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Research Article

Finite-Time H_∞ Filtering for Singular Stochastic Systems

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This paper addresses the problem of finite-time H_∞ filtering for one family of singular stochastic systems with parametric uncertainties and time-varying norm-bounded disturbance. Initially, the definitions of singular stochastic finite-time boundedness and singular stochastic H_∞ finite-time boundedness are presented. Then, the H_∞ filtering is designed for the class of singular stochastic systems with or without uncertain parameters to ensure singular stochastic finite-time boundedness of the filtering error system and satisfy a prescribed H_∞ performance level in some given finite-time interval. Furthermore, sufficient criteria are presented for the solvability of the filtering problems by employing the linear matrix inequality technique. Finally, numerical examples are given to illustrate the validity of the proposed methodology.

1. Introduction

Singular systems also referred to as descriptor systems or generalized state-space systems represent one family of dynamical systems since it generalizes the linear system model and has extensive applications in economics systems, power systems, mechanics systems, chemical processes, and so on; see for more practical examples [1, 2] and the references therein. Many control results in state-space systems have been extended to singular systems, such as stability, stabilization, H_∞ control, and the filtering problems, for instance, see [3–6] and the references therein. Meanwhile, Markovian jump systems are referred to as one special family of hybrid systems and stochastic systems, which are very appropriate to model plants whose structure is subject to random abrupt changes, see the reference [7]. Thus, many attracting results have been studied, such as stochastic stability and stabilization [8, 9], robust control [10–12], guaranteed cost control [13], and other issues. For more details, the readers may be refered to [7, 14] and the references therein. Recently, the problem of state estimation for singular Markovian jump systems has also attracted considerable attention. As far as we know, the traditional Kalman filtering requires the exact knowledge of statistics of the noise

signals. To overcome the limitations regarding the system uncertainties and the statistical properties, the H_∞ filtering problem has been proposed and tackled for both the continuous-time case and the discrete-time one including without or with time-delay and full-order or reduced-order [15–20]. For more details, we refer the readers to [7, 21] and the references therein.

On the other hand, in many practical processes, many concerned problems are the practical ones which described that system state does not exceed some bound during some time interval. Compared with classical Lyapunov asymptotical stability, in order to deal with the transient performance of control systems, finite-time stability or short-time stability was introduced in [22]. Employing linear matrix inequality (LMI) theory and Lyapunov function approach, some appealing results were obtained to ensure finite-time stability, finite-time boundedness, and finite-time stabilization of various systems including linear systems, nonlinear systems, and stochastic systems. For instance, Amato et al. [23] investigated the output feedback finite-time stabilization for continuous linear system. Zhang and An [24] considered finite-time control problems for linear stochastic system. For more details of the literature related to finite-time stability, the reader is referred to [25–34] and the references therein. However, to date and to the best of our knowledge, the H_∞ filtering problem for singular stochastic systems has not investigated in finite-time interval. The problem is important and challenging in many practice applications, which motivates us for this study.

This paper deals with the problem of finite-time H_∞ filtering for one family of singular stochastic systems with parametric uncertainties and time-varying norm-bounded disturbance. Our results are totally different from those previous results, although some studies on H_∞ filtering and finite-time stability for singular stochastic systems have been addressed, see [19–21, 31, 32, 35]. The main aim of this paper is to design an H_∞ filtering which guarantees the filtering error system singular stochastic finite-time boundedness and satisfies a prescribed H_∞ performance level in the given finite-time interval. Sufficient criteria are presented for the solvability of the filtering problems by applying the LMI technique. Finally, simulation examples are presented to demonstrate the validity of the developed theoretical results.

Notations. Throughout the paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the sets of n component real vectors and $n \times m$ real matrices, respectively. The superscript T stands for matrix transposition or vector. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respective to some probability measure $\mathbb{P}$. In addition, the symbol $*$ denotes the term that is induced by symmetry and $\operatorname{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the smallest and the largest eigenvalue of matrix P, respectively. Notations sup. and inf. denote the supremum and infimum, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

In this paper, let us consider the dynamics of continuous-time singular system with Markovian jumps:

$$\begin{aligned} E\dot{x}(t) &= [A(r_t) + \Delta A(r_t)]x(t) + [B(r_t) + \Delta B(r_t)]w(t), \\ y(t) &= C(r_t)x(t) + D(r_t)w(t), \\ z(t) &= L(r_t)x(t) + G(r_t)w(t), \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $y(t) \in \mathbb{R}^{q_1}$ is the measurement output of the system, $z(t) \in \mathbb{R}^{q_2}$ is the signal to be estimated, and E is a singular matrix with $\operatorname{rank}(E) = r < n$; $\{r_t, t \geq 0\}$ is continuous-time Markov stochastic process taking values in a finite space $\mathbb{M} := \{1, 2, \ldots, N\}$ with transition matrix $\Gamma = (\pi_{ij})_{N\times N}$, and the transition probabilities are described as follows:

$$P_{ij} = Pr(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if} \quad i \neq j, \\ 1 + \pi_{ij}\Delta + o(\Delta), & \text{if} \quad i = j, \end{cases} \tag{2.2}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, π_{ij} satisfies $\pi_{ij} \geq 0$ $(i \neq j)$, and $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$ for all $i, j \in \mathbb{M}$; $\Delta A(r_t)$ and $\Delta B(r_t)$ are uncertain matrices and satisfy

$$[\Delta A(r_t), \Delta B(r_t)] = F(r_t)\Delta(r_t)[E_1(r_t), E_2(r_t)], \tag{2.3}$$

where $\Delta(r_t)$ is an unknown, time-varying matrix function and satisfies $\Delta^T(r_t)\Delta(r_t) \leq I$ for all $r_t \in \mathbb{M}$; moreover, the disturbance input $w(t) \in \mathbb{R}^p$ satisfies

$$\int_0^T w^T(t)w(t)dt \leq d^2, \quad d \geq 0, \tag{2.4}$$

and the matrices $A(r_t), B(r_t), C(r_t), D(r_t), L(r_t)$, and $G(r_t)$ are coefficient matrices and of appropriate dimension for all $r_t \in \mathbb{M}$.

