Research Article

New Traveling Wave Solutions by the Extended Generalized Riccati Equation Mapping Method of the (2 + 1)-Dimensional Evolution Equation

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Received 19 September 2012; Accepted 14 October 2012

Academic Editor: Mohamed A. Abdou

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The generalized Riccati equation mapping is extended with the basic \( G'/G \)-expansion method which is powerful and straightforward mathematical tool for solving nonlinear partial differential equations. In this paper, we construct twenty-seven traveling wave solutions for the (2+1)-dimensional modified Zakharov-Kuznetsov equation by applying this method. Further, the auxiliary equation \( G'(\eta) = w + uG(\eta) + vG^2(\eta) \) is executed with arbitrary constant coefficients and called the generalized Riccati equation. The obtained solutions including solitons and periodic solutions are illustrated through the hyperbolic functions, the trigonometric functions, and the rational functions. In addition, it is worth declaring that one of our solutions is identical for special case with already established result which verifies our other solutions. Moreover, some of obtained solutions are depicted in the figures with the aid of Maple.

1. Introduction

The study of analytical solutions for nonlinear partial differential equations (PDEs) has become more imperative and stimulating research fields in mathematical physics, engineering sciences, and other technical arena [1–47]. In the recent past, a wide range of methods have been developed to construct traveling wave solutions of nonlinear PDEs such as, the inverse scattering method [1], the Backlund transformation method [2], the Hirota bilinear transformation method [3], the bifurcation method [4, 5], the Jacobi elliptic function expansion method [6–8], the Weierstrass elliptic function method [9], the direct algebraic method [10], the homotopy perturbation method [11, 12], the Exp-function method [13–17], and others [18–28].

Recently, Wang et al. [29] presented a widely used method, called the \((G'/G)\)-expansion method to obtain traveling wave solutions for some nonlinear evolution equations (NLEEs). Further, in this method, the second-order linear ordinary differential equation
The importance of our present work is, in order to construct many new traveling wave solutions including solitons, periodic, and rational solutions, a (2+1)-dimensional Modified Zakharov-Kuznetsov equation considered by applying the extended generalized Riccati equation mapping method.

2. The Extended Generalized Riccati Equation Mapping Method

Suppose the general nonlinear partial differential equation

\[ H(v, v_t, v_x, v_y, v_{xt}, v_{yt}, v_{xy}, v_{tt}, v_{xx}, v_{yy}, \ldots) = 0, \]  

(2.1)
where \( v = v(x, y, t) \) is an unknown function, \( H \) is a polynomial in \( v(x, y, t) \), and the subscripts indicate the partial derivatives.

The most important steps of the generalized Riccati equation mapping together with the \((G'/G)\)-expansion method [29, 40] are as follows.

**Step 1.** Consider the traveling wave variable:

\[
v(x, y, t) = g(\eta), \quad \eta = x + y - Ct,
\]

where \( C \) is the speed of the traveling wave. Now using (2.2), (2.1) is converted into an ordinary differential equation for \( g(\eta) \):

\[
F(g, g', g'', g''', \ldots) = 0,
\]

where the superscripts stand for the ordinary derivatives with respect to \( \eta \).

**Step 2.** Equation (2.3) integrates term by term one or more times according to possibility and yields constant(s) of integration. The integral constant(s) may be zero for simplicity.

**Step 3.** Suppose that the traveling wave solution of (2.3) can be expressed in the form [29, 40]

\[
g(\eta) = \sum_{j=0}^{n} e_j \left( \frac{G'}{G} \right)^j,
\]

where \( e_j \ (j = 0, 1, 2, \ldots, n) \) and \( e_n \neq 0 \), with \( G = G(\eta) \) is the solution of the generalized Riccati equation:

\[
G' = w + uG + vG^2,
\]

where \( u, v, w \) are arbitrary constants and \( v \neq 0 \).

**Step 4.** To decide the positive integer \( n \), consider the homogeneous balance between the nonlinear terms and the highest order derivatives appearing in (2.3).

**Step 5.** Substitute (2.4) along with (2.5) into the (2.3), then collect all the coefficients with the same order, the left hand side of (2.3) converts into polynomials in \( G^k(\eta) \) and \( G^{-k}(\eta) \), \( (k = 0, 1, 2, \ldots) \). Then equating each coefficient of the polynomials to zero and yield a set of algebraic equations for \( e_j \ (j = 0, 1, 2, \ldots, n) \), \( u, v, w, \) and \( C \).

