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Research Article

Stability Analysis of Predator-Prey System with Fuzzy Impulsive Control

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Having attracted much attention in the past few years, predator-prey system provides a good mathematical model to present the correlation between predators and preys. This paper focuses on the robust stability of Lotka-Volterra predator-prey system with the fuzzy impulsive control model, and Takagi-Sugeno (T-S) fuzzy impulsive control model as well. Via the T-S model and the Lyapunov method, the controlling conditions of the asymptotical stability and exponential stability are established. Furthermore, the numerical simulation for the Lotka-Volterra predator-prey system with impulsive effects verifies the effectiveness of the proposed methods.

1. Introduction

Since Volterra presented the differential equation to solve the issue of the sharp change of the population of the sharks (predator) and the minions (prey) in 1925, the predator-prey system has been applied into many areas and played an important role in the biomathematics. Much attention has been attracted to the stability of the predator-prey system. Brauer and Soudack studied the global behavior of a predator-prey system under constant-rate prey harvesting with a pair of nonlinear ordinary differential equations [1]. Xu and his workmates concluded that a short-time delay could ensure the stability of the predator-prey system [2]. After analyzing the different capability between the mature and immature predator, Wang and his workmates obtained the global stability with the small time-delay system [3]. Li and his partners studied the impulsive control of Lotka-Volterra predator-prey system and established sufficient conditions of the asymptotic stability with the method of Lyapunov functions [4]. Liu and Zhang studied the coexistence and stability of predator-prey model with Beddington-DeAngelis functional response and stage structure [5]. Li did some work on the predator-prey system with Holling II functional response and obtained the existence,

uniqueness and global asymptotic stability of the in random perturbation [6]. Furthermore, Ko and Ryu studied the qualitative behavior of nonconstant positive solutions on a general Gauss-type predator-prey model with constant diffusion rates under homogenous Neumann boundary condition [7]. Additionally, many papers discussed the predator-prey system with other different methods, such as LaSalle's invariance principle method [8], Liu and Chen's impulsive perturbations method [9], and Moghadas and Alexander's generalized Gauss-type predator–prey model [10].

In recent years, fuzzy impulsive theory has been applied to the stability analysis of the non-linear differential equations [11–15]. However, it should be admitted that the stability of fuzzy logic controller (FLC) is still an open problem. It is well-known that the parallel distributed compensation technique has been the most popular controller design approach and belongs to a continuous input control way. It is important to point out that there exist many systems, like the predator-prey system, which cannot commonly endure continuous control inputs, or they have impulsive dynamical behavior due to abrupt jumps at certain instants during the evolving processes. In this sense, it is the same with communication networks, biological population management, chemical control, and so forth [16–23]. Hence, it is necessary to extend FLC and reflect these impulsive jump phenomena in the predator-prey system. Until recently, few papers talk about the stability of Lotka-Volterra predator-prey system with fuzzy impulsive control. In this paper, the writer will study the robustness of the predator-prey system by the fuzzy impulsive control based on the T-S mathematical model.

The rest of this paper is organized as follows. Section 2 describes the Lotka-Volterra predator-prey system and T-S fuzzy system with impulsive control. In Section 3, the theoretic analysis and design algorithm on stability of the impulsive fuzzy system are performed. Numerical simulations for the predator-prey system with impulsive effects are carried out with respect to the proposed method in Section 4. Finally, some conclusions are made in Section 5.

2. Problem Equation

The Lotka-Volterra predator-prey system is expressed with the following differential equation:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\big(\mu_1 - r_{12}x_2(t)\big), \\ \dot{x}_2(t) &= x_2(t)\big(-\mu_2 + r_{21}x_1(t)\big), \end{aligned} \tag{2.1}$$

where $x_1(t), x_2(t)$ $(x_1(t) > 0, x_2(t) > 0)$ denote the species density of the preys and the predators in the group at time t respectively. The coefficient $\mu_1 > 0$ denotes the birth rate of the preys, and $\mu_2 > 0$ denotes the death rate of the predators. The other two coefficients r_{12} and r_{21} (both positive) describe interactions between the species.