In this paper, we construct the following full-order filter:

$$\begin{aligned} E_f\dot{\tilde{x}}(t) &= A_f(r_t)\tilde{x}(t) + B_f(r_t)y(t), \\ \tilde{z}(t) &= C_f(r_t)\tilde{x}(t), \end{aligned} \tag{2.5}$$

where $\tilde{x}(t) \in \mathbb{R}^n$ is the filter state, $\tilde{z}(t) \in \mathbb{R}^{q_2}$ is the filter output, and $E_f, A_f(r_t), B_f(r_t)$, and $C_f(r_t)$ are to design the filter matrices with appropriate dimensions.

Define $\overline{x}(t) = [x^T(t) \;\; x^T(t) - \tilde{x}^T(t)]^T$, $e(t) = z(t) - \tilde{z}(t)$ and combining (2.1) and (2.5), one can obtain the following filtering error dynamics as follows:

$$\begin{aligned} \overline{E}\dot{\overline{x}}(t) &= \overline{A}(r_t)\overline{x}(t) + \overline{B}(r_t)w(t), \\ e(t) &= \overline{L}(r_t)\overline{x}(t) + G(r_t)w(t), \end{aligned} \tag{2.6}$$

where

$$\begin{gathered} \overline{E} = \begin{bmatrix} E & 0 \\ E - E_f & E_f \end{bmatrix}, \qquad \overline{A}(r_t) = \begin{bmatrix} A(r_t) + \Delta A(r_t) & 0 \\ A(r_t) + \Delta A(r_t) - A_f(r_t) - B_f(r_t)C(r_t) & A_f(r_t) \end{bmatrix}, \\ \overline{B}(r_t) = \begin{bmatrix} B(r_t) + \Delta B(r_t) \\ B(r_t) + \Delta B(r_t) - B_f(r_t)D(r_t) \end{bmatrix}, \qquad \overline{L}(r_t) = \left[L(r_t) - C_f(r_t) \;\; C_f(r_t)\right]. \end{gathered} \tag{2.7}$$

For notational simplicity, in the sequel, for each possible $r_t = i,\ i \in \mathbb{M}$, a matrix $K(r_t)$ will be denoted by K_i; for instance, $A(r_t)$ will be denoted by A_i, $B(r_t)$ by B_i, and so on.

Throughout the paper, we need the following definitions and lemmas.

Definition 2.1 (regular and impulse free, see [21]). (i) The singular system with Markovian jumps (2.1) is said to be regular in time interval $[0,T]$ if the characteristic polynomial $\det(sE - A_i - \Delta A_i)$ is not identically zero for all $t \in [0,T]$.

(ii) The singular systems with Markovian jumps (2.1) is said to be impulse free in time interval $[0,T]$, if $\deg(\det(sE - A_i - \Delta A_i)) = \operatorname{rank}(E)$ for all $t \in [0,T]$.

Definition 2.2 (singular stochastic finite-time boundedness (SSFTB)). The singular system with Markovian jumps (2.6) which satisfies (2.4) is said to be SSFTB with respect to $(c_1, c_2, T,\ \overline{R}_i, d)$, with $c_1 < c_2$, $\overline{R}_i > 0$, if the stochastic system (2.6) is regular and impulse free in time interval $[0,T]$ and satisfies

$$\mathbb{E}\left\{\overline{x}^T(0)\overline{E}^T\overline{R}_i\overline{E}\overline{x}(0)\right\} \le c_1^2 \Longrightarrow \mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{R}_i\overline{E}\overline{x}(t)\right\} < c_2^2, \quad \forall t \in [0,T]. \tag{2.8}$$

Remark 2.3. SSFTB implies that not only is dynamical mode of the filtering error system finite-time bounded but also whole mode of the one is finite-time bounded since the static mode is regular and impulse free.

Definition 2.4 (singular stochastic H_∞ finite-time boundedness (SSH_∞FTB)). The singular system with Markovian jumps (2.6) is said to be SSH_∞FTB with respect to $(0, c_2, T,\ \overline{R}_i, \gamma, d)$, if the singular system with Markovian jumps (2.6) is SSFTB with respect to $(c_1, c_2, T,\ \overline{R}_i, d)$ and under the zero-initial condition, the output error $e(t)$ satisfies the cost constrained function

$$\mathbb{E}\left\{\int_0^T e^T(t)e(t)dt\right\} < \gamma^2 \int_0^T w^T(t)w(t)dt, \tag{2.9}$$

for any nonzero $w(t)$ which satisfies (2.4), where γ is a prescribed positive scalar.

Definition 2.5 (see [9]). Let $V(x(t), r_t = i,\ t > 0)$ be the stochastic function, define its weak infinitesimal operator L of stochastic process $\{(x(t), r_t = i), t \ge 0\}$ by

$$LV(x(t), r_t = i, t) = V_t(x(t), i, t) + V_x(x(t), i, t)\dot{x}(t, i) + \sum_{j=1}^{N} \pi_{ij} V\big(x(t), j, t\big). \tag{2.10}$$

Lemma 2.6 (see [36]). *For matrices Y, M, and N of appropriate dimensions, where Y is a symmetric matrix, then*

$$Y + MF(t)N + N^T F^T(t) M^T < 0 \tag{2.11}$$

holds for all matrix $F(t)$ satisfying $F^T(t)F(t) \le I$ for all $t \in \mathbb{R}$, if and only if there exists a positive constant ϵ, such that the following inequality:

$$Y + \epsilon^{-1} M M^T + \epsilon N^T N < 0 \tag{2.12}$$

holds.

