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Research Article

LMI Approach to Stability Analysis of Cohen-Grossberg Neural Networks with p-Laplace Diffusion

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The nonlinear p-Laplace diffusion ($p > 1$) was considered in the Cohen-Grossberg neural network (CGNN), and a new linear matrix inequalities (LMI) criterion is obtained, which ensures the equilibrium of CGNN is stochastically exponentially stable. Note that , if $p = 2$, p-Laplace diffusion is just the conventional Laplace diffusion in many previous literatures. And it is worth mentioning that even if $p = 2$, the new criterion improves some recent ones due to computational efficiency. In addition, the resulting criterion has advantages over some previous ones in that both the impulsive assumption and diffusion simulation are more natural than those of some recent literatures.

1. Introduction and Preparation

It is well known that Cohen-Grossbeg neural network (CGNN) was proposed by Cohen and Grossberg [1] in 1983. Since then there have been a lot of interested results obtained in many literatures (see [2–9]) due to its general applications, such as pattern recognition, image and signal processing, optimization automatic control, and artificial intelligence. Usually, there exist the impulsive effect and time-varying delays phenomenon in various neural networks [3, 5–7, 10–14]. Besides, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields [15–18]. However, diffusion disturbance was always simulated simply by linear Laplace diffusion [15–18]. Few papers

involved the nonlinear reaction-diffusion [19]. So in this paper, we investigate the stability of the following stochastic CGNN with nonlinear p-Laplace diffusion ($p > 1$):

$$\begin{aligned} du(t,x) = {} & \Big\{\nabla\cdot\Big(\widetilde{D}(t,x,u)\circ\nabla_p u\Big) - A(u(t,x)) \\ & \times\big[B(u(t,x)) - Cf(u(t,x)) + Dg(u(t-\tau(t),x))\big]\Big\}dt \\ & + \sigma(u(t,x))dw(t), \quad t\in[t_k,t_{k+1}) \\ & u(t_k^+,x) = Mu(t^-,x), \quad t = t_k \\ & u(t_0+\theta,x) = \varphi(\theta,x), \quad (\theta,x)\in[-\tau,0]\times\partial\Omega \\ & \frac{\partial u_i(t,x)}{\partial\nu} = 0, \quad (t,x)\in[-\tau,+\infty)\times\partial\Omega, \quad i = 1,2,\ldots,n, \end{aligned} \tag{1.1}$$

where Ω is a bounded subset in R^m with smooth boundary $\partial\Omega$, and $\partial u_i(t,x)/\partial\nu = ((\partial u_i(t,x)/\partial x_1),(\partial u_i(t,x)/\partial x_2),\ldots,(\partial u_i(t,x)/\partial x_m))^T$ denotes the outward normal derivative on $\partial\Omega$.

Remark 1.1. If $p = 2$, system (1.1) was studied by [5] though there is a little difference between Dirichlet boundary condition and Neumann boundary condition. However, our impulsive assumption $u(t_k^+,x) = Mu(t_k^-,x)$ is more natural than that of [5], which will result in some difference in methods.

