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Research Article

Self-Similar Solutions of the Compressible Flow in One-Space Dimension

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For the isentropic compressible fluids in one-space dimension, we prove that the Navier-Stokes equations with density-dependent viscosity have neither forward nor backward self-similar strong solutions with finite kinetic energy. Moreover, we obtain the same result for the nonisentropic compressible gas flow, that is, for the fluid dynamics of the Navier-Stokes equations coupled with a transport equation of entropy. These results generalize those in Guo and Jiang's work (2006) where the one-dimensional compressible fluids with constant viscosity are considered.

1. Introduction

Self-similar solutions have attracted much attention in mathematical physics because understanding them is fundamental and important for investigating the well-posedness, regularity, and asymptotic behavior of differential equations in physics. Since the pioneering work of Leray [1], self-similar solutions of the Navier-Stokes equations for incompressible fluids have been widely studied in different settings (e.g., [2, page 207]; [3, page 120]; [4–10]; [11, Chapter 23]; [12–20]). On the contrary, studies on the self-similar solutions of the compressible Navier-Stokes equations have been limited partially due to the complicated nonlinearities in the equations (see [21–24]).

In one-space dimension, the isentropic compressible fluid flow is governed by the Navier-Stokes equations:

$$\begin{gathered}\rho_t + (\rho u)_x = 0,\\ (\rho u)_t + \left(\rho u^2\right)_x + P(\rho)_x = (\mu(\rho) u_x)_x, \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+,\end{gathered} \tag{1}$$

where $\rho = \rho(x,t)$ and $u = u(x,t)$ are the density and velocity of the fluid, $\mu(\rho)$ and $P(\rho)$ denote the density-dependent viscosity and pressure, respectively, and the subscripts mean partial derivations. Guo and Jiang [21] considered (1) with constant viscosity, $\mu(\rho) \equiv \mu > 0$, and linear density-dependent pressure, $P(\rho) = a\rho$, where $a > 0$ is a constant, and proved that there exist neither forward nor backward self-similar solutions with finite total energy. Their investigation generalized the results for 3D incompressible fluids in Nečas et al.'s work [6] to the 1D compressible case with $P(\rho) = \rho^\gamma$, where $\gamma = 1$. The problem with $\gamma > 1$, however, is open. From a physical point of view, one can derive the compressible Navier-Stokes equations from the Boltzmann equations by exploiting the Chapman-Enskog expansion up to the second order and then find that the viscosity depends on the temperature. If considering an isentropic process, this dependence can be translated into that on the density, such as $\mu(\rho) = \rho^\theta$, where $\theta > 0$ is a constant (see [25]). Okada et al. [26] pointed out that, because of the hard sphere interaction, the relation between indices θ and γ is $\theta = (\gamma - 1)/2$. In the first part of this paper, we are concerned with (1) where

$$\mu(\rho) = \rho^\theta, \qquad P(\rho) = \rho^\gamma, \qquad \frac{\gamma-1}{2} \le \theta < \frac{\gamma+1}{2}, \qquad \gamma \ge 1. \tag{2}$$

When considering an ideal compressible gas flow, particularly in the thermodynamic analysis with exergy loss and

entropy generation, both the viscosity and pressure rely on the entropy, so it is necessary to extend the nonisentropic fluid dynamics to include the transport of entropy (see [13, 27–35]). We consider the following coupled system of the Navier-Stokes equations with an entropy transport equation in a pure form:

$$\rho s_t + \rho u s_x = 0, \tag{3}$$

$$\rho_t + (\rho u)_x = 0, \tag{4}$$

$$(\rho u)_t + (\rho u^2)_x + P(\rho, s)_x = (\mu(\rho, s) u_x)_x, \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+, \tag{5}$$

where $s = s(x,t)$ is the entropy of the fluid and $\mu(\rho, s)$ and $P(\rho, s)$ denote the density-entropy-dependent viscosity and pressure, respectively. In this system, we assume that

$$\mu(\rho, s) = \rho^\theta e^s, \qquad P(\rho, s) = \rho^\gamma e^s, \qquad 0 \le \theta < \frac{\gamma+1}{2}, \quad \gamma \ge 1. \tag{6}$$

