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Research Article

Robust Adaptive Finite-Time Synchronization of Two Different Chaotic Systems with Parameter Uncertainties

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This paper is concerned with the finite-time synchronization problem for two different chaotic systems with parameter uncertainties. Using finite-time control approach and robust control method, an adaptive synchronization scheme is proposed to make the synchronization errors of the systems with parameter uncertainties zero in a finite time. On the basis of Lyapunov stability theory, appropriate adaptive laws are derived to deal with the unknown parameters of the systems. And the convergence of the parameter errors is guaranteed in a finite time. The proposed method can be applied to a variety of chaos systems. Numerical simulations are given to demonstrate the efficiency of the proposed control scheme.

1. Introduction

In the past few decades, chaos synchronization has gained much attention from various fields [1–3], since Pecora and carroll [4] introduced a method to synchronize two identical chaotic systems with different initial conditions in 1990. Most of the works on chaos synchronization have focused on two identical chaotic systems [5–11]. However, in many real world applications, there are no exactly two identical chaotic systems. Therefore, the problem of chaos synchronization between two different chaotic systems with uncertainties is an important research issue [12]. Different synchronization control methods for two different chaotic systems, such as adaptive control [13–21], nonlinear feedback control [22], backstepping [23, 24], fuzzy technique [25–27], and sliding mode control [28–30], have been proposed to solve the synchronization problem.

Since some systems' parameters cannot be exactly known in advance, many efforts have been devoted to adaptive synchronization. In [18, 31], Huang discussed the

synchronizations between Lorenz-Stenflo (LS) system and CYQY system, and between LS system and hyperchaotic Chen system with fully uncertain parameters. Wang et al. [15] designed a general adaptive robust controller and parameter update laws which made the drive-response systems with different structures asymptotically synchronized. In [16], the sufficient conditions for achieving synchronization between generalized Henon-Heiles system and hyperchaotic Chen system with unknown parameters were derived based on Lyapunov stability theory. A new adaptive synchronization scheme by pragmatical asymptotically stability theorem was proposed for two different uncertain chaotic systems [17], but the unknown signals were used in the controller. Chaos synchronization between two different chaotic systems with uncertainties in both master and slave chaotic systems remains a challenging problem [30].

Most methods only guarantee the asymptotic stability of the synchronization error dynamics, namely, the trajectories of the slave system approach the trajectories of the master system as $t \to \infty$. From a practical point of view, however, it is more valuable that the synchronization objective is realized in a finite time [28]. In recent years, some researchers have applied finite-time control techniques, such as nonsingular terminal sliding mode control method [32], CLF-based method [33, 34], sliding mode control method [28–30], and the finite-time stability theory-based method [28, 35, 36], to realize synchronization.

Compared with the existing results in the literature, there are three advantages which make our approach attractive. First, based on the finite-time control technique, adaptive control, and robust control, a new synchronization method is presented for a wide class of nonlinear systems. Second, it guarantees that all the errors are driven to zero in a finite time even for the systems with parameter uncertainties. Third, it guarantees that all the parameter errors converge to zero in a finite time.

In this paper, an adaptive finite-time synchronization scheme is proposed for a class of chaotic systems. The rest of the paper is organized as follows. In Section 2, we introduce the chaotic systems considered in this paper and preliminary lemmas. In Section 3, the proposed finite-time controller is designed to synchronize two different chaotic systems. We give the simulation results and the conclusions in Sections 4 and 5, respectively.

2. System Description

Consider the following master chaotic system:

$$\dot{x} = (A_1 + \Delta A_1)x + (B_1 + \Delta B_1)f_1(x), \tag{2.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes a state vector, f_1 is a nonlinear continuous vector function, A_1 and B_1 are $n \times n$ nominal coefficient matrices, ΔA_1 and ΔB_1 are unknown parts of $n \times n$ coefficient matrices.

The slave system is given with

$$\dot{y} = (A_2 + \Delta A_2)y + (B_2 + \Delta B_2)f_2(y) + u, \tag{2.2}$$

where $y = [y_1, y_2, \ldots, y_n]^T \in R^n$ denotes a state vector, f_2 is a nonlinear continuous vector function, A_2 and B_2 are $n \times n$ nominal coefficient matrices, ΔA_2 and ΔB_2 are unknown parts

of $n \times n$ coefficient matrices, and $u = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is a control input vector to be designed.

Subtracting (2.1) from (2.2) yields the error dynamical system as follows:

$$\dot{e} = (A_2 + \Delta A_2)y + (B_2 + \Delta B_2)f_2(y) - (A_1 + \Delta A_1)x - (B_1 + \Delta B_1)f_1(x) + u, \tag{2.3}$$

where $e = y - x$. Note that only a part of elements of the coefficient matrices unknown, without loss of generality, we assume that the number of the unknown elements of the ith row of ΔA_1 is N_{A1i}, that of ΔA_2 is N_{A2i}, that of ΔB_1 is N_{B1i}, and that of ΔB_2 is N_{B2i}. Then (2.3) can be rewritten as

$$\begin{aligned}\dot{e} &= A_2 y + B_2 f_2(y) - A_1 x - B_1 f_1(x) + u \\ &\quad + \begin{bmatrix} \sum_{i=1}^{N_{A21}} \delta a_{21i}\overline{y}_{1i} \\ \vdots \\ \sum_{i=1}^{N_{A2n}} \delta a_{2ni}\overline{y}_{ni} \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{N_{B21}} \delta b_{21i}\overline{f}_{21i} \\ \vdots \\ \sum_{i=1}^{N_{B2n}} \delta b_{2ni}\overline{f}_{2ni} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{N_{A11}} \delta a_{11i}\overline{x}_{1i} \\ \vdots \\ \sum_{i=1}^{N_{A1n}} \delta a_{1ni}\overline{x}_{ni} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{N_{B11}} \delta b_{11i}\overline{f}_{11i} \\ \vdots \\ \sum_{i=1}^{N_{B1n}} \delta b_{1ni}\overline{f}_{1ni} \end{bmatrix},\end{aligned} \tag{2.4}$$

where δa_{*ji} are nonzero elements of the jth row of ΔA_*, $\overline{y}_{ji}$ are corresponding elements of y, δb_{*ji} are nonzero elements of the jth row of ΔB_* and $\overline{f}_{*ji}$ are corresponding elements of f_*, $j = 1, \ldots, n$.

