Research Article

New Traveling Wave Solutions of the Higher Dimensional Nonlinear Partial Differential Equation by the Exp-Function Method

Hasibun Naher,^{1,2} Farah Aini Abdullah,¹ and M. Ali Akbar^{1,3}

¹ School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia

² Department of Mathematics and Natural Sciences, Brac University, 66 Mohakhali, Dhaka 1212, Bangladesh

³ Department of Applied Mathematics, University of Rajshahi, Rajshahi, Bangladesh

Correspondence should be addressed to Hasibun Naher, hasibun06tasau@gmail.com

Received 12 July 2011; Revised 4 October 2011; Accepted 4 October 2011

Academic Editor: A. A. Soliman

Copyright © 2012 Hasibun Naher et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We construct new analytical solutions of the $(3 + 1)$ -dimensional modified KdV-Zakharov-Kuznetsev equation by the Exp-function method. Plentiful exact traveling wave solutions with arbitrary parameters are effectively obtained by the method. The obtained results show that the Exp-function method is effective and straightforward mathematical tool for searching analytical solutions with arbitrary parameters of higher-dimensional nonlinear partial differential equation.

1. Introduction

Nonlinear partial differential equations (NLPDEs) play a prominent role in different branches of the applied sciences. In recent time, many researchers investigated exact traveling wave solutions of NLPDEs which play a crucial role to reveal the insight of complex physical phenomena. In the past several decades, a variety of effective and powerful methods, such as variational iteration method [1–3], tanh-coth method [4], homotopy perturbation method [5–7], Fan subequation method [8], projective Riccati equation method [9], differential transform method [10], direct algebraic method [11], first integral method [12], Hirota's bilinear method [13], modified extended direct algebraic method [14], extended tanh method [15], Backlund transformation [16], bifurcation method [17], Cole-Hopf transformation method [18], sech-tanh method [19], (G'/G) -expansion method [20–22], modified (G'/G) -expansion method [23], multiwave method [24], extended (G'/G) -expansion method [25, 26], and others [27–33] were used to seek exact traveling wave solutions of the nonlinear evolution equations (NLEEs).

Recently, He and Wu [34] have presented a novel method called the Exp-function method for searching traveling wave solutions of the nonlinear evolution equations arising in mathematical physics. The Exp-function method is widely used to many kinds of NLPDEs, such as good Boussinesq equations [35], nonlinear differential equations [36], higher-order boundary value problems [37], nonlinear problems [38], Calogero-Degasperis-Fokas equation [39], nonlinear reaction-diffusion equations [40], 2D Bratu type equation [41], nonlinear lattice differential equations [42], generalized-Zakharov equations [43], $(3 + 1)$ -dimensional Jimbo-Miwa equation [44], modified Zakharov-Kuznetsov equation [45], Brusselator reaction diffusion model [46], nonlinear heat equation [47], and the other important NLPDEs [48–51].

In this article, we apply the Exp-function method [34] to obtain the analytical solutions of the nonlinear partial differential equation, namely, $(3 + 1)$ -dimensional modified KdV-Zakharov-Kuznetsev equation.

2. Description of the Exp-Function Method

Consider the general nonlinear partial differential equation

$$P(u, u_t, u_x, u_y, u_z, u_{tt}, u_{xt}, u_{xx}, u_{xy}, u_{yy}, u_{yt}, u_{zz}, u_{zt}, u_{zx}, u_{zy}, \dots) = 0. \quad (2.1)$$

The main steps of the Exp-function method [34] are as follows.

Step 1. Consider a complex variable as

$$u(x, y, z, t) = u(\eta), \quad \eta = x + y + z - Vt. \quad (2.2)$$

Now using (2.2), (2.1) converts to a nonlinear ordinary differential equation for $u(\eta)$

$$Q(u, u', u'', u''', \dots) = 0, \quad (2.3)$$

where primes denote the ordinary derivative with respect to η .

Step 2. We assume that the traveling wave solution of (2.3) can be expressed in the form [34]

$$u(\eta) = \frac{\sum_{n=-c}^d a_n \exp(n\eta)}{\sum_{m=-p}^q b_m \exp(m\eta)} = \frac{a_{-c} \exp(-c\eta) + \dots + a_d \exp(d\eta)}{b_{-p} \exp(-p\eta) + \dots + b_q \exp(q\eta)}, \quad (2.4)$$

where c , d , p , and q are positive integers to be determined later, and a_n and b_m are unknown constants. Equation (2.4) can be rewritten in the following equivalent form:

$$u(\eta) = \frac{a_c \exp(c\eta) + \dots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \dots + b_{-q} \exp(-q\eta)}. \quad (2.5)$$

Step 3. In order to determine the values of c and p , we balance the highest order linear term with the highest order nonlinear term, and, determining the values of d and q , we balance

Figure 1: Periodic solution for $\alpha = -1$, $y = 0$ and $z = 0$.

the lowest order linear term with the lowest order nonlinear term in (2.3). Thus, we obtain the values of c , d , p , and q .