Here, M is a diagonal matrix, $\Omega\in R^m$ is a bounded compact set with smooth boundary, $u = (u_1,u_2,\ldots,u_n)^T\in R^n$, and $w(t) = (w_1(t),w_2(t),\ldots,w_n(t))^T$ is a n-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},P)$ with the natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by the process $\{w(s) : 0\leq s\leq t\}$. We associate Ω with the canonical space generated by all $\{w_i(t)\}$ and denoted by $\mathcal{F}$ the associated σ-algebra generated by $w(t)$ with the probability measure p. $A(u(t,x))$ presents an amplification function, $B(u(t,x))$ is an appropriately behavior function, and f and g denote the activation function. $\tau(t)$ $(0\leq\tau(t)\leq\tau)$ corresponds to the transmission delays at time, and t_k is called the impulsive moment with $0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty}t_k = +\infty$. We always assume $u(t_k^+,x) = u(t_k,x)$. $\nabla_p u = (\nabla_p u_1,\ldots,\nabla_p u_n)^T$, $\nabla_p u_i = (|\nabla u_i|^{p-2}(\partial u_i/\partial x_1),\ldots,|\nabla u_i|^{p-2}(\partial u_i/\partial x_m))^T$, $\widetilde{D}\circ\nabla_p u = (\widetilde{D}_{ik}|\nabla u_i|^{p-2}(\partial u_i/\partial x_k))_{n\times m}$ is Hadamard product of matrix $\widetilde{D}$ and $\nabla_p u$ [20]. Here, the diffusion parameters matrix $\widetilde{D}(t,x,u)$ is denoted simply as $\widetilde{D} = (\widetilde{D}_{ik})_{n\times m}$. Let $Y_i = (y_{i1},\ldots,y_{im})^T$, $i = 1,2,\ldots,n$, and matrix $\mathbb{Y} = (Y_1,\ldots,Y_n)^T$, and we denote $\nabla\cdot Y_i = \sum_{k=1}^m \partial y_{ik}/\partial x_k$, $\nabla\cdot\mathbb{Y} = (\nabla\cdot Y_1,\nabla\cdot Y_2,\ldots,\nabla\cdot Y_n)^T$. Particularly, $\nabla_p u = \nabla u$ for the case of $p = 2$.

Remark 1.2. Diffusion effects always occur in the neural networks when electrons are moving in asymmetric electromagnetic fields [15–18], and diffusion behavior is so complicated that it cannot always be simulated by linear Laplace diffusion. So in this paper, the nonlinear p-Laplace diffusion is considered in System (1.1).

Assume, in addition, the following.

(H1) $A(u(t,x))$ is a bounded, positive, and continuous diagonal matrix, that is, there exist two positive diagonal matrices $\underline{A}$ and $\overline{A}$ such that $0 < \underline{A} \leq A(u(t,x)) \leq \overline{A}$.

(H2) $B(u(t,x)) = (b_1(u_1(t,x)), b_2(u_2(t,x)), \ldots, b_n(u_n(t,x)))^T$ such that there exists a positive diagonal matrix $B = \operatorname{diag}(B_1, B_2, \ldots, B_n)$ satisfying $(b_i(u_i(t,x))/u_i(t,x)) \geq B_i$ for all i.

(H3) There exist two positive diagonal matrices $F = \operatorname{diag}(F_1, F_2, \ldots, F_n)$ and $G = \operatorname{diag}(G_1, G_2, \ldots, G_n)$ such that

$$0 \leq \frac{f_i(r)}{r} \leq F_i, \qquad 0 \leq \frac{g_i(r)}{r} \leq G_i, \quad \forall i. \tag{1.2}$$

(H4) The null solution is the equilibrium point of system (1.1), that is, the following conditions hold:

$$B(0) - Cf(0) - Dg(0) = 0, \qquad \operatorname{trace}\left[\sigma^T(u(t,x))\sigma(u(t,x))\right] \leq u^T(t,x)Qu(t,x), \tag{1.3}$$

where the symmetrical matrix $Q > 0$.

For convenience's sake, we introduce some standard notations

(i) $L^2(R)(R \times \Omega)$: the space of real Lebesgue measurable functions of $R \times \Omega$, it is a Banach space for the 2-norm $\|u(t)\|_2 = (\sum_{i=1}^n \|u_i(t)\|)^{1/2}$ with $\|u_i(t)\| = (\int_\Omega |u_i(t,x)|^2 dx)^{1/2}$, where $|u_i(t,x)|$ is Euclid norm.

(ii) $L^2_{F_0}([-\tau,0] \times \Omega; R^n)$: the family of all F_0-measurable $C([-\tau,0] \times \Omega; R^n$-) value random variable $\xi = \{\xi(\theta,x) : -\tau \leq \theta \leq 0,\ x \in \Omega\}$ such that $\sup_{-\tau\leq\theta\leq 0} E\|\xi(\theta)\|_2^2 < \infty$, where $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure p.

(iii) $Q = (q_{ij})_{n\times n} > 0$ (<0): a positive (negative) definite symmetrical matrix, that is, $y^T Q y > 0$ (<0) for any $0 \neq y \in R^n$.

