Research Article
New Exact Jacobi Elliptic Function Solutions for the Coupled Schrödinger-Boussinesq Equations

Baojian Hong1,2 and Dianchen Lu1

1 Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, China
2 Department of Basic Courses, Nanjing Institute of Technology, Nanjing 211167, China

Correspondence should be addressed to Dianchen Lu; dclu@ujs.edu.cn

Received 18 July 2013; Accepted 2 September 2013

Copyright © 2013 B. Hong and D. Lu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

A general algebraic method based on the generalized Jacobi elliptic functions expansion method, the improved general mapping deformation method, and the extended auxiliary function method with computerized symbolic computation is proposed to construct more new exact solutions for coupled Schrödinger-Boussinesq equations. As a result, several families of new generalized Jacobi elliptic function wave solutions are obtained by using this method, some of them are degenerated to solitary wave solutions and trigonometric function solutions in the limited cases, which shows that the general method is more powerful than plenty of traditional methods and will be used in further works to establish more entirely new solutions for other kinds of nonlinear partial differential equations arising in mathematical physics.

1. Introduction
Nonlinear partial differential equations (NLPDEs) are widely used to describe complex physical phenomena arising in the world around us and various fields of science. The investigation of exact solutions of NLPDEs plays an important role in the study of these phenomena such as the nonlinear dynamics and the mechanism behind the phenomena. With the development of soliton theory, many powerful methods for obtaining exact solutions of NLPDEs have been presented, such as homotopy perturbation method [1], nonperturbative method [2], homogeneous balance method [3], Bäcklund transformation [4], Darboux transformation [5], extended tanh-function method [6], extended $F$-expansion method [7], $G'/G$ method [8], exp-function method [9], sine-cosine method [10], Jacobi elliptic function method [11], extended Riccati equation rational expansion method [12], extended auxiliary function method [13], and other methods [14, 15].

In [16, 17], Hong proposed a generalized Jacobi elliptic functions expansion method to obtain generalized exact solutions of NLPDEs. In [18], Hong and Lu proposed an improved general mapping deformation method which is more general than many other algebraic expansion methods [19, 20]. The solution procedure of this method, by the help of Matlab or Mathematica, is of utmost simplicity, and this method can be easily extended to all kinds of NLPDEs. In this work, we will propose the general algebraic method which contained the two methods [16–18] to obtain several new families of exact solutions for the coupled Schrödinger-Boussinesq equations.

2. Summary of the General Algebraic Method
Consider a given nonlinear evolution equation with one physical field $u(x,t)$ in two variables $x$ and $t$

$$P (u, u_t, u_{xx}, u_{xxx}, \ldots ) = 0. \quad (1)$$

We seek the following formal solutions of the given system by a new intermediate transformation:

$$u (\xi ) = \sum_{i=0}^{n} A_i \phi^i (\xi ), \quad (2)$$

where $A_i$ are constants to be determined later. $\xi = \xi (x,t)$ are arbitrary functions with the variables $x$ and $t$. The parameter
$n$ can be determined by balancing the highest order derivative terms with the nonlinear terms in (1). And $\phi(\xi)$ is a solution of the following ordinary differential equation:

$$
\phi^{(n)}(\xi) = \sum_{i=0}^{n} a_i \phi^i(\xi).
$$

(3)

Substituting (3) and (2) into (1) and setting the coefficients of $\phi^{(i)}(\xi)$ ($i = 0, 1, 2, \ldots$) and $\phi^{(j)}(\xi) \sqrt{\sum_{i=0}^{n} a_i \phi^i(\xi)}$ ($j = \ldots, -2, -1, 0, 1, 2, \ldots$) to zero yield a set of algebraic equations for $A_i, a_i$, and $\xi$. Using the Mathematica to solve the algebraic equations and substituting each of the solutions of the set, that is, each of the expressions of $\phi(\xi)$ into (2), we can get the solutions of (1). In order to obtain some new general solutions of (3), we assume that (3) has the following solutions:

$$
\phi(\xi) = c_0 + c_1 e(\xi) + c_2 f(\xi) + c_3 g(\xi) + c_4 h(\xi),
$$

(4)

where $c_i = c_i(t)$ ($i = 0, \ldots, 4$) are functions of $t$ to be determined later and the four functions $e = e(\xi)$, $f = f(\xi)$, $g = g(\xi)$, and $h = h(\xi)$ are expressed as follows:

$$
e = \frac{1}{p + qF + rF^2 + Ef^2},
f = \frac{F}{p + qF + rF^2 + Ef^2},
g = \frac{F^2}{p + qF + rF^2 + Ef^2},
h = \frac{F^2}{p + qF + rF^2 + Ef^2},
$$