Assumption 2.1. The unknown parameters are norm-bounded, that is,

$$|\delta a_{*ji}| \le d_{a_*ji}, \qquad |\delta b_{*ji}| \le d_{b_*ji}, \tag{2.5}$$

where d_{a_*ji} and d_{b_*ji} are known positive constants.

Definition 2.2 (see [28]). Consider the master and slave chaotic systems described by (2.1) and (2.2), respectively. If there exists a constant $T = T(e(0)) > 0$, such that

$$\lim_{t \to T} \|e(t)\| = 0 \tag{2.6}$$

and $\|e(t)\| \equiv 0$, if $t \ge T$, then the chaos synchronization between the systems (2.1) and (2.2) is achieved in a finite time.

Lemma 2.3 (see [28]). *Consider the system*

$$\dot{x} = f(x), \qquad f(0) = 0, \quad x \in R^n, \tag{2.7}$$

where $f : D \to R^n$ is continuous on an open neighborhood $D \in R^n$.

Suppose there exists a continuous differential positive-definite function $V(x) : D \to R$, real numbers $p > 0$, $0 < \eta < 1$, such that

$$\dot{V}(x) + pV^{\eta}(x) \leq 0, \quad \forall x \in D. \tag{2.8}$$

Then, the origin of system (2.7) is a locally finite-time stable equilibrium, and the settling time, depending on the initial state $x(0) = x_0$, satisfies

$$T(x_0) \leq \frac{V^{1-\eta}(x_0)}{p(1-\eta)}. \tag{2.9}$$

In addition, if $D = R^n$ and $V(x)$ is also radially unbounded (i.e., $V(x) \to +\infty$ as $\|x\| \to +\infty$), then the origin is a globally finite-time stable equilibrium of system (2.7).

Lemma 2.4 (see [28]). *Suppose $a_1, a_2, \ldots, a_n$, and $0 < q < 2$ are all real numbers, then the following inequality holds:*

$$|a_1|^q + |a_2|^q + \cdots + |a_n|^q \geq \left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^{q/2}. \tag{2.10}$$

3. Synchronization of Two Different Chaotic Systems with Parameter Uncertainties

Consider two different chaotic systems (2.1) and (2.2) from different initial states. The aim of controller design is to determine appropriate u such that

$$\lim_{t \to T} e = 0. \tag{3.1}$$

Now we are ready to give the design steps.

Define Lyapunov function

$$\begin{aligned} V = {} & \frac{1}{2}e^T e + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}}(1+k_{dj})^{-1}\gamma_{a2ji}^{-1}\delta\tilde{a}_{2ji}^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}}(1+k_{dj})^{-1}\gamma_{a1ji}^{-1}\delta\tilde{a}_{1ji}^2 \\ & + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}}(1+k_{dj})^{-1}\gamma_{b2ji}^{-1}\delta\tilde{b}_{2ji}^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}}(1+k_{dj})^{-1}\gamma_{b1ji}^{-1}\delta\tilde{b}_{1ji}^2, \end{aligned} \tag{3.2}$$

where $\delta\tilde{a}_{2ji} = \delta\hat{a}_{2ji} - \delta a_{2ji}$, $\delta\tilde{a}_{1ji} = \delta\hat{a}_{1ji} - \delta a_{1ji}$, $\delta\tilde{b}_{2ji} = \delta\hat{b}_{2ji} - \delta b_{2ji}$, $\delta\tilde{b}_{1ji} = \delta\hat{b}_{1ji} - \delta b_{1ji}$, and $\delta\hat{a}_{2ji}$, $\delta\hat{a}_{1ji}$, $\delta\hat{b}_{2ji}$, $\delta\hat{b}_{1ji}$ are estimation values of δa_{2ji}, δa_{1ji}, δb_{2ji}, δb_{1ji}, respectively, and γ_{a2ji}^{-1}, γ_{a1ji}^{-1}, γ_{b2ji}^{-1}, γ_{b1ji}^{-1}, k_{dj} are constants greater than zero.

Taking the time derivative of (3.2) gives

$$\begin{aligned}\dot{V} = e^T\dot{e} &+ \sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}}(1+k_{dj})^{-1}\gamma_{a2ji}^{-1}\delta\tilde{a}_{2ji}\delta\dot{\hat{a}}_{2ji} + \sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}}(1+k_{dj})^{-1}\gamma_{a1ji}^{-1}\delta\tilde{a}_{1ji}\delta\dot{\hat{a}}_{1ji} \\ &+ \sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}}(1+k_{dj})^{-1}\gamma_{b2ji}^{-1}\delta\tilde{b}_{2ji}\delta\dot{\hat{b}}_{2ji} + \sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}}(1+k_{dj})^{-1}\gamma_{b1ji}^{-1}\delta\tilde{b}_{1ji}\delta\dot{\hat{b}}_{1ji}.\end{aligned} \tag{3.3}$$