(iv) $Q = (q_{ij})_{n\times n} \geq 0$ ($\leq$0): a semipositive (semi-negative) definite symmetrical matrix, that is, $y^T Q y \geq 0$ ($\leq$0) for any $y \in R^n$.

(v) $Q \geq \tilde{Q}$ ($Q \leq \tilde{Q}$): this means $Q - \tilde{Q}$ is a semi-positive (semi-negative) symmetrical definite matrix.

(vi) $Q > \tilde{Q}$ ($Q < \tilde{Q}$): this means $Q - \tilde{Q}$ is a positive (negative) symmetrical definite matrix.

(vii) $\lambda_{\max}(\Phi)$, $\lambda_{\min}(\Phi)$ denotes the largest and smallest eigenvalue of symmetrical matrix Φ, respectively.

(viii) I: identity matrix with compatible dimension.

(ix) Denote $|C| = (|c_{ij}|)_{n\times n}$ for any matrix $C_{n\times n}$; $|u(t,x)| = (|u_1|, |u_2|, \ldots, |u_n|)$ for any $u \in R^n$.

Let $u(t,x;\varphi)$ denote the state trajectory from the initial data $u(t_0+\theta,x;\varphi) = \varphi(\theta,x)$ on $-\tau \leq \theta \leq 0$ in $L^2_{F_0}([-\tau,0] \times \Omega; R^n)$.

Definition 1.3. The null solution of impulsive system (2.2) is globally stochastically exponentially stable in the mean square if for every $\varphi \in L^2_{F_0}([-\tau,0]\times\Omega;R^n)$, there exists scalars $\beta>0$ and $\gamma>0$ such that

$$E\left(\|u(t,\varphi)\|_2^2\right) \le \gamma e^{-\beta t}\sup_{-\tau\le\theta\le 0} E\left(\|\varphi(\theta)\|_2^2\right). \tag{1.4}$$

Lemma 1.4 (see [11]). *Let U, P be any matrices, $\varepsilon>0$ is a positive number and matrix $H=H^T>0$, then*

$$P^TU+U^TP \le \varepsilon P^THP+\varepsilon^{-1}U^TH^{-1}U. \tag{1.5}$$

Lemma 1.5 (Schur complement [3]). *The LMI*

$$\begin{pmatrix} Q(t) & S(t) \\ S^T(t) & R(t)\end{pmatrix} > 0, \tag{1.6}$$

where matrix $S(t)$ and symmetrical matrices $Q(t)$ and $R(t)$ depend on t, is equivalent to any one of the following conditions:

(L_1) $R(t)>0,\ Q(t)-S(t)R^{-1}(t)S^T(t)>0$;

(L_2) $Q(t)>0,\ R(t)-S^T(t)Q^{-1}(t)S^T(t)>0$.

Lemma 1.6 (see [21]). *Consider the following differential inequality:*

$$\begin{gathered} D^+v(t) \le -av(t)+b[v(t)]_\tau, \quad t\ne t_k \\ v(t_k)\le a_kv(t_k^-)+b_k\left[v(t_k^-)\right]_\tau, \end{gathered} \tag{1.7}$$

where $v(t)\ge 0$, $[v(t_k)]_\tau=\sup_{t-\tau\le s\le t}v(s)$, $[v(t_k^-)]_\tau=\sup_{t-\tau\le s\le t}v(s)$ and $v(t)$ is continuous except $t_k, k=1,2,\ldots$, where it has jump discontinuities. The sequence t_k satisfies $0=t_0<t_1<t_2<\cdots<t_k<t_{k+1}<\cdots$, and $\lim_{k\to\infty}t_k=\infty$. Suppose that

(1) $a>b\ge 0$;

(2) $t_k-t_{k-1}>\delta\tau$, *where $\delta>1$, and there exist constants $\gamma>0$, $M>0$ such that*

$$\rho_1\rho_2\cdots\rho_{k+1}\le Me^{\gamma t_k}, \tag{1.8}$$

where $\rho_i=\max\{1,a_i+b_ie^{\lambda\tau}\}$, $\lambda>0$ is the unique solution of equation $\lambda=a-be^{\lambda\tau}$ then

$$v(t)\le M[v(0)]_\tau e^{-(\lambda-\gamma)t}. \tag{1.9}$$

In addition, if $\theta=\sup_{k\in Z}\{1,a_k+b_ke^{\lambda\tau}\}$, then

$$v(t)=\theta[v(0)]_\tau e^{-(\lambda-(\ln(\theta e^{\lambda\tau})/\delta\tau))t}, \quad t\ge 0. \tag{1.10}$$

Computer simulation is shown in Figures 1, 2, 3, and 4.