Step 4. Substituting the values of c , d , p , and q into (2.5), and then substituted (2.5) into (2.3) and simplifying, we obtain

$$\sum_i C_i \exp(\pm i\eta) = 0, \quad i = 0, 1, 2, 3, \dots \quad (2.6)$$

Then each coefficient $C_i = 0$ is to set, yields a system of algebraic equations for a_c 's and b_p 's.

Step 5. We assume that the unknown a_c 's and b_p 's can be determined by solving the system of algebraic equations obtained in Step 4. Putting these values into (2.5), we obtain exact traveling wave solutions of the (2.1).

3. Application of the Method

In this section, we apply the method to construct the traveling wave solutions of the $(3 + 1)$ -dimensional modified KdV-Zakharov-Kuznetsev equation. The obtained solutions will be displayed in Figures 1, 2, 3, 4, 5, and 6 by using the software Maple 13.

We consider the $(3 + 1)$ -dimensional modified KdV-Zakharov-Kuznetsev equation

$$u_t + \alpha u^2 u_x + u_{xxx} + u_{xyy} + u_{xzz} = 0, \quad (3.1)$$

where α is a nonzero constant.

Zayed [52] solved (3.1) using the (G'/G) -expansion method. Later, in article [53], he solved same equation by the generalized (G'/G) -expansion method.

Here, we will solve this equation by the Exp-function method.

Figure 2: Periodic solution for $\beta = 2$, $y = 0$ and $z = 0$.

Figure 3: Solitons solution for $y = 0$ and $z = 0$.

Now, we use the transformation (2.2) into (3.1), which yields

$$-Vu' + \alpha u^2 u' + 3u''' = 0, \quad (3.2)$$

where primes denote the derivatives with respect to η .

According to Step 2, the solution of (3.2) can be written in the form of (2.5). To determine the values of c and p , according to Step 3, we balance the highest order linear term

Figure 4: Solitons solution for $y = 0$ and $z = 0$.

Figure 5: Solitons solution for $y = 0$ and $z = 0$.

of u''' with the highest order nonlinear term of u^2u' in (3.2), that is, u''' and u^2u' . Therefore, we have the following:

$$\begin{aligned} u''' &= \frac{c_1 \exp[(3p+c)\eta] + \dots}{c_2 \exp[4p\eta] + \dots}, \\ u^2u' &= \frac{c_3 \exp[(p+3c)\eta] + \dots}{c_4 \exp[4p\eta] + \dots}, \end{aligned} \quad (3.3)$$

Figure 6: Solitons solution for $y = 0$ and $z = 0$.

where c_j are coefficients only for simplicity; from (3.3), we obtain that

$$3p + c = p + 3c, \quad \text{which leads } p = c. \quad (3.4)$$

To determine the values of d and q , we balance the lowest order linear term of u''' with the lowest order nonlinear term of $u^2 u'$ in (3.2). We have

$$\begin{aligned} u''' &= \frac{\cdots + d_1 \exp[-(d-q)\eta]}{\cdots + d_2 \exp[-4q\eta]}, \\ u^2 u' &= \frac{\cdots + d_3 \exp[-3(d-q)\eta]}{\cdots + d_4 \exp[-4q\eta]}, \end{aligned} \quad (3.5)$$

where d_j are determined coefficients only for simplicity; from (3.5), we obtain

$$-(d-q) = -3(d-q), \quad \text{which leads } q = d. \quad (3.6)$$

Any real values can be considered for c and d , since they are free parameters. But the final solutions of (3.1) do not depend upon the choice of c and d .

