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Research Article

Implicit Schemes for Solving Extended General Nonconvex Variational Inequalities

Muhammad Aslam Noor,[1] Khalida Inayat Noor,[1] Zhenyu Huang,[2] and Eisa Al-Said[3]

[1] *Mathematics Department, COMSATS Institute of Information Technology, Park Road, Islamabad, Pakistan*
[2] *Mathematics Department, Nanjing University, Nanjing 210093, China*
[3] *Mathematics Department, College of Science, King Saud University, Riyadh 11451, Saudi Arabia*

Correspondence should be addressed to Muhammad Aslam Noor, noormaslam@hotmail.com

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We suggest and analyze some implicit iterative methods for solving the extended general nonconvex variational inequalities using the projection technique. We show that the convergence of these iterative methods requires only the gh-pseudomonotonicity, which is a weaker condition than gh-monotonicity. We also discuss several special cases. Our method of proof is very simple as compared with other techniques.

1. Introduction

Variational inequalities, which were introduced and studied in early sixties, contain wealth of new ideas. Variational inequalities can be considered as a natural extension of the variational principles. It is well known that the variational inequalities characterize the optimality conditions of the differentiable convex functions on the convex sets in normed spaces. In recent years, Noor [1–6] has introduced and studied a new class of variational inequalities involving three different operators, which is called the extended general variational inequalities. Noor [1–6] has shown that the minimum of a differentiable nonconvex (gh-convex) function on the nonconvex set (gh-convex) can be characterized by the class of extended general variational inequalities. The class of extended general variational include the general variational inequalities [1–33] and variational inequalities as special cases. This clearly shows that the extended general variational inequalities are more general and unifying ones. For applications, physical formulation, numerical methods, and other aspects of variational inequalities, see [1–35] and the references therein. However, all the work carried out in this direction assumes

that the underlying set is a convex set. In many practical situations, a choice set may not be a convex set so that the existing results may not be applicable. To handle such situations, Noor [20–25] has introduced and considered a new class of variational inequalities, called the general nonconvex variational inequality on the uniformly prox-regular sets. It is well known that uniformly prox-regular sets are nonconvex and include the convex sets as special cases, see [8, 9, 32]. Using the projection operator, Noor [27] proved a new characterization of the projection operator for the prox-regular sets. Using this characterization, one can easily show that nonconvex projection operator is Lipschitz continuous, which is a new result. Using this new characterization of the projection of the prox-regular sets, one can establish the equivalence between the nonconvex variational inequalities and the fixed point problems. This equivalence is useful to study various concepts for the nonconvex variational inequalities.

Motivated and inspired by the recent activities in this dynamic field, we consider the extended general noncomvex variational inequalities on the prox-regular sets. We use the projection technique to establish the equivalence between the extended general nonconvex variational inequalities and the fixed point problems. We use this alternative formulation to some unified implicit and extragradient methods for solving the extended general nonconvex variational inequalities. These new methods include the modified projection method of Noor [27] and the extragradient method of Korpelevič [11] as special cases. The main motivation of this paper is to improve the convergence criteria. We show that the convergence of the implicit iterative methods requires only the gh-pseudomonotonicity, which is weaker condition that gh-monotonicity. It is worth mentioning that we do not need the Lipschitz continuity of the operator. In this sense, our result represents an improvement and refinement of the known results. Our method of proof is very simple.

2. Basic Concepts

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$, respectively. Let K be a nonempty closed convex set in H. The basic concepts and definitions used in this paper are exactly the same as in Noor [20, 22]. We now recall some basic concepts and results from nonsmooth analysis [9, 32].

Definition 2.1 (see [9, 32]). The proximal normal cone of K at $u \in H$ is given by

$$N_K^P(u) := \{\xi \in H : u \in P_K[u + \alpha\xi]\}, \tag{2.1}$$

where $\alpha > 0$ is a constant and

$$P_K[u] = \{u^* \in K : d_K(u) = \|u - u^*\|\}. \tag{2.2}$$

Here $d_K(\cdot)$ is the usual distance function to the subset K, that is,

$$d_K(u) = \inf_{v \in K} \|v - u\|. \tag{2.3}$$

The proximal normal cone $N_K^P(u)$ has the following characterization.