Design the control law as

$$\begin{aligned}u = &-Ke - K_D\dot{e} + \alpha_s - \left[c_1\operatorname{sgn}(e_1)|e_1|^{\alpha},\ldots,c_n\operatorname{sgn}(e_n)|e_n|^{\alpha}\right]^T \\ &-\mu\left[\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}}\left(\left|\delta\hat{a}_{2ji}\right|+d_{a2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}}\left(\left|\delta\hat{a}_{1ji}\right|+d_{a1ji}\right)^{1+\alpha}\right. \\ &\left.+\sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}}\left(\left|\delta\hat{b}_{2ji}\right|+d_{b2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}}\left(\left|\delta\hat{b}_{1ji}\right|+d_{b1ji}\right)^{1+\alpha}\right]\alpha_e \\ &+\left[\sum_{i=1}^{N_{A11}}\delta\hat{a}_{11i}\overline{y}_{1i},\ldots,\sum_{i=1}^{N_{A1n}}\delta\hat{a}_{1ni}\overline{y}_{ni}\right]^T - \left[\sum_{i=1}^{N_{B21}}\delta\hat{b}_{21i}\overline{f}_{21i},\ldots,\sum_{i=1}^{N_{A1n}}\delta\hat{b}_{2ni}\overline{f}_{2ni}\right]^T \\ &-\left[\sum_{i=1}^{N_{A21}}\delta\hat{a}_{21i}\overline{y}_{1i},\ldots,\sum_{i=1}^{N_{A2n}}\delta\hat{a}_{2ni}\overline{y}_{ni}\right]^T + \left[\sum_{i=1}^{N_{B11}}\delta\hat{b}_{11i}\overline{f}_{11i},\ldots,\sum_{i=1}^{N_{B1n}}\delta\hat{b}_{1ni}\overline{f}_{1ni}\right]^T,\end{aligned} \tag{3.4}$$

where $K = \operatorname{diag}\{k_1, k_2, \ldots, k_n\}$, $k_i > 0$, $K_D = \operatorname{diag}\{k_{d1}, k_{d2}, \ldots, k_{dn}\}$, $k_{di} > 0$, and $c_i > 0$ are constants, $0 < \alpha < 1$ is a constant, $\alpha_s = [\,\alpha_{s1} \ldots \alpha_{sn}\,]^T$ and α_{si} is given in (3.5)

$$\alpha_{si} = \begin{cases} -\dfrac{(\partial V_A)/(\partial e_i)f_{Ai} + K_{Ai}\sqrt{\left((\partial V_A)/(\partial e_i)f_{Ai}\right)^2 + \left((\partial V_A)/(\partial e_i)\right)^4}}{(\partial V_A)/(\partial e_i)}, & \text{if } \dfrac{\partial V_A}{\partial e_i} \neq 0 \\ 0, \text{if } \dfrac{\partial V_A}{\partial e_i} = 0, & i = 1,\ldots,n, \end{cases} \tag{3.5}$$

where $V_A = (1/2)e^Te$, $f_A = -A_2y - B_2f_2(y) + A_1x + B_1f_1(x)$ and $K_{Ai} > 0$. And $\alpha_e = [\,\alpha_{e1} \ldots \alpha_{en}\,]^T$ and α_{ei} is given in

$$\alpha_{ei} = \begin{cases} \dfrac{1}{e_i}, & \text{if } |e_i| \geq e_\sigma \\ \dfrac{\operatorname{sgn}(e_i)}{e_\sigma}, & \text{if } |e_i| < e_\sigma, \end{cases} \tag{3.6}$$

where $e_\sigma > 0$ is a small constant.

Substituting (3.4) into (2.4) gives

$$\begin{aligned}
\dot{e} = & -K(I+K_D)^{-1}e - \begin{bmatrix} \sum_{i=1}^{N_{A21}} (1+k_{d1})^{-1}\delta\tilde{a}_{21i}\overline{y}_{1i} \\ \vdots \\ \sum_{i=1}^{N_{A2n}} (1+k_{dn})^{-1}\delta\tilde{a}_{2ni}\overline{y}_{ni} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{N_{B21}} (1+k_{d1})^{-1}\delta\tilde{b}_{21i}\overline{f}_{21i} \\ \vdots \\ \sum_{i=1}^{N_{B2n}} (1+k_{dn})^{-1}\delta\tilde{b}_{2ni}\overline{f}_{2ni} \end{bmatrix} \\
& + f_A + \alpha_s + \begin{bmatrix} \sum_{i=1}^{N_{A11}} (1+k_{d1})^{-1}\delta\tilde{a}_{11i}\overline{x}_{1i} \\ \vdots \\ \sum_{i=1}^{N_{A1n}} (1+k_{dn})^{-1}\delta\tilde{a}_{1ni}\overline{x}_{ni} \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{N_{B11}} (1+k_{d1})^{-1}\delta\tilde{b}_{11i}\overline{f}_{11i} \\ \vdots \\ \sum_{i=1}^{N_{B1n}} (1+k_{dn})^{-1}\delta\tilde{b}_{1ni}\overline{f}_{1ni} \end{bmatrix} \\
& - \mu(I+K_D)^{-1}\left[\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}} \left(\left|\delta\hat{a}_{2ji}\right| + d_{a2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}} \left(\left|\delta\hat{a}_{1ji}\right| + d_{a1ji}\right)^{1+\alpha}\right. \\
& \qquad \left. + \sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}} \left(\left|\delta\hat{b}_{2ji}\right| + d_{b2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}} \left(\left|\delta\hat{b}_{1ji}\right| + d_{b1ji}\right)^{1+\alpha}\right]\alpha_e \\
& - (I+K_D)^{-1}\left[c_1\,\mathrm{sgn}(e_1)|e_1|^{\alpha}, \ldots, c_n\,\mathrm{sgn}(e_n)|e_n|^{\alpha}\right]^T.
\end{aligned} \tag{3.7}$$

