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Research Article

On a Quasi-Neutral Approximation to the Incompressible Euler Equations

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We rigorously justify a singular Euler-Poisson approximation of the incompressible Euler equations in the quasi-neutral regime for plasma physics. Using the modulated energy estimates, the rate convergence of Euler-Poisson systems to the incompressible Euler equations is obtained.

1. Introduction

In this paper, we shall consider the following hydrodynamic system:

$$\begin{gathered}\partial_t n^\lambda + \operatorname{div}\left(n^\lambda \mathbf{u}^\lambda\right) = 0, \quad x \in \mathbb{T}^3,\ t > 0,\\ \partial_t \mathbf{u}^\lambda + \left(\mathbf{u}^\lambda \cdot \nabla\right)\mathbf{u}^\lambda = \nabla\phi^\lambda, \quad x \in \mathbb{T}^3,\ t > 0,\\ \Delta\phi^\lambda = \frac{n^\lambda - 1}{\lambda}, \quad x \in \mathbb{T}^3,\ t > 0\end{gathered} \tag{1.1}$$

for $x \in \mathbb{T}^3$ and $t > 0$, subject to the initial conditions

$$\left(n^\lambda, \mathbf{u}^\lambda\right)(t = 0) = \left(n_0^\lambda, \mathbf{u}_0^\lambda\right) \tag{1.2}$$

for $x \in \mathbb{T}^3$. In the above equations, $\mathbb{T}^3$ is 3-dimensional torus and $\lambda > 0$ is small parameter. Here n^λ, $\mathbf{u}^\lambda$, ϕ^λ denote the electron density, electron velocity, and the electrostatic potential, respectively.

System (1.1) is a model of a collisionless plasma where the ions are supposed to be at rest and create a neutralizing background field. Then the motion of the electrons can be described by using either the kinetic formalism or the hydrodynamic equations of conservation of mass and momentum as we do here. The self-induced electric field is the gradient of a potential that depends on the electron's density n^λ through the linear Poisson equation $\Delta\phi^\lambda = (n^\lambda - 1)/\lambda$.

To solve uniquely the Poisson equation, we add the condition $\int_{\mathbb{T}^3} n^\lambda dx = 1$. Passing to the limit when λ goes to zero, it is easy to see, at least at a very formal level, that $(n^\lambda, \mathbf{u}^\lambda, \phi^\lambda)$ tends to $(n^I, \mathbf{u}^I, \phi^I)$, where $n^I = 1$ and

$$\begin{gathered}\partial_t \mathbf{u}^I + \left(\mathbf{u}^I \cdot \nabla\right)\mathbf{u}^I = \nabla\phi^I, \\ \operatorname{div} \mathbf{u}^I = 0.\end{gathered} \tag{1.3}$$

In other words, $\mathbf{u}^I$ is a solution of the incompressible Euler equations. The aim of this paper is to give a rigorous justification to this formal computation. We shall prove the following result.

Theorem 1.1. *Let $\mathbf{u}^I$ be a solution of the incompressible Euler equations* (1.3) *such that $\mathbf{u}^I \in ([0,T], H^{s+3}(\mathbb{T}^3))$ and $\int_{\mathbb{T}^3} \mathbf{u}^I dx = 0$ for $s > 5/2$. Assume that the initial value $(n_0^\lambda, \mathbf{u}_0^\lambda) \in H^{s+1}$ is such that*

$$\begin{gathered}\int_{\mathbb{T}^3} n_0^\lambda dx = 1, \\ \int_{\mathbb{T}^3} \mathbf{u}_0^\lambda dx = 0, \\ M_s(\lambda) := \left\|\mathbf{u}_0^\lambda - \mathbf{u}_0^I\right\|_{H^{s+1}}^2 + \frac{1}{\lambda}\left\|n_0^\lambda - \lambda\Delta\phi_0^I(t) - 1\right\|_{H^s}^2 \longrightarrow 0 \quad (\textit{when } \lambda \longrightarrow 0),\ \mathbf{u}_0^I = \mathbf{u}^I|_{t=0}.\end{gathered} \tag{1.4}$$

Then, there exist λ_0 and C_T such that for $0 < \lambda \le \lambda_0$ there is a solution $(n^\lambda, \mathbf{u}^\lambda) \in ([0,T], H^{s+1}(\mathbb{T}^3))$ of (1.1) *satisfying*

$$\left\|\mathbf{u}^\lambda(t) - \mathbf{u}^I(t)\right\|_{H^{s+1}}^2 + \frac{1}{\lambda}\left\|n^\lambda(t) - \lambda\Delta\phi^I - 1\right\|_{H^s}^2 \le C_T(\lambda + M_s(\lambda)) \tag{1.5}$$

for any $0 \le t \le T$.