(5)

where $p, q, r$, and $l$ are arbitrary constants which ensure the denominator unequal to zero, and $F = F(\xi)$ is a solution of the following equations:

$$F'' = A + BF + CF^2 + 2DF + 2EF^3,
F''' = BF + 2CF^3 + D + 3EF^2,
$$

(6)

where $''''$ denotes $d^4/d\xi^4$, $'''''$ denotes $d^5/d\xi^5$, $A, B, C, D$, and $E$ are arbitrary constants, and $e, f, g$, and $h$ satisfy the following relations:

$$
e' = -2eh - 2rfh - l \big( De^2 + Bef + 2Cfg + 3Ef^2 \big),
f' = peh - rgh + l \big( Ae^2 + Def - Cg^2 - Ef^2 \big),
g' = qgh + 2pfh + l \big( 2Aef + 3DF^2 + Bfg + Eg^2 \big),
h' = (Dp - Aq) e^2 + (Bp - Dq - 2Ar) ef
+ (2Cp + Eq - Br) fg + 3(Ep - Dr) f^2 + (Cq - Er) g^2,
f^2 = eg,
h^2 = Ae^2 + Bf^2 + Cg^2 + 2Def + 2Efg,
pe + qf + rg + lh = 1.
$$

(7)

And $e, f, g$, and $h$ satisfy one of the following relations at the same time.

**Family 1. When $p = 0$**

$$
(\phi^2 - r^2) h^2 = -C + 2Ch - Br(1 - lh - qf)e
- Ae^2 r^2 - 2Df^2 ef + (2Cq - 2Er) f
+ (2Er - 2Cf) h + (2Efr - Cg^2) f^2.
$$

(8a)

**Family 2. When $q = 0$**

$$
(\phi^2 - r^2) h^2 = 2C (lh + pe - pleh) + 2Er (lh - 1) f
- Br (1 - lh - pe) e - (Cp^2 + Ar^2) e^2
+ (2Ep r - 2Dr^2) ef - C.
$$

(8b)

**Family 3. When $r = 0$**

$$
\phi^2 g^2 = 1 - 2Eff g - 2pe + (p^2 - Al^2) e^2
- 2qf + 2 \big( pq - Dl^2 \big) ef + (q^2 - Bl^2) eg.
$$

(8c)

**Family 4. When $l = 0$**

$$
r^2 h^2 = C - 2Cpe + (2Er - 2Cf) f + (Cp^2 + Ar^2) e^2
+ (Cq^2 - 2Eq r - Br^2) eg
+ (2Cpq - 2Epr + 2Dr^2) ef.
$$

(8d)

Substituting (4), (7) along with (8a)–(8d) into (3) separately yields four families of polynomial equations for $e, f, g$, and $h$. Setting the coefficients of $e', e^2, e^3, e^4, e^5, \phi(\xi)$, and $\phi^{(j)}(\xi)$ to zero yields a set of overdetermined differential equations (ODEs) in $p, q, r, l, a_i$, and $c_i$ ($i = 0, 1, 2, 3, 4$), $A, B, C, D, E$, and $\xi(x, t)$, solving the ODEs by Mathematica and Wu elimination, we can obtain many exact solutions of (3) according to (4), (5), (6). If we let $c_0 = c_1 = c_2 = c_4 = 0, c_3 = 1, p = 1, q = r = l = 0, a_0 = A, a_1 = 2D, a_2 = B, a_3 = 2E, a_4 = C$, we have $\phi(\xi) = F(\xi)$; our method contains the improved general mapping deformation method [18].

**Remark 1.** Our method proposed here is more general than the $G'/G$ method [8], the extended Riccati equation rational expansion method [12], the extended auxiliary function method [13], the generalized Jacobi elliptic functions expansion method [16, 17], and many other algebra expansion methods [6, 7, 11, 18–21].
Remark 2. Equations (2) and (3) can be extended to the following forms:

\[ u(ξ) = \sum_{i=0}^{N} A_i(t) \phi^i(ξ) + \sum_{i=-n}^{N} A_{n-i}(t) \phi^{i}(ξ) \]

\[ + \sum_{i=n}^{N} B_i(t) \phi^i(ξ) \phi'(ξ), \quad (9) \]

\[ \phi'^2(ξ) = \sum_{i=0}^{N} a_i(t) \phi^i(ξ), \]

where \( N \) can be an arbitrary positive integer. \( n \) is usually a positive integer. If \( n \) is a fraction or a negative integer, we make the following transformation:

(a) when \( n = \frac{d}{c} \) is a fraction, we let \( u(ξ) = v^{d/c}(ξ) \), then return to determine the balance constant \( n \) again;

(b) when \( n \) is a negative integer, we suppose \( u(ξ) = v^n(ξ) \), then return to determine the balance constant \( n \) again.

