh)$). In this case, set $M_2$ and $N_2$ are both in the left side of stable center $R$, in the light of the different position of the set $N_2$, we consider the following two cases.

Subcase 2.1 ($0 < h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda - dh)$). In this case, the set $N_2$ is in the right side of $M_1$ (as shown in Figure 5). The trajectory passing through point $A$ which tangents to $N_2$ at point $A$ intersects with $M_2$ at point $P_0(h_2, y_{P_0})$. Since the point $P_0 \in M_2$, then impulse occurs at point $P_0$. Supposing the point $P_0$ is subject to impulsive effects to point $P_0^+ ((1-\alpha)h_2, y_{P_0}^+)$, where $y_{P_0}^+ = (1-\beta)y_{P_0} + q$, the position of $P_0^+$ has the following three cases:

1. $((1-\beta)y_{P_0} + q > a/b)$. Take a point $B_1((1-\alpha)h_2, \varepsilon + a/b) \in N_2$ above $A$, where $\varepsilon > 0$ is small enough. Then there must exist a trajectory passing through the point $B_1$ which intersects with the set $M_2$ at point $P_1(h_2, y_{P_1})$. In view of continuous dependence of the solution on initial value and time, we know $y_{P_1} < y_{P_0}$ and the point $P_1$ is close to $P_0$ enough, so we have the point $P_1^+$ is close to $P_0^+$ enough and $y_{P_1}^+ < y_{P_0}^+$, then we obtain $f(B_1) = y_{P_1}^+ - y_{P_1} > 0$.

2. On the other hand, the trajectory passing through point $B$ tangents to $N_1$ at point $B$. Set $F(S) = P_2(h_2, y_{P_2}) \in M_2$. Denote the coordinates of impulsive point $P_2^+((1-\alpha)h_2, y_{P_2}^+)$ corresponding to the point $P_2(h_2, y_{P_2})$.

If $y_S \geq y_{P_0}^-$ then $y_{P_1} < y_{P_0} < y_S$. So we obtain $f(S) = y_{P_2}^+ - y_S < 0$. There exists an order one periodic solution of system (1.4), whose initial point is between the point $B_1$ and the point $S$ in set $N_2$ (Figure 5).

If $y_S < y_{P_2}^-$ and $y_{P_2}^+ \leq y_S$, we have $f(S) = y_{P_2}^+ - y_S \leq 0$, we conclude that there exists an order one periodic solution of system (1.4).

![Figure 4: $0 < h_1 < (d/b(1-dh))(a/b) > y_{P_0} + \delta$.](image-url)
If \( y_S < y_{P_0} \) and \( y_{P_1} > y_S \), from the vector field of system (1.4), we know the trajectory of system (1.4) with any initiating point on the \( N_2 \) will ultimately stay in \( \Omega_1 = \{(x, y)/0 \leq x \leq h_1, y \geq 0\} \) after one impulsive effect. Therefore there is no an order one periodic solutions of system (1.4).

(2) \(((1 - \beta)y_{P_0} + q < a/b \) (as shown in Figure 6)). In this case, the point \( P_0^+ \) lies below the point \( A \), that is, \((1 - \beta)y_{P_0} + q < a/b \), thus the successor function of the point \( A \) is \( f(A) = (1 - \beta)y_{P_0} + q - a/b < 0 \).

Take another point \( B_1((1 - a)h_2, \varepsilon) \in N_2 \), where \( \varepsilon > 0 \) is small enough. Then there must exist a trajectory passing through the point \( B_1 \) which intersects \( M_2 \) at point \( P_1(h_2, y_{P_1}) \in M_2 \). Suppose the point \( P_1(h_2, y_{P_1}) \) is subject to impulsive effects to point \( P_1^+((1 - a)h_2, y_{P_1}^+) \), then we have \( y_{P_1}^+ > \varepsilon \). So we have \( f(B_1) = y_{P_1}^+ - \varepsilon > 0 \).

From Lemma 2.7, there exists an order one periodic solution of system (1.4), whose initial point is between \( B_1 \) and \( A \) in set \( N_2 \).

(3) \(((1 - \beta)y_{P_0} + q = a/b \). \( P_0^+ \) coincides with \( A \), and the successor function of \( A \) is that \( f(A) = 0 \), so there exists an order one periodic solution of system (1.4) which is just a part of the trajectory passing through the \( A \).

Now, we can summarize the above results as the following theorem.

**Theorem 3.3.** Assuming that \( \lambda > dh \) and \( 0 < h_1 < (1 - a)h_2 < h_2 < d/b(\lambda - dh) \).

If \( y_{P_0}' \leq y_A \), there exists an order one periodic solutions of the system (1.4).

If \( y_{P_0}' > y_A \), \( y_S \geq y_{P_0}' \) or \( y_S < y_{P_0}' \), and \( y_S > y_{P_1}' \), there exists an order one periodic solutions of the system (1.4).

