Research Article

(λ, μ) -Fuzzy Version of Ideals, Interior Ideals, Quasi-Ideals, and Bi-Ideals

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Received 12 July 2011; Accepted 16 December 2011

Academic Editor: Vu Phat

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We introduced (λ, μ) -fuzzy ideals, (λ, μ) -fuzzy interior ideals, (λ, μ) -fuzzy quasi-ideals, and (λ, μ) -fuzzy bi-ideals of an ordered semigroup and studied them. When $\lambda = 0$ and $\mu = 1$, we meet the ordinary fuzzy ones. This paper can be seen as a generalization of Kehayopulu and Tsingelis (2006), Kehayopulu and Tsingelis (2007), and Yao (2009).

1. Introduction and Preliminaries

An ideal of a semigroup is a special subsemigroup satisfying certain conditions. The best way to know an algebraic structure is to begin with a special substructure of it. There are plenty of papers on ideals. After Zadeh' introduction of fuzzy set in 1965 (see [1]), the fuzzy sets have been used in the reconsideration of classical mathematics. Also, fuzzy ideals have been considered by many researchers. For example, Kim [2] studied intuitionistic fuzzy ideals of semigroups, Meng and Guo [3] researched fuzzy ideals of BCK/BCI-algebras, Koguep [4] researched fuzzy ideals of hyperlattices, and Kehayopulu and Tsingelis [5] researched fuzzy interior ideals of ordered semigroups.

Recently, Yuan et al. [6] introduced the concept of fuzzy subfield with thresholds. A fuzzy subfield with thresholds λ and μ is also called a (λ, μ) -fuzzy subfield. Yao continued to research (λ, μ) -fuzzy normal subfields, (λ, μ) -fuzzy quotient subfields, (λ, μ) -fuzzy subrings, and (λ, μ) -fuzzy ideals in [7–10]. Feng et al. researched (λ, μ) -fuzzy sublattices and (λ, μ) -fuzzy subhyperlattices in [11].

An ordered semigroup $(S, \circ, \leq)$ is a poset $(S, \leq)$ equipped with a binary operation $\circ$, such that

- (1) $(S, \circ)$ is a semigroup, and
- (2) if $x, a, b \in S$, then

$$a \leq b \Rightarrow \begin{cases} a \circ x \leq b \circ x \\ x \circ a \leq x \circ b. \end{cases} \quad (1.1)$$

Given an ordered semigroup S , a fuzzy subset of S (or a fuzzy set in S) is an arbitrary mapping $f : S \rightarrow [0, 1]$, where $[0, 1]$ is the usual closed interval of real numbers. For any $\alpha \in [0, 1]$, f_α is defined by $f_\alpha = \{x \in S \mid f(x) \geq \alpha\}$. For $a \in S$, we define that $A_a = \{(y, z) \in S \times S \mid a \leq yz\}$. For two fuzzy subsets f and g of S , we define the multiplication of f and g as the fuzzy subset of S defined by

$$(f * g)(a) = \begin{cases} \sup_{(y,z) \in A_a} (f(y) \wedge g(z)), & \text{if } A_a \neq \emptyset, \\ 0, & \text{if } A_a = \emptyset. \end{cases} \quad (1.2)$$

In the set of fuzzy subsets of S , we define the order relation as follows: $f \subseteq g$ if and only if $f(x) \leq g(x)$ for all $x \in S$. For two fuzzy subsets f and g of S , we define

$$(f \cap g)(x) = f(x) \wedge g(x), \quad (f \cup g)(x) = f(x) \vee g(x). \quad (1.3)$$

Note that we use $a \wedge b$ to denote $\min(a, b)$ and use $a \vee b$ to denote $\max(a, b)$.

For any $\alpha \in [0, 1]$, α can be seen as a fuzzy subset of S which is defined by $\alpha(x) = \alpha$, for all $x \in S$.

In the following, we will use S or $(S, \circ, \leq)$ to denote an ordered semigroup and the multiplication of x, y will be xy instead of $x \circ y$.

In the rest of this paper, we will always assume that $0 \leq \lambda < \mu \leq 1$.