Remark 3. Notice that

\[ F_1(ξ)|_{(A,B,C,D,E)} \rightarrow \phi_1(ξ)|_{(a_0,a_1,a_2,a_3,a_4)} \]

\[ = F_2(ξ)|_{(A,B,C,D,E)} \rightarrow \phi_2(ξ)|_{(a_0,a_1,a_2,a_3,a_4)} \]

\[ = F_3(ξ)|_{(A,B,C,D,E)} \rightarrow \cdots \quad (10) \]

We find a meaningful conclusion that this general method implies a BT of (1) with the compatible conditions (4), (5), (6), (7), and (8a)–(8d).

In the following, we will use this method to solve the Schrödinger-Boussinesq equations.

### 3. Exact Solutions to the Coupled Schrödinger-Boussinesq Equations

We consider the coupled Schrödinger-Boussinesq equations [21–28]

\[ iE_t + E_{xx} + \alpha E - NE = 0, \]

\[ 3N_{tx} - N_{xxxx} + 3(\mathcal{N}^2)_{xx} + \beta N_{xx} - (|E|^2)_{xx} = 0. \quad (11) \]

These equations are known to describe various physical processes in laser and plasma, such as formation, Langmuir field amplitude, intense electromagnetic waves, and modulational instabilities [22–25]. The problem of the complete integrability of this system has been studied by Chowdhury et al. from the point of view of Painlevé analysis [25]. The solitary wave solutions for system (11) have been obtained in [26, 27]. The Jacobi doubly periodic wave solutions and a range of other solutions for this system have been investigated in [21, 28]. We are interested in searching new generalized Jacobi elliptic function solutions for (11) by using our method.

We consider the following transformations:

\[ E(x, t) = E(ξ) e^{(kx + at + \xi_0)}, \quad (12) \]

\[ N(x, t) = N(ξ) = ν(ξ), \]

\[ ξ = ax - 2akt + ξ_0. \quad (13) \]

where \( a \) and \( k \) are constants to be determined later and \( ξ_0 \) and \( ξ_{00} \) are arbitrary constants.

Substituting (12) and (13) into (11) and integrating the second equation of system (11) twice, we have

\[ a^2 u'' + (\alpha - \omega - k^2) u - uv = 0, \quad (14) \]

\[ a^2 ν'' + u^2 - (12k^2 + \beta) ν - 3ν^2 = 0. \]

By balancing the highest derivative term with the nonlinear terms in (14), we obtain \( n = 2 \). Therefore, we assume that (14) have the following solutions:

\[ u(ξ) = A_0 + A_1φ(ξ) + A_2φ^2(ξ), \]

\[ ν(ξ) = B_0 + B_1φ(ξ) + B_2φ^2(ξ). \quad (15) \]

Substituting (3), (13), and (15) into (14) and setting the coefficients of \( φ(ξ) \) \((i = 0, 1, 2, \ldots) \) and \( φ^2(ξ) \sqrt{\sum_{i=0}^{4} a_iφ^i(ξ)} \((j = \ldots, -2, -1, 0, 1, 2, \ldots) \) to zero yield a set of over-determined equations (ODEs) for \( A_i, \) \( k, \) \( ω, \) and \( a_i \). After solving the ODEs by Mathematica, we could determine the following solutions.