If \( y_{P_1}' > y_A \), \( y_S < y_{P_0}' \) and \( y_S < y_{P_1}' \), there is no an order one periodic solutions of the system (1.4). The trajectory of system (1.4) with any initiating point on the \( N_2 \) will ultimately stay in \( \Omega_1 = \{(x, y)/0 \leq x \leq h_1, y \geq 0\} \) after one impulsive effect.
Subcase 2.2 (0 < (1 - a)h₂ < h₁ < h₂ < d/b(λ - dh)). In this case, the set N₂ is on the left side of N₁. Any trajectory from initial point \((x₀^+, y₀^+)\) ∈ N₂ will intersect with M₁ at some point with time increasing. By the analysis of Case 1, the trajectory from initial point \((x₀^+, y₀^+)\) ∈ N₂ on the set N₂ will stay in the region \(\Omega₁ = \{(x, y) \mid x ≥ 0, y ≥ 0, x ≤ h₁\}\). Similarly, any trajectory from initial point \((x₀^+, y₀^+)\) ∈ \(Ω₀ = \{(x, y) \mid x ≥ 0, y ≥ 0, x ≤ h₂\}\) will stay in the region \(\Omega₁ = \{(x, y) \mid x ≥ 0, y ≥ 0, x ≤ h₁\}\) after one impulsive effect or free from impulsive effect.

**Theorem 3.4.** If \(λ > dh, 0 < (1 - a)h₂ < h₁ < h₂ < d/b(λ - dh)\), there is no an order one periodic solutions to the system (1.4), and the trajectory with initial point \((x₀^+, y₀^+)\) ∈ \(Ω₀ = \{(x, y) \mid x ≥ 0, y ≥ 0, x ≤ h₂\}\) will stay in the region \(\Omega₁ = \{(x, y) \mid x ≥ 0, y ≥ 0, x ≤ h₁\}\).

Case 3 \((d/b(λ - dh) < h₂)\). In this case, the set M₂ is on the right side of stable center R. In the light of the different position of N₂, we consider the following two subcases.

Subcase 3.1 \((h₁ < (1 - a)h₂ < d/b(λ - dh) < h₂)\). In this case, the set M₂ is in the right side of R. Then there exists a unique closed trajectory \(Γ₁\) of system (1.4) which contains the point R and is tangent to M₂ at the point A.

Since \(Γ₁\) is closed trajectory, we take their the minimal value \(δ_{min}\) of abscissas at the trajectory \(Γ₁\), namely, \(δ_{min} ≤ x\) holds for any abscissas of \(Γ₁\).

(1) \((h₁ < (1 - a)h₂ < δ_{min} < d/b(λ - dh) < h₂)\). In this case, there is a trajectory, which contains the point R\(d/b(λ - dh), a/b\) and is tangent to the N₂ at the point B intersects M₂ at a point \(P₁(h₂, y₁)\) ∈ M₂. Suppose point \(P₁\) is subject to impulsive effects to point \(P₁⁺((1 - a)h₂, y₁⁺)\), here \(y₁⁺ = (1 - β)y₁ + q\). The position of \(P₁⁺\) has the following three sub-cases.

If \((1 - β)y₁ + q < a/b\) (Figure 7), the point \(P₁⁺\) lies below the point B. Like the analysis of Subcase 2.1(2), we can prove there exists an order one periodic solution to the system (1.4) in this case.
If \((1 - \beta) y_{P_1} + q > a/b\), the point \(P_1^+\) lies above the point \(B\); the trajectory from initiating point \(P_1^+\) intersects with the line \(L_1\) at point \(C\). If \(h_1 \leq y_C\) (Figure 8), we have \(y_{P_1} > y_{P_1}^+\) and \(y_{P_2} > y_{P_2}^+\), then the successor function of \(P_1^+\) is that \(f(P_1^+) = y_{P_1}^+ - y_{P_1} < 0\). Then, we know that there exists an order one periodic solution of system (1.4), whose initial point is between the point \(P_1^+\) and \(B\) in set \(N_2\). If \(h_1 > y_C\) (Figure 9), there is a trajectory which is tangent to the \(N_1\) at a point \(D\) intersects with \(M_2\) at a point \(P_3(h_2, y_{P_3}) \in M_2\), \(P_3\) jumps to \(P_3^+\) after the impulsive effects. If \(y_{P_3^+} \leq y_{B_1}\), we can easily know that there exists an order one periodic solution of system (1.4). If \(y_{P_3^+} > y_{B_1}\), by the qualitative analysis of the system (1.4), we know that trajectory with any initiating point on the \(N_2\) will ultimately stay in \(\Gamma_1\) after a finite number of impulsive effects.

If \((1 - \beta) y_{P_1} + q = a/b\), the point \(P_1^+\) coincides with the point \(B\), and the successor function of the point \(B\) is that \(f(B) = 0\); then there exists an order one periodic solution which is just a part of the trajectory passing through the point \(B\).
Now, we can summarize the above results as the following theorem.

Theorem 3.5. Assuming that $\lambda > dh$ and $h_1 < (1 - \alpha)h_2 < \delta_{\min} < d/(\lambda - h d) < h_2$.