In this paper, we introduced (λ, μ) -fuzzy ideals, (λ, μ) -fuzzy interior ideals, (λ, μ) -fuzzy quasi-ideals and (λ, μ) -fuzzy bi-ideals of an ordered semigroup. We obtained the followings:

- (1) in an ordered semigroup, every (λ, μ) -fuzzy ideal is a (λ, μ) -fuzzy interior ideal;
- (2) in an ordered semigroup, every (λ, μ) -fuzzy right (resp. left) ideal is a (λ, μ) -fuzzy quasi-ideal;
- (3) in an ordered semigroup, every (λ, μ) -fuzzy quasi-ideal is a (λ, μ) -fuzzy bi-ideal;
- (4) in a regular ordered semigroup, the (λ, μ) -fuzzy quasi-ideals and the (λ, μ) -fuzzy bi-ideals coincide.

2. (λ, μ) -Fuzzy Ideals and (λ, μ) -Fuzzy Interior Ideals

Definition 2.1. Let $(S, \cdot, \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a (λ, μ) -fuzzy right ideal (resp. (λ, μ) -fuzzy left ideal) of S if

- (1) $f(xy) \vee \lambda \geq f(x) \wedge \mu$ (resp. $f(xy) \vee \lambda \geq f(y) \wedge \mu$) for all $x, y \in S$, and
 (2) if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$ for all $x, y \in S$.

A fuzzy subset f of S is called a (λ, μ) -fuzzy ideal of S if it is both a (λ, μ) -fuzzy right and a (λ, μ) -fuzzy left ideal of S .

Example 2.2. Let $(S, *, \leq)$ be an ordered semigroup where $S = \{e, a, b\}$ and $e \leq a \leq b$. The multiplication table is defined by the following:

$$\begin{array}{c|ccc}
 * & e & a & b \\
 \hline
 e & e & a & b \\
 a & a & b & e \\
 b & b & e & a
 \end{array} \quad (2.1)$$

A fuzzy set f is defined as follows:

$$\begin{array}{c|ccc}
 S & e & a & b \\
 \hline
 f & 0.1 & 0.2 & 0.3
 \end{array} \quad (2.2)$$

Then, f is a $(0.3, 0.7)$ -fuzzy ideal of S . But it is not a fuzzy ideal of S .

Definition 2.3 (see [12]). If $(S, \circ, \leq)$ is an ordered semigroup, a nonempty subset A of S is called an interior ideal of S if

- (1) $SAS \subseteq A$, and
 (2) if $a \in A, b \in S$, and $b \leq a$, then $b \in A$.

Definition 2.4. If $(S, \circ, \leq)$ is an ordered semigroup, a fuzzy subset f of S is called a (λ, μ) -fuzzy interior ideal of S if

- (1) $f(xay) \vee \lambda \geq f(a) \wedge \mu$ for all $x, a, y \in S$, and
 (2) if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$.

In the previous example, f is also a $(0.3, 0.7)$ -fuzzy interior ideal of S . In fact, every fuzzy ideal of an ordered semigroup is a fuzzy interior.

Theorem 2.5. Let $(S, \circ, \leq)$ be an ordered semigroup and f a (λ, μ) -fuzzy ideal of S , then f is a (λ, μ) -fuzzy interior ideal of S .

Proof. Let $x, a, y \in S$. Since f is a (λ, μ) -fuzzy left ideal of S and $x, ay \in S$, we have

$$f(x(ay)) \vee \lambda \geq f(ay) \wedge \mu. \quad (2.3)$$

Since f is a (λ, μ) -fuzzy right ideal of S , we have

$$f(ay) \vee \lambda \geq f(a) \wedge \mu. \quad (2.4)$$

From (2.3) and (2.4) we know that $f(xay) \vee \lambda = (f(x(ay)) \vee \lambda) \vee \lambda \geq (f(ay) \wedge \mu) \vee \lambda = (f(ay) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(a) \wedge \mu$. $\square$

Theorem 2.6. Let $(S, \circ, \leq)$ be an ordered semigroup, then f is a (λ, μ) -fuzzy interior ideal of S if and only if f_α is an interior ideal of S for all $\alpha \in (\lambda, \mu]$.

Proof. Let f be a (λ, μ) -fuzzy interior ideal of S and $\alpha \in (\lambda, \mu]$.

First of all, we need to show that $xay \in f_\alpha$, for all $a \in f_\alpha$, $x, y \in S$.

From $f(xay) \vee \lambda \geq f(a) \wedge \mu \geq \alpha \wedge \mu = \alpha$ and $\lambda < \alpha$, we conclude that $f(xay) \geq \alpha$, that is, $xay \in f_\alpha$.