**Step 6.** Solve the system of algebraic equations which are found in Step 5 with the aid of algebraic software Maple to obtain values for \( e_j \ (j = 0, 1, 2, \ldots, n) \) and \( C \) then, substitute obtained values in (2.4) along with (2.5) with the value of \( n \), we obtain exact solutions of (2.1).

In the following, we have twenty seven solutions including four different families of (2.5).
Family 1. When \(u^2 - 4vw > 0\) and \(uv \neq 0\) or \(vw \neq 0\), the solutions of (2.5) are:

\[
\begin{align*}
G_1 &= -\frac{1}{2v} \left( u + \sqrt{u^2 - 4vw} \tanh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right), \\
G_2 &= -\frac{1}{2v} \left( u + \sqrt{u^2 - 4vw} \coth \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right), \\
G_3 &= -\frac{1}{2v} \left( u + \sqrt{u^2 - 4vw} \left( \tanh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \pm \text{sech} \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right) \right), \\
G_4 &= -\frac{1}{2v} \left( u + \sqrt{u^2 - 4vw} \left( \coth \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \pm \text{csch} \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right) \right), \\
G_5 &= -\frac{1}{4v} \left( 2u + \sqrt{u^2 - 4vw} \left( \tanh \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) + \coth \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right) \right), \\
G_6 &= \frac{1}{2v} \left( -u + \frac{\pm \sqrt{(D^2 + E^2)(u^2 - 4vw)} - D \sqrt{u^2 - 4vw} \cosh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right)} {D \sinh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) + E} \right), \\
G_7 &= \frac{1}{2v} \left( -u - \frac{\pm \sqrt{(D^2 + E^2)(u^2 - 4vw)} + D \sqrt{u^2 - 4vw} \cosh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right)} {D \sinh \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) + E} \right),
\end{align*}
\]

(2.6)

where \(D\) and \(E\) are two nonzero real constants.

\[
\begin{align*}
G_8 &= \frac{2w \cosh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right)} {\sqrt{u^2 - 4vw} \sinh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right) - u \cosh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right)}, \\
G_9 &= \frac{-2w \sinh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right)} {u \sinh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right) - \sqrt{u^2 - 4vw} \cosh \left( \left( \frac{\sqrt{u^2 - 4vw}}{2} \eta \right) \right)}, \\
G_{10} &= \frac{2w \cosh \left( \sqrt{u^2 - 4vw} \right)} {\sqrt{u^2 - 4vw} \sinh \left( \sqrt{u^2 - 4vw} \right) - u \cosh \left( \sqrt{u^2 - 4vw} \right) \pm i \sqrt{u^2 - 4vw}}, \\
G_{11} &= \frac{2w \sinh \left( \sqrt{u^2 - 4vw} \right)} {-u \sinh \left( \sqrt{u^2 - 4vw} \right) + \sqrt{u^2 - 4vw} \cosh \left( \sqrt{u^2 - 4vw} \right) \pm \sqrt{u^2 - 4vw}}, \\
G_{12} &= \frac{4w \sinh \left( \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right) \cosh \left( \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right)} {-2u \sinh \left( \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right) \cosh \left( \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right) + \Delta_1},
\end{align*}
\]

(2.7)

where \(\Delta_1 = 2 \sqrt{u^2 - 4vw} \cosh^2 \left( \left( \frac{\sqrt{u^2 - 4vw}}{4} \eta \right) \right) - \sqrt{u^2 - 4vw} \).
Family 2. When $u^2 - 4vw < 0$ and $uv \neq 0$ or $vw \neq 0$, the solutions of (2.5) are:

\[
G_{13} = \frac{1}{2v} (-u + \sqrt{4vw - u^2} \tan \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)),
\]

\[
G_{14} = \frac{-1}{2v} \left( u + \sqrt{4vw - u^2} \cot \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) \right),
\]

\[
G_{15} = \frac{1}{2v} (-u + \sqrt{4vw - u^2} \left( \tan \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) \pm \sec \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) \right)),
\]

\[
G_{16} = \frac{-1}{2v} \left( u + \sqrt{4vw - u^2} \left( \cot \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) \pm \csc \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) \right) \right),
\]

\[
G_{17} = \frac{1}{4v} \left( -2u + \sqrt{4vw - u^2} \left( \tan \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right) - \cot \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right) \right) \right),
\]

\[
G_{18} = \frac{1}{2v} \left( -u + \pm \sqrt{(D^2 - E^2)(4vw - u^2) - D\sqrt{4vw - u^2}\cos \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)} \right),
\]

\[
G_{19} = \frac{1}{2v} \left( -u - \frac{\pm \sqrt{(D^2 - E^2)(4vw - u^2) + D\sqrt{4vw - u^2}\cos \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)}}{D\sin \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) + E} \right),
\]

where $D$ and $E$ are two nonzero real constants and satisfy $D^2 - E^2 > 0$.