Lemma 2.2. *Let K be a nonempty, closed and convex subset in H. Then $\zeta \in N_K^P(u)$, if and only if, there exists a constant $\alpha > 0$ such that*

$$\langle \zeta, v - u \rangle \leq \alpha \|v - u\|^2, \quad \forall v \in K. \tag{2.4}$$

Definition 2.3. The Clarke normal cone, denoted by $N_K^C(u)$, is defined as

$$N_K^C(u) = \overline{\mathrm{co}}\left[N_K^P(u)\right], \tag{2.5}$$

where $\overline{\mathrm{co}}$ means the closure of the convex hull. Clearly $N_K^P(u) \subset N_K^C(u)$, but the converse is not true. Note that $N_K^P(u)$ is always closed and convex, whereas $N_K^C(u)$ is convex, but may not be closed [32].

Definition 2.4 (see [29]). For a given $r \in (0, \infty]$, a subset K_r is said to be normalized uniformly r-prox-regular if and only if every nonzero proximal normal to K_r can be realized by an r-ball, that is, for all $u \in K_r$ and $0 \neq \xi \in N_{K_r}^P(u)$, one has

$$\left\langle \frac{(\xi)}{\|\xi\|}, v - u \right\rangle \leq \left(\frac{1}{2r}\right)\|v - u\|^2, \quad \forall v \in K_r. \tag{2.6}$$

It is clear that the class of normalized uniformly prox-regular sets is sufficiently large to include the class of convex sets, p-convex sets, $C^{1,1}$ submanifolds (possibly with boundary) of H, the images under a $C^{1,1}$ diffeomorphism of convex sets, and many other nonconvex sets, see [11, 29]. It is well known [9, 32] that the union of two disjoint intervals $[a, b]$ and $[c, d]$ is a prox regular set with $r = (c - b)/2$. Obviously, for $r = \infty$, the uniformly prox-regularity of K_r is equivalent to the convexity of K. This class of uniformly prox-regular sets have played an important part in many nonconvex applications such as optimization, dynamic systems, and differential inclusions. It is known that if K_r is a uniformly prox-regular set, then the proximal normal cone $N_{K_r}^P(u)$ is closed as a set-valued mapping.

We now recall the well-known proposition which summarizes some important properties of the uniformly prox-regular sets K_r.

Lemma 2.5. *Let K be a nonempty closed subset of H, $r \in (0, \infty]$ and set $K_r = \{u \in H : d_K(u) < r\}$. If K_r is uniformly prox-regular, then*

(i) *for all $u \in K_r$, $P_{K_r}(u) \neq \emptyset$,*

(ii) *for all $r' \in (0, r)$, P_{K_r} is Lipschitz continuous with constant $r/(r - r')$ on $K_{r'}$.*

For given nonlinear operators T, g, h, we consider the problem of finding $u \in H$: $h(u) \in K_r$ such that

$$\langle \rho T u + h(u) - g(u), g(v) - h(u) \rangle + \gamma \|g(v) - h(u)\|^2 \geq 0, \quad \forall v \in H : g(v) \in K_r, \tag{2.7}$$

which is called the *extended general nonconvex variational inequality.* Here $\gamma > 0$ and $\rho > 0$ are constants.

We remark that if $g = h$, then problem (2.7) is equivalent to finding $u \in H : g(u) \in K_r$ such that

$$\langle \rho Tu, g(v) - g(u) \rangle + \gamma \| g(v) - g(u) \|^2 \geq 0, \quad \forall v \in H : g(v) \in K_r, \tag{2.8}$$

which is called the *general nonconvex variational inequality,* introduced and studied by Noor [27].

We note that, if $K_r \equiv K$, the convex set in H, then problem (2.7) is equivalent to finding $u \in H : h(u) \in K$ such that

$$\langle Tu + h(u) - g(u), g(v) - h(u) \rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{2.9}$$

which is known as the extended general variational inequality, introduced and studied by Noor [1–6]. For the applications, numerical methods, formulation, and other aspects of the extended general variational inequalities (2.8), see [1–6, 30, 31] and the references therein.

If $g \equiv h \equiv I$, the identity operator, then both problems (2.7) and (2.8) are equivalent to finding $u \in K_r$ such that

$$\langle \rho Tu, v - u \rangle + \gamma \| v - u \|^2 \geq 0, \quad \forall v \in K_r, \tag{2.10}$$

which is called the nonconvex variational inequality. For the formulation and numerical methods for the nonconvex variational inequalities, see [3, 8, 18–27].