If $(1 - \beta)y_B + q < a/b$, then there exists an order one periodic solution to the system (1.4).

If $(1 - \beta)y_B + q > a/b$ and $h_1 = y_C$, then there exists an order one periodic solution to the system (1.4).

If $(1 - \beta)y_B + q > a/b$, $h_1 > y_C$, and $y_B > y_A$, then there exists an order one periodic solution to the system (1.4).

(2) $h_1 < \delta_{\min} < (1 - \alpha)h_2 < d/b(\lambda - dh) < h_2$. In this case, denote the closed trajectory $\Gamma_1$ of system (1.4) intersects with $N_2$ two points $A_1 = ((1 - \alpha)h_2, y_{A_1})$ and $A_2 = ((1 - \alpha)h_2, y_{A_2})$ (as shown in Figure 10). Since $A \in M_2$, impulse occurs at the point $A$. Suppose point $A$ is subject to impulsive effects to point $P_0^+((1 - \alpha)h_2, y_{P_0^+})$, here $y_{P_0^+} = (1 - \beta)(a/b) + q$.

If $(1 - \beta)(a/b) + q = y_{A_1}$ or $(1 - \beta)(a/b) + q = y_{A_2}$, then $P_0^+$ coincides with $A_1$ or $P_0^+$ coincides with $A_2$, and the successor function of $A_1$ or $A_2$ is that $f(A_1) = 0$ or $f(A_2) = 0$. So, there exists an order one periodic solution of system (1.4) which is just a part of the trajectory $\Gamma_1$.

If $(1 - \beta)(a/b) + q < y_{A_1}$, the point $P_0^+$ lies below the point $A_2$. Like the analysis of Subcase 2.1(2), we can prove there exists an order one periodic solution to the system (1.4) in this case.

If $(1 - \beta)(a/b) + q > y_{A_1}$ (as shown in Figure 11), the point $P_0^+$ is above the point $A_1$. Like the analysis of Subcase 3.1(1), we obtain sufficient conditions of existence of order one periodic solution to the system (1.4).

Theorem 3.6. Assuming that $\lambda > dh$, $h_1 < (1 - \alpha)h_2 < \delta_{\min} < d/b(\lambda - dh) < h_2$.

If $(1 - \beta)(a/b) + q < y_{A_2}$, there exists an order one periodic solution to the system (1.4).

If $(1 - \beta)(a/b) + q > y_{A_1}$ and $h_1 \leq y_C$, then there exists an order one periodic solution to the system (1.4).
If \((1 - \beta)(a/b) + q > y_{A_1}, (1 - \beta)y_{P_1} + q > a/b, h_1 > y_C, \) and \(y_{P_0}^* \leq y_{B_1},\) then there exists an order one periodic solution to the system (1.4).

(3) \((y_{A_1} < (1 - \beta)(a/b) + q < y_{A_2}).\) In this case, we note that the point \(P_0^*\) must lie between the point \(A_1\) and the point \(A_2\) (As shown in Figure 12). Taking a point \(B_1 \in M_2\) such that \(B_1\) jumps to \(A_2\) after the impulsive effect, denote \(A_2 = B_1^*\). Since \(y_{P_0}^* > y_{B_1}\), we have \(y_A > y_{B_1}\). Let \(R(B_1) = B_2^* \in N_2\), take a point \(B_2 \in M_2\) such that \(B_2\) jumps to \(B_1^*\) after the impulsive effects, then we have \(y_{B_1} > y_{B_2}\), \(y_{B_2} > y_{B_1}\). This process continues until there exists a \(B_k^* \in N_2\) \((K \in Z_+\) satisfying \(y_{B_k^*} < q\). So we obtain a sequence \(\{B_k^*\}_{k=1,2,\ldots,K}\) of the set \(M_2\) and a sequence \(\{B_k\}_{k=1,2,\ldots,K}\) of set \(N_2\) satisfying \(R(B_{k-1}^*) = B_k^* \in N_2, y_{B_k} > y_{B_{k-1}}^*\). In the following, we will prove the trajectory of system (1.4) with any initiating point of set \(N_2\) will ultimately stay in \(\Gamma_1\).

From the vector field of system (1.4), we know the trajectory of system (1.4) with any initiating point between the point \(A_1\) and \(A_2\) will be free from impulsive effect and ultimately will stay in \(\Gamma_1\).
For any point below $A_2$, it must lie between $B_{k+1}^+$ and $B_{k}^-$, here $k = 2, 3, \ldots, K + 1$ and $A_2 = B_1^+$. After $k$ times' impulsive effects, the trajectory with this initiating point will arrive at some point of the set $N_2$ which must be between $A_1$ and $A_2$, and then ultimately stay in $\Gamma_1$.

Denote the intersection of the trajectory passing through the point $B$ which tangents to $N_1$ at point $B$ with the set $N_2$ at $S((1-\alpha)h_2, y_S)$. With time increasing, the trajectory of system (1.4) from any initiating point on segment $A_1S$ intersect with the set $N_2$ at some point which is below $A_2$; so just like the analysis above we obtain, it will ultimately stay in $\Gamma_1$. So for any point below $S$, will ultimately stay in region $\Gamma_1$ with time increasing.