\[
G_{20} = \frac{-2w \cos \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)}{\sqrt{4vw - u^2} \sin \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) + u \cos \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)},
\]

\[
G_{21} = \frac{2w \sin \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)}{\sin \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right) + \sqrt{4vw - u^2} \cos \left( \frac{\sqrt{4vw - u^2}}{2} \eta \right)},
\]

\[
G_{22} = \frac{-2w \cos \left( \sqrt{4vw - u^2} \eta \right)}{\sqrt{4vw - u^2} \sin \left( \sqrt{4vw - u^2} \eta \right) + u \cos \left( \sqrt{4vw - u^2} \eta \right) \pm \sqrt{4vw - u^2}},
\]

\[
G_{23} = \frac{2w \sin \left( \sqrt{4vw - u^2} \eta \right)}{-u \sin \left( \sqrt{4vw - u^2} \eta \right) + \sqrt{4vw - u^2} \cos \left( \sqrt{4vw - u^2} \eta \right) \pm \sqrt{4vw - u^2}},
\]

\[
G_{24} = \frac{4w \sin \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right) \cos \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right)}{-2u \sin \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right) \cos \left( \frac{\sqrt{4vw - u^2}}{4} \eta \right) + \Delta_2},
\]

where $\Delta_2 = 2\sqrt{4vw - u^2}\cos^2 \left( \sqrt{\frac{4vw - u^2}{4}} \eta \right) - \sqrt{4vw - u^2}$.
Family 3. When $w = 0$ and $uv \neq 0$, the solution (2.5) becomes:

\[
G_{25} = \frac{-uf_1}{v(f_1 + \cosh(u\eta) - \sinh(u\eta))},
\]

\[
G_{26} = \frac{-u(\cosh(u\eta) + \sinh(u\eta))}{v(f_1 + \cosh(u\eta) + \sinh(u\eta))},
\]

where $f_1$ is an arbitrary constant.

Family 4. when $v \neq 0$ and $w = u = 0$, the solution of (2.5) becomes:

\[
G_{27} = \frac{-1}{v\eta + l_1},
\]

where $l_1$ is an arbitrary constant.

3. Applications of the Method

In this section, we have constructed new traveling wave solutions for the $(2 + 1)$-dimensional modified Zakharov-Kuznetsov equation by using the method.

3.1. The $(2 + 1)$-Dimensional Modified Zakharov-Kuznetsov Equation

We consider the $(2 + 1)$-dimensional Modified Zakharov-Kuznetsov equation followed by Bekir [47]

\[
\frac{du}{dt} + u^2 \frac{du}{dx} + u_{xxx} + u_{xyy} = 0.
\]

(3.1)

Now, we use the wave transformation (2.2) into the (3.1), which yields:

\[-CG' + u^2G' + 2g'' = 0.
\]

(3.2)

Equation (3.2) is integrable, therefore, integrating with respect $\eta$ once yields:

\[Q - Cg + \frac{1}{3}g^3 + 2g'' = 0,
\]

(3.3)

where $Q$ is an integral constant which is to be determined later.

Taking the homogeneous balance between $g^3$ and $g''$ in (3.3), we obtain $n = 1$.

Therefore, the solution of (3.3) is of the form:

\[g(\eta) = e_1 \left( \frac{G'}{G} \right) + e_0, \quad e_1 \neq 0.
\]

(3.4)
Using (2.5), (3.4) can be rewritten as

\[ g(\eta) = e_1 \left( u + \omega G^{-1} + \nu G \right) + e_0, \tag{3.5} \]

where \( u, \nu, \) and \( \omega \) are free parameters.

