Research Article

On Fuzzy Corsini’s Hyperoperations

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We generalize the concept of C-hyperoperation and introduce the concept of F-C-hyperoperation. We list some basic properties of F-C-hyperoperation and the relationship between the concept of C-hyperoperation and the concept of F-C-hyperoperation. We also research F-C-hyperoperations associated with special fuzzy relations.

1. Introduction and Preliminaries

Hyperstructures and binary relations have been studied by many researchers, for instance, Chvalina [1, 2], Corsini and Leoreanu [3], Feng [4], Hort [5], Rosenberg [6], Spartalis [7], and so on.

A partial hypergroupoid \( \langle H, \ast \rangle \) is a nonempty set \( H \) with a function from \( H \times H \) to the set of subsets of \( H \).

A hypergroupoid is a nonempty set \( H \), endowed with a hyperoperation, that is, a function from \( H \times H \) to \( P(H) \), the set of nonempty subsets of \( H \).

If \( A, B \in P(H) \setminus \{\emptyset\} \), then we define \( A \ast B = \bigcup \{ a \ast b \mid a \in A, b \in B \} \), \( x \ast B = \{ x \} \ast B \) and \( A \ast \{ y \} = A \ast \{ y \} \).

A Corsini’s hyperoperation was first introduced by Corsini [8] and studied by many researchers; for example, see [3, 8–15].

Definition 1.1 (see [8]). Let \( \langle H, R \rangle \) be a a pair of sets where \( H \) is a nonempty set and \( R \) is a binary relation on \( H \). Corsini’s hyperoperation (briefly, C-hyperoperation) \( \ast_R \) associated with


\[ *_R : H \times H \rightarrow P(H) : x *_R y = \{ z \in H \mid xRz, zRy \}, \]  

(1.1)

where \( P(H) \) denotes the family of all the subsets of \( H \).

A fuzzy subset \( A \) of a nonempty set \( H \) is a function \( A : H \rightarrow [0, 1] \). The family of all the fuzzy subsets of \( H \) is denoted by \( F(H) \).

We use \( \emptyset \) to denote a special fuzzy subset of \( H \) which is defined by \( \emptyset(x) = 0 \), for all \( x \in H \).

For any \( A, B \in F(H) \), we will use the following definitions:

\[
A \subseteq B \equiv A(x) \leq B(x), \quad \forall x \in H,
\]

\[
A = B \equiv A(x) = B(x), \quad \forall x \in H,
\]

\[
(A \cup B)(x) \equiv A(x) \lor B(x), \quad \forall x \in H,
\]

\[
(A \cap B)(x) \equiv A(x) \land B(x), \quad \forall x \in H.
\]

A partial fuzzy hypergroupoid \((H, *)\) is a nonempty set endowed with a fuzzy hyperoperation \( * : H \times H \rightarrow F(H) \). Moreover, \((H, *)\) is called a fuzzy hypergroupoid if for all \( x, y \in H \), there exists at least one \( z \in H \), such that \((x * y)(z) \neq \emptyset\) holds.

For any \( A, B \in F(H) \), we will use the following definitions:

\[
A \subseteq B \equiv A(x) \leq B(x), \quad \forall x \in H,
\]

\[
A = B \equiv A(x) = B(x), \quad \forall x \in H,
\]

\[
(a \ast b)(x) \equiv \lor_{B(b) > \emptyset}(a \ast b)(x).
\]

(1.3)

For any \( a, b \in H \), \( A \ast B \) can be defined similarly. When \( B \) is a crisp subset of \( H \), we treat \( B \) as a fuzzy subset by treating it as \( B(x) = 1 \), for all \( x \in B \) and \( B(x) = 0 \), for all \( x \in H - B \).

2. Fuzzy Corsini’s Hyperoperation

In this section, we will generalize the concept of Corsini’s hyperoperation and introduce the fuzzy version of Corsini’s hyperoperation.