Now, we can summarize the above results as the following theorem.

**Theorem 3.7.** Assuming that $\lambda > dh$, $h_1 < \delta_{\min} < (1-\alpha)h_2 < d/b(\lambda - dh) < h_2$, and $y_{A_2} < (1-\beta)(a/b) + q < y_{A_1}$, there is no periodic solution in system (1.4), and the trajectory with any initiating point on the set $N_2$ will stay in $\Gamma_1$ or in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$.

Subcase 3.2 ($0 < (1-\alpha)h_2 < h_1 < d/b(\lambda - dh) < h_2$). In this case, the set $N_2$ is on the left side of the set $N_1$ and $M_2$ in the right side of $R$. Like the analysis of Subcase 2.2, we can know that any trajectory with initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$ will stay in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$ after one impulsive effect or free from impulsive effect.

### 4. Attractiveness of the Order One Periodic Solution

In this section, under the condition of existence of order one periodic solution to system (1.4) and the initial value of pest population $x(0) \leq h_2$, we discuss its attractiveness. We focus on Case 1 and Case 2; by similar method, we can obtain similar results about Case 3.
Theorem 4.1. Assuming that $\lambda > d(h_1 \cdot h_2 < d/(b(\lambda - dh))$ and $\delta \geq a/b$.

If $y_{P^*} > y_{P^*} > y_{P^*}$ or $y_{P^*} < y_{P^*} < y_{P^*}$ (Figure 14), then

(I) there exists a unique order one periodic solution of system (1.4),

(II) if $(1 - a)h_2 < h_1$, order one periodic solution of system (1.4) is attractive in the region

$$\Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}.$$ 

Proof. By the derivation of Theorem 3.1, we know there exists an order one periodic solution of system (1.4). We assume trajectory $PP^*$ and segment $PP^*$ formulate an order one periodic solution of system (1.4), that is, there exists a $P^* \in N_2$ such that the successor function of $P^*$ satisfies $f(P^*) = 0$. First, we will prove the uniqueness of the order one periodic solution.

We take any two points $C_1(h_1, y_{C_1}) \in N_1$, $C_2(h_1, y_{C_2}) \in N_1$ satisfying $y_{C_2} > y_{C_1} > y_{A'}$, then we obtain two trajectories whose initiate points are $C_1$ and $C_2$ intersects the set $M_1$ two points $D_1(h_1, y_{D_1})$ and $D_2(h_1, y_{D_2})$, respectively, (Figure 13). In view of the vector field of system (1.4) and the disjointness of any two trajectories without impulse, we know $y_{D_1} > y_{D_2}$.

Suppose the points $D_1$ and $D_2$ are subject to impulsive effect to points $D_1(h_1, y_{D_1})$ and $D_2(h_1, y_{D_2})$, respectively, then we have $y_{D_1} > y_{D_2}$ and $f(C_1) = y_{D_1} - y_{C_1}$, $f(C_2) = y_{D_2} - y_{C_2}$, so we get $f(C_1) = f(C_2) < 0$, thus we obtain the successor function $f(x)$ is decreasing monotonously of $N_1$, so there is a unique point $P^* \in N_1$ satisfying $f(P^*) = 0$, and the trajectory $P^*PP^*$ is a unique order one periodic solution of system (1.4).

Next, we prove the attractiveness of the order one periodic solution $P^*PP^*$ in the region $\Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$. We focus on the case $y_{P^*} > y_{P^*} > y_{P^*}$; by similar method, we can obtain similar results about case $y_{P^*} < y_{P^*} < y_{P^*}$ (Figure 14).

Take any point $P_0^*(h_1, y_{P_0}) \in N_1$ above $P^*$. Denote the first intersection point of the trajectory from initiating point $P_0^*(h_1, y_{P_0})$ with the set $M_1$ at $P_1(h_1, y_{P_1})$, and the corresponding consecutive points are $P_2(h_1, y_{P_2}), P_3(h_1, y_{P_3}), P_4(h_1, y_{P_4}), \ldots$, respectively.

![Figure 13: There is a unique order one periodic solution (Theorem 4.1).](image-url)
Consequently, under the effect of impulsive function, the corresponding points after pulse are $P^+_1(h_1, y_{P^+_1}), P^+_2(h_1, y_{P^+_2}), P^+_2(h_1, y_{P^+_1}), \ldots$.