By substituting (3.5) into (3.3), the left hand side is converted into polynomials in \( G^k \) and \( G^{-k} \) \((k = 0, 1, 2, \ldots)\). Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for \( e_0, e_1, u, \nu, \omega, Q, \) and \( C \) (algebraic equations are not shown, for simplicity). Solving the system of algebraic equations with the help of algebraic software Maple, we obtain

\[ e_0 = \mp u \sqrt{3}, \quad e_1 = \pm 2i \sqrt{3}, \quad C = -u^2 - 8\nu \omega, \quad Q = 8\nu \omega i \sqrt{3}. \tag{3.6} \]

**Family 5.** The soliton and soliton-like solutions of (3.1) (when \( u^2 - 4\nu \omega > 0 \) and \( u\nu \neq 0 \) or \( \nu \omega \neq 0 \)) are:

\[ g_1 = \pm 2i \sqrt{3} \frac{2\Psi \sech^2(\Psi \eta)}{u + 2\Psi \tanh(\Psi \eta)} \mp u \sqrt{3}, \tag{3.7} \]

where \( \Psi = (1/2)\sqrt{u^2 - 4\nu \omega}, \eta = x + y + (u^2 + 8\nu \omega)t \) and \( u, \nu, \omega \) are arbitrary constants.

\[ g_2 = \mp 2i \sqrt{3} \frac{2\Psi \csch^2(\Psi \eta)}{u + 2\Psi \coth(\Psi \eta)} \mp u \sqrt{3}, \]

\[ g_3 = \pm 2i \sqrt{3} \frac{4\Psi \sech^2(\Psi \eta)(1 \mp i \sinh(2\Psi \eta))}{u \cosh(2\Psi \eta) + 2\Psi \sinh(2\Psi \eta) \pm 2\Psi} \mp u \sqrt{3}, \]

\[ g_4 = \mp 2i \sqrt{3} \frac{2\Psi \csch(\Psi \eta)}{u \sinh(\Psi \eta) + 2\Psi \cosh(\Psi \eta)} \mp u \sqrt{3}, \]

\[ g_5 = \mp 2i \sqrt{3} \frac{4\Psi \csch(2\Psi \eta)}{u \tanh(\Psi \eta) + 2\Psi} \mp u \sqrt{3}, \]

\[ g_6 = \mp 2i \sqrt{3} \frac{4 D \Psi^2 \left( D - E \sinh(2\Psi \eta) - \sqrt{(D^2 + E^2) \cosh(2\Psi \eta)} \right)}{(D \sinh(2\Psi \eta) + E) \Omega_1} \mp u \sqrt{3}, \]

\[ g_7 = \mp 2i \sqrt{3} \frac{4 D \Psi^2 \left( D - E \sinh(2\Psi \eta) + \sqrt{(D^2 + E^2) \cosh(2\Psi \eta)} \right)}{(D \sinh(2\Psi \eta) + E) \Omega_2} \mp u \sqrt{3}, \]
where \( \Omega_1 = uD \sinh(2\Psi \eta) + uE - 2\Psi \sqrt{(D^2 + E^2)} + 2D \Psi \cosh(2\Psi \eta), \Omega_2 = uD \sinh(2\Psi \eta) + uE + 2\Psi \sqrt{(D^2 + E^2)} + 2D \Psi \cosh(2\Psi \eta), D \) and \( E \) are two nonzero real constants.

\[
g_8 = \mp 2i \sqrt{3} \frac{2\Psi^2 \text{sech}(\Psi \eta)}{2\Psi \sinh(\Psi \eta) - u \cosh(\Psi \eta)} \mp ui \sqrt{3},
\]

\[
g_9 = \mp 2i \sqrt{3} \frac{2\Psi^2 \text{csch}(\Psi \eta)}{2\Psi \cosh(\Psi \eta) - u \sinh(\Psi \eta)} \mp ui \sqrt{3},
\]

\[
g_{10} = \pm 2i \sqrt{3} \frac{4\Psi^2 \text{sech}(2\Psi \eta)}{u \cosh(2\Psi \eta) - 2\Psi \sinh(2\Psi \eta) \pm 2\Psi \mp i2\Psi} \mp ui \sqrt{3},
\]

\[
g_{11} = \pm 2i \sqrt{3} \frac{4\Psi^2 \text{csch}(2\Psi \eta)}{2\Psi \cosh(2\Psi \eta) - u \sinh(2\Psi \eta) \mp 2\Psi} \mp ui \sqrt{3},
\]

\[
g_{12} = \pm 2i \sqrt{3} \frac{2\Psi^2 \text{csch}(\Psi \eta)}{2\Psi \cosh(\Psi \eta) - u \sinh(\Psi \eta)} \mp ui \sqrt{3}.
\]