Case 1. We set $p = c = 1$ and $q = d = 1$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{b_1 e^\eta + b_0 + b_{-1} e^{-\eta}}. \quad (3.7)$$

Since, $b_1 \neq 0$, (3.7) can be simplified

$$u(\eta) = \frac{a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{e^\eta + b_0 + b_{-1} e^{-\eta}}. \quad (3.8)$$

By substituting (3.8) into (3.2) and equating the coefficients of $\exp(\pm n\eta)$, $n = 0, 1, 2, 3, \dots$, with the aid of Maple 13, we obtain a set of algebraic equations in terms of $a_{-1}, a_0, a_1, b_{-1}, b_0$, and V

$$\frac{1}{A} (C_3 e^{3\eta} + C_2 e^{2\eta} + C_1 e^\eta + C_0 + C_{-1} e^{-\eta} + C_{-2} e^{-2\eta} + C_{-3} e^{-3\eta}) = 0. \quad (3.9)$$

And, setting each coefficient of $\exp(\pm n\eta)$, $n = 0, 1, 2, 3, \dots$, to zero, we obtain

$$C_3 = 0, \quad C_2 = 0, \quad C_1 = 0, \quad C_0 = 0, \quad C_{-1} = 0, \quad C_{-2} = 0, \quad C_{-3} = 0. \quad (3.10)$$

For determining unknowns, we solve the obtained system of algebraic (3.10) with the aid of Maple 13, and we obtain four different sets of solutions.

Set 1. We obtain that

$$b_{-1} = b_{-1}, \quad a_{-1} = \mp \frac{6b_{-1}}{\sqrt{-2\alpha}}, \quad a_0 = 0, \quad a_1 = \pm \frac{6}{\sqrt{-2\alpha}}, \quad b_0 = 0, \quad V = -6, \quad (3.11)$$

where b_{-1} is free parameter.

Set 2. We obtain that

$$\begin{aligned} a_0 &= a_0, \quad b_0 = b_0, \quad a_{-1} = \mp \frac{1}{12\sqrt{-2\alpha}} (2\alpha a_0^2 + 9b_0^2), \\ a_1 &= \pm \frac{3}{\sqrt{-2\alpha}}, \quad b_{-1} = \frac{1}{18} \alpha a_0^2 + \frac{1}{4} b_0^2, \quad V = -\frac{3}{2}, \end{aligned} \quad (3.12)$$

where a_0 and b_0 are free parameters.

Set 3. We obtain that

$$\begin{aligned} a_1 &= a_1, \quad b_0 = b_0, \quad a_{-1} = \frac{b_0^2 (2\alpha a_1^2 + 9)}{8\alpha a_1}, \\ a_0 &= \frac{b_0 (\alpha a_1^2 + 9)}{\alpha a_1}, \quad b_{-1} = \frac{b_0^2 (2\alpha a_1^2 + 9)}{8\alpha a_1^2}, \quad V = 3 + \alpha a_1^2, \end{aligned} \quad (3.13)$$

where a_1 and b_0 are free parameters.

Set 4. We obtain that

$$a_0 = a_0, \quad a_{-1} = 0, \quad a_1 = 0, \quad b_{-1} = \frac{1}{72} \alpha a_0^2, \quad b_0 = 0, \quad V = 3, \quad (3.14)$$

where a_0 is free parameter.

Now, substituting (3.11) into (3.8), we obtain traveling wave solution

$$u(\eta) = \frac{\pm 6 e^\eta \mp 6 b_{-1} e^{-\eta}}{\sqrt{-2\alpha} (e^\eta + b_{-1} e^{-\eta})}. \quad (3.15)$$

Equation (3.15) can be simplified as

$$u(\eta) = \frac{\pm 6}{\sqrt{-2\alpha}} \left[1 - \frac{2b_{-1}(\cosh \eta - \sinh \eta)}{(1 + b_{-1}) \cosh \eta + (1 - b_{-1}) \sinh \eta} \right], \quad (3.16)$$

where $\eta = x + y + z + 6t$.

If $b_{-1} = 1$ from (3.16), we obtain

$$u(\eta) = \frac{\pm 6i}{\sqrt{2\alpha}} \tanh \eta. \quad (3.17)$$

Substituting (3.12) into (3.8) and simplifying, we get traveling wave solution

$$u(\eta) = \frac{\pm 3}{\sqrt{-2\alpha}} \left[1 + \frac{12(\pm a_0 \sqrt{-2\alpha} + 3b_0) - 2(2\alpha a_0^2 + 9b_0^2)(\cosh \eta - \sinh \eta)}{(36 + 2\alpha a_0^2 + 9b_0^2) \cosh \eta + (36 - 2\alpha a_0^2 - 9b_0^2) \sinh \eta + 36b_0} \right], \quad (3.18)$$

where $\eta = x + y + z + (3/2)t$.