Lemma 2.7 (see [36]). *The linear matrix inequality* $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$ *is equivalent to* $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$, *where* $S_{11} = S_{11}^T$ *and* $S_{22} = S_{22}^T$.

Lemma 2.8. *The following items are true.*

(i) *Assume that* $\operatorname{rank}(E) = r$, *there exist two orthogonal matrices* $\mathcal{U}$ *and* V *such that* E *has the decomposition as*

$$E = \mathcal{U}\begin{bmatrix} \Sigma_r & 0 \\ * & 0 \end{bmatrix} V^T = \mathcal{U}\begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix} \mathcal{V}^T, \tag{2.13}$$

where $\Sigma_r = \operatorname{diag}\{\delta_1, \delta_1, \ldots, \delta_r\}$ *with* $\delta_k > 0$ *for all* $k = 1, 2, \ldots, r$. *Partition* $\mathcal{U} = [\mathcal{U}_1 \;\; \mathcal{U}_2]$, $V = [V_1 \;\; V_2]$, *and* $\mathcal{V} = [V_1\Sigma_r \;\; V_2]$ *with* $EV_2 = 0$ *and* $\mathcal{U}_2^T E = 0$.

(ii) *If* P *satisfies*

$$E^T P = P^T E \geq 0, \tag{2.14}$$

then $\widetilde{P} = \mathcal{U}^T P \mathcal{V}^{-T}$ *with* $\mathcal{U}$ *and* $\mathcal{V}$ *satisfying* (2.13) *if and only if*

$$\widetilde{P} = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}, \tag{2.15}$$

with $P_{11} \geq 0 \in \mathbb{R}^{r\times r}$. *In addition, when* P *is nonsingular, one has* $P_{11} > 0$ *and* $\det(P_{22}) \neq 0$. *Furthermore,* P *satisfying* (2.14) *can be parameterized as*

$$P = \mathcal{U}X\mathcal{U}^T E + \mathcal{U}_2 Y \mathcal{V}^T, \tag{2.16}$$

where $X = \operatorname{diag}\{P_{11}, \Lambda\}$, $Y = [P_{21} \;\; P_{22}]$, *and* $\Lambda \in \mathbb{R}^{(n-r)\times(n-r)}$ *is an arbitrary parameter matrix.*

(iii) *If* P *is a nonsingular matrix,* R *and* Λ *are two symmetric positive definite matrices,* P *and* E *satisfy* (2.14), X *is a diagonal matrix from* (2.16), *and the following equality holds:*

$$E^T P = E^T R^{1/2} Q R^{1/2} E. \tag{2.17}$$

Then the symmetric positive definite matrix $Q = R^{-1/2}\mathcal{U}X\mathcal{U}^T R^{-1/2}$ *is a solution of* (2.17).

Proof. One only requires to prove that (ii) and (iii) hold. Let

$$\widetilde{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \tag{2.18}$$

Then by (2.13) and (2.14), it follows that condition $\widetilde{P} = U^T P \mathcal{V}^{-T}$ if and only if $P_{12} = 0$ and $P_{11} \geq 0 \in \mathbb{R}^{r\times r}$. In addition, when P is nonsingular, it follows that $P_{11} > 0$ and $\det(P_{22}) \neq 0$. Noting that (2.13) and U is an orthogonal matrix, thus we have

$$\begin{aligned} P &= U\begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}\mathcal{V}^T \\ &= \left(U\begin{bmatrix} P_{11} & 0 \\ * & \Lambda \end{bmatrix}U^T\right)\left(U\begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix}\mathcal{V}^T\right) + U\begin{bmatrix} 0 & 0 \\ P_{21} & P_{22} \end{bmatrix}\mathcal{V}^T \\ &= U\begin{bmatrix} P_{11} & 0 \\ * & \Lambda \end{bmatrix}U^T E + [U_1 \;\; U_2]\begin{bmatrix} 0 & 0 \\ P_{21} & P_{22} \end{bmatrix}\mathcal{V}^T \\ &= UXU^T E + U_2 Y \mathcal{V}^T, \end{aligned} \tag{2.19}$$

where $X = \operatorname{diag}\{P_{11}, \Lambda\}$, $Y = [P_{21} \;\; P_{22}]$ with a parameter matrix $\Lambda \in \mathbb{R}^{(n-r)\times(n-r)}$. Thus (ii) is true.

By (i) and (ii), noticing $U_2^T E = 0$ and $P = UXU^T E + U_2 Y \mathcal{V}^T$, we have

$$E^T P = E^T\left(UXU^T E + U_2 Y \mathcal{V}^T\right) = E^T UXU^T E. \tag{2.20}$$

Thus, $Q = R^{-1/2}UXU^T R^{-1/2}$ is a solution of (2.17). This completes the proof of the lemma. □

In the paper, our main objective is to concentrate on designing the filter of system (2.1) which guarantees the resulting filtering error dynamic system (2.6) SSH_∞FTB.

3. Main Results

In this section, firstly we give SSH_∞FTB analysis results of the filtering problem for nominal system (2.1). Then these results will be extended to the uncertain systems. Linear matrix inequality conditions are established to show the nominal system or the uncertain system (2.6) is finite-time boundedness, and the output error $e(t)$ and disturbance $w(t)$ satisfy the constrain condition (2.9).

