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Research Article

A New Pressure Regularity Criterion of the Three-Dimensional Micropolar Fluid Equations

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This paper concerns the regularity criterion of the weak solutions to the three-dimensional (3D) micropolar fluid equations in terms of the pressure. It is proved that if one of the partial derivatives of pressure satisfies $\partial_3\pi \in L^p(0,T;L^q(\mathbf{R}^3))$ with $2/p+3/q \le 2, 3 < q < \infty, 1 < p < \infty$, then the weak solution of the micropolar fluid equations becomes regular on $(0,T]$.

1. Introduction

In the past ten years, the mathematical models of fluid dynamics attract more and more attention. As a classic fluid dynamical model, Naiver-Stokes equations [1] are proved as an accurate model in many practical situations, which presume that the derivatives of the components of the velocity are small. However, for certain anisotropic fluids, for example, liquid crystals, which are made up of dumbbell molecules, and some polymeric fluids or fluids containing certain additives in narrow films [2], the constructive relations do not satisfy Stoke Law. In 1960s, Eringen [3] introduced viscous incompressible micropolar fluid flows, a non-Newtonian fluid model with asymmetric stress tensor. From the viewpoint of mathematics, micropolar fluid model is coupled with the incompressible Navier-Stokes equations, microrotational effects, and microrotational inertia. The three-dimensional (3D) viscous incompressible micropolar fluid equations are written as

$$\begin{gathered}\partial_t u-(\nu+\kappa)\Delta u-2\kappa\nabla\times w+\nabla\pi+u\cdot\nabla u=0,\\ \partial_t w-\gamma\Delta w-(\alpha+\beta)\nabla\nabla\cdot w+4\kappa w-2\kappa\nabla\times u+u\cdot\nabla w=0,\\ \nabla\cdot u=0,\end{gathered}\tag{1}$$

associated with the initial prescribed data

$$u(x,t)|_{t=0}=u_0,\qquad w(x,t)|_{t=0}=w_0.\tag{2}$$

Here $u=(u_1,u_2,u_3)$, π, and $w=(w_1,w_2,w_3)$ stand for the divergence-free velocity vector field, the scalar pressure field, and nondivergence free microrotation vector field, respectively. $\nu>0$ is the Newtonian kinetic viscosity, $\kappa>0$ is the dynamics microrotation viscosity, and $\alpha,\beta,\gamma>0$ are the angular viscosity.

Due to their importance in mathematics, there is large literature on the well-posedness and large time behaviors for weak solutions of micropolar fluid equations [4–10]. However, the question of global regularity or uniqueness of weak solutions of three-dimensional micropolar fluid equations is still a challenge open problem. Therefore, it is interesting and natural to consider the regularity criteria for weak solutions of micropolar fluid equations by imposing some growth conditions on the velocity or the pressure. As for the velocity regularity criteria, Dong and Chen [11] (see also [12]) obtained that if the velocity fields satisfy one of the following conditions:

$$\begin{aligned}&\text{(i)}\quad u\in L^p\left(0,T;L^{q,\infty}\left(\mathbf{R}^3\right)\right),\quad \frac{2}{p}+\frac{3}{q}=1,\quad 3<q\le\infty,\\ &\text{(ii)}\quad \nabla u\in L^p\left(0,T;L^{q,\infty}\left(\mathbf{R}^3\right)\right),\quad \frac{2}{p}+\frac{3}{q}=2,\quad \frac{3}{2}<q\le\infty,\\ &\text{(iii)}\quad u\in L^{2/(1-r)}\left(0,T;\dot{X}^{-r}\left(\mathbf{R}^3\right)\right),\quad r\in(0,1],\end{aligned}\tag{3}$$

then the weak solutions of the micropolar fluid equations (1) and (2) are regular. Here the critical spaces $L^{q,\infty}$, X^{-r} are Lorentz spaces and Multiplier space. The results were further refined by many authors [13–15] to some large critical spaces such as Besov spaces and Triebel-Lizorkin spaces.