In order to discuss the stability of the system, a matrix differential equation is presented as follows:

$$\dot{x} = Ax + \Phi(x), \quad \text{where } A = \begin{bmatrix} \mu_1 & 0 \\ 0 & -\mu_2 \end{bmatrix}, \ \Phi(x) = \begin{bmatrix} -r_{12}x_1x_2 \\ r_{21}x_1x_2 \end{bmatrix}. \tag{2.2}$$

Lemma 2.1. *$\dot{x} = f(x(t))$, where $x(t) \in R^n$ is the state variable, and $f \in C[R^n, R^n]$ satisfies $f(0) = 0$, is a vector field defined over a compact region $W \subseteq R^n$. By using the methods introduced in [24], one can construct fuzzy model for system (2.1) as follows.*

Control Rule i $(i = 1, 2, \ldots, r)$: IF $z_1(t)$ is $M_{i1}, z_2(t)$ is $M_{i2} \ldots$, and $z_p(t)$ is M_{ip}, THEN $\dot{x}(t) = A_i x(t)$, where r is the number of T-S fuzzy rules, and $z_1(t), z_2(t), \ldots, z_p(t)$ are the premise variables, each M_{ij} $(j = 1, 2, \ldots, p)$ is a fuzzy set, and $A_i \in R^{n\times n}$ is a constant matrix.

Thus, the nonlinear equation can be transformed to the following linear equation.

If $x_2(t)$ is M_i

$$\begin{gathered} \dot{x} = A_i x(t), \quad t \neq \tau_j, \\ \Delta x|_{t=\tau_j} = K_{i,j} x(t), \quad t = \tau_j, \\ i = 1, 2, \ldots, r, \quad j = 1, 2 \ldots, \end{gathered} \tag{2.3}$$

where

$$A_i = \begin{bmatrix} \mu_1 - d_i r_{12} & 0 \\ d_i r_{21} & -\mu_2 \end{bmatrix}, \tag{2.4}$$

and d_i is related to the value of $x_2(t)$ (here, $d_i = x_2(t)$). $M_i, x(t), A_i \in R^{2\times 2}, r$ is the number of the IF-THEN rules, $K_{i,j} \in R^{2\times 2}$ denotes the control of the j th impulsive instant, $\Delta x|_{t=\tau_j} \equiv x(\tau_j^+ - \tau_j^-)$.

Correspondently, with center-average defuzzifier, the overall T-S fuzzy impulsive system can be represented as follows:

$$\begin{gathered} \dot{x}(t) = \sum_{i=1}^{r} h_i(x_2(t))(A_i x(t)), \quad t \neq \tau_j, \\ \Delta x|_{t=\tau_j} = \sum_{i=1}^{r} h_i(x_2(t)) K_{i,j} x, \quad t = \tau_j, \end{gathered} \tag{2.5}$$

where $h_i(x_2(t)) = \omega_i(x_2(t)) / \sum_{i=1}^{r} \omega_i(x_2(t))$ and $\omega_i(x_2(t)) = \prod_{j=1}^{p} M_{i,j}(x_2(t))$.

Obviously, $h_i(x_2(t)) \geq 0, \sum_{i=1}^{r} h_i(x_2(t)) = 1, i = 1, 2, \ldots, r$.

Lemma 2.2. *If P is a real semipositive matrix, then a real matrix C exists, making $P = C^T C$.*

3. Stability Analysis

Theorem 3.1. *Assume that λ_i is the maximum eigenvalue of $[A_i^T + A_i]$ $(i = 1, 2, \ldots, r)$, let $\lambda(\alpha) = \max_i\{\lambda_i\}$, $0 < \delta_j = \tau_j - \tau_{j-1} < \infty$ is impulsive distance [25]. If $\lambda(\alpha) \geq 0$ and there exists a constant scalar $\varepsilon > 1$ and a semipositive matrix P, such that*

$$\ln(\varepsilon\beta_j) + \lambda(\alpha)\delta_j \leq 0, \qquad PA_i = A_i P, \tag{3.1}$$

where

$$P = C^T C, \qquad \beta_j = \max_i \left\| C\left(I + K_{i,j}\right) \right\|. \tag{3.2}$$