Then, we need to show that $b \in f_\alpha$ for all $a \in f_\alpha$, $b \in S$ such that $b \leq a$.

From $b \leq a$ we know that $f(b) \vee \lambda \geq f(a) \wedge \mu$ and from $a \in f_\alpha$ we have $f(a) \geq \alpha$. Thus, $f(b) \vee \lambda \geq \alpha \wedge \mu = \alpha$. Notice that $\lambda < \alpha$, then we conclude that $f(b) \geq \alpha$, that is, $b \in f_\alpha$.

Conversely, let f_α be an interior ideal of S for all $\alpha \in (\lambda, \mu]$.

If there are $x_0, a_0, y_0 \in S$, such that $f(x_0a_0y_0) \vee \lambda < \alpha = f(a_0) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $f(a_0) \geq \alpha$ and $f(x_0a_0y_0) < \alpha$. That is $a_0 \in f_\alpha$ and $x_0a_0y_0 \notin f_\alpha$. This is a contradiction with that f_α is an interior ideal of S . Hence $f(xay) \vee \lambda \geq f(a) \wedge \mu$ holds for all $x, a, y \in S$.

If there are $x_0, y_0 \in S$ such that $x_0 \leq y_0$ and $f(x_0) \vee \lambda < \alpha = f(y_0) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $f(y_0) \geq \alpha$, and $f(x_0) < \alpha$, that is, $y_0 \in f_\alpha$ and $x_0 \notin f_\alpha$. This is a contradiction with that f_α is an interior ideal of S . Hence if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$. $\square$

3. (λ, μ) -Fuzzy Quasi-Ideals and (λ, μ) -Fuzzy Bi-Ideals

Definition 3.1. Let $(S, \circ, \leq)$ be an ordered semigroup. A subset A of S is called a quasi-ideal of S if

- (1) $AS \cap SA \subseteq S$, and
- (2) if $x \in S$ and $x \leq y \in A$, then $x \in A$.

Definition 3.2. A nonempty subset A of an ordered semigroup S is called a bi-ideal of S if it satisfies

- (1) $ASA \subseteq A$, and
- (2) $x \in S$ and $x \leq y \in A$, then $x \in A$.

Definition 3.3. Let $(S, \circ, \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a (λ, μ) -fuzzy quasi-ideal of S if

- (1) $f \cup \lambda \supseteq (f * 1) \cap (1 * f) \cap \mu$, and
- (2) if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$ for all $x, y \in S$.

Definition 3.4. Let $(S, \circ, \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a (λ, μ) -fuzzy bi-ideal of S if for all $x, y, z \in S$,

- (1) $f(xyz) \vee \lambda \geq (f(x) \wedge f(z)) \wedge \mu$, and
- (2) if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$.

Remark 3.5. It is easy to see that a fuzzy quasi-ideal [13] of S is a $(0, 1)$ -fuzzy quasi-ideal of S , and a fuzzy bi-ideal [13] of S is a $(0, 1)$ -fuzzy bi-ideal of S .

Theorem 3.6. Let $(S, \circ, \leq)$ be an ordered semigroup, then f is a (λ, μ) -fuzzy quasi-ideal of S if and only if f_α is a quasi-ideal of S for all $\alpha \in (\lambda, \mu]$.

Proof. Let f be a (λ, μ) -fuzzy quasi-ideal of S and $\alpha \in (\lambda, \mu]$.

First of all, we need to show that $Sf_\alpha \cap f_\alpha S \subseteq f_\alpha$.

If $x \in Sf_\alpha \cap f_\alpha S$, then $x = st_1 = t_2s$ for some $t_1, t_2 \in f_\alpha$ and $s \in S$.

From $f \cup \lambda \supseteq (f * 1) \cap (1 * f) \cap \mu$, we conclude that $f(x) \vee \lambda \geq (f * 1)(x) \wedge (1 * f)(x) \wedge \mu \geq f(t_1) \wedge f(t_2) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Thus, $f(x) \geq \alpha$, and so $x \in f_\alpha$. Hence, $S * f_\alpha \cap f_\alpha * S \subseteq f_\alpha$.

Next, we need to show that $b \in f_\alpha$ for all $a \in f_\alpha, b \in S$ such that $b \leq a$.