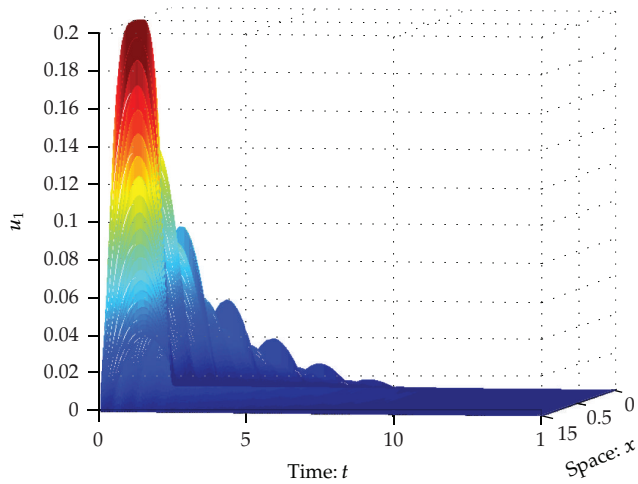

Figure 1: Computer simulation of the state $u_1(t, x)$.

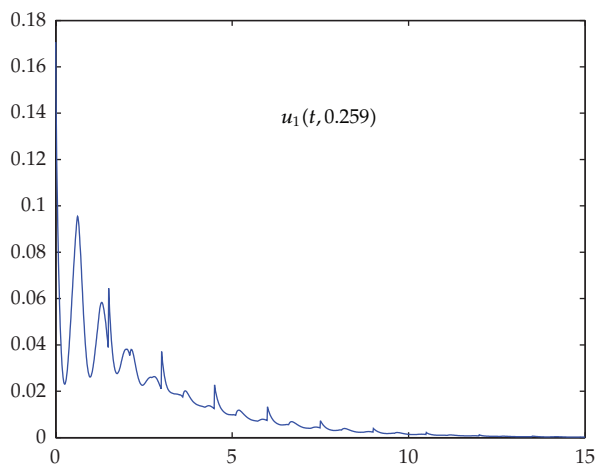

Figure 2: Sectional curve of the state variable $u_1(t, x)$.

2. Main Results

Theorem 2.1. *If assumptions (H1)–(H4) hold, in addition, the following conditions are satisfied:*

(C1) *there exists diagonal matrices* $P_1 = \mathrm{diag}(p_{11}, p_{12}, \ldots, p_{1n}) > 0$ *and* $P_2 > 0$ *such that*

$$\begin{pmatrix} -2P_1\underline{A}B + F^2 + P_1Q + P_2 & P_1\overline{A}|C| & P_1\overline{A}|D| \\ |C^T|\overline{A}P_1 & -I & 0 \\ |D^T|\overline{A}P_1 & 0 & -I \end{pmatrix} < 0; \tag{2.1}$$

(C2) $\min\{(\lambda_{\min}\Theta/\lambda_{\max}P_1), (1-\mu)\} > (\lambda_{\max}G^2/\lambda_{\min}P_2) \geq 0$, *where* $\Theta = 2P_1\underline{A}B - P_1\overline{A}|C||C|^T\overline{A}P_1 - P_1\overline{A}|D||D|^T\overline{A}P_1 - F^2 - P_1Q - P_2$, *where* $\tau'(t) \leq \mu < 1$ *for all* t;