**Family 1.**

\[ A_0 = \pm a^2 \frac{\sqrt{3a_1a_3} - 8a_1a_4}{2}, \]

\[ A_1 = \pm \frac{2a^2a_4}{a_3} \sqrt{-2a_1a_3 - 16a_0a_4}, \quad A_2 = 0, \]

\[ B_0 = 0, \quad B_1 = a^2 a_3, \quad B_2 = 2a^2 a_4, \]

\[ ω = -\frac{a^2}{48a_1^2 a_4} \frac{(a_1^4 - 80a_1a_3a_4^2 - 128a_0a_4^2)}{3a_4^2} + \frac{a + \beta}{12}, \quad (16) \]

\[ k = \frac{e}{4} \left[a^2 \left(a_1^4 + 16a_1a_3a_4^2 - 128a_0a_4^2\right) - 4a_1^2a_4^2\beta\right] \frac{3a_4^2}{3a_1^2a_4}. \]

\[ e^2 = 1, \quad a_2 = \frac{a^2}{4a_4} + \frac{2a_1a_4}{a_3}, \quad (17) \]

**Family 2.**

\[ A_0 = \pm \sqrt{2} B_0, \quad A_1 = 0, \quad A_2 = \pm 6 \sqrt{2} a^2 a_4, \]

\[ B_0 = \text{const}, \quad B_1 = 0, \quad B_2 = 6a^2 a_4, \]

\[ a_1 = a_3 = 0, \quad ω = \frac{1}{12} \left(44a_1^2 a_4^2 - 22B_0 + 12a + \beta\right), \quad (17) \]

\[ k = \frac{e}{4} \left[4a_1^2 a_4^2 - 2B_0 - \beta\right], \quad e^2 = 1. \]
Substituting (4), (5), (6), (7) along with (8a)–(8d) and (16) into (14) separately yields an ODEs; after solving the ODEs by Mathematica and Wu elimination, we can obtain the following solutions of (14) according to (4), (5), (6), (16).

**Case 1.**

\[ A = a_0 = \frac{1}{4} \left( 1 - m^2 \right), \quad B = a_1 = 2 - m^2, \]
\[ C = a_2 = 1, \quad D = \frac{1}{2} a_3 = \frac{1}{2} (m^2 - 1), \]
\[ E = \frac{1}{2} a_4 = -1, \quad 0 \leq m \leq 1, \]
\[ p = 1, \quad q = r = l = 0, \]
\[ c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_1 (\xi_1) = F_1 (\xi_1) = \frac{\cn \xi_1}{\pm 1 \pm \sn \xi_1 + \cn \xi_1}, \quad \xi_1 = ax - 2ak_1t + \xi_0. \]

**Case 2.**

\[ A = a_0 = \frac{1}{4}, \quad B = a_1 = 2 - m^2, \]
\[ C = a_2 = 1 - m^2, \quad D = \frac{1}{2} a_3 = \frac{1}{2}, \]
\[ E = \frac{1}{2} a_4 = 1 - m^2, \quad 0 \leq m \leq 1, \]
\[ p = 1, \quad q = r = l = 0, \]
\[ c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_2 (\xi_2) = F_2 (\xi_2) = \frac{\sn \xi_2}{\pm 1 \pm \sn \xi_2 + \cn \xi_2}, \quad \xi_2 = ax - 2ak_2t + \xi_0. \]

**Case 3.**

\[ A = a_0 = -\left( c_0^2 - 1 \right) \left( 1 + (c_0^2 - 1) m^2 \right), \]
\[ B = a_1 = \left( 2 - 6c_0^2 \right) m^2 - 1, \quad C = a_2 = -m^2, \]
\[ D = \frac{1}{2} a_3 = c_0 - 2c_0^2 m^2 + 2c_0^3 m^2, \]
\[ E = \frac{1}{2} a_4 = 2c_0 m^2, \quad 0 \leq m \leq 1, \]
\[ p = 1, \quad q = r = l = 0, \]
\[ c_0 = \text{const}, \quad c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_3 (\xi_3) = F_3 (\xi_3) = c_0 + \cn ^2 \xi_3, \quad \xi_3 = ax - 2ak_3t + \xi_0. \]
Figure 3: The modulus of $E_6$ when $\alpha = 1, \beta = -1, \xi_0 = 0$, and $m = 0.9$ and a plane graph when $t = 0$.

Case 4.

\[ A = 1, \quad B = -m^2 - 1, \quad C = m^2, \]
\[ D = E = 0, \quad F = \text{sn} \xi, \quad 0 \leq m \leq 1, \]
\[ a_0_4 = 1, \quad a_1_4 = \pm 4 \sqrt{2(1 + m) \sqrt{m}}, \]
\[ a_2_4 = 12(1 + m) \sqrt{m} - 6m - m^2 - 1, \]
\[ a_3_4 = 8m(m + 1)^2 - 2(1 + m)(1 + m^2 + 6m) \sqrt{m}, \]
\[ p = 1, \quad q = \pm \sqrt{2(1 + m) \sqrt{m}}, \quad r = m, \]
\[ l = 0, \quad c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_4(\xi_4) = \text{sn} \xi_4 \frac{\text{sn} \xi_4 + \text{sn} \xi_4 \sqrt{\text{dn} \xi_4}}{1 + \sqrt{2(1 + m) \sqrt{m} \text{sn} \xi_4 + \text{sn} \xi_4 \sqrt{\text{dn} \xi_4}}}, \]
\[ \xi_4 = ax - 2ak_4t + \xi_0. \]  