Due to conditions $y_{P^+_1} > y_{P^+_2} > y_{P^+} > y_{P^+_k} = y_{P_k} + \delta$, $\delta \geq a/b$ and disjointness of any two trajectories, then we get a sequence $\{P^+_k\}_{k=1,2,\ldots}$, of the set $N_1$ satisfying

$$y_{P^+_1} < y_{P^+_2} < \cdots < y_{P^+_k} < y_{P^+_{2k+1}} < \cdots < y_{P^+_{2k}} < \cdots < y_{P^+_2} < y_{P^+_1},$$

(4.1)

So the successor function $f(P^+_k) = y_{P^+_k} - y_{P^+_{2k+1}} > 0$ and $f(P^+_k) = y_{P^+_{2k+1}} - y_{P^+_{2k}} < 0$ hold. Series $\{y_{P^+_k}\}_{k=1,2,\ldots}$ increases monotonously and has upper bound, so $\lim_{k \to \infty} y_{P^+_k}$ exists. Next, we will prove $\lim_{k \to \infty} y_{P^+_k} = y_{P^+}$. Set $\lim_{k \to \infty} P_{2k-1} = C^+$, we will prove $P^+ = C^+$. Otherwise $P^+ \neq C^+$, then there is a trajectory passing through the point $C^+$ which intersects the set $M_1$ at point $\tilde{C}$, then we have $y_{\tilde{C}} < y_{P^+}, y_{\tilde{C}^+} > y_{P^+}$. Since $f(C^+) \geq 0$ and $P^+ \neq C^+$, according to the uniqueness of the periodic solution, then we have $f(C^+) = y_{\tilde{C}^+} - y_{\tilde{C}} > 0$, thus $y_{\tilde{C}} < y_{P^+} < y_{\tilde{C}^+}$ hold. Analogously, let trajectory passing through the point $C^+$ which intersects the set $M_1$ at point $\tilde{C}$, and the corresponding consecutive points is $\tilde{C}$, then $y_{\tilde{C}} > y_{\tilde{C}^+} > y_{P^+} > y_{\tilde{C}^+} > y_{\tilde{C}}$, then we have $f(\tilde{C}^+) = y_{\tilde{C}^+} - y_{\tilde{C}} > 0$, this is, contradict to the fact that $C^+$ is a limit of sequence $\{P^+_k\}_{k=1,2,\ldots}$, so we obtain $P^+ = C^+$. So, we obtain $\lim_{k \to \infty} y_{P^+_{2k-1}} = y_{P^+}$. Similarly, we can prove $\lim_{k \to \infty} y_{P^+_{2k}} = y_{P^+}$.

From above analysis, we know there exists a unique order one periodic solution in system (1.4), and the trajectory from initiating any point of the $N_1$ will ultimately tend to be order one periodic solution $P^+P^+$. 

\textbf{Figure 14:} Order one periodic solution is attractive (Theorem 4.1).
Any trajectory from initial point \((x^*, y^*) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}\) will intersect with \(N_1\) at some point with time increasing on the condition that \((1 - \alpha)h_2 < h_1 < h_2 < d/b(\lambda - dh);\) therefore the trajectory from initial point on \(N_1\) ultimately tends to be order one periodic solution \(\hat{P} PP^+\). Therefore, order one periodic solution \(\hat{P} PP^+\) is attractive in the region \(\Omega_0\). This completes the proof.

Remark 4.2. Assuming that \(\lambda > dh, h_1 < h_2 < d/b(\lambda - dh)\) and \(\delta \geq a/b,\) if \(y_{P^+} < y_{P_0} < y_{P_2}\) or \(y_{P^+} > y_{P_0} > y_{P_2}\) then the order one periodic solution is unattractive.

**Theorem 4.3.** Assuming that \(\lambda > dh, h_1 < (1 - \alpha)h_2 < h_2 < d/b(\lambda - dh)\) and \(y_{P_0} < y_A\) (as shown in Figure 15), then

- (I) There exists an odd number of order one periodic solutions of system (1.4) with initial value between \(C^+_1\) and \(A\) in set \(N_2\).

- (II) If the periodic solution is unique, then the periodic solution is attractive in region \(\Omega_2\), here \(\Omega_2\) is open region which is constituted by trajectory \(\overline{GB}\), segment \(\overline{BH}\), segment \(\overline{HE}\), and segment \(\overline{EG}\).

**Proof.** (I) According to the Subcase 2.1(2), \(f(A) < 0\) and \(f(C^+_1) > 0\), and the continuous successor function \(f(x)\), there exists an odd number of root satisfying \(f(x) = 0\), then we can get there exists an odd number of order one periodic solutions of system (1.4) with initial value between \(C^+_1\) and \(A\) in set \(N_2\).

(II) By the derivation of Theorem 3.3, we know there exists an order one periodic solution of system (1.4) whose initial point is between \(C^+_1\) and \(P^+_0\) in the set \(N_2\). Assume
trajectory \( \hat{P} \) and segment \( \hat{PP}^+ \) formulate the unique order one periodic solution of system (1.4) with initial point \( P^* \in N_2 \).