Then the system (2.5) *is stable globally and asymptotically.*

Proof. Let the candidate Lyapunov function be in the form of

$$V(x) = \frac{1}{2} x^T P x. \tag{3.3}$$

Clearly, for $t \neq \tau_j$,

$$\begin{aligned}
\dot{V}(x) &= \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) x^T \left[A_i^T P + P A_i\right] x \\
&= \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) x^T P \left[P^{-1} A_i^T P + A_i\right] x \\
&\leq \frac{1}{2}\lambda(\alpha) x^T P \sum_{i=1}^{r} h_i(x_2(t)) x \\
&= \frac{1}{2}\lambda(\alpha) x^T P x \\
&= \lambda(\alpha) V(x(t)),
\end{aligned} \tag{3.4}$$

where $t \in (\tau_{j-1}, \tau_j]$ $(j = 1, 2, \ldots)$.

For $t = \tau_j$, we have

$$\begin{aligned}
V\left(x\left(\tau_j^+\right)\right) &= \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) \left[\left(I + K_{i,j}\right) x(\tau_j)\right]^T P \left[\left(I + K_{i,j}\right) x(\tau_j)\right] \\
&= \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) \left[\left(I + K_{i,j}\right) x(\tau_j)\right]^T C^T C \left[\left(I + K_{i,j}\right) x(\tau_j)\right] \\
&= \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) \left\| C\left(I + K_{i,j}\right) x(\tau_j) \right\| \\
&\leq \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) \left\| C\left(I + K_{i,j}\right) \right\| \left\| x(\tau_j) \right\| \\
&\leq \frac{1}{2}\sum_{i=1}^{r} h_i(x_2(t)) \beta_j \left\| x(\tau_j) \right\| \\
&= \beta_j V\left(x(\tau_j)\right), \quad j \in N.
\end{aligned} \tag{3.5}$$

Let $j = 1$, for any $t \in (\tau_0, \tau_1]$, by (3.4), we obtain

$$V(x(t)) \leq V(x(\tau_0)) \exp(\lambda(\alpha)(t - \tau_0)). \tag{3.6}$$

Then

$$V(x(\tau_1)) \leq V(x(\tau_0)) \exp(\lambda(\alpha)(\tau_1 - \tau_0)). \tag{3.7}$$

From (3.5) and (3.7), we obtain

$$V\left(x\left(\tau_1^+\right)\right) \leq \beta_1 V(x(\tau_1)) \leq \beta_1 V(x(\tau_0)) \exp(\lambda(\alpha)(\tau_1 - \tau_0)). \tag{3.8}$$

In the same way, for any $t \in (\tau_1, \tau_2]$, we have

$$V(t, x) \leq V\left(\tau_1^+, x\right) \exp(\lambda(\alpha)(t - \tau_1)) \leq \beta_1 V(\tau_0, x) \exp(\lambda(\alpha)(t - \tau_0)). \tag{3.9}$$

Similarly, for all k and $t \in (\tau_k, \tau_{k+1}]$, we obtain

$$V(t, x) \leq \beta_k \cdots \beta_2 \beta_1 V(\tau_0, x) \exp(\lambda(\alpha)(t - \tau_0)). \tag{3.10}$$

From (3.2), we obtain

$$\beta_k \exp(\lambda(\alpha)\delta_k) \leq \frac{1}{\varepsilon}, \quad k \in N. \tag{3.11}$$

Thus, for $t \in (\tau_k, \tau_{k+1}]$, $k \in N$, we have

$$\begin{aligned} V(x(t)) &\leq V(x(\tau_0))\beta_1\beta_2 \cdots \beta_k \exp(\lambda(\alpha)(t - \tau_0)) \\ &= V(x(\tau_0))\left[\beta_1 \exp(\lambda(\alpha)\delta_1)\right] \cdots \left[\beta_k \exp(\lambda(\alpha)\delta_k)\right] \exp(\lambda(\alpha)(t - \tau_k)) \\ &\leq V(x(\tau_0)) \frac{1}{\varepsilon^k} \exp(\lambda(\alpha)(t - \tau_k)). \end{aligned} \tag{3.12}$$