From $b \leq a$ we know that $f(b) \vee \lambda \geq f(a) \wedge \mu$ and from $a \in f_\alpha$ we have $f(a) \geq \alpha$. Thus, $f(b) \vee \lambda \geq \alpha \wedge \mu = \alpha$. Notice that $\lambda < \alpha$, we conclude that $f(b) \geq \alpha$, that is, $b \in f_\alpha$.

Conversely, let f_α be a quasi-ideal of S for all $\alpha \in (\lambda, \mu]$. Then, $f_\alpha S \cap S f_\alpha \subseteq f_\alpha$.

If there is $x_0 \in S$, such that $f(x_0) \vee \lambda < \alpha = (f * 1)(x) \wedge (1 * f)(x) \wedge \mu$, then $\alpha \in (\lambda, \mu], f(x_0) < \alpha, (f * 1)(x_0) \geq \alpha$ and $(1 * f)(x_0) \geq \alpha$. That is $x_0 \notin f_\alpha, \sup_{x_0 \leq x_1 x_2} f(x_1) \geq \alpha$ and $\sup_{x_0 \leq x_1 x_2} f(x_2) \geq \alpha$.

From $f_\alpha S \cap S f_\alpha \subseteq f_\alpha$ and $x_0 \notin f_\alpha$, we obtain that $x_0 \notin f_\alpha S \cap S f_\alpha$.

From $\sup_{x_0 \leq x_1 x_2} f(x_1) \geq \alpha$ and $\alpha \neq 0$, we know that there exists at least one pair $(x_1, x_2) \in S \times S$ such that $x_0 \leq x_1 x_2$ and $f(x_1) \geq \alpha$. Thus, $x_0 \leq x_1 x_2 \in f_\alpha S$. Hence, $x_0 \in f_\alpha S$.

Similarly, we can prove that $x_0 \in S f_\alpha$.

So $x_0 \in f_\alpha S \cap S f_\alpha$. This is a contradiction.

Hence, $f \cup \lambda \supseteq (f * 1) \cap (1 * f) \cap \mu$ holds.

If there are $x_0, y_0 \in S$ such that $x_0 \leq y_0$ and $f(x_0) \vee \lambda < \alpha = f(y_0) \wedge \mu$, then $\alpha \in (\lambda, \mu], f(y_0) \geq \alpha$ and $f(x_0) < \alpha$, that is, $y_0 \in f_\alpha$ and $x_0 \notin f_\alpha$. This is a contradiction with that f_α is a quasi-ideal of S . Hence if $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$. $\square$

Theorem 3.7. Let $(S, \circ, \leq)$ be an ordered semigroup, then f is a (λ, μ) -fuzzy bi-ideal of S if and only if f_α is a bi-ideal of S for all $\alpha \in (\lambda, \mu]$.

Proof. The proof of this theorem is similar to the proof of the previous theorem. $\square$

Theorem 3.8. Let $(S, \circ, \leq)$ be an ordered semigroup, then the (λ, μ) -fuzzy right (resp. left) ideals of S are (λ, μ) -fuzzy quasi-ideals of S .

Proof. Let f be a (λ, μ) -fuzzy right ideal of S and $x \in S$. First we have

$$((f * 1) \cap (1 * f))(x) = (f * 1)(x) \wedge (1 * f)(x). \quad (3.1)$$

If $A_x = \emptyset$, then we have $(f * 1)(x) = 0 = (1 * f)(x)$. So $f(x) \vee \lambda \geq 0 = (f * 1)(x) \wedge (1 * f)(x) \wedge \mu$. Thus, $f \cup \lambda \supseteq (f * 1) \cap (1 * f) \cap \mu$.

If $A_x \neq \emptyset$, then

$$(f * 1)(x) = \sup_{(u,v) \in A_x} (f(u) \wedge 1(v)). \quad (3.2)$$

On the other hand, $f(x) \vee \lambda \geq f(u) \wedge 1(v) \wedge \mu$, for all $(u, v) \in A_x$.

Indeed, if $(u, v) \in A_x$, then $x \leq uv$, thus $f(x) \vee \lambda = f(x) \vee \lambda \vee \lambda \geq (f(uv) \wedge \mu) \vee \lambda = (f(uv) \vee \lambda) \wedge (\lambda \vee \mu) \geq (f(u) \wedge \mu) \wedge \mu = f(u) \wedge \mu = f(u) \wedge 1(v) \wedge \mu$.