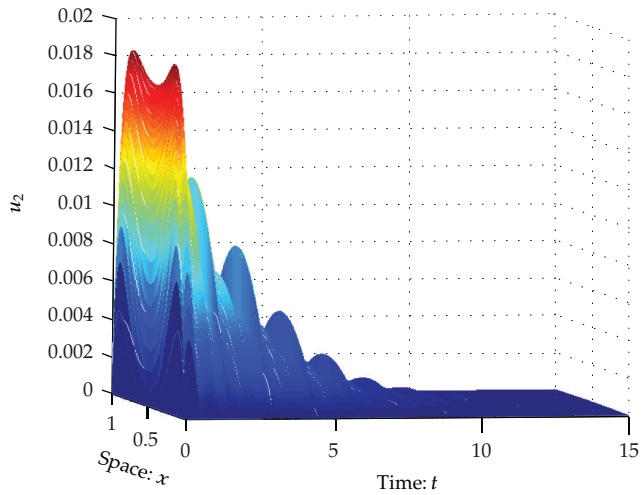

Figure 3: Computer simulation of the state $u_2(t,x)$.

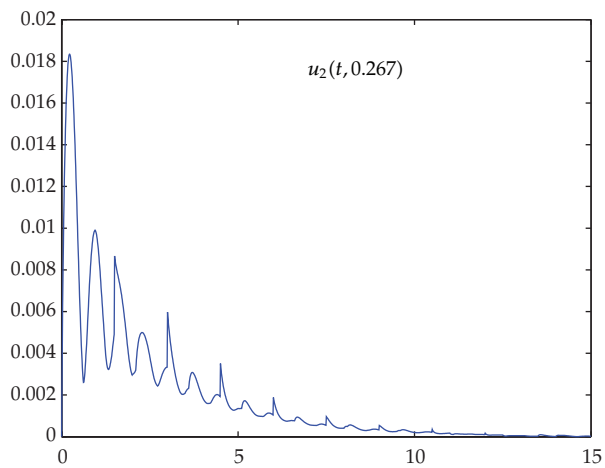

Figure 4: Sectional curve of the state variable $u_2(t,x)$.

(C3) *there exists a constant* $\delta > 1$ *such that* $\inf_{k\in Z}(t_k - t_{k-1}) > \delta\tau$, $\delta^2\tau > \ln(\rho e^{\lambda\tau})$ *and* $\lambda - (\ln(\rho e^{\lambda\tau})/\delta\tau) > 0$, *where* $\lambda > 0$ *is the unique solution of the equation* $\lambda = a - be^{\lambda\tau}$, *and* $\rho = \max\{1, (\lambda_{\max}(MP_1M)/\lambda_{\min}P_1) + e^{\lambda\tau}\}$, $a = \min\{(\lambda_{\min}\Theta/\lambda_{\max}P_1), (1-\mu)\}$, $b = \lambda_{\max}G^2/\lambda_{\min}P_2$, *then the null solution of system* (1.1) *is stochastically exponentially stable with convergence rate* $(1/2)(\lambda - (\ln(\rho e^{\lambda\tau})/\delta\tau))$.

Proof. First, we can get by Guass formula (see [20, Lemma 2.3])

$$\int_\Omega u^T P_1\left(\nabla\cdot\left(\tilde{D}(t,x,u)\circ\nabla_p u\right)dx\right.$$

$$= \int_\Omega u^T P_1\left(\sum_{k=1}^m \frac{\partial}{\partial x_k}\left(\tilde{D}_{1k}|\nabla u_1|^{p-2}\frac{\partial u_1}{\partial x_k}\right),\ldots,\sum_{k=1}^m \frac{\partial}{\partial x_k}\left(\tilde{D}_{nk}|\nabla u_n|^{p-2}\frac{\partial u_n}{\partial x_k}\right)\right)^T dx$$

$$
\begin{aligned}
&= \int_{\Omega} \sum_{j=1}^{n} p_{1j} u_j \sum_{k=1}^{m} \frac{\partial}{\partial x_k} \left(\widetilde{D}_{jk} \left| \nabla u_j \right|^{p-2} \frac{\partial u_j}{\partial x_k} \right) dx \\
&= -\sum_{k=1}^{m} \sum_{j=1}^{n} \int_{\Omega} p_{1j} \widetilde{D}_{jk} \left| \nabla u_j \right|^{p-2} \left(\frac{\partial u_j}{\partial x_k} \right)^2 dx.
\end{aligned}
\tag{2.2}
$$