Navier-Stokes equations enjoy a scaling property: if (ρ, u) solves (1)-(2), then

$$\left(\rho^{(\lambda)}, u^{(\lambda)}\right) = \left(\lambda^a \rho\left(\lambda^c x, \lambda^d t\right), \lambda^b u\left(\lambda^c x, \lambda^d t\right)\right) \tag{7}$$

does so for any $\lambda > 0$, by setting $a = 1/(\gamma - \theta)$, $b = (\gamma - 1)/2(\gamma - \theta)$, $c = (\gamma + 1 - 2\theta)/2(\gamma - \theta)$, and $d = 1$. Note that, from (2), $a \ge c > 0$ and $b \ge 0$. Solution (ρ, u) is called forward self-similar if

$$(\rho, u) = \left(\rho^{(\lambda)}, u^{(\lambda)}\right), \quad \text{for every } \lambda > 0. \tag{8}$$

In that case, $\rho(x,t)$ and $u(x,t)$ are decided by their values at the instant of $t = 1/\lambda$:

$$\rho(x,t) = \frac{1}{t^a} Q\left(\frac{x}{t^c}\right), \quad u(x,t) = \frac{1}{t^b} U\left(\frac{x}{t^c}\right), \quad t > 0, \tag{9}$$

where $Q(y) = \rho(y, 1)$ and $U(y) = u(y, 1)$ are defined on $\mathbb{R}$. In the same manner, the backward self-similar solutions are of the form:

$$\rho(x,t) = \frac{1}{(T-t)^a} Q\left(\frac{x}{(T-t)^c}\right), \quad u(x,t) = \frac{1}{(T-t)^b} U\left(\frac{x}{(T-t)^c}\right), \quad 0 < t < T, \tag{10}$$

where $Q(y) = \rho(y, T-1)$ and $U(y) = u(y, T-1)$ for $T > 1$. Substitution of (9) or (10) into (1) gives

$$(QU)' - c(yQ)' + (c-a) Q = 0, \tag{11}$$

$$\left(Q^\theta U'\right)' - \left(QU^2\right)' + c(yQU)' - (Q^\gamma)' + (a+b-c) QU = 0, \tag{12}$$

for forward self-similar solutions, or

$$(QU)' + c(yQ)' - (c-a) Q = 0, \tag{13}$$

$$\left(Q^\theta U'\right)' - \left(QU^2\right)' - c(yQU)' - (Q^\gamma)' - (a+b-c) QU = 0, \tag{14}$$

for backward self-similar solutions, respectively. In comparison with those in Guo and Jiang [21], forward (backward) self-similar equations above process necessary modifications and additional difficulties. For instance, (11) and (13) have solutions with an additional integral term, and thus the modified blow-up analysis needs an L^∞ estimate on the density and a new large-scale argument on the energy. In addition, conditions on θ and γ proposed in (2) are directly related to the energy estimate.

Mellet and Vasseur [25] obtained the global existence of strong solutions for the Cauchy problem of (1) with positive initial density having (possibly different) positive limits at $x = \pm\infty$. Precisely, fix constant positive density $\rho_+ > 0$ and $\rho_- > 0$, and let $\overline{\rho}(x)$ be a smooth monotone function satisfying

$$\overline{\rho}(x) = \rho_\pm \quad \text{when } \pm x \ge 1, \ \overline{\rho}(x) > 0, \ \forall x \in \mathbb{R}. \tag{15}$$

Assume that the initial data $\rho(x,t)|_{t=0} = \rho_0(x)$ and $u(x,t)|_{t=0} = u_0(x)$ satisfy

$$0 < \underline{\kappa}_0 \le \rho_0(x) \le \overline{\kappa}_0 < \infty, \quad \rho_0 - \overline{\rho} \in H^1(\mathbb{R}), \quad u_0 \in H^1(\mathbb{R}), \tag{16}$$

for some constants $\underline{\kappa}_0$ and $\overline{\kappa}_0$. Assume also that $\mu(\rho)$ and $P(\rho)$ verify (2). Mellet and Vasseur [25] proved that there exists a global strong solution (ρ, u) of (1) on $\mathbb{R} \times \mathbb{R}_+$ such that for every $T > 0$:

$$\rho - \overline{\rho} \in L^\infty\left(0, T; H^1(\mathbb{R})\right), \quad u \in L^\infty\left(0, T; H^1(\mathbb{R})\right) \cap L^2\left(0, T; H^2(\mathbb{R})\right). \tag{17}$$