Concerning the quasi-neutral limit, there are some results for various specific models. In particular, this limit has been performed for the Vlasov-Poisson system [1, 2], for the drift-diffusion equations and the quantum drift-diffusion equations [3, 4], for the one-dimensional and isothermal Euler-Poisson system [5], for the multidimensional Euler-Poisson equations [6, 7], for the bipolar Euler-Poisson system [8, 9], for the Vlasov-Maxwell system [10], and for Euler-Maxwell equations [11]. We refer to [12–15] and references therein for more recent contributions.

The main focus in the present note is on the use of the modulated energy techniques for studying incompressible fluids. We will mostly restrict ourselves to the case of well-prepared initial data. Our result gives a more general rate of convergence in strong H^s norm

of the solution of the singular system towards a smooth solution of the incompressible Euler equation. We noticed that the quasi-neutral limit with pressure is treated in [5, 6]. But the techniques used there do not apply here.

It should be pointed that the model that we considered is a collisionless plasma while the model in [6, 7] includes the pressure. Our proof is based on the modulated energy estimates and the curl-div decomposition of the gradient while the proof in [6, 7] is based on formal asymptotic expansions and iterative methods. Meanwhile, the model that we considered in this paper is a different scaling from that of [16]. Furthermore, our convergence result is different from the convergence result in [16].

2. Proof of Theorem 1.1

First, let us set

$$(n, \mathbf{u}, \phi) = \left(n^{\lambda} - 1 - \lambda \Delta \phi^{I}, \mathbf{u}^{\lambda} - \mathbf{u}^{I}, \phi^{\lambda} - \phi^{I}\right). \tag{2.1}$$

Then, we know the vector $(n, \mathbf{u}, \phi)$ solves the system

$$\begin{gathered}
\partial_t \mathbf{u} + \left(\mathbf{u} + \mathbf{u}^{I}\right) \cdot \nabla \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u}^{I} = \nabla \phi, \\
\partial_t n + \left(\mathbf{u} + \mathbf{u}^{I}\right) \cdot \nabla n = -(n+1) \operatorname{div} \mathbf{u} - \lambda\left(\partial_t \Delta \phi^{I} + \operatorname{div}\left(\Delta \phi^{I}\left(\mathbf{u} + \mathbf{u}^{I}\right)\right)\right), \\
\Delta \phi = \frac{n}{\lambda},
\end{gathered} \tag{2.2}$$

where $\nabla \mathbf{u} : \nabla \mathbf{v} = \sum_{i,j=1}^{3} (\partial_{x_i} \mathbf{u} / \partial_{x_j})(\partial_{x_j} \mathbf{v} / \partial_{x_i})$. In fact, from (1.3), we get $\Delta \phi^{I} = \nabla \mathbf{u}^{I} : \nabla \mathbf{u}^{I}$.

As in [16], we make the following change of unknowns:

$$(d, \mathbf{c}) = (\operatorname{div} \mathbf{u}, \operatorname{curl} \mathbf{u}). \tag{2.3}$$

By using the last equation in (2.2), we get the following system:

$$\begin{gathered}
\partial_t d + \left(\mathbf{u} + \mathbf{u}^{I}\right) \cdot \nabla d = \frac{n}{\lambda} - \nabla\left(\mathbf{u} + 2\mathbf{u}^{I}\right) : \nabla \mathbf{u}, \\
\partial_t \mathbf{c} + \left(\mathbf{u} + \mathbf{u}^{I}\right) \cdot \nabla \mathbf{c} = \mathbf{c} \cdot \nabla\left(\mathbf{u} + \mathbf{u}^{I}\right) + \operatorname{curl}\left(\nabla \mathbf{u}^{I} \cdot \mathbf{u}\right) - d\mathbf{c} - \operatorname{curl}\left((\mathbf{u} \cdot \nabla)\mathbf{u}^{I}\right), \\
\partial_t n + \left(\mathbf{u} + \mathbf{u}^{I}\right) \cdot \nabla n = -(n+1)d - \lambda\left(\partial_t \Delta \phi^{I} + \operatorname{div}\left(\Delta \phi^{I}\left(\mathbf{u} + \mathbf{u}^{I}\right)\right)\right).
\end{gathered} \tag{2.4}$$