Hence, we have that $f(x) \vee \lambda \geq (\sup_{(u,v) \in A_x} (f(u) \wedge 1(v))) \wedge \mu = (f * 1)(x) \wedge \mu \geq (f * 1)(x) \wedge (1 * f)(x) \wedge \mu$. Thus, $f \cup \lambda \supseteq (f * 1) \cap (1 * f) \cap \mu$.

Therefore, f is a (λ, μ) -fuzzy quasi-ideal of S . $\square$

Theorem 3.9. Let $(S, \circ, \leq)$ be an ordered semigroup, then the (λ, μ) -fuzzy quasi-ideals of S are (λ, μ) -fuzzy bi-ideals of S .

Proof. Let f be a (λ, μ) -fuzzy quasi-ideal of S and $x, y, z \in S$. Then we have that

$$f(xyz) \vee \lambda \geq (f * 1)(xyz) \wedge (1 * f)(xyz) \wedge \mu. \quad (3.3)$$

From $(x, yz) \in A_{xyz}$, we have that $(f * 1)(xyz) \geq f(x) \wedge 1(yz) = f(x)$.

From $(xy, z) \in A_{xyz}$, we have that $(1 * f)(xyz) \geq 1(xy) \wedge f(z) = f(z)$.

Thus, $f(xyz) \vee \lambda \geq f(x) \wedge f(z) \wedge \mu$.

Therefore, f is a (λ, μ) -fuzzy bi-ideal of S . $\square$

Definition 3.10 (see [5]). An ordered semigroup $(S, \circ, \leq)$ is called regular if for all $a \in S$ there exists $x \in S$ such that $a \leq axa$.

Theorem 3.11. In a regular ordered semigroup S , the (λ, μ) -fuzzy quasi-ideals and the (λ, μ) -fuzzy bi-ideals coincide.

Proof. Let f be a (λ, μ) -fuzzy bi-ideal of S and $x \in S$. We need to prove that

$$f(x) \vee \lambda \geq (f * 1)(x) \wedge (1 * f) \wedge \mu. \quad (3.4)$$

If $A_x = \emptyset$, it is easy to verify that condition (3.4) is satisfied.

Let $A_x \neq \emptyset$.

(1) If $(f * 1)(x) \wedge \mu \leq f(x) \vee \lambda$, then we have that $f(x) \vee \lambda \geq (f * 1)(x) \wedge \mu \geq (f * 1)(x) \wedge (1 * f)(x) \wedge \mu$. Thus, condition (3.4) is satisfied.

(2) If $(f * 1)(x) \wedge \mu > f(x) \vee \lambda$, then there exists at least one pair $(z, w) \in A_x$ such that $f(z) \wedge 1(w) \wedge \mu > f(x) \vee \lambda$. That is $z, w \in S$, $x \leq zw$ and $f(z) \wedge \mu > f(x) \vee \lambda$.

We will prove that $(1 * f)(x) \wedge \mu \leq f(x) \vee \lambda$. Then, $f(x) \vee \lambda \geq (1 * f)(x) \wedge \mu \geq (f * 1)(x) \wedge (1 * f)(x) \wedge \mu$, and condition (3.4) is satisfied.

For any $(u, v) \in A_x$, we need to show that $1(u) \wedge f(v) \wedge \mu \leq f(x) \vee \lambda$.

Let $(u, v) \in A_x$, then $x \leq uv$ for some $u, v \in S$. Since S is regular, there exists $s \in S$ such that $x \leq xsx$.

From $x \leq xsx$, $x \leq zw$ and $x \leq uv$, we obtain that $x \leq zwsuv$. Since f is a (λ, μ) -fuzzy bi-ideal of S , we have that

$$f(x) \vee \lambda \geq (f(zwsuv) \wedge \mu) \vee \lambda = (f(zwsuv) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(z) \wedge f(v) \wedge \mu. \quad (3.5)$$

Note that $f(z) \wedge \mu > f(x) \vee \lambda$. Thus, $f(x) \vee \lambda \geq f(v) \wedge \mu = 1(u) \wedge f(v) \wedge \mu$. $\square$

Acknowledgments

This paper is prepared before the first author's visit to Università degli Studi di Udine, the first author wishes to express his gratitude to Professor Corsini, Dr. Paronuzzi, and Professor Russo for their hospitality. The first author is highly grateful to CMEC (KJ091104, KJ111107), CSTC, and CTGU(10QN-27) for financial support.

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