Moreover, for every $T > 0$, there exist uniform bounds away from zero with respect to all strong solutions having the same initial data. Precisely, there exist some constants $C(T)$, $\underline{\kappa}(T)$, and $\overline{\kappa}(T)$ depending only on T, $\rho_0(x)$, and $u_0(x)$ such that the following bounds hold uniformly for any strong solution (ρ, u):

$$\left\|\rho - \overline{\rho}\right\|_{L^\infty(0,T;H^1(\mathbb{R}))} \le C(T), \quad \|u\|_{L^\infty(0,T;H^1(\mathbb{R}))} \le C(T), \tag{18}$$

$$0 < \underline{\kappa}(T) \le \rho(x,t) \le \overline{\kappa}(T), \quad \forall (x,t) \in \mathbb{R} \times [0,T]. \tag{19}$$

Define

$$p(\rho \mid \overline{\rho}) = \frac{1}{\gamma-1}\rho^\gamma - \frac{1}{\gamma-1}\overline{\rho}^\gamma - \frac{\gamma}{\gamma-1}\overline{\rho}^{\gamma-1}(\rho - \overline{\rho}) \tag{20}$$

as the relative potential energy density of (1), and

$$E(t) = \int_{\mathbb{R}} \frac{1}{2}\rho(x,t) u^2(x,t)\, dx \tag{21}$$

as the kinetic energy. Note that, since p is strictly convex, $p(\rho \mid \overline{\rho})$ is nonnegative for every ρ, and $p(\rho \mid \overline{\rho}) = 0$ if and only if $\rho = \overline{\rho}$. Mellet and Vasseur [25] also showed that, if the initial total energy is finite, that is, the sum of the kinetic and potential energy at time 0 satisfies

$$\int_{\mathbb{R}} \left[\frac{1}{2}\rho_0 u_0^2 + p\left(\rho_0 \mid \overline{\rho}\right)\right] dx < +\infty, \tag{22}$$

then the following global-energy estimate on $(-\infty,\infty) \times [0,T]$ holds uniformly with respect to all strong solutions; that is, for every $T > 0$, there exists a positive constant $C(T)$ depending only on T, $\rho_0(x)$, and $u_0(x)$ such that

$$\begin{aligned} &\sup_{[0,T]} \int_{\mathbb{R}} \left[\frac{1}{2}\rho u^2 + p\left(\rho \mid \overline{\rho}\right)\right] dx \\ &\quad + \int_0^T \int_{\mathbb{R}} \mu(\rho) \left|u_x\right|^2 dx\, dt \le C(T) \end{aligned} \tag{23}$$

holds for any strong solution (ρ, u). Correspondingly, for $R > 0$, $0 < t_1 < T$, and some constant $C(R, t_1, T)$, we call

$$\begin{aligned} &\sup_{t_1 \le t \le T} \int_{-R}^{R} \left[\frac{1}{2}\rho u^2 + p\left(\rho \mid \overline{\rho}\right)\right] dx \\ &\quad + \int_{t_1}^{T} \int_{-R}^{R} \mu(\rho) \left|u_x\right|^2 dx\, dt \le C(R, t_1, T) \end{aligned} \tag{24}$$

the local-energy estimate on $[-R, R] \times [t_1, T]$. Note that the global-energy estimate implies the local-energy estimate.

The main result for the self-similar solutions of the isentropic compressible Navier-Stokes equations is as follows.

Theorem 1. *Assume that $\mu(\rho)$ and $P(\rho)$ in* (1) *verify* (2). *Then the following statements are true.*

(1) *There is no self-similar strong solution satisfying the global-energy estimate* (23).

(2) *If there is a forward (backward) self-similar strong solution satisfying the local-energy estimate* (24), *then its kinetic energy* (21) *blows up as $t \downarrow 0^+$ ($t \uparrow T^-$).*

For the self-similar solutions of the coupled system of the nonisentropic compressible Navier-Stokes equations with an entropy transport equation, the main result is as follows.

Theorem 2. *Assume that $\mu(\rho, s)$ and $P(\rho, s)$ in* (3)–(5) *verify* (6). *Then the following statements are true.*

(1) *There is no self-similar strong solution satisfying the global-energy estimate* (23).

(2) *If there is a forward (backward) self-similar strong solution satisfying the local-energy estimate* (24), *then its kinetic energy* (21) *blows up as $t \downarrow 0^+$ ($t \uparrow T^-$).*

Theorem 1 is proved in Section 2 and Theorem 2 in Section 3.

2. Proof of Theorem 1

Any self-similar solution of (1) is either forward or backward, so we first prove Theorem 1 for forward and then for backward self-similar solutions.