This last system can be written as a singular perturbation of a symmetrizable hyperbolic system:

$$\partial_t \mathbf{v} + \sum_{j=1}^{3} \left(\mathbf{u} + \mathbf{u}^{I}\right)^{j} \partial_{x_j} \mathbf{v} = \frac{1}{\lambda} \mathcal{K}^{\lambda} \mathbf{v} + \mathcal{L}(\mathbf{v}) + \mathcal{S}(\mathbf{v}) + \lambda \mathcal{R}(\mathbf{v}), \tag{2.5}$$

where $(\mathbf{u}+\mathbf{u}^I)^j$ denotes the ith component of $(\mathbf{u}+\mathbf{u}^I)$ and where

$$\mathbf{v}=\begin{pmatrix} d \\ \mathbf{c} \\ n \end{pmatrix}, \qquad \mathcal{K}^\lambda=\begin{pmatrix} 0 & \mathbf{0} & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\lambda & \mathbf{0} & 0 \end{pmatrix}, \qquad \mathcal{L}(\mathbf{v})=\begin{pmatrix} 0 \\ -d\mathbf{c} \\ -dn \end{pmatrix},$$
$$\mathcal{S}(\mathbf{v})=\begin{pmatrix} -\nabla\left(\mathbf{u}+2\mathbf{u}^I\right):\nabla\mathbf{u} \\ \mathbf{c}\cdot\nabla\left(\mathbf{u}+\mathbf{u}^I\right)+\operatorname{curl}\left(\nabla\mathbf{u}^I\cdot\mathbf{u}\right)-\operatorname{curl}\left((\mathbf{u}\cdot\nabla)\mathbf{u}^I\right) \\ 0 \end{pmatrix}, \tag{2.6}$$
$$\mathcal{R}(\mathbf{v})=\begin{pmatrix} 0 \\ \mathbf{0} \\ -\left(\partial_t\Delta\phi^I+\operatorname{div}\left(\Delta\phi^I\left(\mathbf{u}+\mathbf{u}^I\right)\right)\right) \end{pmatrix}.$$

Now, let us set $\mathcal{A}_0^\lambda=\begin{pmatrix} 1 & \mathbf{0} & 0 \\ \mathbf{0} & I_d & \mathbf{0} \\ 0 & \mathbf{0} & 1/\lambda \end{pmatrix}$ and for $|\alpha|\le s$ with $s>d/2$,

$$E_{\alpha,s}^\lambda(t)=\frac{1}{2}\left(\mathcal{A}_0^\lambda\partial_x^\alpha\mathbf{v},\partial_x^\alpha\mathbf{v}\right)=\frac{1}{2}\left(\|\partial_x^\alpha d\|^2+\|\partial_x^\alpha\mathbf{c}\|^2+\frac{1}{\lambda}\|\partial_x^\alpha n\|^2\right),$$
$$E_s^\lambda(t)=\sum_{|\alpha|\le s}E_{\alpha,s}^\lambda(t). \tag{2.7}$$

It is easy to know that system (2.5) is a hyperbolic system. Consequently, for $\lambda>0$ fixed, we have a result of local existence and uniqueness of strong solutions in $C([0,T],H^s)$, see [17]. This allows us to define T^λ as the largest time such that

$$E_s^\lambda(t)\le M_\lambda, \quad \forall t\in\left[0,T^\lambda\right], \tag{2.8}$$

where M_λ which is such that $M_\lambda\to 0$ when λ goes to zero will be chosen carefully later. To achieve the proof of Theorem 1.1, and in particular inequality (1.5), it is sufficient to establish that $T^\lambda\ge T$, which will be proved by showing that in (2.8) the equality cannot be reached for $T^\lambda<T$ thanks to a good choice of M_λ.