**Family 6.** The periodic form solutions of (3.1) (when \( u^2 - 4vw < 0 \) and \( uv \neq 0 \) or \( vw \neq 0 \)) are:

\[
g_{13} = \pm 2i \sqrt{3} \frac{2\Theta^2 \sec^2(\Theta \eta)}{-u + 2\Theta \tan(\Theta \eta)} \mp ui \sqrt{3},
\]

where \( \Theta = (1/2)\sqrt{4 \ v w - u^2}, \ \eta = x + y + (u^2 + 8vw)t \) and \( u, v, w \) are arbitrary constants.

\[
g_{14} = \mp 2i \sqrt{3} \frac{2\Theta^2 \csc^2(\Theta \eta)}{u + 2\Theta \cot(\Theta \eta)} \mp ui \sqrt{3},
\]

\[
g_{15} = \pm 2i \sqrt{3} \frac{4\Theta^2 \sec(2\Theta \eta) (1 \mp \sin(2\Theta \eta))}{-u \cos(2\Theta \eta) + 2\Theta \sin(2\Theta \eta) \pm 2\Theta} \mp ui \sqrt{3},
\]

\[
g_{16} = \mp 2i \sqrt{3} \frac{2\Theta^2 \sec(\Theta \eta)}{u \cos(\Theta \eta) + 2\Theta \sin(\Theta \eta)} \mp ui \sqrt{3},
\]

\[
g_{17} = \mp 2i \sqrt{3} \frac{2\Theta^2 \csc(\Theta \eta)}{u \sin(\Theta \eta) + 2\Theta \cos(\Theta \eta)} \mp ui \sqrt{3},
\]

\[
g_{18} = \mp 2i \sqrt{3} \frac{4\Theta^2 \left(\sqrt{(D^2 - E^2) \cos(2\Theta \eta) - E \sin(2\Theta \eta) - D}\right)}{(D \sin(2\Theta \eta) + E)\Omega_3} \mp ui \sqrt{3},
\]

\[
g_{19} = \mp 2i \sqrt{3} \frac{4\Theta^2 \left(\sqrt{(D^2 - E^2) \cos(2\Theta \eta) + E \sin(2\Theta \eta) + D}\right)}{(D \sin(2\Theta \eta) + E)\Omega_4} \mp ui \sqrt{3},
\]
where $\Omega_3 = uD \sin(2\Theta \eta) + 2D \Theta \cos(2\Theta \eta) + uE - 2\Theta \sqrt{(D^2 - E^2)}$, $\Omega_4 = uD \sin(2\Theta \eta) + 2D \Theta \cos(2\Theta \eta) + uE + 2\Theta \sqrt{(D^2 - E^2)}$, $D$ and $E$ are two nonzero real constants and satisfies $D^2 - E^2 > 0$.

$$g_{20} = \mp 2i\sqrt{3} \frac{4\Theta^2 \csc(2\Theta \eta)}{u \cot(\Theta \eta) + 2\Theta} \mp ui\sqrt{3},$$

$$g_{21} = \mp 2i\sqrt{3} \frac{4\Theta^2 \csc(2\Theta \eta)}{u \tan(\Theta \eta) + 2\Theta} \mp ui\sqrt{3},$$

$$g_{22} = \mp 2i\sqrt{3} \frac{2\Theta^2 \sec(2\Theta \eta)(1 \pm \sin(2\Theta \eta))(u \cos(2\Theta \eta) + 2\Theta \sin(2\Theta \eta) \pm 2\Theta)}{(u^2 - 2\nu \omega) \cos^2(2\Theta \eta) + 2\Theta(1 \pm \sin(2\Theta \eta))(2\Theta \pm u \cos(2\Theta \eta))} \mp ui\sqrt{3},$$

$$g_{23} = \pm 2i\sqrt{3} \frac{2\Theta^2 \csc(2\Theta \eta)(-u \sin(2\Theta \eta) + 2\Theta \cos(2\Theta \eta) \pm 2\Theta)}{(2\nu \omega - u^2) \cos(2\Theta \eta) - 2u\Theta \sin(2\Theta \eta) \pm 2\nu \omega} \mp ui\sqrt{3},$$

$$g_{24} = \mp 2i\sqrt{3} \frac{2\Theta^2 \csc(\Theta \eta)}{u \sin(\Theta \eta) + 2\Theta \cos(\Theta \eta)} \mp ui\sqrt{3}. \quad (3.12)$$