Case 1 ($|e_i| \geq e_\sigma$). Substituting (3.7) into (3.3) yields

$$\begin{aligned}
\dot{V} \leq & -e^T K(I+K_D)^{-1}e - \sum_{i=1}^{n} c_i(1+k_{di})^{-1}|e_i|^{1+\alpha} \\
& - \mu k_d\left[\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}} \left(\left|\delta\hat{a}_{2ji}\right| + d_{a2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}} \left(\left|\delta\hat{a}_{1ji}\right| + d_{a1ji}\right)^{1+\alpha}\right. \\
& \qquad \left. + \sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}} \left(\left|\delta\hat{b}_{2ji}\right| + d_{b2ji}\right)^{1+\alpha} + \sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}} \left(\left|\delta\hat{b}_{1ji}\right| + d_{b1ji}\right)^{1+\alpha}\right] \\
& - \sum_{j=1}^{n} e_j \sum_{i=1}^{N_{A2j}} (1+k_{dj})^{-1}\delta\tilde{a}_{2ji}\overline{y}_{ji} + \sum_{j=1}^{n} e_j \sum_{i=1}^{N_{A1j}} (1+k_{dj})^{-1}\delta\tilde{a}_{1ji}\overline{x}_{ji} \\
& - \sum_{j=1}^{n} e_j \sum_{i=1}^{N_{B2j}} (1+k_{dj})^{-1}\delta\tilde{b}_{2ji}\overline{f}_{2ji} + \sum_{j=1}^{n} e_j \sum_{i=1}^{N_{B1j}} (1+k_{dj})^{-1}\delta\tilde{b}_{1ji}\overline{f}_{1ji}
\end{aligned}$$

$$+\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}}(1+k_{dj})^{-1}\gamma_{a2ji}^{-1}\delta\tilde{a}_{2ji}\delta\dot{\hat{a}}_{2ji}+\sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}}(1+k_{dj})^{-1}\gamma_{a1ji}^{-1}\delta\tilde{a}_{1ji}\delta\dot{\hat{a}}_{1ji}$$
$$+\sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}}(1+k_{dj})^{-1}\gamma_{b2ji}^{-1}\delta\tilde{b}_{2ji}\delta\dot{\hat{b}}_{2ji}+\sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}}(1+k_{dj})^{-1}\gamma_{b1ji}^{-1}\delta\tilde{b}_{1ji}\delta\dot{\hat{b}}_{1ji}, \tag{3.8}$$

where $k_d=\min\{(1+k_{d1})^{-1},(1+k_{d2})^{-1},\ldots,(1+k_{dn})^{-1}\}$. Choosing the updating law as

$$\begin{aligned}
\delta\dot{\hat{a}}_{2ji}&=\begin{cases}\gamma_{a2ji}e_j\overline{y}_{ji}, & \text{if } |\hat{a}_{2ji}|<d_{a2ji}\\ 0, & \text{otherwise,}\end{cases}\\
\delta\dot{\hat{a}}_{1ji}&=\begin{cases}-\gamma_{a1ji}e_j\overline{x}_{ji}, & \text{if } |\hat{a}_{1ji}|<d_{a1ji}\\ 0, & \text{otherwise,}\end{cases}\\
\delta\dot{\hat{b}}_{2ji}&=\begin{cases}\gamma_{b2ji}e_j\overline{f}_{2ji}, & \text{if } |\hat{b}_{2ji}|<d_{b2ji}\\ 0, & \text{otherwise,}\end{cases}\\
\delta\dot{\hat{b}}_{1ji}&=\begin{cases}-\gamma_{b1ji}e_j\overline{f}_{1ji}, & \text{if } |\hat{b}_{1ji}|<d_{b1ji}\\ 0, & \text{otherwise.}\end{cases}
\end{aligned} \tag{3.9}$$

Substituting (3.9) into (3.8) yields

$$\begin{aligned}
\dot{V}\le&-e^TK(I+K_D)^{-1}e-\sum_{i=1}^{n}c_i(1+k_{di})^{-1}|e_i|^{1+\alpha}\\
&-\mu k_d\left[\sum_{j=1}^{n}\sum_{i=1}^{N_{A2j}}(|\delta\hat{a}_{2ji}|+d_{a2ji})^{1+\alpha}+\sum_{j=1}^{n}\sum_{i=1}^{N_{A1j}}(|\delta\hat{a}_{1ji}|+d_{a1ji})^{1+\alpha}\right.\\
&\left.+\sum_{j=1}^{n}\sum_{i=1}^{N_{B2j}}\left(\left|\delta\hat{b}_{2ji}\right|+d_{b2ji}\right)^{1+\alpha}+\sum_{j=1}^{n}\sum_{i=1}^{N_{B1j}}\left(\left|\delta\hat{b}_{1ji}\right|+d_{b1ji}\right)^{1+\alpha}\right].
\end{aligned} \tag{3.10}$$

Since

$$\begin{aligned}
|\delta\hat{a}_{2ji}-\delta a_{2ji}|&\le|\delta\hat{a}_{2ji}|+|\delta a_{2ji}|\le|\delta\hat{a}_{2ji}|+d_{a2ji},\\
|\delta\hat{a}_{1ji}-\delta a_{1ji}|&\le|\delta\hat{a}_{1ji}|+|\delta a_{1ji}|\le|\delta\hat{a}_{1ji}|+d_{a1ji},\\
\left|\delta\hat{b}_{2ji}-\delta b_{2ji}\right|&\le\left|\delta\hat{b}_{2ji}\right|+|\delta b_{2ji}|\le\left|\delta\hat{b}_{2ji}\right|+d_{b2ji},\\
\left|\delta\hat{b}_{1ji}-\delta b_{1ji}\right|&\le\left|\delta\hat{b}_{1ji}\right|+|\delta b_{1ji}|\le\left|\delta\hat{b}_{1ji}\right|+d_{b1ji}
\end{aligned} \tag{3.11}$$

hold, one can conclude that $-(|\delta\hat{a}_{2ji}| + d_{a2ji})^{1+\alpha} \leq -|\delta\hat{a}_{2ji} - \delta a_{2ji}|^{1+\alpha}$,

$$\begin{aligned} -\left(\left|\delta\hat{a}_{1ji}\right| + d_{a1ji}\right)^{1+\alpha} &\leq -\left|\delta\hat{a}_{1ji} - \delta a_{1ji}\right|^{1+\alpha}, \\ -\left(\left|\delta\hat{b}_{2ji}\right| + d_{b2ji}\right)^{1+\alpha} &\leq -\left|\delta\hat{b}_{2ji} - \delta b_{2ji}\right|^{1+\alpha}, \end{aligned} \tag{3.12}$$

and $-(|\delta\hat{b}_{1ji}| + d_{b1ji})^{1+\alpha} \leq -|\delta\hat{b}_{1ji} - \delta b_{1ji}|^{1+\alpha}$. Therefore, the inequality (3.10) can be rewritten as