2.1. Forward Self-Similar Solutions

Lemma 3. *If (Q, U) solves* (11)-(12), *then the corresponding strong solution (ρ, u) defined by* (9) *of* (1) *does not satisfy the global-energy estimate* (23).

Proof. From (11),

$$Q(y)U(y) = cyQ(y) + (a-c)\int_0^y Q(z)\, dz + C_0, \tag{25}$$

where C_0 is an arbitrary constant. From (19),

$$0 < \underline{\kappa}(1) \le \rho(y, 1) = Q(y) \le \overline{\kappa}(1), \quad \forall y \in \mathbb{R}. \tag{26}$$

Since $a \ge c > 0$, (25) and (26) imply that, for $y \ge Y_0 > 0$ where Y_0 is large enough,

$$\begin{aligned} Q(y)U(y) &= \frac{c}{2} yQ(y) + c\frac{y}{2}Q(y) + (a-c) \\ &\quad \times \int_0^y Q(z)\, dz + C_0 \\ &\ge \frac{c}{2} yQ(y) + \frac{c}{2} yQ(y) + C_0 \\ &\ge \frac{c}{2} yQ(y) + \frac{c}{2} Y_0 \underline{\kappa}(1) + C_0 \\ &\ge \frac{c}{2} yQ(y). \end{aligned} \tag{27}$$

Thus, from (9) and (21), for any $t > 0$,

$$\begin{aligned} E(t) &= \int_{\mathbb{R}} \frac{1}{2} \cdot \frac{1}{t^a} Q\left(\frac{x}{t^c}\right) \cdot \frac{1}{t^{2b}} U^2\left(\frac{x}{t^c}\right) dx \\ &= \frac{1}{2t^{a+2b-c}} \int_{\mathbb{R}} \frac{1}{Q(y)} \left[Q(y)U(y)\right]^2 dy \\ &\ge \frac{1}{2t^{a+2b-c}} \int_{Y_0}^{+\infty} \frac{1}{Q(y)} \left[\frac{c}{2} yQ(y)\right]^2 dy \\ &\ge \frac{c^2 \underline{\kappa}(1)}{8t^{a+2b-c}} \int_{Y_0}^{+\infty} y^2 dy \\ &= +\infty. \end{aligned} \tag{28}$$

This proves the lemma. □

Lemma 4. *If (Q, U) solves* (11)-(12) *and the corresponding strong solution (ρ, u) defined by* (9) *of* (1) *satisfies the local-energy estimate* (24), *then as $t \downarrow 0$, the kinetic energy* (21) *must blow up.*

Proof. Similar to the proof of Lemma 3, for any $t > 0$ and $R > Y_0 t^c > 0$,

$$\begin{aligned} E(t) &\ge \frac{c^2 \underline{\kappa}(1)}{8t^{a+2b-c}} \int_{Y_0}^{R/t^c} y^2 dy \\ &= \frac{c^2 \underline{\kappa}(1)}{24t^{a+2b-c}} \left(\frac{R^3}{t^{3c}} - Y_0^3\right) \\ &= \frac{c^2 \underline{\kappa}(1)}{24t^{a+2b+2c}} \left[R^3 - \left(Y_0 t^c\right)^3\right] \longrightarrow +\infty, \quad \text{as } t \downarrow 0. \end{aligned} \tag{29}$$

This proves the lemma. □

2.2. Backward Self-Similar Solutions

Lemma 5. *If* (Q,U) *solves* (13)-(14), *then the corresponding strong solution* (ρ,u) *defined by* (10) *of* (1) *does not satisfy the global-energy estimate* (23).

Proof. Fix $T>1$. From (13),

$$Q(y)U(y) = -cyQ(y) - (a-c)\int_0^y Q(z)\,dz + C_0, \quad (30)$$

where C_0 is an arbitrary constant. From (19),

$$0 < \underline{\kappa}(T) \le \rho(y, T-1) = Q(y) \le \overline{\kappa}(T), \quad \forall y \in \mathbb{R}. \quad (31)$$

Since $a \ge c > 0$, (30) and (31) imply that, for $y \ge Y_0 > 0$ where Y_0 is large enough,

$$\begin{aligned} Q(y)U(y) &= -\frac{c}{2}yQ(y) - c\frac{y}{2}Q(y) - (a-c) \\ &\quad \times \int_0^y Q(z)\,dz + C_0 \\ &\le -\frac{c}{2}yQ(y). \end{aligned} \quad (32)$$