Lemma 3.1. *The filtering error system* (2.6) *is SSFTB with respect to* $(c_1, c_2, T, \overline{R}_i, d)$, *if there exists a scalar* $\alpha \geq 0$, *a set of nonsingular matrices* $\{\overline{P}_i, i \in \mathbb{M}\}$ *with* $\overline{P}_i \in \mathbb{R}^{2n\times 2n}$, *two sets of symmetric positive definite matrices* $\{\overline{Q}_{1i}, i \in \mathbb{M}\}$ *with* $\overline{Q}_{1i} \in \mathbb{R}^{2n\times 2n}$, $\{\overline{Q}_{2i}, i \in \mathbb{M}\}$ *with* $\overline{Q}_{2i} \in \mathbb{R}^{p\times p}$, *and for all* $i \in \mathbb{M}$ *such that the following inequalities hold:*

$$\overline{E}^T \overline{P}_i = \overline{P}_i^T \overline{E} \geq 0, \tag{3.1a}$$

$$\begin{bmatrix} \overline{A}_i^T\overline{P} + \overline{P}_i^T\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}\overline{E}^T\overline{P}_j - \alpha\overline{E}^T\overline{P}_i & \overline{P}_i^T\overline{B}_i \\ * & -\overline{Q}_{2i} \end{bmatrix} < 0, \tag{3.1b}$$

$$\overline{E}^T\overline{P}_i = \overline{E}^T\overline{R}_i^{1/2}\overline{Q}_{1i}\overline{R}_i^{1/2}\overline{E}, \tag{3.1c}$$

$$c_1^2\sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{1i}\right)\right\} + d^2\sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{2i}\right)\right\} < c_2^2e^{-\alpha T}\inf_{i\in\mathbb{M}}\left\{\lambda_{\min}\left(\overline{Q}_{1i}\right)\right\}. \tag{3.1d}$$

Proof. Firstly, one proves the filtering error system (2.6) is regular and impulse free in time interval $[0,T]$. By Lemma 2.7 and noting that condition (3.1b), one has

$$\overline{A}_i^T\overline{P}_i + \overline{P}_i^T\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}\overline{E}^T\overline{P}_j - \alpha\overline{E}^T\overline{P}_i < 0. \tag{3.2}$$

Now, we choose two orthogonal matrices $\overline{U}$ and $\overline{V}$ such that $\overline{E}$ has the decomposition as

$$\overline{E} = \overline{U}\begin{bmatrix} \Sigma_{\overline{r}} & 0 \\ * & 0 \end{bmatrix}\overline{V}^T = \overline{U}\begin{bmatrix} I_{\overline{r}} & 0 \\ * & 0 \end{bmatrix}\overline{\mathcal{V}}^T, \tag{3.3}$$

where $\Sigma_{\overline{r}} = \operatorname{diag}\{\delta_1,\delta_2,\ldots,\delta_{\overline{r}}\}$ with $\delta_k > 0$ for all $k = 1,2,\ldots,r$. Partition $\overline{U} = [\overline{U}_1 \ \ \overline{U}_2]$, $\overline{V} = [\overline{V}_1 \ \ \overline{V}_2]$ and $\overline{\mathcal{V}} = [\overline{V}_1\Sigma_{\overline{r}} \ \ \overline{V}_2]$ with $\overline{E}\,\overline{V}_2 = 0$ and $\overline{U}_2^T\overline{E} = 0$. Denote

$$\overline{U}^T\overline{A}_i\overline{\mathcal{V}}^{-T} = \begin{bmatrix} \overline{A}_{11i} & \overline{A}_{12i} \\ \overline{A}_{21i} & \overline{A}_{22i} \end{bmatrix}, \qquad \overline{U}^T\overline{P}_i\overline{\mathcal{V}}^{-T} = \begin{bmatrix} \overline{P}_{11i} & \overline{P}_{12i} \\ \overline{P}_{21i} & \overline{P}_{22i} \end{bmatrix}. \tag{3.4}$$

Noting that condition (3.1a) and $\overline{P}_i$ is a nonsingular matrix, by Lemma 2.8, we have $\overline{P}_{12i} = 0$ and $\det(\overline{P}_{22i}) \neq 0$. Pre and postmultiplying by $\overline{\mathcal{V}}^{-1}$ and $\overline{\mathcal{V}}^{-T}$, it can easily obtain $\overline{A}_{22i}^T\overline{P}_{22i} + \overline{P}_{22i}^T\overline{A}_{22i} < 0$. Therefore $\overline{A}_{22i}$ is nonsingular, which implies that system (2.6) is regular and impulse free in time interval $[0,T]$.

Let us consider the quadratic Lyapunov function candidate $V(\overline{x}(t),i) = \overline{x}^T(t)\overline{E}^T\overline{P}_i\overline{x}(t)$ for system (2.6). Computing LV, the derivative of $V(\overline{x}(t),i)$ along the solution of system (2.6), we obtain

$$LV(\overline{x}(t),i) = \xi^T(t)\begin{bmatrix} \overline{A}_i^T\overline{P}_i + \overline{P}_i^T\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}\overline{E}^T\overline{P}_j & \overline{P}_i^T\overline{B}_i \\ * & 0 \end{bmatrix}\xi(t), \tag{3.5}$$

where $\xi(t) = [\overline{x}^T(t), w^T(t)]^T$. From (3.1b) and (3.5), we obtain

$$\mathbb{E}\{LV(\overline{x}(t), i)\} < \alpha\mathbb{E}\{V(\overline{x}(t), i)\} + w^T(t)\overline{Q}_{2i}w(t). \tag{3.6}$$

Further, (3.6) can be rewritten as

$$\mathbb{E}\{L[e^{-\alpha t}V(\overline{x}(t), i)]\} < e^{-\alpha t}w^T(t)\overline{Q}_{2i}w(t). \tag{3.7}$$

Integrating (3.7) from 0 to t, with $t \in [0, T]$, we obtain

$$e^{-\alpha t}\mathbb{E}\{V(\overline{x}(t), i)\} < E\{V(\overline{x}(0), i = r_0)\} + \int_0^t e^{-\alpha\tau}w^T(\tau)\overline{Q}_{2i}w(\tau)d\tau. \tag{3.8}$$