$$\begin{aligned} \dot{V} \leq & -e^T K(I + K_D)^{-1} e - \sum_{i=1}^{n} c_i (1 + k_{di})^{-1} |e_i|^{1+\alpha} \\ & - \mu k_d \left[\sum_{j=1}^{n} \sum_{i=1}^{N_{A2j}} \left|\delta\hat{a}_{2ji} - \delta a_{2ji}\right|^{1+\alpha} + \sum_{j=1}^{n} \sum_{i=1}^{N_{A1j}} \left|\delta\hat{a}_{1ji} - \delta a_{1ji}\right|^{1+\alpha} \right. \\ & \left. + \sum_{j=1}^{n} \sum_{i=1}^{N_{B2j}} \left|\delta\hat{b}_{2ji} - \delta b_{2ji}\right|^{1+\alpha} + \sum_{j=1}^{n} \sum_{i=1}^{N_{B1j}} \left|\delta\hat{b}_{1ji} - \delta b_{1ji}\right|^{1+\alpha} \right] \\ \leq & -c_\mu \left[\sum_{i=1}^{n} |e_i|^2 + \sum_{j=1}^{n} \sum_{i=1}^{N_{A2j}} \left|\delta\hat{a}_{2ji} - \delta a_{2ji}\right|^2 + \sum_{j=1}^{n} \sum_{i=1}^{N_{A1j}} \left|\delta\hat{a}_{1ji} - \delta a_{1ji}\right|^2 \right. \\ & \left. + \sum_{j=1}^{n} \sum_{i=1}^{N_{B2j}} \left|\delta\hat{b}_{2ji} - \delta b_{2ji}\right|^2 + \sum_{j=1}^{n} \sum_{i=1}^{N_{B1j}} \left|\delta\hat{b}_{1ji} - \delta b_{1ji}\right|^2 \right]^{(1+\alpha)/2} \\ \leq & -c_\mu V^{(1+\alpha)/2}, \end{aligned} \tag{3.13}$$

where $c_\mu = \min\{c_i(1 + k_{di})^{-1}, k_i(1 + k_{di})^{-1}, \mu k_d, i = 1, \ldots, n\}$. According to Lemma 2.3, $e \to B_{e_\sigma}$ in a finite time, where $B_{e_\sigma} \triangleq \{e \| e_i| \leq e_\sigma, i = 1, \ldots, n\}$.

Case 2 ($|e_i| < e_\sigma$). Using (3.3)–(3.7) and (3.9), it is easy to show that

$$\dot{V} \leq -e^T K(I + K_D)^{-1} e - \sum_{i=1}^{n} c_i (1 + k_{di})^{-1} |e_i|^{1+\alpha} \tag{3.14}$$

holds. According to Barbalat's lemma [37], we can conclude that $e \to 0$ as $t \to \infty$.

From the discussion above, we have the following result.

Theorem 3.1. *For the systems* (2.1) *and* (2.2), *under Assumption 2.1, if the control law is designed as* (3.4), *updating laws are chosen as* (3.9), *then e will converge to* B_{e_σ} *in finite time,* $e \to 0$ *as* $t \to \infty$, *and* $\delta\tilde{a}_{2ji}$, $\delta\tilde{a}_{1ji}$, $\delta\tilde{b}_{2ji}$, *and* $\delta\tilde{b}_{1ji}$ *remain bounded.*

Remark 3.2. Since the control signal (3.4) contains the discontinuous sign functions, as a hard switcher, it may cause undesirable chattering. In order to avoid the chattering, the "sgn" function can be replaced by a continuous function (tanh) to remove discontinuity.

4. Numerical Simulation

In this section, we present numerical results to verify the proposed synchronization approach.

Consider the following master chaotic system:

$$\begin{aligned}
\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} &= \begin{bmatrix}a(x_2-x_1)\\ bx_1-cx_1x_3\\ -gx_3+hx_1^2\end{bmatrix} = \begin{bmatrix}-a & a & 0\\ b & 0 & 0\\ 0 & 0 & -g\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ 0 & -c & 0\\ 0 & 0 & h\end{bmatrix}\begin{bmatrix}0\\ x_1x_3\\ x_1^2\end{bmatrix}\\
&= \underbrace{\begin{bmatrix}-a_0 & a_0 & 0\\ b_0 & 0 & 0\\ 0 & 0 & -g_0\end{bmatrix}}_{A_1}\underbrace{\begin{bmatrix}x_1\\ x_2\\ x_2\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}-\delta a_0 & \delta a_0 & 0\\ \delta b_0 & 0 & 0\\ 0 & 0 & -\delta g_0\end{bmatrix}}_{\Delta A_1}\underbrace{\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}}_{x}\\
&\quad + \underbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & -c_0 & 0\\ 0 & 0 & h_0\end{bmatrix}}_{B_1}\underbrace{\begin{bmatrix}0\\ x_1x_3\\ x_1^2\end{bmatrix}}_{f_1(x)} + \underbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & -\delta c_0 & 0\\ 0 & 0 & \delta h_0\end{bmatrix}}_{\Delta B_1}\underbrace{\begin{bmatrix}0\\ x_1x_3\\ x_1^2\end{bmatrix}}_{f_1(x)},
\end{aligned} \tag{4.1}$$

where $a = a_0 + \delta a_0$, $b = b_0 + \delta b_0$, $c = c_0 + \delta c_0$, $g = g_0 + \delta g_0$, $h = h_0 + hg_0$, $a_0 = 8$, $\delta a_0 = 2$, $b_0 = 35$, $\delta b_0 = 5$, $c_0 = 0.7$, $\delta c_0 = 0.3$, $g_0 = 2.0$, $\delta g_0 = 0.5$, $h_0 = 0.8$, and $\delta h_0 = 0.2$.