Before performing the energy estimate, we apply the operator ∂_x^α for $\alpha\in\mathbb{N}^3$ with $|\alpha|\le s$ to (2.5), to obtain

$$\partial_t\partial_x^\alpha\mathbf{v}+\sum_{j=1}^3\left(\mathbf{u}+\mathbf{u}^I\right)^j\partial_{x_j}\partial_x^\alpha\mathbf{v}=\frac{1}{\lambda}\mathcal{K}^\lambda\partial_x^\alpha\mathbf{v}+\partial_x^\alpha\mathcal{L}(\mathbf{v})+\partial_x^\alpha\mathcal{S}(\mathbf{v})+\lambda\partial_x^\alpha\mathcal{R}(\mathbf{v})+\Sigma_\alpha^\lambda, \tag{2.9}$$

where

$$\Sigma_\alpha^\lambda=\sum_{j=1}^3\left[\left(\mathbf{u}+\mathbf{u}^I\right)^j\partial_{x_j}\partial_x^\alpha\mathbf{v}-\partial_x^\alpha\left(\left(\mathbf{u}+\mathbf{u}^I\right)^j\partial_{x_j}\mathbf{v}\right)\right]. \tag{2.10}$$

Along the proof, we shall denote by C a number independent of λ, which actually may change from line to line, and by $C(\cdot)$ a nondecreasing function. Moreover $(\cdot,\cdot)$ and $\|\cdot\|$ stand

for the usual L^2 scalar product and norm, $\|\cdot\|_s$ is the usual H^s Sobolev norm, and $\|\cdot\|_{s,\infty}$ is the usual $W^{s,\infty}$ norm.

Now, we proceed to perform the energy estimates for (2.9) in a classical way by taking the scalar product of system (2.9) with $\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}$. Then, we have

$$\begin{aligned}\frac{d}{dt}E_{\alpha,s}^\lambda(t) &= -\left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \sum_{j=1}^{3}\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j}\partial_x^\alpha \mathbf{v}\right)\\ &\quad + \frac{1}{\lambda}\left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \mathcal{K}^\lambda \partial_x^\alpha \mathbf{v}\right) + \left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \partial_x^\alpha \mathcal{L}(\mathbf{v})\right)\\ &\quad + \left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \partial_x^\alpha \mathcal{S}(\mathbf{u})\right) + \lambda\left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \partial_x^\alpha \mathcal{R}(\mathbf{u})\right) + \left(\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \Sigma_\alpha^\lambda\right)\\ &= \sum_{i=1}^{6}\mathcal{J}_i.\end{aligned} \tag{2.11}$$

Let us start the estimate of each term in the above equation. For $\mathcal{J}_1$, since $\mathcal{A}_0^\lambda$ is symmetric and $\operatorname{div}\mathbf{u}^I = 0$, by Cauchy-Schwartz's inequality and Sobolev's lemma, we have that

$$\mathcal{J}_1 = \frac{1}{2}\left(\operatorname{div}\mathbf{u}\mathcal{A}_0^\lambda \partial_x^\alpha \mathbf{v}, \partial_x^\alpha \mathbf{v}\right) \le \|\operatorname{div}\mathbf{u}\|_{0,\infty} E_s^\lambda(t) \le C\left(E_s^\lambda(t)\right)^{3/2}. \tag{2.12}$$

Next, since $\mathcal{A}_0^\lambda \mathcal{K}^\lambda$ is skew-symmetric, we have that

$$\mathcal{J}_2 = 0. \tag{2.13}$$

For $\mathcal{J}_3$, by a direct calculation, one gets

$$\begin{aligned}\mathcal{J}_3 &= -(\partial_x^\alpha \mathbf{c}, \partial_x^\alpha(d\mathbf{c})) - \frac{1}{\lambda}(n, \partial_x^\alpha(dn))\\ &\le \|\partial_x^\alpha \mathbf{c}\|\|\partial_x^\alpha(d\mathbf{c})\| + \frac{1}{\lambda}\|\partial_x^\alpha n\|^2\|\partial_x^\alpha d\|\\ &\le C\left(E_s^\lambda(t)\right)^{3/2}.\end{aligned} \tag{2.14}$$

Here, we have used the basic Moser-type calculus inequalities [18].

To give the estimate of the term $\mathcal{J}_4$, we split it in two terms. Specifically, we can deduce that

$$\begin{aligned}\mathcal{J}_4 &= -\left(\partial_x^\alpha d, \partial_x^\alpha\left(\nabla\left(\mathbf{u}+2\mathbf{u}^I\right) : \nabla\mathbf{u}\right)\right)\\ &\quad + \left(\partial_x^\alpha \mathbf{c}, \partial_x^\alpha\left(\mathbf{c}\cdot\nabla\left(\mathbf{u}+\mathbf{u}^I\right) + \operatorname{curl}\left(\nabla\mathbf{u}^I\cdot\mathbf{u}\right) - \operatorname{curl}\left((\mathbf{u}\cdot\nabla)\mathbf{u}^I\right)\right)\right)\end{aligned}$$