**Definition 2.1.** Let \((H, R)\) be a pair of sets where \( H \) is a non-empty set and \( R \) is a fuzzy relation on \( H \). We define a fuzzy hyperoperation \( R : H \times H \rightarrow F(H) \), for any \( x, y, z \in H \), as follows:

\[
(x \ast_R y)(z) \equiv R(x, z) \land R(z, y).
\]

(2.1)
*R is called a fuzzy Corsini’s hyperoperation (briefly, F-C-hyperoperation) associated with R. The fuzzy hyperstructure \( (H, *R) \) is called a partial F-C-hypergroupoid.

**Remark 2.2.** It is obvious that the concept of F-C-hyperoperation is a generalization of the concept of C-hyperoperation.

**Example 2.3.** Letting \( H = \{a, b\} \) be a non-empty set, R is a fuzzy relation on H as described in Table 1.

From the previous definition, by calculating, for example, \( (a *R a)(a) = R(a, a) \wedge R(a, a) = 0.1 \wedge 0.1 = 0.1, R(a * b)(a) = R(a, a) \wedge R(a, b) = 0.1 \wedge 0.2 = 0.1 \), we can obtain Table 2 which is a partial F-C-hypergroupoid.

**Definition 2.4.** Supposing R, S are two fuzzy relations on a non-empty set H, the composition of R and S is a fuzzy relation on H and is defined by \( (R \circ S)(x, y) \equiv \bigvee_{z \in H} (R(x, z) \wedge S(z, y)) \), for all \( x, y \in H \).

**Proposition 2.5.** A partial F-C-hypergroupoid \( (H, *R) \) is a F-C-hypergroupoid if and only if \( \text{supp}(R \circ R) = H \times H \), where \( \text{supp}(R \circ R) = \{(x, y) \mid (R \circ R)(x, y) \neq 0\} \).

**Proof.** Suppose that \( (H, *R) \) is a hypergroupoid. For any \( x, y \in H \), there exists at least one \( z \in H \), such that \( (x *R y)(z) \neq 0 \) holds.

So \( (R \circ R)(x, y) = \bigvee_{z \in H} (R(x, z) \wedge R(z, y)) \neq 0 \). Thus \( (x, y) \in \text{supp}(R \circ R) \). And we conclude that \( H \times H \subseteq \text{supp}(R \circ R) \).

\( \text{supp}(R \circ R) \subseteq H \times H \) is obvious. And so \( \text{supp}(R \circ R) = H \times H \).

Conversely, if \( \text{supp}(R \circ R) = H \times H \), then for any \( x, y \in H \), \( (x, y) \in H \times H \Rightarrow \text{supp}(R \circ R) \).

So \( (R \circ R)(x, y) = \bigvee_{z \in H} (R(x, z) \wedge R(z, y)) \neq 0 \). That is, there exists at least one \( z \in H \) such that \( (x *R y)(z) \neq 0 \) holds. And so \( (H, *R) \) is a hypergroupoid.

Thus we complete the proof. \( \square \)

**Definition 2.6.** Letting \( H \) be a non-empty set, \( * \) is a fuzzy hyperoperation of \( H \), the hyperoperation \( *_p \) is defined by \( x *_p y = (x * y)_p \), for all \( x, y \in H, p \in [0, 1] \). \( *_p \) is called the p-cut of \( * \).
Definition 2.7. Letting $R$ be a fuzzy relation on a non-empty set $H$, we define a binary relation $R_p$ on $H$, for all $p \in (0, 1]$, as follows:

$$xR_py \equiv R(x, y) \geq p.$$  \hfill (2.2)

$R_p$ is called the $p$-cut of the fuzzy relation $R$.

Proposition 2.8. Let $(H, \star_R)$ be a partial F-C-hypergroupoid. Then $(\star_R)_p$ is a C-hyperoperation associated with $R_p$, for all $0 < p \leq 1$.

Proof. For any $0 < p \leq 1$ and for any $x, y \in H$, we have

$$x(\star_R)_p y = (x \star_R y)_p = \{ z \in H \mid (x \star_R y)(z) \geq p \} = \{ z \in H \mid R(x, z) \land R(z, y) \geq p \}$$

$$= \{ z \in H \mid R(x, z) \geq p, R(z, y) \geq p \} = \{ z \in H \mid xR_p z, zR_p y \}.$$ \hfill (2.3)

From the definition of C-hyperoperation, we conclude that $(\star_R)_p$ is a C-hyperoperation associated with $R_p$.