We note that if $K_r \equiv K$, the convex set in H, then problem (2.8) is equivalent to finding $u \in H : g(u) \in K$ such that

$$\langle Tu, g(v) - g(u) \rangle \geq 0, \quad \forall v \in H : g(v) \in K. \tag{2.11}$$

Inequality of type (2.11) is called the *general variational inequality* involving two operators, which was introduced and studied by Noor [14] in 1988. It has been shown that the minimum of the differentiable g-convex function on a g-convex set can be characterized by the general variational inequality of type (3.4). It has been shown that a wide class of odd-order and nonsymmetric problems can be studied via the general variational inequalities. For the numerical method, sensitivity analysis, dynamical systems, and other aspects of general variational inequalities, see [7, 15, 16]. For $g = I$, the identity operator, one can obtain the original variational inequality, which was introduced and studied by Stampacchia [33] in 1964. It turned out that a number of unrelated obstacle, free, moving, unilateral, and equilibrium problems arising in various branches of pure and applied sciences can be studied via variational inequalities, see [1–35] and the references therein.

If K_r is a nonconvex (uniformly prox-regular) set, then problem (2.7) is equivalent to finding $u \in H : g(u) \in K_r$ such that

$$0 \in \rho Tu + h(u) - g(u) + N^P_{K_r}(h(u)), \tag{2.12}$$

which is called the extended general nonconvex variational inclusion problem associated with general nonconvex variational inequality (2.7). Here $N^P_{K_r}(h(u))$ denotes the normal

cone of K_r at $h(u)$ in the sense of nonconvex analysis. This equivalent formulation plays a crucial and basic part in this paper. We would like to point out that this equivalent formulation allows us to use the projection operator technique for solving the general nonconvex variational inequalities of the type (2.7).

We now prove that the projection operator P_{K_r} has the following characterization for the prox-regular sets. This result is due to Noor [27]. We include its proof for the sake of completeness and to convey an idea of the technique.

Lemma 2.6 (see [27]). *Let K_r be a prox-regular and closed set in H. Then, for a given $z \in H$, $u \in K_r$ satisfies the inequality*

$$\langle u - z, v - u \rangle + \gamma \|v - u\|^2, \quad \forall v \in K_r, \tag{2.13}$$

if and only if,

$$u = P_{K_r}[z], \tag{2.14}$$

where P_{K_r} is the projection of H onto the prox-regular set K_r.

Proof. Let $u \in K_r$. Then, for given $z \in H$, we have

$$\begin{aligned} (2.13) &\Longleftrightarrow u - z \in N_{K_r}^P(u) \\ &\Longleftrightarrow z \in \left(I + N_{K_r}^P\right)^{-1}(u) \\ &\Longleftrightarrow u = \left(I + N_{K_r}^P\right)^{-1}[z] = P_{K_r}[z], \end{aligned} \tag{2.15}$$

where $P_{K_r} = (I + N_{K_r}^P)^{-1}$ is the projection operator. □

We note that, if $K_r \equiv K$, the closed convex set, then Lemma 2.6 is a well-known result, see [10]. Using Lemma 2.6, one can easily prove that the nonconvex projection operator P_{K_r} is Lipschitz continuous.

Definition 2.7. An operator $T : H \to H$ with respect to the arbitrary operators g, h is said to be *gh-pseudomonotone,* if and only if,

$$\langle \rho T u, g(v) - h(u) \rangle + \rho\gamma \|g(v) - h(v)\|^2 \geq 0 \Longrightarrow -\langle Tv, h(v) - g(u) \rangle + \gamma \|g(u) - h(v)\|^2 \geq 0, \quad \forall u, v \in H. \tag{2.16}$$

3. Main Results

It is known that the extended general nonconvex variational inequalities (2.7) are equivalent to the fixed point problem. One can also prove this result using Lemma 2.6.

Lemma 3.1. *$u \in H : h(u) \in K_r$ is a solution of the general nonconvex variational inequality* (2.7) *if and only if $u \in H : h(u) \in K_r$ satisfies the relation*

$$h(u) = P_{K_r}\left[g(u) - \rho T u\right], \tag{3.1}$$

where P_{K_r} is the projection of H onto the uniformly prox-regular set K_r.

Lemma 3.1 implies that (2.7) is equivalent to the fixed point problem (3.1). This alternative equivalent formulation is very useful from the numerical and theoretical points of view. Using the fixed point formulation (3.1), we suggest and analyze the following iterative methods for solving the extended general nonconvex variational inequality (2.7).