On the one hand, take a point \( C_1 = ((1-\alpha)h_2, y_{C_1}) \in N_2 \) satisfying \( y_{C_1} = \varepsilon < q \) and \( y_{C_1} < y_P \). The trajectory passing through the point \( C_1 = ((1-\alpha)h_2, \varepsilon) \) which intersects with the set \( M_2 \) at point \( C_2(h_2, y_{C_1}) \), that is, \( F(C_1) = C_2 \in M_2 \), then we have \( y_{C_2} < y_P \), thus \( y_{C_2} < y_{C_1} \), since \( y_{C_2} = (1-\beta)y_{C_2} + q > \varepsilon \). So, we obtain \( f(C_1) = y_{C_1} - y_{C_1} = y_{C_2} - \varepsilon > 0 \); Set \( F(C_2) = C_3 \in M_2 \), because \( y_{C_1} < y_{C_2} < y_P \), we know \( y_{C_1} < y_{C_3} < y_P \), then we have \( y_{C_3} < y_{C_1} < y_P \) and \( f(C_1) = y_{C_3} - y_{C_1} > 0 \). This process is continuing, then we get a sequence \( \{C_k\}_{k=1,2,...} \) of the set \( N_2 \) satisfying

\[
y_{C_1} < y_{C_2} < \cdots < y_{C_k} < \cdots < y_P .
\]

(4.2)

and \( f(C_k) = y_{C_1} - y_{C_k} > 0 \). Series \( \{y_{C_k}\}_{k=1,2,...} \) increase monotonously and have upper bound, so \( \lim_{k \to \infty} y_{C_k} \) exists. Like the proof of Theorem 4.1, we can prove \( \lim_{k \to \infty} y_{C_k} = y_P \).

On the other hand, set \( F(P_0) = D_1 \in M_2 \), then \( D_1 \) jumps to \( D_1^+ \in N_2 \) under the impulsive effects. Since \( y_P < y_{P_2} < y_A \), we have \( y_P < y_{D_0} < y_{P_2} \), thus we obtain \( y_P < y_{D_1} < y_{D_0} \), \( f(P_0) = y_{D_1} - y_{P_0} < 0 \). Set \( F(D_1) = D_2 \in M_2 \), then \( D_2 \) jumps to \( D_2^+ \in N_2 \) under the impulsive effects. We have \( y_P < y_{D_2} < y_{D_1} \); this process is continuing, we can obtain a sequence \( \{D_k\}_{k=1,2,...} \) of the set \( N_2 \) satisfying

\[
y_{P_0} > y_{D_1} > y_{D_2} > \cdots > y_{D_k} > \cdots > y_P
\]

(4.3)

and \( f(D_k) = y_{D_k} - y_{D_1} < 0 \). Series \( \{y_{D_k}\}_{k=1,2,...} \) decreases monotonously and has lower bound, so \( \lim_{k \to \infty} y_{D_k} \) exists. Similarly, we can prove \( \lim_{k \to \infty} y_{D_k} = y_P \).

Any point \( Q \in N_2 \) below \( A \) must be in some interval \( [y_{D_{k+1}}, y_{D_k})_{k=1,2,...}, [y_{D_{k+1}}, y_{P_0})_{k=1,2,...}, [y_P, y_A), [y_{C_k}, y_{C_{k+1}})_{k=1,2,...} \). Without loss of generality, we assume the point \( Q \in [y_{D_{k+1}}, y_{D_k})_{k=1,2,...} \).

The trajectory with initiating point \( Q \) moves between trajectory \( D_k \) and \( D_{k+1} \) and intersects with \( M_2 \) at some point between \( D_{k+1} \) and \( D_k \); under the impulsive effects, it jumps to the point of \( N_2 \) which is between \( [y_{D_{k+1}}, y_{D_k}) \), then trajectory \( \hat{P}(Q, t) \) continues to move between trajectory \( D_k \) and \( D_{k+1} \). This process can be continued unlimitedly. Since \( \lim_{k \to \infty} y_{D_k} = y_P \), the intersection sequence of trajectory \( \hat{P}(Q, t) \), and the set \( N_2 \) will ultimately tend to the point \( P^* \). Similarly, if \( Q \in [y_{C_k}, y_{C_{k+1}}) \), we also can get the intersection sequence of trajectory \( \hat{P}(Q, t) \) and the set \( N_2 \) will ultimately tend to point \( P^* \). Thus, the trajectory initiating any point below \( A \) ultimately tend to the unique order one periodic solution \( P^* \).

Denote the intersection of the trajectory passing through the point \( B \) which tangents to \( N_1 \) at the point \( B \) with the set \( N_2 \) at a point \( S(1-\alpha)h_2, y_S) \). The trajectory from any initiating point on segment \( AS \) will intersect with the set \( N_2 \) at some point below \( A \) with time increasing. So like the analysis above, we obtain the trajectory from any initiating point on segment \( AS \) will ultimately tend to be the unique order one periodic solution \( P^* \).

Since the trajectory with any initiating point of the \( \Omega_2 \) will certainly intersect with the set \( N_2 \), then from the above analysis, we know the trajectory with any initiating point on segment \( \hat{AS} \) will ultimately tend to be order one periodic solution \( P^*/P^* \). Therefore, the unique order one periodic solution \( P^* \) is attractive in the region \( \Omega_2 \). This completes the proof. \( \square \)
Remark 4.4. Assuming that $\lambda > dh$, $h_1 < (1 - \alpha)h_2 < h_2 < d/b(\lambda - dh)$, and $y_C < y_A < y_P$, then the order one periodic solution with initial point between $A$ and $P_0$ is unattractive.