Thus we complete the proof. \hfill \Box

From the previous proposition and the construction of the F-C-hyperoperation, we can easily conclude that a fuzzy hyperoperation is a F-C-hyperoperation if and only if every $p$-cut of the F-C-hyperoperation is a C-hyperoperation. That is, consider the following.

Proposition 2.9. Let $H$ be a non-empty set and let $\star$ be a fuzzy hyperoperation of $H$, then the fuzzy hyperoperation $\star$ is an F-C-hyperoperation associated with a fuzzy relation $R$ on $H$ if and only if $\star_p$ is a C-hyperoperation associated with $R_p$, for any $0 < p \leq 1$.

3. Basic Properties of F-C-Hyperoperations

In this section, we list some basic properties of F-C-hyperoperations.

Proposition 3.1. Let $(H, \star_R)$ be a partial or nonpartial F-C-hypergroupoid defined on $H \neq \emptyset$. Then, for all $x, y, a, b \in H$, we have

$$x \star_R y \cap a \star_R b = x \star_R b \cap a \star_R y.$$ \hfill (3.1)

Proof. For any $x, y, a, b, z \in H$, we have that $(x \star_R y \cap a \star_R b)(z) = (x \star_R y)(z) \land (a \star_R b)(z) = R(x, z) \land R(z, y) \land R(a, z) \land R(z, b) = R(x, z) \land R(z, b) \land R(a, z) \land R(z, y) = (x \star_R b \cap a \star_R y)(z)$.

So

$$x \star_R y \cap a \star_R b = x \star_R b \cap a \star_R y,$$ \hfill (3.2)

for all $x, y, a, b \in H$. \hfill \Box
Proposition 3.2. Let \( \langle H, \star_R \rangle \) be a partial F-C-hypergroupoid and \( x, y \in H \), \( x \star_R y = \emptyset \). Then,

1. \( x \star_R H \cap H \star_R y = \emptyset \);
2. If \( H = x \star_R H \) then \( H \star_R y = \emptyset \);
3. If \( H = H \star_R x \) then \( y \star_R H = \emptyset \).

Proof. (1) Supposing \( x \star_R H \cap H \star_R y \neq \emptyset \), then there exist \( a, b \in H \), such that \( x \star_R a \cap b \star_R y \neq \emptyset \). So from the previous proposition, we have \( x \star_R y \cap b \star_R a \neq \emptyset \). This is a contradiction.

(2) From \( H = x \star_R H \) and \( x \star_R H \cap H \star_R y = \emptyset \), we have that \( H \cap H \star_R y = \emptyset \), and so, \( H \star_R y = \emptyset \).

(3) is proved similar to (2).

Proposition 3.3. Letting \( \star_R \) be the F-C-hyperoperation defined on the non-empty set \( H \), \( p \in (0, 1] \), then the following are equivalent:

1. for some \( a \in H \), \( (a \star_R a)_p = H \);
2. for all \( x, y \in H \), \( a \in (x \star_R y)_p \).

Proof. Let \( (a \star_R a)_p = H \). Then, for all \( x, y \in H \), we have that \( (a \star_R a)(x) \geq p, (a \star_R a)(y) \geq p \), that is \( R(a, x) \geq p, R(x, a) \geq p, R(y, a) \geq p, R(y, a) \geq p \), and so \( R(x, a) \land R(a, y) \geq p \). Thus \( a \in (x \star_R y)_p \), for all \( x, y \in H \).

Conversely, let \( a \in (x \star_R y)_p \), for all \( x, y \in H \). Specially, we have \( a \in (a \star_R x)_p \) and \( a \in (x \star_R a)_p \). Thus, \( R(a, x) \geq p \) and \( R(x, a) \geq p \). And so \( x \in (a \star_R a)_p \).