$$
\begin{aligned}
&\leq \|d\|_s \left\| \nabla\left(\mathbf{u}+2\mathbf{u}^I\right) : \nabla \mathbf{u} \right\|_s \\
&\quad + \|\mathbf{c}\|_s \left(\left\| \mathbf{c}\cdot\nabla\left(\mathbf{u}+\mathbf{u}^I\right)\right\|_s + \left\|\operatorname{curl}\left(\nabla\mathbf{u}^I\cdot\mathbf{u}\right)\right\|_s + \left\|\operatorname{curl}\left((\mathbf{u}\cdot\nabla)\mathbf{u}^I\right)\right\|_s \right) \\
&\leq C\left(E_s^\lambda + \left(E_s^\lambda\right)^{3/2}\right).
\end{aligned}
\tag{2.15}
$$

Here, we have used the curl-div decomposition inequality

$$
\|\nabla\mathbf{u}\|_s \leq C(\|d\|_s + \|\mathbf{c}\|_s). \tag{2.16}
$$

For $\mathcal{J}_5$, we have that

$$
\begin{aligned}
\mathcal{J}_5 &\leq \|n\|_s \left\| \partial_t \Delta\phi^I + \operatorname{div}\left(\Delta\phi^I\left(\mathbf{u}+\mathbf{u}^I\right)\right)\right\|_s \\
&\leq C\|n\|_s(1+\|d\|_s+\|\mathbf{c}\|_s) \\
&\leq C\lambda + E_s^\lambda.
\end{aligned}
\tag{2.17}
$$

To estimate the last term, that is, $\mathcal{J}_5$, by using basic Moser-type calculus inequalities and Sobolev's lemma, we have

$$
\begin{aligned}
\mathcal{J}_6 &= \left(\partial_x^\alpha d, \sum_{j=1}^3 \left[\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j}\partial_x^\alpha d - \partial_x^\alpha\left(\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j} d\right)\right]\right) \\
&\quad + \left(\partial_x^\alpha \mathbf{c}, \sum_{j=1}^3 \left[\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j}\partial_x^\alpha \mathbf{c} - \partial_x^\alpha\left(\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j} \mathbf{c}\right)\right]\right) \\
&\quad + \frac{1}{\lambda}\left(\partial_x^\alpha n, \sum_{j=1}^3 \left[\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j}\partial_x^\alpha n - \partial_x^\alpha\left(\left(\mathbf{u}+\mathbf{u}^I\right)^j \partial_{x_j} n\right)\right]\right) \\
&\leq C\|d\|_s \left(\left\|\nabla\left(\mathbf{u}+\mathbf{u}^I\right)\right\|_{0,\infty} \|\partial_x^\alpha d\| + \|\nabla d\|_{0,\infty} \left\|\partial_x^\alpha\left(\mathbf{u}+\mathbf{u}^I\right)\right\|\right) \\
&\quad + C\|\mathbf{c}\|_s \left(\left\|\nabla\left(\mathbf{u}+\mathbf{u}^I\right)\right\|_{0,\infty} \|\partial_x^\alpha \mathbf{c}\| + \|\nabla \mathbf{c}\|_{0,\infty} \left\|\partial_x^\alpha\left(\mathbf{u}+\mathbf{u}^I\right)\right\|\right) \\
&\quad + \frac{C}{\lambda}\|n\|_s \left(\left\|\nabla\left(\mathbf{u}+\mathbf{u}^I\right)\right\|_{0,\infty} \|\partial_x^\alpha n\| + \|\nabla n\|_{0,\infty} \left\|\partial_x^\alpha\left(\mathbf{u}+\mathbf{u}^I\right)\right\|\right) \\
&\leq C\left(\|d\|_s^2 + \|\mathbf{c}\|_s^2 + \frac{1}{\lambda}\|n\|_s^2\right)(\|d\|_s + \|\mathbf{c}\|_s + 1) \\
&\leq C\left(E_s^\lambda + \left(E_s^\lambda\right)^{3/2}\right).
\end{aligned}
\tag{2.18}
$$

Now, we collect all the previous estimates (2.12)–(2.18) and we sum over α to find

$$
\frac{d}{dt}E_s^\lambda \leq C\lambda + CE_s^\lambda + \left(E_s^\lambda\right)^2. \tag{2.19}
$$

By using (2.8), we get with $M_\lambda \ll 1$ that

$$\frac{d}{dt}E_s^\lambda \leq C\lambda + CE_s^\lambda, \quad \forall t \in \left[0, T^\lambda\right]. \tag{2.20}$$

Hence, by the Gronwall inequality, we get that

$$E_s^\lambda(t) \leq (M_s(\lambda) + Ct\lambda)e^{Ct}, \quad \forall t \in \left[0, T^\lambda\right]. \tag{2.21}$$

Consequently, if we choose $M_\lambda = (M_s(\lambda) + Ct\lambda)^{1/2}$, we see that we cannot reach equality in (2.8) for $T^\lambda < T$. This proves that $T^\lambda > T$ and that (2.21) is valid on $[0, T]$.