Noting that $\alpha \geq 0,\ t \in [0, T]$ and condition (3.1c), we have

$$\begin{aligned}\mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{P}_i\overline{x}(t)\right\} &= \mathbb{E}\{V(\overline{x}(t), i)\} \\ &< e^{\alpha t}\mathbb{E}\{V(\overline{x}(0), i = r_0)\} + e^{\alpha t}\int_0^t e^{-\alpha\tau}w^T(\tau)\overline{Q}_{2i}w(\tau)d\tau \\ &\leq e^{\alpha t}\left[\sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{1i}\right)\right\}c_1^2 + \sup_{i\in\mathbb{M}}\{\lambda_{\max}(Q_{2i})\}d^2\right].\end{aligned} \tag{3.9}$$

Taking into account that

$$\begin{aligned}\mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{P}_i\overline{x}(t)\right\} &= \mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{R}_i^{1/2}\overline{Q}_{1i}\overline{R}_i^{1/2}\overline{E}\overline{x}(t)\right\} \\ &\geq \inf_{i\in\mathbb{M}}\left\{\lambda_{\min}\left(\overline{Q}_{1i}\right)\right\}\mathbb{E}\left\{x^T(t)\overline{E}^T\overline{R}_i\overline{E}x(t)\right\},\end{aligned} \tag{3.10}$$

we obtain

$$\begin{aligned}\mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{R}_i\overline{E}\overline{x}(t)\right\} &\leq \sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{1i}^{-1}\right)\right\}\mathbb{E}\left\{\overline{x}^T(t)\overline{E}^T\overline{P}_i\overline{x}(t)\right\} \\ &< e^{\alpha T}\frac{\sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{1i}\right)\right\}c_1^2 + \sup_{i\in\mathbb{M}}\left\{\lambda_{\max}\left(\overline{Q}_{2i}\right)\right\}d^2}{\inf_{i\in\mathbb{M}}\left\{\lambda_{\min}\left(\overline{Q}_{1i}\right)\right\}}.\end{aligned} \tag{3.11}$$

Therefore, it follows that condition (3.1d) implies $\mathbb{E}\{x^T(t)\overline{E}^T\overline{R}_i\overline{E}x(t)\} < c_2^2$ for all $t \in [0, T]$. This completes the proof of the lemma. □

Lemma 3.2. *The filtering error system* (2.6) *is* $SSH_\infty FTB$ *with respect to* $(0, c_2, T, \overline{R}_i, \gamma, d)$, *if there exists a scalar* $\alpha \geq 0$, *a set of nonsingular matrices* $\{\overline{P}_i, i \in \mathbb{M}\}$ *with* $\overline{P}_i \in \mathbb{R}^{2n\times 2n}$, *a set of symmetric*

positive definite matrices $\{\overline{Q}_{1i}, i \in \mathbb{M}\}$ *with* $\overline{Q}_{1i} \in \mathbb{R}^{2n\times 2n}$, *and for all* $i \in \mathbb{M}$ *such that* (3.1a), (3.1c) *and the following inequalities hold:*

$$\begin{bmatrix} \overline{A}_i^T\overline{P}_i + \overline{P}_i^T\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}\overline{E}^T\overline{P}_j + \overline{L}_i^T\overline{L}_i - \alpha\overline{E}^T\overline{P}_i & \overline{L}_i^T G_i + \overline{P}_i^T\overline{B}_i \\ * & G_i^T G_i - \gamma^2 e^{-\alpha T} I \end{bmatrix} < 0, \tag{3.12a}$$

$$d^2\gamma^2 < c_2^2 \inf_{i\in\mathbb{M}}\left\{\lambda_{\min}\left(\overline{Q}_{1i}\right)\right\}. \tag{3.12b}$$

Proof. Noting that

$$\begin{bmatrix} \overline{L}_i^T\overline{L}_i & \overline{L}_i^T G_i \\ * & G_i^T G_i \end{bmatrix} = \begin{bmatrix} \overline{L}_i^T \\ G_i^T \end{bmatrix}\begin{bmatrix} \overline{L}_i & G_i \end{bmatrix} \geq 0. \tag{3.13}$$

Thus, condition (3.12a) implies that

$$\begin{bmatrix} \overline{A}_i^T\overline{P}_i + \overline{P}_i^T\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}\overline{E}^T\overline{P}_j - \alpha\overline{E}^T\overline{P}_i & \overline{P}_i^T\overline{B}_i \\ * & -\gamma^2 e^{-\alpha T} I \end{bmatrix} < 0. \tag{3.14}$$

Let $\overline{Q}_{2i} = -\gamma^2 e^{-\alpha T} I$ for all $i \in \mathbb{M}$, by Lemma 3.1, conditions (3.1a), (3.1c), (3.12b), and (3.14) guarantee that system (2.6) is SSFTB with respect to $(0, c_2, T, \overline{R}_i, d)$. Therefore, we only need to prove that (2.9) holds. Let $V(\overline{x}(t), i) = \overline{x}^T(t)\overline{E}^T\overline{P}_i\overline{x}(t)$ and noting that (3.5) and (3.14), we obtain

$$\mathbb{E}\{LV(\overline{x}(t), i)\} < \alpha\mathbb{E}\{V(\overline{x}(t), i)\} + \gamma^2 e^{-\alpha T} w^T(t)w(t) - \mathbb{E}\{e^T(t)e(t)\}. \tag{3.15}$$

Then using the similar proof as Lemma 3.1, condition (2.9) can be easily obtained and thus is omitted. Therefore, the proof of the lemma is completed. □

Denote $\overline{P}_i = \operatorname{diag}\{P_i, P_i\}$, $\overline{Q}_{1i} = \operatorname{diag}\{Q_{1i}, Q_{1i}\}$, $M_i = P_i^T A_{fi}$, $N_i = P_i^T B_{fi}$, and $E_f = E$. Using Lemmas 2.7 and 3.2, we obtain the following theorem.