If α is negative, that is, $\alpha = -\beta$, $\beta > 0$, $b_0 = 2$ and $a_0 = 0$, then from (3.18), we obtain

$$u(\eta) = \frac{\pm 3}{\sqrt{2\beta}} \tanh \frac{\eta}{2}. \quad (3.19)$$

Substituting (3.13) into (3.8) and simplifying, we obtain

$$u(\eta) = a_1 \left[1 + \frac{72b_0}{\{8\alpha a_1^2 + b_0^2(2\alpha a_1^2 + 9)\} \cosh \eta + \{8\alpha a_1^2 - b_0^2(2\alpha a_1^2 + 9)\} \sinh \eta + 8\alpha a_1^2 b_0} \right], \quad (3.20)$$

where $\eta = x + y + z - (3 + \alpha a_1^2)t$.

If $b_0 = 1$, $\alpha = 6$, and $a_1 = 1/2$, (3.20) becomes

$$u(\eta) = \frac{1}{2} + \frac{3}{1 + 2 \cosh \eta}. \quad (3.21)$$

Substituting (3.14) into (3.8) and simplifying, we obtain

$$u(\eta) = \frac{72a_0}{(72 + \alpha a_0^2) \cosh \eta + (72 - \alpha a_0^2) \sinh \eta}, \quad (3.22)$$

where $\eta = x + y + z - 3t$.

If $a_0 = 3$ and $\alpha = 8$, (3.22) becomes

$$u(\eta) = \frac{3}{2} \operatorname{sech} \eta. \quad (3.23)$$

Case 2. We set $p = c = 2$ and $q = d = 1$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_2 e^{2\eta} + a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{b_2 e^{2\eta} + b_1 e^\eta + b_0 + b_{-1} e^{-\eta}}. \quad (3.24)$$

Since, there are some free parameters in (3.24), for simplicity, we may consider that $b_2 = 1$ and $b_{-1} = 0$. Then the solution (3.24) is simplified as

$$u(\eta) = \frac{a_2 e^{2\eta} + a_1 e^\eta + a_0 + a_{-1} e^{-\eta}}{e^{2\eta} + b_1 e^\eta + b_0}. \quad (3.25)$$

Performing the same procedure as described in Case 1, we obtain four sets of solutions.

Set 1. We obtain that

$$b_0 = b_0, \quad a_{-1} = 0, \quad a_0 = \mp \frac{6b_0}{\sqrt{-2\alpha}}, \quad a_1 = 0, \quad a_2 = \pm \frac{6}{\sqrt{-2\alpha}}, \quad b_1 = 0, \quad V = -6, \quad (3.26)$$

where b_0 is free parameter.

Set 2. We obtain that

$$\begin{aligned} a_1 = a_1, \quad b_1 = b_1, \quad a_{-1} = 0, \quad a_0 = \mp \frac{1}{12\sqrt{-2\alpha}} (2\alpha a_1^2 + 9b_1^2), \\ a_2 = \pm \frac{3}{\sqrt{-2\alpha}}, \quad b_0 = \frac{1}{18} \alpha a_1^2 + \frac{1}{4} b_1^2, \quad V = \frac{-3}{2}, \end{aligned} \quad (3.27)$$

where a_1 and b_1 are free parameters.

Set 3. We obtain that

$$\begin{aligned} a_2 &= a_2, & b_1 &= b_1, & a_{-1} &= 0, & a_0 &= \frac{b_1^2(2\alpha a_2^2 + 9)}{8\alpha a_2}, \\ a_1 &= \frac{b_1(\alpha a_2^2 + 9)}{\alpha a_2}, & b_0 &= \frac{b_1^2(2\alpha a_2^2 + 9)}{8\alpha a_2^2}, & V &= \alpha a_2^2 + 3, \end{aligned} \quad (3.28)$$

where a_2 and b_1 are free parameters.

Set 4. We obtain that

$$a_1 = a_1, \quad a_{-1} = 0, \quad a_0 = 0, \quad a_2 = 0, \quad b_0 = \frac{\alpha a_1^2}{72}, \quad b_1 = 0, \quad V = 3, \quad (3.29)$$

where a_1 is a free parameter.

Using (3.26) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{\pm 6}{\sqrt{-2\alpha}} \left[1 - \frac{2b_0(\cosh \eta - \sinh \eta)}{(1+b_0)\cosh \eta + (1-b_0)\sinh \eta} \right]. \quad (3.30)$$

If $b_0 = 1$, from (3.30), we obtain that

$$u(\eta) = \frac{\pm 6i}{\sqrt{2\alpha}} \tanh \eta, \quad (3.31)$$

where $\eta = x + y + z + 6t$.