Proposition 3.4. Let \( \langle H, \star_R \rangle \) be a partial or nonpartial F-C-hypergroupoid defined on \( H \neq \emptyset \). Then, for all \( a, b \in H \), \( p \in (0, 1] \), we have

\[ a \in (b \star_R b)_p \iff b \in (a \star_R a)_p . \tag{3.3} \]

Proof. For any \( a, b \in H \), we have that

\[ a \in (b \star_R b)_p \implies (b \star_R b)(a) \geq p \implies R(b, a) \land R(a, b) \geq p \]

\[ \implies R(a, b) \land R(b, a) \geq p \implies (a \star_R a)(b) \geq p \implies b \in (a \star_R a)_p . \tag{3.4} \]

The remaining part can be proved similarly.

4. F-C-Hyperoperations Associated with p-Fuzzy Reflexive Relations

In this section, we will assume that \( R \) is a p-fuzzy reflexive relation on a non-empty set.

Definition 4.1. A fuzzy relation \( R \) on a non-empty set \( H \) is called p-fuzzy reflexive if for any \( x \in H \),

\[ R(x, x) \geq p . \tag{4.1} \]

Example 4.2. The fuzzy relation \( R \) introduced in Example 2.3 is 0.1-fuzzy reflexive. Of course, it is p-fuzzy reflexive, where \( 0 \leq p \leq 0.1 \).
Proposition 4.3. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is \( p \)-fuzzy reflexive. Then, for all \( a, b \in H \), \( p \in (0, 1] \), the following are equivalent:

1. \( R(a, b) \geq p \);
2. \( a \in (a \ast_R b)_p \);
3. \( b \in (a \ast_R b)_p \).

Proof. “(1)\( \Rightarrow \) (2)”
From \( R(a, a) \geq p \) and \( R(a, b) \geq p \) we have that \( R(a, a) \land R(a, b) \geq p \) which shows that \( a \in (a \ast_R b)_p \).

“(2)\( \Rightarrow \) (3)”
From \( a \in (a \ast_R b)_p \) we have that \( R(a, b) \geq p \). Since \( R(b, b) \geq p \), so \( R(a, b) \land R(b, b) \geq p \) which implies that \( b \in (a \ast_R b)_p \).

“(3)\( \Rightarrow \) (1)”
It is obvious. \( \square \)

Proposition 4.4. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is \( p \)-fuzzy reflexive. Then, for any \( a \in H \), we have that

\[
a \in (a \ast_R a)_p.
\] (4.2)

Proof. From \( R(a, a) \geq p \) we have \( R(a, a) \land R(a, a) \geq p \). That is \( a \in (a \ast_R a)_p \). \( \square \)

Proposition 4.5. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is \( p \)-fuzzy reflexive. Then, for any \( a, b \in H \), \( p \in (0, 1] \), we have that

\[
b \in (a \ast_R a)_p \iff a \in (a \ast_R b \land b \ast_R a)_p.
\] (4.3)

Proof. From \( b \in (a \ast_R a)_p \) we have that \( R(a, b) \land R(b, a) \geq p \). So \( R(a, b) \geq p \) and \( R(b, a) \geq p \). Thus \( R(a, a) \land R(a, b) \geq p \) and \( R(b, a) \land R(a, a) \geq p \). That is \( (a \ast_R b)(a) \geq p \) and \( (b \ast_R a)(a) \geq p \). So \( (a \ast_R b \land b \ast_R a)(a) \geq p \). Thus \( a \in (a \ast_R b \land b \ast_R a)_p \).

Conversely, suppose that \( a \in (a \ast_R b \land b \ast_R a)_p \). Then \( (a \ast_R b)(a) \land (b \ast_R a)(a) \geq p \). Thus \( R(a, a) \land R(a, b) \land R(b, a) \land R(a, a) \geq p \). So \( R(a, b) \land R(b, a) \geq p \). That is \( b \in (a \ast_R a)_p \). \( \square \)

Corollary 4.6. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is \( p \)-fuzzy reflexive. Then, for any \( a, b \in H \), \( p \in (0, 1] \), we have that

\[
b \in (a \ast_R a)_p \iff a \in (b \ast_R b)_p \iff a \in (a \ast_R b \land b \ast_R a)_p \iff b \in (a \ast_R b \land b \ast_R a)_p.
\] (4.4)

Proposition 4.7. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is \( p \)-fuzzy reflexive. Then, for any \( a, b \in H \), we have that

\[
c \in (a \ast_R b)_p \iff c \in (a \ast_R c \land c \ast_R b)_p.
\] (4.5)