**Family 7.** The soliton and soliton-like solutions of (3.1) (when $\nu = 0$ and $uv \neq 0$) are:

$$g_{25} = \pm 2i\sqrt{3} \frac{u(f \cosh(u) - \sinh(u))}{f_1 + \cosh(u) - \sinh(u)} \mp ui\sqrt{3},$$

$$g_{26} = \pm 2i\sqrt{3} \frac{uf_1}{f_1 + \cosh(u) + \sinh(u)} \mp ui\sqrt{3}, \quad (3.13)$$

where $f_1$ is an arbitrary constant, $\eta = x + y + (u^2 + 8\nu \omega)t$. 
Family 8. The rational function solution (when \( v \neq 0 \) and \( w = u = 0 \)) is:

\[
g_{2^7} = \mp \frac{2i v \sqrt{3}}{v \eta + l_1},
\]

(3.14)

where \( l_1 \) is an arbitrary constant and \( \eta = x + y + (u^2 + 8v \omega)t \).
4. Results and Discussion

It is significant to mention that one of our solutions is coincided for some special case with already published results which are presented in Table 1. Furthermore, some of newly constructed solutions are illustrated in Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.
Table 1: Comparison between Bekir [47] solutions and newly obtained solutions.

<table>
<thead>
<tr>
<th>Bekir [47] solutions</th>
<th>New solutions</th>
</tr>
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<tbody>
<tr>
<td>(i) If $C_1 = 1$, $C_2 = \frac{1}{2}$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \pm i\sqrt{3} \frac{2}{2+x}$.</td>
<td>(i) If $\nu = \frac{1}{2}$, $l_1 = 1$, $y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution $g_{27}$ becomes: $u_{5,6}(\xi) = \pm i\sqrt{3} \frac{2}{2+x}$.</td>
</tr>
<tr>
<td>(ii) $C_1 = 1$, $C_2 = -\frac{1}{2}$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \mp i\sqrt{3} \frac{2}{2-x}$.</td>
<td>(ii) If $\nu = -\frac{1}{2}$, $l_1 = 1$, $y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution $g_{27}$ becomes: $u_{5,6}(\xi) = \mp i\sqrt{3} \frac{2}{2-x}$.</td>
</tr>
<tr>
<td>(iii) $C_1 = 0$, $C_2 = 1$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \mp i\sqrt{3} \left( \frac{2}{x} \right)$.</td>
<td>(iii) If $\nu = 1$, $l_1 = 0$, $y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution $g_{27}$ becomes: $u_{5,6}(\xi) = \mp i\sqrt{3} \left( \frac{2}{x} \right)$.</td>
</tr>
</tbody>
</table>

Figure 6: Solitons solution for $u = 1$, $v = 0.45$, $w = 3$.

As in Table 1, we have newly constructed traveling wave solutions $g_1$ to $g_{26}$ which are not being stated in the earlier literature.

4.1. Graphical Depictions of Newly Obtained Traveling Wave Solutions

The graphical descriptions of some solutions are represented in Figures 1–12 with the aid of commercial software Maple.
Figure 7: Solitons solution for $u = 2, v = 2.5 \times 10^{-4}, w = 2.5 \times 10^{-3}$.

Figure 8: Solitons solution for $u = 1, v = 1, w = 1$.

5. Conclusions

In this paper, we have investigated the $(2 + 1)$-dimensional modified Zakharov-Kuznetsov equation via the extended generalized Riccati equation mapping method. Twenty seven exact traveling wave solutions are constructed including solitons and periodic wave solutions by applying this powerful method. In addition, newly obtained solutions are depicted in terms of the hyperbolic, the trigonometric, and the rational functional form. The obtained solutions
reveal that this method is a promising mathematical tool because it can establish a variety of new solutions of dissimilar physical structures if compared with existing methods. The correctness of newly constructed solutions is verified to be compared with already published results. Consequently, nonlinear evolution equations which regularly arise in many scientific real-time application fields can be studied by applying the extended generalized Riccati equation mapping method.
Figure 11: Solitons solution for \( u = 3, \ v = 25 \times 10^{-4}, \ w = 25 \times 10^{-3}, \).

Figure 12: Periodic solution for \( u = 0.25, \ v = -5, \ w = 0, \ f_1 = 0.25. \)

Acknowledgments

This paper is supported by the USM short-term Grant (Ref. no. 304/PMATHS/6310072), and the authors would like to express their thanks to the School of Mathematical Sciences, USM for providing related research facilities.

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