Substituting (3.27) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{\pm 3}{\sqrt{-2\alpha}} \left[1 + \frac{12(\pm a_1\sqrt{-2\alpha} + 3b_1) - 2(2\alpha a_1^2 + 9b_1^2)(\cosh \eta - \sinh \eta)}{(36 + 2\alpha a_1^2 + 9b_1^2)\cosh \eta + (36 - 2\alpha a_1^2 - 9b_1^2)\sinh \eta + 36b_1} \right]. \quad (3.32)$$

If α is negative, that is, $\alpha = -\beta$, $\beta > 0$, $b_1 = 2$ and $a_1 = 0$, (3.32) can be simplified as

$$u(\eta) = \frac{\pm 3}{\sqrt{2\beta}} \tanh \frac{\eta}{2}, \quad (3.33)$$

where $\eta = x + y + z + (3/2)t$.

Substituting (3.28) into (3.25) and simplifying, we obtain that

$$u(\eta) = a_2 \left[1 + \frac{72b_1}{\{8\alpha a_2^2 + b_1^2(2\alpha a_2^2 + 9)\}\cosh \eta + \{8\alpha a_2^2 - b_1^2(2\alpha a_2^2 + 9)\}\sinh \eta + 8\alpha a_2^2 b_1} \right]. \quad (3.34)$$

If $b_1 = 1$, $\alpha = 6$, and $a_2 = 1/2$, (3.34) becomes

$$u(\eta) = \frac{1}{2} + \frac{3}{1 + 2 \cosh \eta}, \quad (3.35)$$

where $\eta = x + y + z - (3 + \alpha a_2^2)t$.

Using (3.29) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{72a_1}{(72 + \alpha a_1^2) \cosh \eta + (72 - \alpha a_1^2) \sinh \eta}. \quad (3.36)$$

If $a_1 = 3$, and $\alpha = 8$, (3.36) becomes

$$u(\eta) = \frac{3}{2} \operatorname{sech} \eta, \quad (3.37)$$

where $\eta = x + y + z - 3t$.

Case 3. We set $p = c = 2$ and $q = d = 2$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_2 e^{2\eta} + a_1 e^\eta + a_0 + a_{-1} e^{-\eta} + a_{-2} e^{-2\eta}}{b_2 e^{2\eta} + b_1 e^\eta + b_0 + b_{-1} e^{-\eta} + b_{-2} e^{-2\eta}}. \quad (3.38)$$

Since, there are some free parameters in (3.38), we may consider $b_2 = 1$, $a_{-2} = 0$, $b_{-2} = 0$, and $b_{-1} = 0$ so that the (3.38) reduces to the (3.25). This indicates that the Case 3 is equivalent to the Case 2. Equation (3.38) can be rewritten as

$$u(\eta) = \frac{a_2 e^\eta + a_1 + a_0 e^{-\eta} + a_{-1} e^{-2\eta} + a_{-2} e^{-3\eta}}{b_2 e^\eta + b_1 + b_0 e^{-\eta} + b_{-1} e^{-2\eta} + b_{-2} e^{-3\eta}}. \quad (3.39)$$

If we put $a_{-2} = 0$, $a_{-1} = 0$, $b_2 = 1$, $b_{-2} = 0$, and $b_{-1} = 0$ into (3.39), we obtain the solution form as (3.8). This implies that the Case 3 is equivalent to the Case 1.

Also, if we consider $p = c = 3$ and $q = d = 3$, it can be shown that this Case is also equivalent to the Cases 1 and 2.

Therefore, we think that no need to find the solutions again.

It is noted that the solution (3.17) and (3.31) are identical, solution (3.19) and (3.33) are identical, solution (3.21) and (3.35) are identical, and solution (3.23) and (3.37) are identical.

Beyond Table 1, Zayed [52] obtained another solution (3.39). But, we obtain two more new solutions (3.21) and (3.23).

Graphical Representations of the Solutions

The above solutions are shown with the aid of Maple 13 in the graphs.

Table 1: Comparison between Zayed [52] solutions and our solutions.