The slave system is given with

$$\begin{aligned}
\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{bmatrix} &= \begin{bmatrix}a_1(y_2-y_1+y_2y_3)\\ b_1y_2-c_1y_1y_3\\ g_1y_2-h_1y_3\end{bmatrix} = \begin{bmatrix}-a_1 & a_1 & 0\\ 0 & b_1 & 0\\ 0 & g_1 & -h_1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} + \begin{bmatrix}a_1 & 0 & 0\\ 0 & -c_1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}y_2y_3\\ y_1y_3\\ 0\end{bmatrix}\\
&= \underbrace{\begin{bmatrix}-a_{10} & a_{10} & 0\\ 0 & b_{10} & 0\\ 0 & g_{10} & -h_{10}\end{bmatrix}}_{A_2}\underbrace{\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}}_{y} + \underbrace{\begin{bmatrix}-\delta a_{10} & \delta a_{10} & 0\\ 0 & \delta b_{10} & 0\\ 0 & \delta g_{10} & -\delta h_{10}\end{bmatrix}}_{\Delta A_2}\underbrace{\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}}_{y}\\
&\quad + \underbrace{\begin{bmatrix}a_{10} & 0 & 0\\ 0 & -c_{10} & 0\\ 0 & 0 & 0\end{bmatrix}}_{B_2}\underbrace{\begin{bmatrix}y_2y_3\\ y_1y_3\\ 0\end{bmatrix}}_{f_2(y)} + \underbrace{\begin{bmatrix}\delta a_{10} & 0 & 0\\ 0 & -\delta c_{10} & 0\\ 0 & 0 & 0\end{bmatrix}}_{\Delta B_2}\underbrace{\begin{bmatrix}y_2y_3\\ y_1y_3\\ 0\end{bmatrix}}_{f_2(y)} + \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix},
\end{aligned} \tag{4.2}$$

where $a_1 = a_{10} + \delta a_{10}$, $b_1 = b_{10} + \delta b_{10}$, $c_1 = c_{10} + \delta c_{10}$, $g_1 = g_{10} + \delta g_{10}$, $h_1 = h_{10} + hg_{10}$, $a_{10} = 0.8$, $\delta a_{10} = 0.2$, $b_{10} = 2.0$, $\delta b_{10} = 0.5$, $c_{10} = 0.7$, $\delta c_{10} = 0.3$, $g_{10} = 0.7$, $\delta g_{10} = 0.3$, $h_{10} = 3.0$, $\delta h_{10} = 1.0$. $x(0) = 1.8$, $y(0) = -1.2$, and $z(0) = 1.5$.

The initial states in master system (4.1) are $x_1(0) = 1.8$, $x_2(0) = -1.2$, $x_3(0) = 1.5$. The initial states in slave system (4.2) are $y_1(0) = 1.5$, $y_2(0) = 1.2$, $y_3(0) = 1.1$. The initial parameter estimation values of the systems (2.1) and (2.2) are $\delta\hat{a}_0 = 0$, $\delta\hat{b}_0 = 0$, $\delta\hat{c}_0 = 0$, $\delta\hat{g}_0 = 0$, $\delta\hat{h}_0 = 0$, $\delta\hat{a}_{10} = 0$, $\delta\hat{b}_{10} = 0$, $\delta\hat{c}_{10} = 0$, $\delta\hat{g}_{10} = 0$, and $\delta\hat{h}_{10} = 0$.

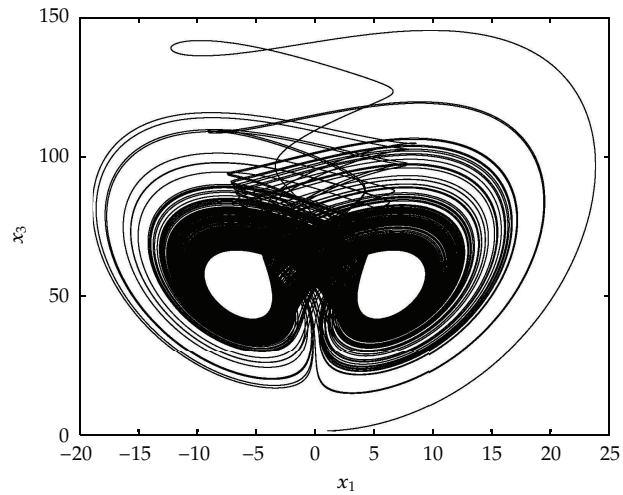

Figure 1: Chaotic behavior of the master chaotic system under the proposed parameters.

According to Remark 3.2, the control law (3.4) is modified as follows:

$$\begin{aligned} u = & -Ke - K_D\dot{e} + \alpha_s - C_{A2} - C_{B2} + C_{A1} + C_{B1} \\ & - \mu\Big[2(|\delta\hat{a}_0| + d_{a0})^{1+\alpha} + \left(\left|\delta\hat{b}_0\right| + d_{b0}\right)^{1+\alpha} + (|\delta\hat{c}_0| + d_{c0})^{1+\alpha} \\ & \quad + \left(\left|\delta\hat{g}_0\right| + d_{g0}\right)^{1+\alpha} + \left(\left|\delta\hat{h}_0\right| + d_{h0}\right)^{1+\alpha} + 3(|\delta\hat{a}_{10}| + d_{a10})^{1+\alpha} + \left(\left|\delta\hat{b}_{10}\right| + d_{b10}\right)^{1+\alpha} \\ & \quad + (|\delta\hat{c}_{10}| + d_{c10})^{1+\alpha} + \left(\left|\delta\hat{g}_{10}\right| + d_{g10}\right)^{1+\alpha} + \left(\left|\delta\hat{h}_{10}\right| + d_{h10}\right)^{1+\alpha}\Big]\alpha_e \\ & - \left[c_1 \tanh(\varepsilon e_1)|e_1|^{\alpha}, \ldots, c_n \tanh(\varepsilon e_n)|e_n|^{\alpha}\right]^T, \end{aligned} \tag{4.3}$$