So, if $t \to \infty$, then $k \to \infty$ and $V(t, x) \to 0$. So the system (2.5) is stable globally and asymptotically. □

Theorem 3.2. *Assume that λ_i is the maximum eigenvalue of $[A_i + A_i^T]$ $(i = 1, 2, \ldots, r)$, let $\lambda(\alpha) = \max_i\{\lambda_i\}$, $0 < \delta_j = \tau_j - \tau_{j-1} < \infty$ is impulsive distance. If $\lambda(\alpha) < 0$ and a constant scalar $0 \leq \varepsilon < -\lambda(\alpha)$ exists, such that*

$$\ln(\beta) - \varepsilon\delta_j \leq 0, \quad PA_i = A_i P, \tag{3.13}$$

where $P = C^T C$ and $\beta_j = \max_i \|C(I + K_{i,j})\|$.

Then the system (2.5) is stable globally and exponentially.

Proof. Let the candidate Lyapunov function be in the form of

$$V(x) = \frac{1}{2}x^T P x. \tag{3.14}$$

Firstly, (3.4)–(3.10) hold.

From (3.13), we obtain

$$\beta_k \exp(-\varepsilon\sigma_k) \le 1, \quad k \in N. \tag{3.15}$$

Thus, for $t \in (\tau_k, \tau_{k+1}], k \in N$,

$$\begin{aligned} V(x(t)) &\le V(x(t_0))\beta_1\beta_2\cdots\beta_k \exp(\lambda(\alpha)(t-t_0)) \\ &= V(x(t_0))\beta_1\beta_2\cdots\beta_k \exp((-\varepsilon)(t-t_0)) \exp((\lambda(\alpha)+\varepsilon)(t-t_0)) \\ &= V(x(t_0))\left[\beta_1 \exp(-\varepsilon(t_1-t_0))\right]\cdots\left[\beta_j \exp(-\varepsilon(t-t_k))\right] \exp((\lambda(\alpha)+\varepsilon)(t-t_0)) \\ &\le V(x(t_0)) \exp((\lambda(\alpha)+\varepsilon)(t-t_0)). \end{aligned} \tag{3.16}$$

Note that $0 \le \varepsilon < -\lambda(\alpha)$, thus $\lambda(\alpha) + \varepsilon < 0$. So the system (2.5) is stable globally and exponentially.

Next, we consider some special cases of the two theorems. Assume that $K = K_{i,j}$ and $\sigma = \sigma_j$ in the two theorems above, so we can have the following corollary. □

Corollary 3.3. *Let λ_i be the largest eigenvalue of $[A + A^T], (i = 1, 2, \ldots, r), \lambda(\alpha) = \max_i\{\lambda_i\} > 0$. If there exists a constant $\varepsilon > 1$ and a real semi-positive P such that*

$$\ln(\varepsilon\beta) + \lambda(a)\delta \le 0, \quad PA_i = A_i P, \tag{3.17}$$

where $P = C^T C$, $\beta_j = \max_i\|C(I + K_{i,j})\|$, and $0 < \delta = \tau_j - \tau_{j-1} < \infty$ $(j \in N)$ is impulsive distance. Then the system (2.5) *is stable globally and asymptotically.*

Corollary 3.4. *Let λ_i be the largest eigenvalue of $[A + A^T](i = 1, 2, \ldots, r), \lambda(\alpha) = \max_i\{\lambda_i\} < 0$. If there exists a constant $0 \le \varepsilon < -\lambda(\alpha)$ and a real semi-positive P such that*

$$\ln(\beta) - \varepsilon\delta \le 0, \quad PA_i = A_i P, \tag{3.18}$$

where $P = C^T C, \beta_j = \max_i\|C(I + K_{i,j})\|$, and $0 < \delta = \tau_j - \tau_{j-1} < \infty (j \in N)$ is impulsive distance. Then the system (2.5) *is stable globally and exponentially.*

4. Numerical Simulation

In this section, we present a design example to show how to perform the impulsive fuzzy control on the Lotka-Volterra predator-prey systems with impulsive effects. Especially, the biological systems are very complex, nonlinear, and uncertain. As a result, they should be represented by fuzzy logical method with linguistic description.