Zayed [52] solutions	Our solutions
(i) If $\lambda = 2$, $\mu = 0$, equation (3.40) becomes $u(\eta) = \pm \frac{6i}{\sqrt{2\alpha}} \tanh \eta.$	(i) Solution (3.17) is $u(\eta) = \pm \frac{6i}{\sqrt{2\alpha}} \tanh \eta.$
(ii) If $1 + \lambda^2 = 4\mu$ and α is replaced with β , equation (3.38) becomes $u(\eta) = \pm \frac{3i}{\sqrt{2\beta}} \tan \frac{\eta}{2}.$	(ii) If η is replaced with $i\eta$, solution (3.19) becomes $u(\eta) = \pm \frac{3i}{\sqrt{2\beta}} \tan \frac{\eta}{2}.$

4. Conclusions

Using the Exp-function method, with the aid of symbolic computation software Maple 13, new exact traveling wave solutions of the (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsev equation are constructed. It is important that some of the obtained solutions are identical to the solutions available in the literature and some are new. These solutions can be used to describe the insight of the complex physical phenomena.

Acknowledgment

This paper is supported by USM short-term Grant no. 304/PMATHS/6310072, and the authors would like to express their thanks to the School of Mathematical Sciences, USM for providing related research facilities. The authors are also grateful to the referee(s) for their valuable comments and suggestions.

References

- [1] M. A. Abdou and A. A. Soliman, "Variational iteration method for solving Burger's and coupled Burger's equations," *Journal of Computational and Applied Mathematics*, vol. 181, no. 2, pp. 245–251, 2005.
- [2] M. A. Abdou and A. A. Soliman, "New applications of variational iteration method," *Physica D*, vol. 211, no. 1-2, pp. 1–8, 2005.
- [3] C. A. Gómez and A. H. Salas, "Exact solutions for the generalized BBM equation with variable coefficients," *Mathematical Problems in Engineering*, vol. 2010, Article ID 498249, 10 pages, 2010.
- [4] A. M. Wazwaz, "The tanh-coth method for solitons and kink solutions for nonlinear parabolic equations," *Applied Mathematics and Computation*, vol. 188, no. 2, pp. 1467–1475, 2007.
- [5] T. Öziş and A. Yildirim, "Traveling wave solution of Korteweg-de vries equation using He's Homotopy Perturbation Method," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 8, no. 2, pp. 239–242, 2007.
- [6] E. M. E. Zayed, T. A. Nofal, and K. A. Gepreel, "On using the homotopy perturbation method for finding the travelling wave solutions of generalized nonlinear Hirota-Satsuma coupled KdV equations," *International Journal of Nonlinear Science*, vol. 7, no. 2, pp. 159–166, 2009.
- [7] S. T. Mohyud-Din, A. Yildirim, and G. Demirli, "Traveling wave solutions of Whitham-Broer-Kaup equations by homotopy perturbation method," *Journal of King Saud University (Science)*, vol. 22, no. 3, pp. 173–176, 2010.
- [8] D. Feng and K. Li, "Exact traveling wave solutions for a generalized Hirota-Satsuma coupled KdV equation by Fan sub-equation method," *Physics Letters. A*, vol. 375, no. 23, pp. 2201–2210, 2011.
- [9] A. Salas, "Some exact solutions for the Caudrey-Dodd-Gibbon equation," arXiv: 0805.2969v2 [math-ph] 21 May 2008.
- [10] J. Biazar, M. Eslami, and M. R. Islam, "Differential transform method for nonlinear parabolic-hyperbolic partial differential equations," *Applications and Applied Mathematics*, vol. 5, no. 10, pp. 1493–1503, 2010.