Theorem 4.5. Assuming that $\lambda > dh$, $h_1 < (1 - \alpha)h_2 < h_2 < d/b(\lambda - dh)$, $y_{P_0} > y_{P_1} > y_A$ (Figure 16) then, there exists a unique order one periodic solution of system (1.4) which is attractive in the region $\Omega_2$, here $\Omega_2$ is open region which enclosed by trajectory $\overline{GB}$, segment $B\overline{H}$, segment $\overline{HE}$ and segment $\overline{EC}$.

Proof. By the derivation of Theorem 3.3, we know there exists an order one periodic solution of system (1.4). We assume trajectory $P^+P$ and segment $PP^+$ formulate an order one periodic solution of system (1.4), that is, $P^+ \in N_2$ is its initial point satisfying $f(P^+) = 0$. Like the proof of Theorem 4.1, we can prove the uniqueness of the order one periodic solution of system (1.4).

Next, we prove the attractiveness of the order one periodic solution $P^+PP^+$ in the region $\Omega_2$.

Denote the first intersection point of the trajectory from initiating point $P_0^+$ with the impulsive set $M_2$ at $P_1(h, y_B)$, and the corresponding consecutive points are $P_2(h, y_B), P_3(h, y_B), P_4(h, y_B), \ldots$ respectively. Consequently, under the effect of impulsive function $I$, the corresponding points after pulse are $P_2^+(h, y_B^+), P_3^+(h, y_B^+), P_4^+(h, y_B^+), \ldots$. In view of $y_{P_0} > y_1 > y_A$ and disjointness of any two trajectories, we have

$$y_{P_1} < y_{P_2} < \cdots < y_{P_{2k-1}} < y_{P_{2k}} < y_{P_{2k+1}} < \cdots < y_{P_{2k+1}} < y_{P_{2k+2}} < y_{P_{2k+3}} < y_{P_0}. \tag{4.4}$$

So $f(P_{2k}^+) = y_{P_{2k}} - y_{P_{2k+1}} > 0$ and $f(P_{2k}^+) = y_{P_{2k+1}} - y_{P_{2k}} < 0$ hold. Like the proof of Theorem 4.1, we can prove $\lim_{k \to \infty} y_{P_{2k}} = \lim_{k \to \infty} y_{P_{2k}} = y_P$.
The trajectory from initiating point between $B_0^+$ and $P_0^+$ will intersect with impulsive set $N_2$ with time increasing, under the impulsive effects it arrives at a point of $N_2$ which is between $[y_{P_2^{k-1}}, y_{P_2^{k+1}}]$ or $[y_{P_2^{k}}, y_{P_2^{k-2}}]$. Then like the analysis of Theorem 4.3, we know the trajectory from any initiating point between $B_0^+$ and $P_0^+$ will ultimately tend to be order one periodic solution $\hat{P}_{PP}^+$. Denote the intersection of the trajectory passing through point $B$ which tangents to $N_1$ at point $B$ with the set $N_2$ at $S$. Since the trajectory from initiating any point below $S$ of the set $N_2$ will certain intersect with set $N_2$, next we only need to prove the trajectory with any initiating point below $S$ of the set $N_2$ will ultimately tend to be order one periodic solution $\hat{P}_{PP}^+$. 

**Figure 17**: The time series and phase diagram for system (1.4) starting from initial value $(0.85, 0.2)$ (red), $(0.8, 0.5)$ (green), and $(0.75, 0.11)$ (blue), $\delta = 0.6$, $h_1 = 1 < x^*$. 
Figure 18: The time series and phase diagram for system (1.4) starting from initial value (0.8,0.1) (red), (0.7,0.5) (green), and (0.75,0.3) (blue) $\alpha = 0.6, \beta = 0.3, q = 0.8, h_2 = 1.8, h_1 < h_2 < x^*$.

Assume a point $B_0$ of set $M_2$ jumps to $B_0^+$ under the impulsive effect. Set $R(B_0) = B_1^+ \in N_2$. Assume point $B_1$ of set $N_2$ jumps to $B_1^+$ under the impulsive effect. Set $R(B_1) = B_2^+ \in N_2$. This process is continuing until there exists a $B_{K_0}^+ \in N_2$ satisfying $y_{B_{K_0}^+} \leq q$. So we obtain a sequence $\{B_k\}_{k=0,1,2,\ldots,K_0}$ of set $M_2$ and a sequence $\{B_k^+\}_{k=0,1,2,\ldots,K_0}$ of set $N_2$ satisfying $R(B_{k-1}^+) = B_k^+, y_{B_k^+} \leq y_{B_{k-1}^+}$. For any point of set $N_2$ below $B_0^+$, it must lie between $B_k^+$ and $B_{k+1}^+$ here $k = 1, 2, \ldots, K_0$. After $K_0$ times’ impulsive effects, the trajectory with this initiating point will arrive at some point of the set $N_2$ which must be between $B_0^+$ and $P_0^+$, and then will ultimately tend to order one periodic solution $\hat{P}PP^+$. There is no order one periodic solution with the initial point below $B_0^+$. 
The trajectory with any initiating point in segment $\overline{AS}$ will intersect with the set $N_2$ at some point below $B_0^+$ with time increasing. Like the analysis above, we obtain the trajectory initiating any point on segment $\overline{AS}$ will ultimately tend to be the unique order one periodic solution $\hat{P}^\ast$. 