Construct the Lyapunov functional as follows:

$$
V = V_1 + V_2, \tag{2.3}
$$

where

$$
\begin{aligned}
V_1 &= \int_{\Omega} u^T(t,x) P_1 u(t,x) dx = \int_{\Omega} \left| u^T(t,x) \right| P_1 |u(t,x)| dx \\
V_2 &= \int_{\Omega} \int_{t-\tau(t)}^{t} u^T(s,x) P_2 u(s,x) ds\, dx.
\end{aligned}
\tag{2.4}
$$

Then

$$
\begin{aligned}
LV_1 = {} & \int_{\Omega} -\sum_{k=1}^{m} \sum_{j=1}^{n} 2 p_{1j} \widetilde{D}_{jk} \left| \nabla u_j \right|^{p-2} \left(\frac{\partial u_j}{\partial x_k} \right)^2 dx \\
& - 2 \int_{\Omega} u^T(t,x) P_1 A(u(t,x)) B(u(t,x)) dx \\
& + 2 \int_{\Omega} u^T(t,x) P_1 A(u(t,x)) C f(u(t,x)) dx \\
& + 2 \int_{\Omega} u^T(t,x) P_1 A(u(t,x)) D g(u(t-\tau(t),x)) dx \\
& + \int_{\Omega} \operatorname{trace}\left[\sigma^T(u(t,x)) P_1 \sigma(u(t,x)) \right] dx \\
\leq {} & -2 \int_{\Omega} \left| u^T(t,x) \right| P_1 \underline{A} B |u(t,x)| dx \\
& + \int_{\Omega} \left[\left| u^T(t,x) \right| P_1 \overline{A} |C| |C|^T \overline{A} P_1 |u(t,x)| + \left| f^T(u(t,x)) \right| \left| f(u(t,x)) \right| \right] dx \\
& + \int_{\Omega} \left[\left| u^T(t,x) \right| P_1 \overline{A} |D| |D|^T \overline{A} P |u(t,x)| + g^T(u(t-\tau(t),x)) g(u(t-\tau(t),x)) \right] dx \\
& + \int_{\Omega} u^T(t,x) P_1 Q u(t,x) dx
\end{aligned}
$$

$$
\begin{aligned}
&\leq -\int_{\Omega}\left|u^T(t,x)\right|\left[2P_1\underline{A}B - P_1\overline{A}|C||C|^T\overline{A}P_1 - P_1\overline{A}|D||D|^T\overline{A}P_1 - F^2 - P_1Q\right]|u(t,x)|dx \\
&\quad + \int_{\Omega} u^T(t-\tau(t),x)G^2u(t-\tau(t),x)dx \\
LV_2 &= \int_{\Omega}\left[u^T(t,x)P_2u(t,x) - \left(1-\tau'(t)\right)u^T(t-\tau(t))P_2u(t-\tau(t))\right]dx \\
&\leq \int_{\Omega}\left|u^T(t,x)\right|P_2|u(t,x)|dx + (\mu-1)\int_{\Omega} u^T(t-\tau(t))P_2u(t-\tau(t))dx.
\end{aligned}
\tag{2.5}
$$

And then we have

$$
LV \leq -\int_{\Omega}\left|u^T(t,x)\right|\Theta|u(t,x)|dx + \int_{\Omega} u^T(t-\tau(t),x)\left[G^2 + (\mu-1)P_2\right]u(t-\tau(t),x)dx. \tag{2.6}
$$

Next, we use the method similar as that of [22]. Since $u(t,x)$ is the solution of system, and $V(u(t,x)) \in C^2[R^m, R^+]$ for all t, we can get by $It\hat{o}$ formula

$$
V(t) = V(t_k) + \int_{t_k}^{t} LV(u(s,x))ds + \int_{t_k}^{t} \frac{\partial V}{\partial u}\sigma(u(s,x))dw(s). \tag{2.7}
$$

Then we have

$$
EV(u(t,x)) = EV(u(t_k,x)) + \int_{t_k}^{t} ELV(s,x)ds, \quad t \in [t_k, t_{k+1}). \tag{2.8}
$$