Theorem 3.3. *The nominal filtering error system* (2.6) *is SS*H_∞*FTB with respect to* $(0, c_2, T, \overline{R}_i, \gamma, d)$ *with* $\overline{R}_i = \operatorname{diag}\{R_i, R_i\}$, *if there exists a scalar* $\alpha \geq 0$, *a set of nonsingular matrices* $\{P_i, i \in \mathbb{M}\}$ *with* $P_i \in \mathbb{R}^{n\times n}$, *a set of positive definite matrices* $\{Q_{1i}, i \in \mathbb{M}\}$ *with* $Q_{1i} \in \mathbb{R}^{n\times n}$, *three sets of matrices* $\{M_i, i \in \mathbb{M}\}$ *with* $M_i \in \mathbb{R}^{n\times n}$, $\{N_i, i \in \mathbb{M}\}$ *with* $N_i \in \mathbb{R}^{n\times q_1}$, $\{C_{fi}, i \in \mathbb{M}\}$ *with* $C_{fi} \in \mathbb{R}^{q_2\times n}$, *for all* $i \in \mathbb{M}$ *such that*

$$E^T P_i = P_i^T E \geq 0, \tag{3.16a}$$

$$\begin{bmatrix} \Theta_{1i} & * & * & * \\ P_i^T A_i - M_i - N_i C_i & \Theta_{2i} & P_i^T B_i - N_i D_i & * \\ B_i^T P_i & * & -\gamma^2 e^{-\alpha T} I & * \\ L_i - C_{fi} & C_{fi} & G_i & -I \end{bmatrix} < 0, \tag{3.16b}$$

$$E^T P_i = E^T R_i^{1/2} Q_{1i} R_i^{1/2} E, \tag{3.16c}$$

$$d^2 \gamma^2 < c_2^2 \inf_{i \in \mathbb{M}} \{\lambda_{\min}(Q_{1i})\} \tag{3.16d}$$

hold, where $\Theta_{1i} = P_i^T A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} E^T P_j - \alpha E^T P_i$, *and* $\Theta_{2i} = M_i + M_i^T + \sum_{j=1}^N \pi_{ij} E^T P_j - \alpha E^T P_i$.

In addition, the desired filter parameters can be chosen by

$$A_{fi} = P_i^{-T} M_i, \qquad B_{fi} = P_i^{-T} N_i, \qquad C_{fi} = C_{fi}, \qquad E_f = E. \tag{3.17}$$

Noting that P_i *is nonsingular matrix, by Lemma 2.8, there exist two orthogonal matrices* U *and* V, *such that* E *has the decomposition as*

$$E = U \begin{bmatrix} \Sigma_r & 0 \\ * & 0 \end{bmatrix} V^T = U \begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix} \mathcal{V}^T, \tag{3.18}$$

where $\Sigma_r = \operatorname{diag}\{\delta_1, \delta_2, \ldots, \delta_r\}$ *with* $\delta_k > 0$ *for all* $k = 1, 2, \ldots, r$. *Partition* $U = [U_1 \;\; U_2]$, $V = [V_1 \;\; V_2]$, *and* $\mathcal{V} = [V_1 \Sigma_r \;\; V_2]$ *with* $EV_2 = 0$ *and* $U_2^T E = 0$. *Let* $\widetilde{P}_i = U^T P_i \mathcal{V}^{-T}$, *from* (3.16a), $\widetilde{P}_i$ *is of the following form* $\begin{bmatrix} P_{11i} & 0 \\ P_{21i} & P_{22i} \end{bmatrix}$, *and* P_i *can be expressed as*

$$P_i = U X_i U^T E + U_2 Y_i \mathcal{V}^T, \tag{3.19}$$

where $X_i = \operatorname{diag}\{P_{11i}, \Lambda_i\}$ *and* $Y_i = [P_{21i} \;\; P_{22i}]$ *with a parameter matrix* Λ_i. *If we choose* Λ_i *being a symmetric positive definite matrix, then* X_i *is a symmetric positive definite matrix. Furthermore, the symmetric positive definite matrix* $Q_{1i} = R_i^{-1/2} U X_i U^T R_i^{-1/2}$ *is a solution of* (3.16c), *and* P_i *satisfies*

$$E^T P_i = P_i^T E = E^T U X_i U^T E. \tag{3.20}$$

From the above discussion, we have the following theorem.

Theorem 3.4. *The nominal filtering error system* (2.6) *is* $SSH_\infty FTB$ *with respect to* $(0, c_2, T, \overline{R}_i, \gamma, d)$ *with* $\overline{R}_i = \operatorname{diag}\{R_i, R_i\}$, *if there exists a scalar* $\alpha \geq 0$, *a set of positive definite matrices* $\{X_i, i \in \mathbb{M}\}$

with $X_i \in \mathbb{R}^{n\times n}$*, four sets of matrices* $\{Y_i, i \in \mathbb{M}\}$ *with* $Y_i \in \mathbb{R}^{(n-r)\times n}$*,* $\{M_i, i \in \mathbb{M}\}$ *with* $M_i \in \mathbb{R}^{n\times n}$*,* $\{N_i, i \in \mathbb{M}\}$ *with* $N_i \in \mathbb{R}^{n\times q_1}$*, and* $\{C_{fi}, i \in \mathbb{M}\}$ *with* $C_{fi} \in \mathbb{R}^{q_2\times n}$*, for all* $i \in \mathbb{M}$ *such that* (3.16d) *and the following linear matrix inequality*

$$\begin{bmatrix} \Xi_{1i} & * & * & * \\ P_i^T A_i - M_i - N_i C_i & \Xi_{2i} & P_i^T B_i - N_i D_i & * \\ B_i^T P_i & * & -\gamma^2 e^{-\alpha T} I & * \\ L_i - C_{fi} & C_{fi} & G_i & -I \end{bmatrix} < 0 \tag{3.21}$$

hold, where $\Xi_{1i} = P_i^T A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} E^T P_j - \alpha E^T P_i$*,* $\Xi_{2i} = M_i + M_i^T + \sum_{j=1}^N \pi_{ij} E^T P_j - \alpha E^T P_i$*,* $P_i = \mathcal{U} X_i \mathcal{U}^T E + \mathcal{U}_2 Y_i \mathcal{V}^T$*,* X_i *and* Y_i *are from the form* (3.19)*; Moreover, other matrical variables are the same as Theorem 3.3.*

By Theorems 3.3 and 3.4 and applying Lemmas 2.6–2.8, one can obtain the results stated as follows.