- [11] S. M. Taheri and A. Neyrameh, "Complex solutions of the regularized long wave equation," *World Applied Sciences Journal*, vol. 12, no. 9, pp. 1625–1628, 2011.
- [12] P. Sharma and O. Y. Kushel, "The first integral method for Huxley equation," *International Journal of Nonlinear Science*, vol. 10, no. 1, pp. 46–52, 2010.
- [13] A.-M. Wazwaz, "Non-integrable variants of Boussinesq equation with two solitons," *Applied Mathematics and Computation*, vol. 217, no. 2, pp. 820–825, 2010.
- [14] A. A. Soliman and H. A. Abdo, "New exact solutions of nonlinear variants of the RLW, and PHI-four and Boussinesq equations based on modified extended direct algebraic method," *International Journal of Nonlinear Science*, vol. 7, no. 3, pp. 274–282, 2009.
- [15] A. M. Wazwaz, "New travelling wave solutions to the Boussinesq and the Klein-Gordon equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 13, no. 5, pp. 889–901, 2008.
- [16] L. Jianming, D. Jie, and Y. Wenjun, "Backlund transformation and new exact solutions of the Sharma-Tasso-Olver equation," *Abstract and Applied Analysis*, vol. 2011, Article ID 935710, 8 pages, 2011.
- [17] M. Song, S. Li, and J. Cao, "New exact solutions for the (2+1)-dimensional Broer-Kaup-Kupershmidt equations," *Abstract and Applied Analysis*, vol. 2010, Article ID 652649, 9 pages, 2010.
- [18] A. H. Salas and C. A. Gómez S., "Application of the Cole-Hopf transformation for finding exact solutions to several forms of the seventh-order KdV equation," *Mathematical Problems in Engineering*, vol. 2010, Article ID 194329, 14 pages, 2010.
- [19] S. M. Sayed, O. O. Elhamahmy, and G. M. Gharib, "Travelling wave solutions for the KdV-Burgers-Kuramoto and nonlinear Schrödinger equations which describe pseudospherical surfaces," *Journal of Applied Mathematics*, vol. 2008, Article ID 576783, 10 pages, 2008.
- [20] X. Liu, L. Tian, and Y. Wu, "Application of (G'/G)-expansion method to two nonlinear evolution equations," *Applied Mathematics and Computation*, vol. 217, no. 4, pp. 1376–1384, 2010.
- [21] B. Zheng, "Travelling wave solutions of two nonlinear evolution equations by using the (G'/G) -expansion method," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5743–5753, 2011.
- [22] J. Feng, W. Li, and Q. Wan, "Using (G'/G) -expansion method to seek the traveling wave solution of Kolmogorov-Petrovskii-Piskunov equation," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5860–5865, 2011.
- [23] X. J. Miao and Z. Y. Zhang, "The modified (G'/G) -expansion method and traveling wave solutions of nonlinear the perturbed nonlinear Schrodinger's equation with Kerr law nonlinearity," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, pp. 4259–4267, 2011.
- [24] Y. Shi, Z. Dai, S. Han, and L. Huang, "The multi-wave method for nonlinear evolution equations," *Mathematical & Computational Applications*, vol. 15, no. 5, pp. 776–783, 2010.
- [25] E. M. E. Zayed and M. A. S. El-Malky, "The Extended (G'/G) -expansion method and its applications for solving the (3+1)-dimensional nonlinear evolution equations in mathematical physics," *Global Journal of Science Frontier Research*, vol. 11, no. 1, 2011.
- [26] E. M. E. Zayed and S. Al-Joudi, "Applications of an extended (G'/G) -expansion method to find exact solutions of nonlinear PDEs in mathematical physics," *Mathematical Problems in Engineering*, vol. 2010, Article ID 768573, 19 pages, 2010.
- [27] L. Ling-xiao, L. Er-qiang, and W. Ming-Liang, "The (G'/G,1/G)-expansion method and its application to travelling wave solutions of the Zakharov equations," *Applied Mathematics*, vol. 25, no. 4, pp. 454–462, 2010.
- [28] M. Abdollahzadeh, M. Hosseini, M. Ghanbarpour, and H. Shirvani, "Exact travelling solutions for fifth order Caudrey-Dodd-Gibbon equation," *International Journal of Applied Mathematics and Computation*, vol. 2, no. 4, pp. 81–90, 2010.
- [29] K. A. Gepreel, "Exact solutions for nonlinear PDEs with the variable coefficients in mathematical physics," *Journal of Information and Computing Science*, vol. 6, no. 1, pp. 003–014, 2011.
- [30] S. A. El-Wakil, A. R. Degheidy, E. M. Abulwafa, M. A. Madkour, M. T. Attia, and M. A. Abdou, "Exact travelling wave solutions of generalized Zakharov equations with arbitrary power nonlinearities," *International Journal of Nonlinear Science*, vol. 7, no. 4, pp. 455–461, 2009.
- [31] Z. Zhao, Z. Dai, and C. Wang, "Extend three-wave method for the (1+2)-dimensional Ito equation," *Applied Mathematics and Computation*, vol. 217, no. 5, pp. 2295–2300, 2010.
- [32] J. Zhou, L. Tian, and X. Fan, "Soliton and periodic wave solutions to the osmosis K(2,2) equation," *Mathematical Problems in Engineering*, vol. 2009, Article ID 509390, 10 pages, 2009.
- [33] Y. Khan, N. Faraz, and A. Yildirim, "New soliton solutions of the generalized Zakharov equations using He's variational approach," *Applied Mathematics Letters*, vol. 24, no. 6, pp. 965–968, 2011.