(21)

Case 5.

\[ A = 1 - m^2, \quad B = 2m^2 - 1, \]
\[ C = -m^2, \quad D = E = 0, \quad 0 \leq m \leq 1, \]
\[ F = \text{cn} \xi, \quad a_0_5 = 1, \quad a_1_5 = -4, \]
\[ a_2_5 = 8 - 4m^2, \quad a_3_5 = 8m^2 - 8, \quad a_4_5 = 4 - 4m^2, \]
\[ p = 0, \quad q = 1, \quad r = 0, \quad l = \pm 1, \]
\[ c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_5(\xi_5) = \text{cn} \xi_5 \frac{\text{sn} \xi_5 + \text{cn} \xi_5 \sqrt{\text{sn} \xi_5}}{\text{cn} \xi_5 + \text{sn} \xi_5 \sqrt{\text{sn} \xi_5}}, \]
\[ \xi_5 = ax - 2ak_5t + \xi_0. \]  

(22)

Case 6.

\[ A = m^2 - 1, \quad B = 2 - m^2, \quad C = -1, \]
\[ D = E = 0, \quad 0 \leq m \leq 1, \]
\[ F = \text{dn} \xi, \quad a_0_6 = 1, \quad a_1_6 = -4m, \]
\[ a_2_6 = 8m^2 - 4, \quad a_3_6 = 8m - 8m^3, \quad a_4_6 = 4m^4 - 4m^2, \]
\[ p = 0, \quad q = m, \quad r = 0, \quad l = \pm 1, \]
\[ c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1, \]
\[ \varphi_6(\xi_6) = \frac{\text{dn} \xi_6}{\text{dn} \xi_6 + m^2 \text{sn} \xi_6 \text{cn} \xi_6}, \quad \xi_6 = ax - 2ak_6t + \xi_0. \]  

(23)

Case 7.

\[ A = \frac{C_1C_3q - 5C_3^2q^3 + \varepsilon \left(C_1 + 3C_3q^2\right) \sqrt{C_3 \left(3C_3q^2 - 2C_1\right)}}{4C_3}, \]
\[ B = 0, \quad C = 0, \quad 2D = C_3, \quad 2E = C_3, \]
\[ F = \rho \left(\frac{\sqrt{C_3^2 \xi} - \frac{4C_1}{C_3}, -\frac{4M}{C_3}}{2}, \right), \]
\[ a_0_7 = 0, \quad a_1_7 = C_3, \quad a_2_7 = -3C_3q, \quad a_3_7 = C_1 + 3C_3q^2, \quad a_4_7 = \left(-3C_1C_3q - 9C_3^2q^3 \right) \left(C_1 + 3C_3q^2\right)^1 \left(3C_3q^2 - 2C_1\right) \times (4C_3)^{-1}, \]

(24)
\[
\varepsilon = \text{sgn} \left[ C_1 + 3C_3q^2 \right], \quad p = 0, \quad q = \text{const}, \quad r = 1, \quad l = 0, \quad c_0 = c_1 = c_3 = c_4 = 0, \quad c_2 = 1,
\]
\[
\varphi_7(\xi_7) = \frac{1}{q + \varphi \left( \left( \sqrt{C_3}/2 \right) \xi_7, -4C_1/C_3, -4M/C_3 \right)}.
\]
\[
E_i(x,t) = \begin{cases} \pm a^2 \sqrt{\frac{-a_1a_3 - 8a_0a_4}{2}} \\ \pm \frac{2a_1a_4}{a_3} \sqrt{-2a_1a_3 - 160a_4q_0(\xi_7)} \end{cases}
\times e^{i(kt_0 + t + \xi_0)}, \quad N_j(x,t) = a^2 a_3 q_0(\xi) + 2a^2 a_0 q_0^2(\xi), \quad \xi_i = ax - 2ak_i t + \xi_0, \quad (i = 1, \ldots, 7),
\]
where \( a \) is an arbitrary constant, \( a_0, a_1, a_2, a_3, \) and \( a_4 \) are defined as Cases 1–7, and \( \omega_i \) and \( k_i \) are defined as follows:
\[
\omega_i = -\frac{a^2 \left( a_4^2 - 80a_1a_3a_5 - 128a_0a_4^2 \right)}{48a_2^2a_4}, \quad k_i = \frac{e}{4} \sqrt{\frac{a^2 \left( a_3^2 + 16a_1a_2a_5 - 128a_0a_4^2 \right)}{3a_2^2a_4}}, \quad \varepsilon = 1, \quad (i = 1, \ldots, 7).
\]