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References

[1] Y. Brenier, "Convergence of the Vlasov-Poisson system to the incompressible Euler equations," *Communications in Partial Differential Equations*, vol. 25, no. 3-4, pp. 737–754, 2000.
[2] E. Grenier, "Oscillations in quasineutral plasmas," *Communications in Partial Differential Equations*, vol. 21, no. 3-4, pp. 363–394, 1996.
[3] A. Jüngel and Y. J. Peng, "A hierarchy of hydrodynamic models for plasmas: quasineutral limits in the drift-diffusion equations," *Asymptotic Analysis*, vol. 28, no. 1, pp. 49–73, 2001.
[4] A. Jüngel and I. Violet, "The quasineutral limit in the quantum drift-diffusion equations," *Asymptotic Analysis*, vol. 53, no. 3, pp. 139–157, 2007.
[5] S. Cordier and E. Grenier, "Quasineutral limit of an Euler-Poisson system arising from plasma physics," *Communications in Partial Differential Equations*, vol. 25, no. 5-6, pp. 1099–1113, 2000.
[6] S. Wang, "Quasineutral limit of Euler-Poisson system with and without viscosity," *Communications in Partial Differential Equations*, vol. 29, no. 3-4, pp. 419–456, 2004.
[7] Y. J. Peng and Y. G. Wang, "Convergence of compressible Euler-Poisson equations to incompressible type Euler equations," *Asymptotic Analysis*, vol. 41, no. 2, pp. 141–160, 2005.
[8] S. Jiang, Q. Ju, H. Li, and Y. Li, "Quasi-neutral limit of the full bipolar Euler-Poisson system," *Science China. Mathematics*, vol. 53, no. 12, pp. 3099–3114, 2010.
[9] Q. Ju, H. Li, Y. Li, and S. Jiang, "Quasi-neutral limit of the two-fluid Euler-Poisson system," *Communications on Pure and Applied Analysis*, vol. 9, no. 6, pp. 1577–1590, 2010.
[10] M. Puel and L. Saint-Raymond, "Quasineutral limit for the relativistic Vlasov-Maxwell system," *Asymptotic Analysis*, vol. 40, no. 3-4, pp. 303–352, 2004.
[11] Y. J. Peng and S. Wang, "Convergence of compressible Euler-Maxwell equations to incompressible Euler equations," *Communications in Partial Differential Equations*, vol. 33, no. 1–3, pp. 349–376, 2008.
[12] S. Wang and S. Jiang, "The convergence of the Navier-Stokes-Poisson system to the incompressible Euler equations," *Communications in Partial Differential Equations*, vol. 31, no. 4–6, pp. 571–591, 2006.
[13] E. Feireisl and P. Zhang, "Quasi-neutral limit for a model of viscous plasma," *Archive for Rational Mechanics and Analysis*, vol. 197, no. 1, pp. 271–295, 2010.
[14] M. H. Vignal, "A boundary layer problem for an asymptotic preserving scheme in the quasi-neutral limit for the Euler-Poisson system," *SIAM Journal on Applied Mathematics*, vol. 70, no. 6, pp. 1761–1787, 2010.
[15] R. Natalini and F. Rousset, "Convergence of a singular Euler-Poisson approximation of the incompressible Navier-Stokes equations," *Proceedings of the American Mathematical Society*, vol. 134, no. 8, pp. 2251–2258, 2006.

[16] G. Loeper, "Quasi-neutral limit of the Euler-Poisson and Euler-Monge-Ampère systems," *Communications in Partial Differential Equations*, vol. 30, no. 7–9, pp. 1141–1167, 2005.
[17] M. E. Taylor, *Partial Differential Equations. III. Nonlinear Equations*, Springer, New York, NY, USA, 1997.
[18] A. Majda, *Compressible Fluid Flow and Systems of Conservation Laws in Several Space Variables*, Springer, New-York, NY, USA, 1984.