From above analysis, we know there exists a unique order one periodic solution in system (1.4), and the trajectory from any initiating point below $S$ will ultimately tend to be order one periodic solution $P^+P^+$ and $P^\ast P^\ast$. Therefore, order one periodic solution $P^\ast P^\ast$ is attractive in the region $\Omega_2$. This completes the proof.

\begin{figure}[h]
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{figure19a}
\caption{}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{figure19b}
\caption{}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{figure19c}
\caption{}
\end{subfigure}
\caption{The time series and phase diagram for system (1.4) starting from initial value $(1, 0.7)$ (red), $(1.4, 0.5)$ (green), and $(1.2, 1)$ (blue) $\alpha = 0.6, \beta = 0.3, q = 0.8, h_2 = 3.5, h_1 < x^* < h_2$.}
\end{figure}
5. Conclusion

In this paper, a state-dependent impulsive dynamical model concerning different control methods at different thresholds is proposed, we find a new method to study existence and attractiveness of order one periodic solution of such system. We define semicontinuous dynamical system and successor function, demonstrate the sufficient conditions that system (1.4) exists order one periodic solution with differential geometry theory and successor function. Besides, we successfully prove the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis. The method can be also extended to mechanical dynamical systems with impacts, for example [19, 20].

These results show that the state-dependent impulsive effects contribute significantly to the richness of the dynamics of the model. The conditions of existence of order one periodic solution in this paper have more extensively applicable scope than the conditions given in [14]. Our results show that, in theory, a pest can be controlled such that its population size is no larger than its ET by applying effects impulsively once, twice, or at most, a finite number of times, or according to a periodic regime. The methods of the theorems are proved to be new in this paper, and these methods are more efficient and easier to operate than the existing research methods which have been applied the models with impulsive state feedback control [16–18, 21], so they are deserved further promotion. In this paper, according to the integrated pest management strategies, we propose a model for pest control which adopts different control methods at different thresholds, the corresponding control is exerted, which leads to the two state impulses in model. Certainly, many biological systems will always be described by three or more state variables, which are the main work in the future.

In order to testify the validity of our results, we consider the following example.

\[
\begin{align*}
x'(t) &= x(t)(0.4 - 0.5y(t)), \\
y'(t) &= y(t)\left(\frac{0.25x(t)}{1 + 0.1x(t)} - 0.6\right), \\
\Delta x(t) &= 0, \\
\Delta y(t) &= \delta, \quad x = h_1, \ y \leq y^*, \\
\Delta x(t) &= -\alpha x(t), \\
\Delta y(t) &= -\beta y(t) + q, \quad x = h_2,
\end{align*}
\]

where \(\alpha, \beta \in (0, 1), \delta > 0, q > 0, 0 < h_1 < h_2\). Now, we consider the impulsive effects influences on the dynamics of system (5.1).

Example 5.1. Existence and attractiveness of order one periodic solution.

We set \(h_1 = 1, \alpha = 0.6, \beta = 0.8, q = 0.8, h_2 = 1.8\), initiating points are (0.85, 0.2) (red), (0.8, 0.5) (green), and (0.75, 0.11) (blue), respectively. Figure 17 shows that the conditions of Theorems 3.1 and 4.1 hold, system (5.1) exists order one periodic solution. The trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

Example 5.2. Existence and attractiveness of positive periodic solution.

We set \(h_1 = 0.7, \alpha = 0.6, \beta = 0.8, q = 0.8, h_2 = 1.8, h_1 < (1 - \alpha)h_2 < x^*\), initiating points are (0.8, 0.1) (red), (0.7, 0.5) (green), and (0.75, 0.3) (blue), respectively. Figure 18 shows that
the conditions of Theorems 3.3 and 4.3 hold, there exists order one periodic solution of the system (5.1), and the trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

**Example 5.3.** Existence and attractive of positive periodic solutions.

We set \( h_1 = 0.7, \alpha = 0.6, \beta = 0.8, q = 0.8, h_3 = 3.5, h_1 < (1 - \alpha)h_2 < x^* < h_2, \)

initiating points are \((1,0.7)\) (red), \((1.4,0.5)\) (green), and \((1.2,1)\) (blue) as shown in Figure 19.

Therefore, the conditions of Theorems 3.6 and 4.5 hold, then system (5.1) exists order one periodic solution, and it is attractive.

**Acknowledgment**

This Project supported by the National Natural Science Foundation of China (no. 10872118).

**References**


Submit your manuscripts at
http://www.hindawi.com