With the similar process, substituting (4), (5), (6), and (7) along with (8a)–(8d) and (17) into (14) separately yields ODEs; after solving the ODEs by Mathematica and Wu elimination, we can obtain the following solutions of (11) according to (4), (5), (12), (13), (15), and (16) and Cases 1–7:
\[
E_j(x,t) = \begin{cases} \pm \sqrt{2}B_0 \pm 6 \sqrt{2}a^2 a_4 q_0^2(\xi_j) \end{cases} e^{i(\omega_j t + \xi_0)}, \quad (j = 0, 1, 2, \ldots), \quad N_j(x,t) = B_0 + 6a^2 a_4 q_0^2(\xi_j), \quad \xi_j = ax - 2ak_j t + \xi_0,
\]
\[
\omega_j = \frac{1}{12} \left( 44a^2 a_2 - 22B_0 + 12a + \beta \right), \quad k_j = \frac{e}{4} \sqrt{\frac{4a^2 a_2 - 22B_0 - \beta}{12}}, \quad \varepsilon = 1.
\]

Remark 4. If we let \( \alpha_{0,1} = 1 - m^2, \alpha_{2,1} = 2m^2 - 1, \alpha_{4,1} = -m^2, \alpha_{1,1} = \alpha_{3,1} = 0, \), \( \delta_{1,1} \) and \( \phi_{1,1}(\xi_{1,1}) = cn \xi_{1,1} \), then we have
\[
E_{1,1}(x,t) = \begin{cases} \pm \sqrt{2}B_0 \pm 6 \sqrt{2}a^2 m^2 \sin^2 \left( \xi_{1,1} \right) \end{cases} e^{i(\omega_j t + \xi_0)}, \quad N_{1,1}(x,t) = B_0 - 6a^2 m^2 \sin^2 \left( \xi_{1,1} \right), \quad \xi_{1,1} = ax - 2ak_{1,1} t + \xi_0,
\]
\[
\omega_{1,1} = \frac{1}{12} \left( 44a^2 \left( 2m^2 - 1 \right) - 22B_0 + 12a + \beta \right), \quad k_{1,1} = \frac{e}{4} \sqrt{\frac{4a^2 \left( 2m^2 - 1 \right) - 22B_0 - \beta}{12}}, \quad \varepsilon = 1.
\]

Remark 5. The seven types of explicit solutions we obtained here to (II) are not shown in the previous literature to our knowledge. They are new exact solutions of (II). Notice that \( sn \xi \to \tanh \xi, \) \( cn \xi \to \sech \xi, \) and \( dn \xi \to \sech \xi \) when the modulus \( m \to 1, \) and \( sn \xi \to \sin \xi, cn \xi \to \cos \xi, \) and \( dn \xi \to 1 \) when the modulus \( m \to 0. \) Solutions \( E_j, N_j, (i = 1, \ldots, 6) \) are degenerated to solitary wave solutions when the modulus \( m \to 1 \) and to triangular functions solutions when the modulus \( m \to 0. \)

We can give the numerical simulation of \( E_5 \) to show their physical properties (see Figures 1, 2, and 3).

4. Conclusion

In this paper, we succeed to propose a general algebraic method approach for finding new exact solutions of the nonlinear evolution equations. By using this method and computerized symbolic computation, we have found abundant new exact solutions for the coupled Schrödinger-Boussinesq equations (II). More importantly, our method is much simpler and powerful to find new solutions to various kinds of nonlinear evolution equations, such as KdV equation, Boussinesq equation, and Zakharov equation. We believe that this method should play an important role for finding exact solutions in the mathematical physics.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 61072031), the Outstanding Personal Program in Six Fields of Jiangsu (Grant no. 2009188), the Graduate Student Innovation Project of Jiangsu Province (Grant no. CXLX13_673), and the General Program of Innovation Foundation of Nanjing Institute of Technology (Grant no. CKJB201218).

References