Thus, for small enough $\Delta t > 0$, we have

$$
EV(u(t+\Delta t,x)) = EV(u(t_k,x)) + \int_{t_k}^{t+\Delta t} ELV(s,x)ds, \quad t \in [t_k, t_{k+1}), \tag{2.9}
$$

and then

$$
\begin{aligned}
&EV(u(t+\Delta t,x)) - EV(u(t,x)) \\
&\quad = \int_{t}^{t+\Delta t} ELV(s,x)ds \\
&\quad \leq E\int_{t}^{t+\Delta t}\left[-\int_{\Omega}\left|u^T(s,x)\right|\Theta|u(s,x)|dx\right. \\
&\qquad\qquad \left. + \int_{\Omega} u^T(s-\tau(s),x)\left[G^2 + (\mu-1)P_2\right]u(s-\tau(s),x)dx\right]ds, \quad t \in [t_k, t_{k+1}).
\end{aligned}
\tag{2.10}
$$

Since

$$\begin{aligned}
&\int_\Omega \left|u^T(t,x)\right|\Theta|u(t,x)|dx + \int_\Omega u^T(t-\tau(t),x)(1-\mu)P_2u(t-\tau(t),x)dx \\
&\quad \geq \frac{\lambda_{\min}\Theta}{\lambda_{\max}P_1}\int_\Omega \left|u^T(t,x)\right|P_1|u(t,x)|dx + (1-\mu)\int_\Omega u^T(t-\tau(t),x)P_2u(t-\tau(t),x)dx \\
&\quad \geq \min\left\{\frac{\lambda_{\min}\Theta}{\lambda_{\max}P_1},(1-\mu)\right\}V \\
&\int_\Omega u^T(t-\tau(t),x)G^2u(t-\tau(t),x)dx \leq \frac{\lambda_{\max}G^2}{\lambda_{\min}P_2}\int_\Omega u^T(t-\tau(t),x)P_2u(t-\tau(t),x)dx,
\end{aligned} \tag{2.11}$$

Then,

$$D^+EV(u(t,x)) \leq -\min\left\{\frac{\lambda_{\min}\Theta}{\lambda_{\max}P_1},(1-\mu)\right\}EV(t) + \frac{\lambda_{\max}G^2}{\lambda_{\min}P_2}[EV(t)]_\tau, \quad t\in[t_k,t_{k+1}). \tag{2.12}$$

Next, we have

$$\begin{aligned}
V(t_k) &= \int_\Omega u^T(t_k,x)P_1u(t_k,x)dx + \int_\Omega u^T(t_k-\tau(t_k),x)P_2u(t_k-\tau(t_k),x)dx \\
&= \int_\Omega \left[u^T(t_k^-,x)MP_1Mu(t_k^-,x)dx + \int_\Omega u^T(t_k-\tau(t_k),x)P_2u(t_k-\tau(t_k),x)dx\right. \\
&\leq \frac{\lambda_{\max}(MP_1M)}{\lambda_{\min}P_1}V(t_k^-) + \left[V(t_k^-)\right]_\tau.
\end{aligned} \tag{2.13}$$

Now the conditions (C1)–(C3) and Lemma 1.6 deduce

$$EV(t) \leq \rho[V(0)]_\tau e^{-(\lambda-(\ln(\rho e^{\lambda\tau})/\delta\tau))t} \tag{2.14}$$

or

$$E\|u(t,x)\|^2 \leq \left(\rho\frac{\lambda_{\max}P}{\lambda_{\min}P}\sup_{-\tau\leq s\leq 0}E\|\phi(s)\|_2^2\right)e^{-(\lambda-(\ln(\rho e^{\lambda\tau})/\delta\tau))t} \tag{2.15}$$

which together with Definition 1.3 implies the accomplishment of the proof. □

Remark 2.2. The nonlinear p-Laplace diffusion ($p > 1$) brings a great difficulties in judging the stability. However, even if $p = 2$, Theorem 2.1 has more computational efficiency than [15, Theorem 3.1] due to LMI criterion.