Theorem 3.5. *The uncertain filtering error system* (2.6) *is* $SSH_\infty FTB$ *with respect to* $(0, c_2, T, \overline{R}_i, \gamma, d)$ *with* $\overline{R}_i = \{R_i, R_i\}$*, if there exists a scalar* $\alpha \geq 0$*, a set of positive definite matrices* $\{X_i, i \in \mathbb{M}\}$ *with* $X_i \in \mathbb{R}^{n\times n}$*, four sets of matrices* $\{Y_i, i \in \mathbb{M}\}$ *with* $Y_i \in \mathbb{R}^{(n-r)\times n}$*,* $\{M_i, i \in \mathbb{M}\}$ *with* $M_i \in \mathbb{R}^{n\times n}$*,* $\{N_i, i \in \mathbb{M}\}$ *with* $N_i \in \mathbb{R}^{n\times q_1}$*,* $\{C_{fi}, i \in \mathbb{M}\}$ *with* $C_{fi} \in \mathbb{R}^{q_2\times n}$*, and a set of positive scalars* $\{\epsilon_i, i \in \mathbb{M}\}$*, for all* $i \in \mathbb{M}$ *such that* (3.16d) *and the following linear matrix inequality*

$$\begin{bmatrix} \Upsilon_{1i} & * & * & * & * \\ P_i^T A_i - M_i - N_i C_i & \Upsilon_{2i} & P_i^T B_i - N_i D_i & * & * \\ B_i^T P_i + \epsilon_i E_{2i}^T E_{1i} & * & \epsilon_i E_{2i}^T E_{2i} - \gamma^2 e^{-\alpha T} I & * & * \\ F_i^T P_i & F_i^T P_i & 0 & -\epsilon_i I & * \\ L_i - C_{fi} & C_{fi} & G_i & 0 & -I \end{bmatrix} < 0 \tag{3.22}$$

hold, where $\Upsilon_{1i} = P_i^T A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} E^T P_j + \epsilon_i E_{1i}^T E_{1i} - \alpha E^T P_i$*,* $\Upsilon_{2i} = M_i + M_i^T + \sum_{j=1}^N \pi_{ij} E^T P_j - \alpha E^T P_i$*,* $P_i = \mathcal{U} X_i \mathcal{U}^T E + \mathcal{U}_2 Y_i \mathcal{V}^T$*,* X_i *and* Y_i *are from the form* (3.19)*; Moreover, other matrical variables are the same as Theorem 3.3.*

Remark 3.6. Theorems 3.4 and 3.5 extend the H_∞ filtering problem of singular stochastic systems to the finite-time H_∞ filtering problem of singular stochastic systems. In fact, if we fix $\alpha = 0$ without condition (3.16d), we can obtain sufficient conditions of the H_∞ filtering of singular stochastic systems.

Let $I < Q_{1i} < \eta I$, then one can check that condition (3.16d) can be guaranteed by imposing the conditions

$$I < R_i^{-1/2} \mathcal{U} X_i \mathcal{U}^T R_i^{-1/2} < \eta I, \qquad d^2\gamma^2 - c_2^2 < 0. \tag{3.23}$$

Remark 3.7. The feasibility of conditions stated in Theorem 3.4 and Theorem 3.5 can be turned into the following LMIs-based feasibility problem with a fixed parameter α, respectively:

$$\begin{gathered}\min_{X_i, Y_i, M_i, N_i, C_{fi}, \eta} \left(\gamma^2 + c_2^2\right) \\ \text{s.t.} \quad (3.21) \text{ and } (3.23), \\ \min_{X_i, Y_i, M_i, N_i, C_{fi}, \epsilon_i, \eta} \left(\gamma^2 + c_2^2\right) \\ \text{s.t.} \quad (3.22) \text{ and } (3.23).\end{gathered} \tag{3.24}$$

4. Simulation Examples

In this section, numerical results are given to illustrate the effectiveness of the suggested method.

Example 4.1. Consider a two-mode singular stochastic system (2.1) with uncertain parameters as follows:

(i) Mode #1,

$$\begin{gathered} A_1 = \begin{bmatrix} 0.2 & 1 & 1 \\ 3 & 2.5 & 1 \\ 0.1 & 3 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 1 \\ 0.1 \\ 1 \end{bmatrix}^T, \quad C_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T, \\ F_1 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0.01 & 0 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0.2 & 0 & 0.03 \\ 0.03 & 0.02 & 0 \\ 0.05 & 0 & 0.01 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.03 \\ 0.02 \\ 0.05 \end{bmatrix}, \end{gathered} \tag{4.1}$$

(ii) Mode #2,

$$\begin{gathered} A_2 = \begin{bmatrix} -4 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.7 \\ 1 \\ 2 \end{bmatrix}^T, \quad C_2 = \begin{bmatrix} 0.8 \\ 4 \\ 1 \end{bmatrix}^T, \\ F_2 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0.01 & 0.02 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 0.02 & 0 & 0.03 \\ 0.03 & 0.03 & 0 \\ 0.02 & 0 & 0.1 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0.03 \\ 0.02 \\ 0.02 \end{bmatrix}, \end{gathered} \tag{4.2}$$

and $E = \operatorname{diag}\{1,1,0\}$, $D_1 = 0.2$, $G_1 = 0.3$, $D_2 = 0.1$, $G_2 = 0.5$, $d = 0.6$, $\Delta_i = \operatorname{diag}\{r_1(i), r_2(i), r_3(i)\}$, where $r_j(i)$ satisfies $|r_j(i)| \leq 1$ for all $i = 1,2$ and $j = 1,2,3$. In addition, the switching between the two modes is described by the transition rate matrix $\Gamma = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$.