Algorithm 3.2. For a given $u_0 \in H$, find the approximate solution u_{n+1} by the iterative scheme

$$h(u_{n+1}) = P_{K_r}\left[g(u_n) - \rho T u_n\right], \quad n = 0, 1, \ldots, \tag{3.2}$$

which is called the explicit iterative method. For the convergence analysis of Algorithm 3.2, see Noor [19].

We again use the fixed point formulation is used to suggest and analyze the following iterative method for solving (2.7).

Algorithm 3.3. For a given $u_0 \in H$, find the approximate solution u_{n+1} by the iterative scheme

$$h(u_{n+1}) = P_{K_r}\left[g(u_n) - \rho T u_{n+1}\right], \quad n = 0, 1, \ldots. \tag{3.3}$$

Algorithm 3.3 is an implicit iterative method for solving the extended general nonconvex variational inequalities (2.7) and is a new one. Using Lemma 2.6, one can rewrite Algorithm 3.3 in the following equivalent form.

Algorithm 3.4. For a given $u_0 \in H$, find the approximate solution u_{n+1} by the iterative schemes

$$\left\langle \rho T u_{n+1} + h(u_{n+1}) - g(u_n), g(v) - h(u_{n+1}) \right\rangle + \gamma \left\| g(v) - h(u_{n+1}) \right\|^2 \geq 0, \quad \forall v \in H : g(v) \in K_r. \tag{3.4}$$

To implement Algorithm 3.3, we use the predictor-corrector technique. We use Algorithm 3.2 as predictor and Algorithm 3.3 as a corrector to obtain the following predictor-corrector method for solving the extended general nonconvex variational inequality (2.7).

Algorithm 3.5. For a given $u_0 \in H$, find the approximate solution u_{n+1} by the iterative schemes

$$\begin{aligned} g(w_n) &= P_{K_r}\left[g(u_n) - \rho T u_n\right], \\ h(u_{n+1}) &= P_{K_r}\left[g(u_n) - \rho T w_n\right], \quad n = 0, 1, \ldots. \end{aligned} \tag{3.5}$$

Algorithm 3.5 is known as the extended extragradient method. This method includes the extragradient method of Korpelevič [11] for $h = g = I$. Here we would like to point out

that the implicit method (Algorithm 3.3) and the extragradient method (Algorithm 3.5) are equivalent.

We now consider the convergence analysis of Algorithm 3.3, and this is the main motivation of our next result.

Theorem 3.6. *Let $u \in H : h(u) \in K_r$ be a solution of* (2.7) *and let u_{n+1} be the approximate solution obtained from Algorithm 3.3. If the operator T is gh-pseudomonotone, then*

$$\|g(u) - h(u_{n+1})\|^2 \leq \|g(u) - g(u_n)\|^2 - \|(g(u_n) - h(u_{n+1})\|^2. \tag{3.6}$$

Proof. Let $u \in H : h(u) \in K_r$ be a solution of (2.7). Then

$$\langle \rho T v, h(v) - g(u) \rangle + \gamma \|g(u) - h(v)\|^2 \geq 0, \quad \forall v \in H : g(v) \in K_r, \tag{3.7}$$

since the operator T is gh-pseudomonotone. Take $v = u_{n+1}$ in (3.7), we have

$$\langle \rho T u_{n+1}, h(u_{n+1}) - g(u) \rangle + \gamma \|g(u) - h(u_{n+1})\|^2 \geq 0. \tag{3.8}$$

Taking $v = u$ in (3.4), we have

$$\langle \rho T u_{n+1} + h(u_{n+1}) - g(u_n), g(u) - h(u_{n+1}) \rangle + \gamma \|g(u) - h(u_{n+1})\|^2 \geq 0. \tag{3.9}$$

From (3.8) and (3.9), we have

$$\langle h(u_{n+1}) - g(u_n), g(u) - h(u_{n+1}) \rangle + \langle \rho T u_{n+1}, g(u) - h(u_{n+1}) \rangle + \|g(u) - h(u_{n+1})\|^2 \geq 0. \tag{3.10}$$

It is well known that

$$2\langle v, u \rangle = \|u + v\|^2 - \|v\|^2 - \|u\|^2, \quad \forall u, v \in H. \tag{3.11}$$

Using (3.11), from (3.10), one can easily obtain

$$\|g(u) - h(u_{n+1})\|^2 \text{les} \|g(u) - g(u_n)\|^2 - \|(g(u_n) - h(u_{n+1})\|^2, \tag{3.12}$$

the required result (3.6). □

Theorem 3.7. *Let $u \in H : h(u) \in K_r$ be a solution of* (2.7) *and let u_{n+1} be the approximate solution obtained from Algorithm 3.3. Let H be a finite dimensional space. Then* $\lim_{n\to\infty}(h(u_{n+})) = g(u)$.