Proof. If \( c \in (a \ast_R b)_p \), then \( R(a, c) \geq p \) and \( R(c, b) \geq p \). Thus \( c \in (a \ast_R c)_p \) and \( c \in (c \ast_R b)_p \). So \( c \in (a \ast_R c \land c \ast_R b)_p \).
Conversely, if \( c \in (a \ast_R c \cap c \ast_R b)_p \), then \((a \ast_R c)(c) \land (c \ast_R b)(c) \geq p\). Thus \(R(a, c) \land R(c, c) \land R(c, b) \geq p\). And so \(R(c, c) \land R(c, b) \geq p\). Thus \(c \in (a \ast_R b)_p\). 

\[ \text{Proposition 4.8.} \text{ Letting } (H, \ast_R) \text{ be a partial F-C-hypergroupoid defined on } H \neq \emptyset, R \text{ is p-fuzzy reflexive. Then, for any } a, b, c \in H, p \in (0, 1], \text{ the following are equivalent:} \]

1. \(c \in (a \ast_R b)_p\);
2. \(a \in (a \ast_R c)_p \text{ and } b \in (c \ast_R b)_p\);
3. \(a \in (a \ast_R c)_p \text{ and } c \in (c \ast_R b)_p\).

\[ \text{Proof. } "(1) \Rightarrow (2)" \]

Suppose that \(c \in (a \ast_R b)_p\). Then \(R(a, c) \geq p \text{ and } R(c, b) \geq p\). So \(R(a, a) \land R(a, c) \geq p \text{ and } R(c, b) \land R(b, b) \geq p\). Thus \(a \in (a \ast_R c)_p \text{ and } b \in (c \ast_R b)_p\).

\[ "(2) \Rightarrow (3)" \]

Suppose that \(b \in (c \ast_R b)_p\). Then \(R(c, b) \geq p\). Thus \(R(c, c) \land R(c, b) \geq p\). And so \(c \in (c \ast_R b)_p\).

\[ "(3) \Rightarrow (1)" \]

From \(a \in (a \ast_R c)_p\) and \(c \in (c \ast_R b)_p\), we have that \(R(a, c) \geq p \text{ and } R(c, b) \geq p\). Thus \(R(a, c) \land R(c, b) \geq p\). So \(c \in (a \ast_R b)_p\). 

\[ \]

5. F-C-Hyperoperations Associated with p-Fuzzy Symmetric Relations

In this section, we will assume that \(R\) is a p-fuzzy symmetric relation on a non-empty set.

\[ \text{Definition 5.1. A fuzzy binary relation } R \text{ on a non-empty set } H \text{ is called } p\text{-fuzzy symmetric if for any } x, y \in H, \]

\[ R(x, y) \geq p \implies R(y, x) \geq p. \quad (5.1) \]

\[ \text{Example 5.2. The fuzzy relation } R \text{ introduced in Example 2.3 is } 0.2\text{-fuzzy symmetric. Of course, it is } p\text{-fuzzy reflexive, where } 0 \leq p \leq 0.2. \]

\[ \text{Proposition 5.3. Letting } (H, \ast_R) \text{ be a partial F-C-hypergroupoid defined on } H \neq \emptyset, R \text{ is } p\text{-fuzzy symmetric relation. Then, for all } a, b \in H, \text{ we have that} \]

\[ (a \ast_R b)_p = (b \ast_R a)_p. \quad (5.2) \]

\[ \text{Proof. For all } a, b \in H, \text{ two cases are possible.} \]

1. If \((a \ast_R b)_p = \emptyset\), then \((a \ast_R b)_p \subseteq (b \ast_R a)_p\).
2. If \((a \ast_R b)_p \neq \emptyset\), let \(x \in (a \ast_R b)_p\). Then \(R(a, x) \geq p \text{ and } R(x, b) \geq p\).

Since \(R\) is p-fuzzy symmetric, so \(R(x, a) \geq p \text{ and } R(b, x) \geq p\). Thus \((b \ast_R a)(x) = R(b, x) \land R(x, a) \geq p\). So \(x \in (b \ast_R a)_p\). And in this case, we also have that \((a \ast_R b)_p \subseteq (b \ast_R a)_p\).