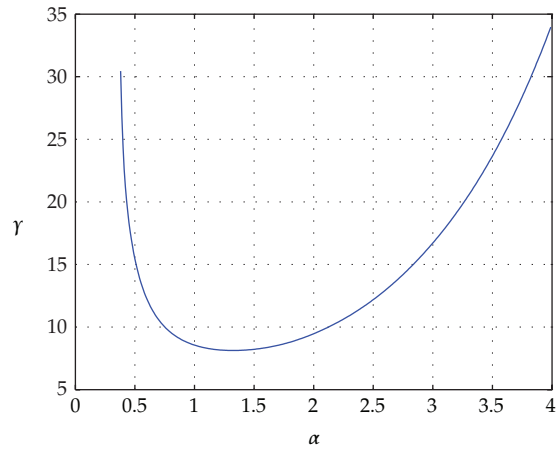

Figure 1: The local optimal bound of γ.

Then, we choose $R_1 = R_2 = I_3$, $T = 2$, by Theorem 3.5, the optimal bound with minimum value of $\gamma^2 + c_2^2$ relies on the parameter α. We can find feasible solution when $0.34 \leq \alpha \leq 11.02$. Figures 1 and 2 show the optimal values with different value of α. Noting that when $\alpha = 2$, it yields the optimal values $\gamma = 9.4643$ and $c_2 = 5.6786$. Then, by using the program *fminsearch* in the optimization toolbox of Matlab starting at $\alpha = 2$, the locally convergent solution can be derived as

$$A_{f1} = \begin{bmatrix} -0.8906 & 1.5044 & 0.3958 \\ 64.1537 & 47.0692 & 51.0207 \\ 14.9048 & 15.6431 & 14.4887 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} 2.2274 \\ -63.2281 \\ -13.4543 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 0.9315 & -0.3546 & 0.6824 \end{bmatrix}, \tag{4.3}$$

$$A_{f2} = \begin{bmatrix} 25.0076 & 170.6684 & 40.7148 \\ -149.7287 & -851.1311 & -192.8675 \\ 20.5802 & 107.7329 & 25.9671 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -44.9909 \\ 223.8116 \\ -27.9168 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 0.3916 & 0.3722 & 1.6748 \end{bmatrix}, \tag{4.4}$$

with $\alpha = 1.3209$, and the optimal values $\gamma = 8.1261$, $c_2 = 4.8758$.

Remark 4.2. From the above example and Remark 3.7, condition (3.22) in Theorem 3.5 is not strict in LMI form, however, one can find the parameter α by an unconstrained nonlinear optimization approach, which a locally convergent solution can be obtained by using the program *fminsearch* in the optimization toolbox of Matlab.

Example 4.3. Consider a two-mode singular stochastic system (2.1) with uncertain parameters as follows:

$$A_1 = \begin{bmatrix} -3 & 2 & 0 \\ -3 & -2.5 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 3 & 0 \\ -1 & -2.5 & 0 \\ -1 & 0 & -4.8 \end{bmatrix}. \tag{4.5}$$

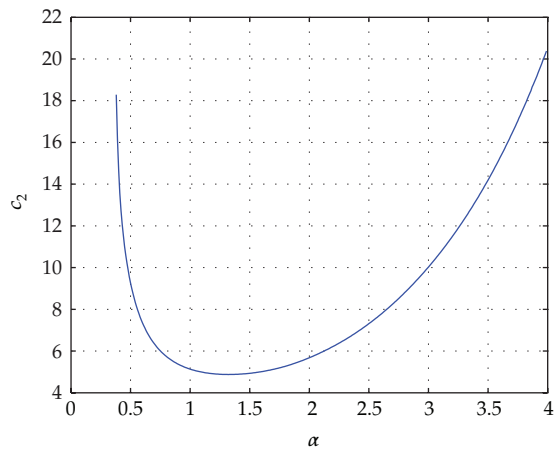

Figure 2: The local optimal bound of c_2.

Moreover, other matrical variables and the transition rate matrix are defined similarly as Example 4.1.

Let $R_1 = R_2 = I_3$, then the feasible solution of the above filtering error system can be found when $\alpha = 0$, Theorem 3.5 yields the optimal values $\gamma = 3.7770$, $c_2 = 2.2663$, and

$$A_{f1} = \begin{bmatrix} 35.9923 & 47.8632 & 42.1297 \\ -125.9936 & -141.5997 & -122.0307 \\ 31.9978 & 34.4604 & 31.8623 \end{bmatrix}, \qquad B_{f1} = \begin{bmatrix} -47.6632 \\ 137.2043 \\ -33.8527 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 0.8294 & 0.1136 & 0.8294 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} 16.8765 & 73.6696 & 25.7772 \\ -137.4627 & -607.3156 & -225.4097 \\ -17.9446 & -72.5666 & -36.9766 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -15.8299 \\ 141.9862 \\ 17.5990 \end{bmatrix},$$
$$C_{f2} = \begin{bmatrix} 1.1290 & 1.5975 & 3.6412 \end{bmatrix}. \tag{4.6}$$

Thus, the above filtering error system is stochastically stable and the calculated minimum H_∞ performance γ satisfies $\|T_{wz}\| < 3.7770$.

5. Conclusion

In this paper, we deal with the problem of finite-time H_∞ filtering for a class of singular stochastic systems with parametric uncertainties and time-varying norm-bounded disturbance. Designed algorithms are provided to guarantee the filtering error system SSFTB and satisfy a prescribed H_∞ performance level in a given finite-time interval, which can be reduced to feasibility problems involving restricted linear matrix equalities with a fixed parameter. Numerical examples are given to demonstrate the validity of the proposed methodology.

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