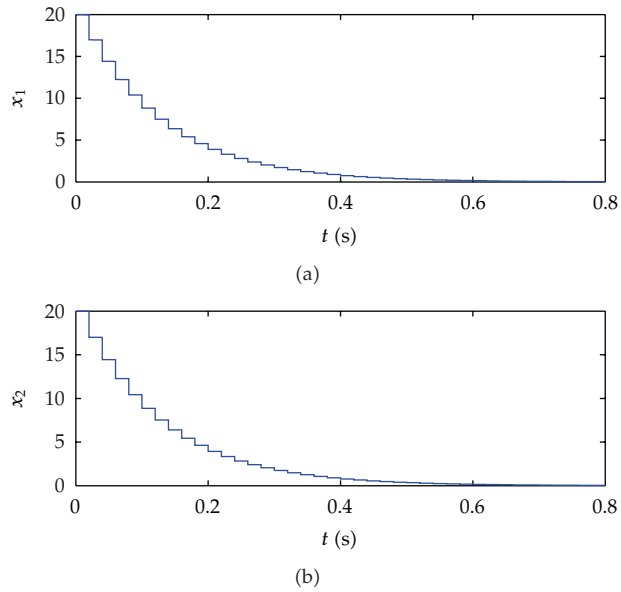

Figure 1: The phase portrait of the system with fuzzy impulsive control.

Now, consider a predator-prey system with impulsive effects as follows:

$$\dot{x} = Ax + \Phi(x), \tag{4.1}$$

where,

$$A = \begin{bmatrix} \mu_1 & 0 \\ 0 & -\mu_2 \end{bmatrix}, \quad \Phi(x) = \begin{bmatrix} -r_{12}x_1x_2 \\ r_{21}x_1x_2 \end{bmatrix}. \tag{4.2}$$

Solving

From (2.3), we have the following impulsive fuzzy control for the above predator-prey model.

Rule i

IF $x_2(t)$ is M_i, then

$$\begin{cases} \dot{x}(t) = A_i x(t) & t \neq \tau_j, \\ \Delta x = K_{i,j} x(t) & t = \tau_j, \end{cases} \quad i = 1, 2, \quad j \in N, \tag{4.3}$$

where,

$$A_1 = \begin{bmatrix} \mu_1 - dr_{12} & 0 \\ dr_{21} & -\mu_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} \mu_1 - (1/2)dr_{12} & 0 \\ (1/2)dr_{21} & -\mu_2 \end{bmatrix}, \tag{4.4}$$

due to $x_2(t) \in [0, d] = [0, 0.12]$, and $M_1(x_2(t)) = x_2(t)/d$, $M_2(x_2(t)) = -x_2(t)/d$.

Let $\varepsilon = 1.2$, $\delta = 0.05$, $P = I$, $\mu_1 = 0.2$, $\mu_2 = 0.16$, $r_{12} = 0.10$, $r_{21} = 0.31$.
From Theorem 3.1 and Corollary 3.3, we can get that $\lambda(\alpha) = 0.194$.
Thus, we have chosen diag $([-0.82, \ -0.82])$ as impulsive control matrix, such that

$$\beta = \|I + K\| = 0.18, \qquad \ln(\varepsilon\beta) + \lambda(\alpha)\delta = -1.316 \leq 0. \tag{4.5}$$

Thus, from Theorem 3.1 and Corollary 3.3, we can conclude that the numerical example is globally stable. The phase portrait of the system with impulsive control is shown in Figure 1.

5. Conclusions

The impulsive control technique, which was proved to be suitable for complex and nonlinear system with impulsive effects, was applied to analyzing the framework of the fuzzy systems based on T-S model and the proposed design approach. First, the robustness of the Lotka-Volterra predator-prey system based on the fuzzy impulsive control was carefully analyzed. Then, the overall impulsive fuzzy system was obtained by blending local linear impulsive system. Meanwhile, the asymptotical stability and exponential stability of the impulsive fuzzy system were derived by Lyapunov method. Finally, a numerical example for predator-prey systems with impulsive effects was given to illustrate the application of impulsive fuzzy control. The simulation results show that the proposed method was effective.

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