3. Examples

Consider the following impulsive CGNN:

$$
\begin{aligned}
du(t,x) = {} & \left\{ \nabla \cdot \left(\begin{pmatrix} 0.0005 & 0 \\ 0 & 0.0005 \end{pmatrix} \circ \nabla_p u \right) - \begin{pmatrix} 1.65 + 0.5\sin u_1 & 0 \\ 0 & 1.58 + 0.5\sin u_2 \end{pmatrix} \right. \\
& \times \left[\begin{pmatrix} 5.19u_1 - 0.02u_1\cos u_1 \\ 5.18u_2 - 0.01u_2\cos u_2 \end{pmatrix} - \begin{pmatrix} 0.1 & -0.03 \\ -0.03 & 0.1 \end{pmatrix} f(u(t,x)) \right. \\
& \left. \left. - \begin{pmatrix} 0.1 & -0.03 \\ -0.03 & 0.1 \end{pmatrix} g(u(t-\tau(t),x)) \right] \right\} dt + \sigma(u(t,x))dw(t), \quad t \in [t_k, t_{k+1}) \\
& u(t_k^+) = \begin{pmatrix} 1.350 & 0 \\ 0 & 1.350 \end{pmatrix} u(t^-,x), \quad t = t_k \\
& u(t_0+\theta,x) = \phi(\theta,x), \quad (\theta,x) \in [-0.65,0] \times \partial\Omega \\
& \frac{\partial u_i(t,x)}{\partial \nu} = 0, \quad (t,x) \in [-\tau,+\infty) \times \partial\Omega, \quad i = 1, 2,
\end{aligned}
\tag{3.1}
$$

where $\Omega = \{(x_1,x_2)^T \in R^2 \mid -\sqrt{2} \le x_1,\ x_2 \le \sqrt{2}\}$, and the corresponding matrices

$$
F = \begin{pmatrix} 0.100 & 0 \\ 0 & 0.100 \end{pmatrix} = G, \qquad Q = \begin{pmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{pmatrix}. \tag{3.2}
$$

We might as well assume that $t_0 = 0$, $t_k - t_{k-1} = 0.525$, $\tau(t) = 0.65$, $\tau'(t) \le \mu = 0.99$ for all $t \ge t_0$, and

$$
\begin{aligned}
& f(u) = g(u) = \begin{pmatrix} \dfrac{|u_1+1| - |u_1-1|}{20} \\ \dfrac{|u_2+1| - |u_2-1|}{20} \end{pmatrix}, \\
& \phi(s,x) = \begin{pmatrix} x(1-\cos(5\pi x))\cos^{189}(x-0.25)e^{-100s} \\ (1-x)\sin^2(4\pi x)\cos^{201}(x-0.55)e^{-100s} \end{pmatrix}, \quad -0.65 \le s \le 0, \\
& \underline{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \overline{A} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 5.160 & 0 \\ 0 & 5.160 \end{pmatrix}.
\end{aligned}
\tag{3.3}
$$

By way of MATLAB LMI Control Toolbox, we can solve the LMI condition in (C1) and get

$$
P_1 = \begin{pmatrix} 0.2392 & 0 \\ 0 & 0.2392 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 1.2291 & 0 \\ 0 & 1.2291 \end{pmatrix}. \tag{3.4}
$$

Next, we will prove that such P_1 and P_2 make (C2) and (C3) hold. Indeed, by computing directly, we can obtain $a = 0.01$, $b = 0.0081$, and then (C2) holds. Moreover, we might as well

assume $\delta = 82$, and then we have $\lambda = 0.0019$, $\rho = 2.8235$, thus (C3) is satisfied. Now from Theorem 2.1 we can compute the convergence $9.3809 * 10^{-0.004}$.

4. Conclusions

In this paper, we investigate the influence of impulse, time-delays and diffusion behaviors on the stability of stochastic Cohen-Grossberg neural network (CGNN). The LMI conditions of stochastic exponential stability of impulsive CGNN with p-Laplace reaction-diffusion terms was given, and an illustrate example was also given to show the effectiveness of the obtained result. Besides, the result obtained in this paper is also valid to the Laplace reaction-diffusion (in the case of $p = 2$) and has more computational efficiency due to the LMI approach even if $p = 2$ (Remark 2.2).

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