The remaining part can be proved by exchanging \(a\) and \(b\). 

\[ \]
**Proposition 5.4.** Let \( \langle H, \ast_R \rangle \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( p \in (0, 1] \), if

1. for all \( a, b \in H \), \( (a \ast_R b)_p = (b \ast_R a)_p \),
2. for any \( x \in H \), there exists a \( y \in H \), such that \( R(x, y) \geq p \).

Then \( R \) is a \( p \)-fuzzy symmetric binary relation on \( H \).

**Proof.** For all \( a, b \in H \), suppose that \( R(a, b) \geq p \). We need to show that \( R(b, a) \geq p \).

Since for \( b \in H \), there exists a \( x \in H \), such that \( R(b, x) \geq p \). So \( R(a, b) \wedge R(b, x) \geq p \). That is, \( b \in (a \ast_R x)_p = (x \ast_R a)_p \). And so \( R(x, b) \wedge R(b, a) \geq p \). And finally we have that \( R(b, a) \geq p \). \( \square \)

### 6. F-C-Hyperoperations Associated with \( p \)-Fuzzy Transitive Relations

In this section, we will assume that \( R \) is a \( p \)-fuzzy transitive relation on a non-empty set.

**Definition 6.1.** A fuzzy binary relation \( R \) on a non-empty set \( H \) is called \( p \)-fuzzy transitive if for any \( x, y, z \in H \),

\[
R(x, y) \geq p, R(y, z) \geq p \Rightarrow R(x, z) \geq p.
\] (6.1)

**Example 6.2.** The fuzzy relation \( R \) introduced in Example 2.3 is 0.1-fuzzy transitive. Of course, it is \( p \)-fuzzy transitive, where \( 0 \leq p \leq 0.1 \).

**Proposition 6.3.** Letting \( \langle H, \ast_R \rangle \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is a \( p \)-fuzzy transitive relation on \( H \), \( p \in (0, 1] \). Then for all \( x, y \in H \), we have that

\[
R(x, y) \geq p \Rightarrow (x \ast_R x \cup y \ast_R y)_p \subseteq (x \ast_R y)_p.
\] (6.2)

**Proof.**

1. If \( (x \ast_R x)_p = \emptyset \), then obviously \( (x \ast_R x)_p \subseteq (x \ast_R y)_p \).

Supposing that \( (x \ast_R x)_p \neq \emptyset \), then for any \( w \in (x \ast_R x)_p \), we have that \( R(x, w) \wedge R(w, x) \geq p \), that is, \( R(x, w) \geq p \) and \( R(w, x) \geq p \). From \( R(w, x) \geq p \) and \( R(x, y) \geq p \) we have that \( R(w, y) \geq p \). From \( R(x, w) \geq p \) and \( R(w, y) \geq p \) we conclude that \( w \in (x \ast_R y)_p \).

So \( (x \ast_R x)_p \subseteq (x \ast_R y)_p \).

2. If \( (y \ast_R y)_p = \emptyset \), then obviously \( (y \ast_R y)_p \subseteq (x \ast_R y)_p \).

Supposing that \( (y \ast_R y)_p \neq \emptyset \), then for any \( w \in (y \ast_R y)_p \), we have that \( R(y, w) \wedge R(w, y) \geq p \), that is, \( R(y, w) \geq p \) and \( R(w, y) \geq p \). From \( R(y, w) \geq p \) and \( R(x, y) \geq p \) we have that \( R(x, w) \geq p \). From \( R(x, w) \geq p \) and \( R(w, y) \geq p \) we conclude that \( w \in (x \ast_R y)_p \).

So \( (y \ast_R y)_p \subseteq (x \ast_R y)_p \). \( \square \)

**Proposition 6.4.** Letting \( \langle H, \ast_R \rangle \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is a \( p \)-fuzzy transitive binary relation. For any \( a, b, c \in H \), we have that

1. \( ((a \ast_R b)_p \ast_R c)_p \subseteq (a \ast_R c)_p \),
2. \( (a \ast_R (b \ast_R c)_p) \subseteq (a \ast_R c)_p \).
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References

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References


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