Proof. Let $\overline{u} \in H : h(\overline{u}) \in K_r$ be a solution of (2.7). Then, the sequence $\{\|h(u_n) - g(\overline{u})\|\}$ is nonincreasing and bounded and

$$\sum_{n=0}^{\infty} \|h(u_{n+1}) - g(u_n)\|^2 \leq \|g(u_0) - g(u)\|^2, \tag{3.13}$$

which implies

$$\lim_{n\to\infty}\left\|h(u_{n+1})-g(u_n)\right\|=0. \tag{3.14}$$

Let $\widehat{u}$ be a cluster point of $\{u_n\}$. Then, there exists a subsequence $\{u_{n_i}\}$ such that $\{u_{n_i}\}$ converges to $\widehat{u}$. Replacing u_{n+1} by u_{n_i} in (3.4), taking the limits in (3.4) and using (3.14), we have

$$\left\langle \rho T\widehat{u}, g(v)-h(\widehat{u})\right\rangle+\gamma\left\|g(v)-h(\widehat{u})\right\|^2\geq 0, \quad \forall v\in H: g(v)\in K_r. \tag{3.15}$$

This shows that $\widehat{u}\in H: h(\widehat{u})\in K_r$ solves the extended general nonconvex variational inequality (2.7) and

$$\left\|h(u_{n+1})-g(\widehat{u})\right\|^2\leq\left\|g(u_n)-g(\widehat{u})\right\|^2, \tag{3.16}$$

which implies that the sequence $\{u_n\}$ has a unique cluster point, and $\lim_{n\to\infty}(h(u_{n+1}))=g(\widehat{u})$ is the solution of (2.7), the required result. □

We again use the fixed point formulation (3.1) to suggest the following method for solving (2.7).

Algorithm 3.8. For a given $u_0\in H$, find the approximate solution u_{n+1} by the iterative schemes

$$h(u_{n+1})=P_{K_r}\left[g(u_{n+1})-\rho Tu_{n+1}\right], \quad n=0,1,2,\ldots, \tag{3.17}$$

which is also known an implicit method. To implement this method, we use the prediction-correction technique. We use Algorithm 3.2 as the predictor and Algorithm 3.8 as the corrector. Consequently, we obtain the following iterative method.

Algorithm 3.9. For a given $u_0\in H$, find the approximate solution u_{n+1} by the iterative schemes:

$$\begin{aligned} h(y_n)&=P_{K_r}\left[g(u_n)-\rho Tu_n\right],\\ h(u_{n+1})&=P_{K_r}\left[g(y_n)-\rho Ty_n\right], \quad n=0,1,2,\ldots. \end{aligned} \tag{3.18}$$

Algorithm 3.9 is called the two-step or predictor-corrector method for solving the extended general nonconvex variational inequality (2.7).

For a given step size $\eta>0$, one can suggest and analyze the following two-step iterative method.

Algorithm 3.10. For a given $u_0\in H$, find the approximate solution by the iterative schemes:

$$\begin{aligned} h(y_n)&=P_{K_r}\left[g(u_n)-\rho Tu_n\right],\\ h(u_{n+1})&=P_{K_r}\left[g(u_n)-\eta\{g(u_n)-g(y_n)+\rho Ty_n\}\right], \quad n=0,1,2,\ldots. \end{aligned} \tag{3.19}$$

Note that for $\eta = 1$, Algorithm 3.10 reduces to Algorithm 3.9. Using the technique of Noor [16], one may study the convergence analysis of Algorithms 3.9 and 3.10.

4. Conclusion

In this paper, we have introduced and considered a new class of general variational inequalities, which is called the general nonconvex variational inequalities. Some new characterizations of the nonconvex projection operator are proved. We have established the equivalent between the general nonconvex variational inequalities and fixed point problem using the technique of the projection operator. This equivalence is used to suggest and analyze some iterative methods for solving the nonconvex general variational inequalities. Several special cases are also discussed. Results proved in this paper can be extended for multivalued and system of general nonconvex variational inequalities using the technique of this paper. The comparison of the iterative method for solving nonconvex general variational inequalities is an interesting problem for future research. We hope that the ideas and technique of this paper may stimulate further research in this field.

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