where

$$\begin{gathered} C_{A2} = \left[\delta\hat{a}_{10}(y_2 - y_1), \delta\hat{b}_{10}y_2, \delta\hat{g}_{10}y_2 - \delta\hat{h}_{10}y_3\right]^T, \\ C_{B2} = \left[\delta\hat{a}_{10}y_2y_3, -\delta\hat{c}_{10}y_1y_3, 0\right]^T, \qquad C_{A1} = \left[\delta\hat{a}_0(x_2 - x_1), \delta\hat{b}_0x_1, -\delta\hat{g}_0x_3\right]^T, \\ C_{B1} = \left[0, -\delta\hat{c}_0x_1x_3, \delta\hat{h}_0x_1^2\right]^T, \qquad K = \operatorname{diag}\{70, 54, 30\}, \\ K_D = \operatorname{diag}\{0.93, 0.75, 0.1\}, \qquad \mu = \operatorname{diag}\{1, 0.2, 0.01\}, \\ K_A = \operatorname{diag}\{2.3, 2.1, 2.3\} d_{a0} = 2, \quad d_{b0} = 10, \quad d_{c0} = 1, \quad d_{g0} = 1, \quad d_{h0} = 2, \\ d_{a10} = 1, \quad d_{b10} = 2, \quad d_{c10} = 2, \quad d_{g10} = 2, \quad d_{h10} = 2, \quad \varepsilon = 40, \quad c_1 = 1, \quad c_2 = 3, \ c_3 = 1. \end{gathered} \tag{4.4}$$

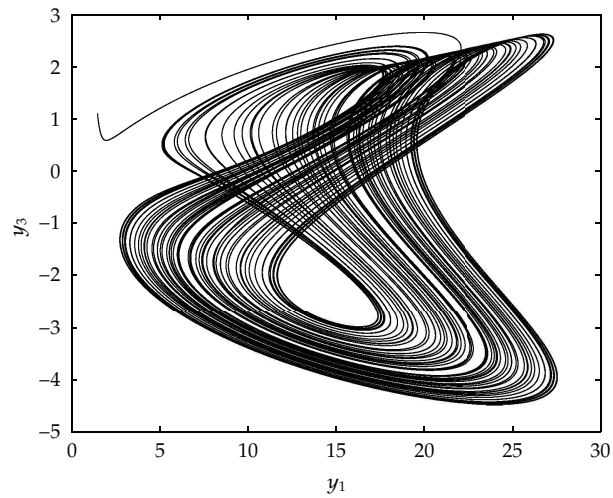

Figure 2: Chaotic behavior of the slave chaotic system under the proposed parameters.

Choosing the updating law as

$$\delta\dot{\hat{a}}_0 = \begin{cases} 0.30e_1(x_1 - x_2), & \text{if } |\hat{a}_0| < d_{a0} \\ 0, & \text{otherwise} \end{cases}, \qquad \delta\dot{\hat{b}}_0 = \begin{cases} -0.90e_2x_1, & \text{if } \left|\hat{b}_0\right| < d_{b0} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta\dot{\hat{c}}_0 = \begin{cases} -0.001e_2x_1x_3, & \text{if } |\hat{c}_0| < d_{c0} \\ 0, & \text{otherwise} \end{cases}, \qquad \delta\dot{\hat{g}}_0 = \begin{cases} 0.002e_3x_3, & \text{if } \left|\hat{g}_0\right| < d_{g0} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta\dot{\hat{h}}_0 = \begin{cases} -0.018e_3x_1^2, & \text{if } \left|\hat{h}_0\right| < d_{h0} \\ 0, & \text{otherwise} \end{cases}, \qquad \delta\dot{\hat{a}}_{10} = \begin{cases} 0.0006e_1(-y_1 + y_2 + y_2y_3), & \text{if } |\hat{a}_{10}| < d_{a10} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta\dot{\hat{b}}_{10} = \begin{cases} -0.08e_2y_2, & \text{if } \left|\hat{b}_{10}\right| < d_{b10} \\ 0, & \text{otherwise} \end{cases}, \qquad \delta\dot{\hat{c}}_{10} = \begin{cases} -0.0015e_2y_1y_3, & \text{if } |\hat{c}_{10}| < d_{c10} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta\dot{\hat{g}}_0 = \begin{cases} 0.1e_3y_2, & \text{if } \left|\hat{g}_{10}\right| < d_{g10} \\ 0, & \text{otherwise} \end{cases}, \qquad \delta\dot{\hat{h}}_{10} = \begin{cases} -0.017e_3y_3, & \text{if } \left|\hat{h}_{10}\right| < d_{h10} \\ 0, & \text{otherwise.} \end{cases} \tag{4.5}$$

Chaotic behavior of the master chaotic system under the proposed parameters is shown in Figure 1. Chaotic behavior of the slave chaotic system under the proposed parameters is shown in Figure 2. From Figures 1 and 2, we know that the two systems are still chaotic under adopted uncertain parameters. The synchronization errors between two different chaotic systems are illustrated in Figures 3, 4, and 5, where the control inputs are activated at $t = 1$s. One can see that the synchronization errors converge to the zero in a finite time, which implies that the chaos synchronization between the two different chaotic systems

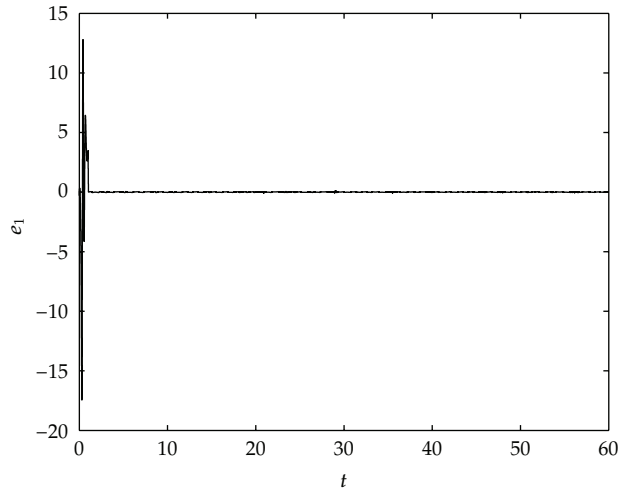

Figure 3: Synchronization error e_1.