- [34] J.-H. He and X.-H. Wu, "Exp-function method for nonlinear wave equations," *Chaos, Solitons and Fractals*, vol. 30, no. 3, pp. 700–708, 2006.
- [35] S. T. Mohyud-Din, M. A. Noor, and A. Waheed, "Exp-function method for generalized traveling solutions of good Boussinesq equations," *Journal of Applied Mathematics and Computing*, vol. 30, no. 1-2, pp. 439–445, 2009.
- [36] S. T. Mohyud-Din, "Solution of nonlinear differential equations by exp-function method," *World Applied Sciences Journal*, vol. 7, pp. 116–147, 2009.
- [37] S. T. Mohyud-Din, M. A. Noor, and K. I. Noor, "Exp-function method for solving higher-order boundary value problems," *Bulletin of the Institute of Mathematics. Academia Sinica. New Series*, vol. 4, no. 2, pp. 219–234, 2009.
- [38] S. T. Mohyud-Din, M. A. Noor, and K. I. Noor, "Some relatively new techniques for nonlinear problems," *Mathematical Problems in Engineering*, vol. 2009, Article ID 234849, 25 pages, 2009.
- [39] S. T. Mohyud-Din, M. A. Noor, and A. Waheed, "Exp-function method for generalized travelling solutions of calogero-degasperis-fokas equation," *Zeitschrift fur Naturforschung: Section A Journal of Physical Sciences*, vol. 65, no. 1, pp. 78–84, 2010.
- [40] A. Yildirim and Z. Pnar, "Application of the exp-function method for solving nonlinear reaction-diffusion equations arising in mathematical biology," *Computers and Mathematics with Applications*, vol. 60, no. 7, pp. 1873–1880, 2010.
- [41] E. Misirli and Y. Gurefe, "Exp-function method for solving nonlinear evolution equations," *Computers and Mathematics with Applications*, vol. 16, no. 1, pp. 258–266, 2011.
- [42] I. Aslan, "Application of the exp-function method to nonlinear lattice differential equations for multi-wave and rational solutions," *Mathematical Methods in the Applied Sciences*, vol. 34, no. 14, pp. 1707–1710, 2011.
- [43] Y. Z. Li, K. M. Li, and C. Lin, "Exp-function method for solving the generalized-Zakharov equations," *Applied Mathematics and Computation*, vol. 205, no. 1, pp. 197–201, 2008.
- [44] T. Öziş and I. Aslan, "Exact and explicit solutions to the (3+1)-dimensional Jimbo-Miwa equation via the Exp-function method," *Physics Letters, Section A*, vol. 372, no. 47, pp. 7011–7015, 2008.
- [45] S. T. Mohyud-Din, M. A. Noor, and K. I. Noor, "Exp-function method for traveling wave solutions of modified Zakharov-Kuznetsov equation," *Journal of King Saud University*, vol. 22, no. 4, pp. 213–216, 2010.
- [46] F. Khani, F. Samadi, and S. Hamed-Nezhad, "New exact solutions of the Brusselator reaction diffusion model using the exp-function method," *Mathematical Problems in Engineering*, vol. 2009, Article ID 346461, 9 pages, 2009.
- [47] W. Zhang, "The extended tanh method and the exp-function method to solve a kind of nonlinear heat equation," *Mathematical Problems in Engineering*, Article ID 935873, 12 pages, 2010.
- [48] K. Parand, J. A. Rad, and A. Rezaei, "Application of Exp-function method for a class of nonlinear PDE's arising in mathematical physics," *Journal of Applied Mathematics & Informatics*, vol. 29, no. 3-4, pp. 763–779, 2011.
- [49] L. Yao, L. Wang, and X.-W. Zhou, "Application of exp-function method to a Huxley equation with variable coefficient," *International Mathematical Forum*, vol. 4, no. 1–4, pp. 27–32, 2009.
- [50] A. H. Salas and C. A. Gómez S, "Exact solutions for a third-order KdV equation with variable coefficients and forcing term," *Mathematical Problems in Engineering*, vol. 2009, Article ID 737928, 13 pages, 2009.
- [51] M. A. Akbar and N. H. M. Ali, "Exp-function method for duffing equation and new solutions of (2+1) dimensional dispersive long wave equations," *Progress in Applied Mathematics*, vol. 1, no. 2, pp. 30–42, 2011.
- [52] E. M. E. Zayed, "Traveling wave solutions for higher dimensional nonlinear evolution equations using the (G'/G) -expansion method," *Journal of Applied Mathematics & Informatics*, vol. 28, no. 1-2, pp. 383–395, 2010.
- [53] E. M. E. Zayed, "New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G) -expansion method," *Journal of Physics A: Mathematical and Theoretical*, vol. 42, no. 19, article 195202, 2009.