Thus, from (10) and (21), for any $t<T$,

$$\begin{aligned} E(t) &= \int_{\mathbb{R}} \frac{1}{2}\cdot\frac{1}{(T-t)^a}Q\left(\frac{x}{(T-t)^c}\right)\cdot\frac{1}{(T-t)^{2b}}U^2 \\ &\quad \times\left(\frac{x}{(T-t)^c}\right)dx \\ &\ge \frac{c^2\underline{\kappa}(T)}{8(T-t)^{a+2b-c}}\int_{Y_0}^{+\infty} y^2 dy \\ &= +\infty. \end{aligned} \quad (33)$$

This proves the lemma. □

Lemma 6. *If* (Q,U) *solves* (13)-(14) *and the corresponding strong solution* (ρ,u) *defined by* (10) *of* (1) *satisfies the local-energy estimate* (24), *then as* $t\uparrow T$, *the kinetic energy* (21) *must blow up.*

Proof. Recalling the proofs of Lemmas 4 and 5, for $R > Y_0(T-t)^c > 0$,

$$\begin{aligned} E(t) &\ge \frac{c^2\underline{\kappa}(T)}{8(T-t)^{a+2b-c}}\int_{Y_0}^{R/(T-t)^c} y^2 dy \\ &= \frac{c^2\underline{\kappa}(T)}{24(T-t)^{a+2b+2c}} \\ &\quad \times\left[R^3 - Y_0^3(T-t)^{3c}\right] \longrightarrow +\infty, \quad \text{as } t\uparrow T. \end{aligned} \quad (34)$$

This proves the lemma. □

Now, Theorem 1 follows from the four lemmas above.

3. Proof of Theorem 2

If (ρ,u,s) solves (3)–(6), then

$$\begin{aligned} \left(\rho^{(\lambda)}, u^{(\lambda)}\right) = \left(\lambda^a\rho\left(\lambda^c x, \lambda^d t\right), \lambda^b u\left(\lambda^c x, \lambda^d t\right),\right. \\ \left.\lambda^l s\left(\lambda^c x, \lambda^d t\right)\right) \end{aligned} \quad (35)$$

does so for any $\lambda>0$, by setting $a = 1/(\gamma-\theta)$, $b = (\gamma-1)/2(\gamma-\theta)$, $c = (\gamma+1-2\theta)/2(\gamma-\theta)$, $d=1$, and $l=0$. Note that, from (6), $a>0$, $b\ge 0$, and $c>0$. The forward self-similar solutions have the following form:

$$\begin{aligned} \rho(x,t) &= \frac{1}{t^a}Q\left(\frac{x}{t^c}\right), \\ u(x,t) &= \frac{1}{t^b}U\left(\frac{x}{t^c}\right), \\ s(x,t) &= S\left(\frac{x}{t^c}\right), \\ t &> 0, \end{aligned} \quad (36)$$

where $Q(y)=\rho(y,1)$, $U(y)=u(y,1)$, and $S(y)=s(y,1)$. The backward self-similar solutions are

$$\begin{aligned} \rho(x,t) &= \frac{1}{(T-t)^a}Q\left(\frac{x}{(T-t)^c}\right), \\ u(x,t) &= \frac{1}{(T-t)^b}Q\left(\frac{x}{(T-t)^c}\right), \\ s(x,t) &= S\left(\frac{x}{(T-t)^c}\right), \\ 0 &< t < T, \end{aligned} \quad (37)$$

where $Q(y)=\rho(y,T-1)$, $U(y)=u(y,T-1)$, and $S(y)=s(y,T-1)$ for $T>1$.

Lions [29] investigated the coupled system of the Navier-Stokes equations with an entropy transport equation in a pure form and obtained the existence of weak solutions satisfying (19) and (23).

Proof of Theorem 2. Suppose that (ρ,u,s) is a forward self-similar solution. Inserting (36) into (3), one gets $U(y)=cy$, and thus $u(x,t)=cx/t$. Therefore, for any $t>0$, (36), (19), and (21) yield

$$E(t) = \int_{\mathbb{R}} \frac{1}{2}\cdot\frac{1}{t^a}Q\left(\frac{x}{t^c}\right)\cdot\left(\frac{cx}{t}\right)^2 \ge \frac{c^2\underline{\kappa}(1)}{2t^{a+2}}\int_{\mathbb{R}} x^2 dx = +\infty. \quad (38)$$

This means that the global-energy estimate (23) does not hold and that the kinetic energy (21) blows up as $t\downarrow 0$.

The case of backward self-similar solutions can be proved similarly, so Theorem 2 is proved. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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