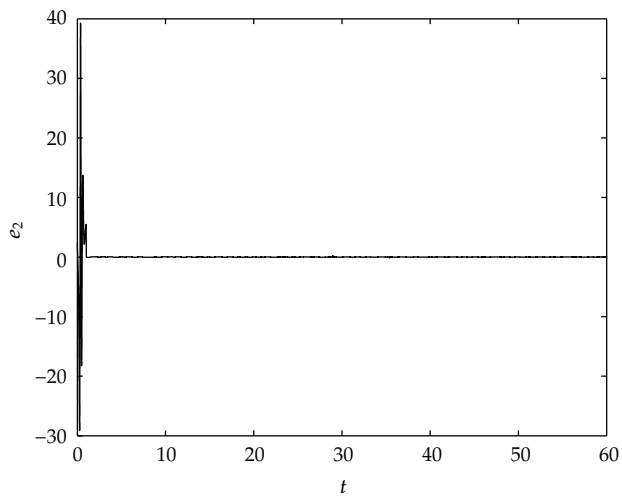

Figure 4: Synchronization error e_2.

is realized. The time responses of parameter estimations $\widehat{a}_0$, $\widehat{b}_0$, $\widehat{c}_0$, $\widehat{g}_0$, and $\widehat{h}_0$ are depicted in Figure 6. The time responses of parameter estimations $\widehat{a}_{10}$, $\widehat{b}_{10}$, $\widehat{c}_{10}$, $\widehat{g}_{10}$, and $\widehat{h}_{10}$ are depicted in Figure 7.

According to the simulations, it has been shown that the proposed control algorithm provides stable behavior when using online adaptive laws. The control performance is satisfactory and the chattering phenomenon has been successfully improved by using tanh functions. In addition, it is easy to see that the parameter estimation values approach their real values in a finite time.

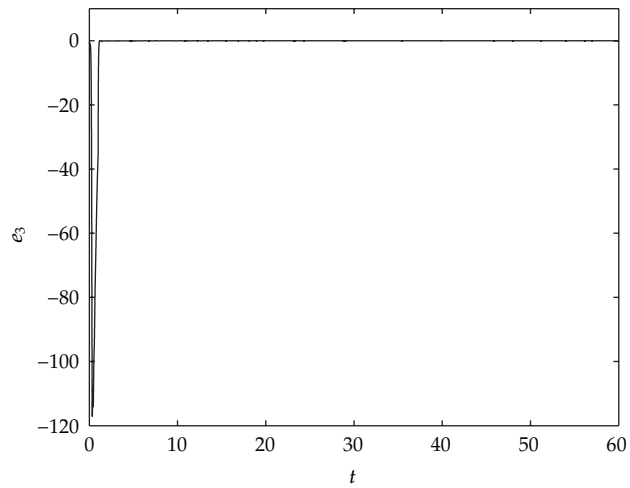

Figure 5: Synchronization error e_3.

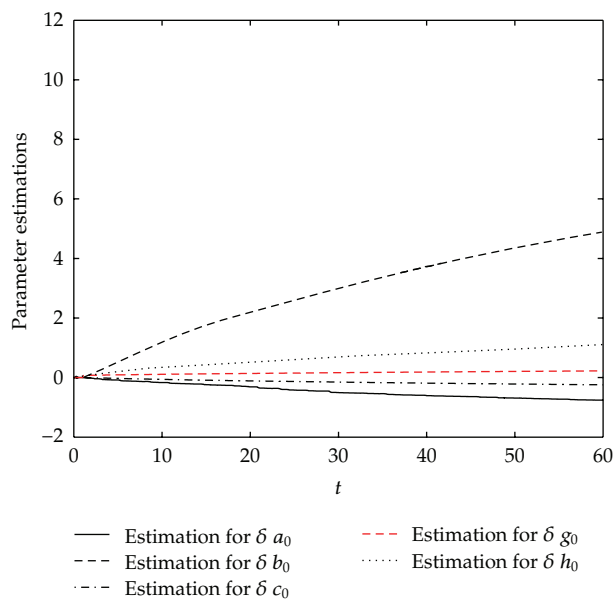

Figure 6: Curves of parameter estimations for the master system.

5. Conclusions

In this paper, we have studied chaos synchronization of two different chaotic systems with parameter uncertainties. The two different chaotic systems with parameter uncertainties are synchronized via robust adaptive control based on the Lyapunov stability theory and finite-time theory. The proposed method can be applied to a variety of chaos systems. It guarantees that all the error states are driven to zero in a finite time. Numerical simulations are given to show the proposed synchronization approach works well for synchronizing two different

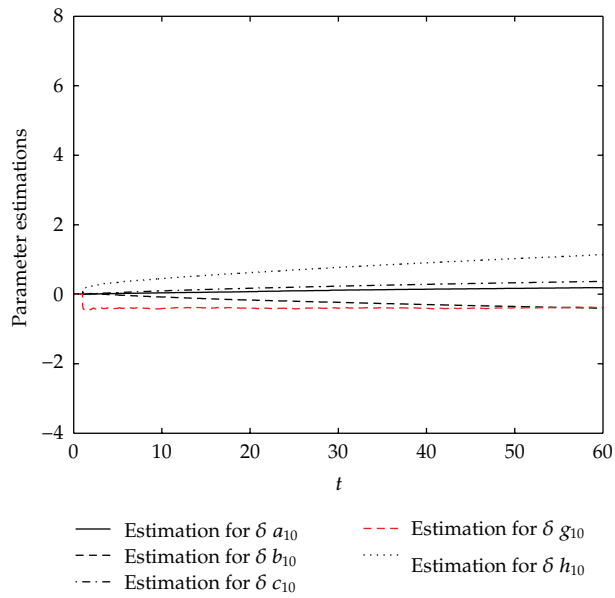

Figure 7: Curves of parameter estimations for the slave system.

chaotic systems in a finite time, even when the parameters of both the master and slave systems are unknown.

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