On the other hand, as for the pressure regularity criteria of the micropolar fluid equations, Yuan [12] showed that the weak solution becomes regular if the pressure satisfies

$$\nabla\pi \in L^p\left(0,T;L^{q,\infty}\left(\mathbf{R}^3\right)\right), \quad \text{for } \frac{2}{p}+\frac{3}{q}\le 3, \quad 1<q\le\infty. \tag{4}$$

Dong et al. [16, 17] improved the regularity criteria by imposing the growth conditions in the critical Besov spaces

$$\pi \in L^1\left(0,T;B^0_{\infty,\infty}\left(\mathbf{R}^3\right)\right). \tag{5}$$

Recently, some interesting logarithmical pressure regularity criteria [18, 19] of micropolar fluid equations are studied. In particular, Jia et al. [20] refined this question by imposing the following regularity criterion condition:

$$\int_0^T \frac{\|\partial_3\pi\|_{L^q}^p}{1+\ln\left(e+\|w\|_{L^4}\right)}ds<\infty, \quad \frac{2}{p}+\frac{3}{q}=\frac{7}{4}, \quad \frac{12}{7}<q\le\infty. \tag{6}$$

Compared with the results (4) and (6), it is natural to consider whether or not the growth condition of the partial derivative of the pressure $\partial_3\pi$ can be released. It should be mentioned that the optimal result is that the regularity for weak solutions of three-dimensional micropolar fluid equations is valid if

$$\partial_3\pi \in L^p\left(0,T;L^q\left(\mathbf{R}^3\right)\right), \quad \frac{2}{p}+\frac{3}{q}\le 3, \quad 1<q\le\infty. \tag{7}$$

This is an open problem on the pressure regularity criterion of micropolar fluid equations. The aim of this paper is to understand this challenge problem. More precisely, we will show the regularity of weak solutions to three-dimensional micropolar fluid equations if one of the partial derivatives of the pressure, say, $\partial_3\pi$, satisfies

$$\partial_3\pi \in L^p\left(0,T;L^q\left(\mathbf{R}^3\right)\right), \quad \frac{2}{p}+\frac{3}{q}\le 2, \quad 3<q<\infty,\ 1<p<\infty. \tag{8}$$

One may also refer to some important results on the regularity criteria of some mathematical models in fluid dynamics. For example, Cao and Titi [21], Chen and Zhang [22], Fan et al. [23], and Zhou [24] investigated the regularity criteria for the classic Navier-Stokes equations, Chen et al. [25], He and Xin [26], and Jia and Zhou [27, 28] for MHD equations, Dong et al. [29, 30] for quasigeostrophic equation, and so on.

2. Preliminaries and Main Results

In this paper, we use the following usual notations. C is the abstract constant which may change from line to line. $L^p(\mathbf{R}^3)$ $(1\le p\le\infty)$ is the scalar or vector Lebesgue space of all L^p integral functions associated with the norm

$$\|f\|_{L^p}=\begin{cases}\left(\int_{\mathbf{R}^3}|f(x)|_{L^p}dx\right)^{1/p}, & 1\le p<\infty,\\ \operatorname*{ess\,sup}_{x\in\mathbf{R}^3}|f(x)|, & p=\infty.\end{cases} \tag{9}$$

The following anisotropic Sobolev inequality is due to Cao and Wu [31].

Lemma 1 (Cao and Wu [31]). *Suppose $f\in H^1(\mathbf{R}^3)$, $\partial_1 f,\partial_2 f\in L^\lambda(\mathbf{R}^3)$, and $\partial_3 f\in L^\mu(\mathbf{R}^3)$ and for three constants $1\le\mu,\lambda,\gamma<\infty$ satisfy*

$$1+\frac{3}{\gamma}=\frac{1}{\mu}+\frac{2}{\lambda}, \tag{10}$$

then there exists a constant $C=C(\mu,\lambda)$ such that the following anisotropic Sobolev inequality:

$$\|f\|_{L^\gamma}\le C\|\partial_1 f\|_{L^\lambda}^{1/3}\|\partial_2 f\|_{L^\lambda}^{1/3}\|\partial_3 f\|_{L^\mu}^{1/3} \tag{11}$$

is valid.

To aid the introduction of our main results, let us recall the definition of the weak solutions of the 3D micropolar fluid flows (1) and (2) (see Łukaszewicz [9]).

Definition 2. Let $(u_0,w_0)\in L^2(\mathbf{R}^3)$ and $\nabla\cdot u_0=0$. A pair of vector fields $(u(x,t),w(x,t))$ is termed as a weak solution to the 3D micropolar fluid equations (1) and (2) on $(0,T)$ if (u,w) satisfies the following properties:

(i) $(u,w)\in L^\infty(0,T;L^2(\mathbf{R}^3))\cap L^2(0,T;H^1(\mathbf{R}^3))$;

(ii) (u,w) verifies (1) in the sense of distribution.

The following existence result of micropolar fluid equations is useful for our results.

Lemma 3 (Dong et al. [16]). *Assume $3<p<\infty$ and $(u_0,w_0)\in L^p(\mathbf{R}^3)$ with $\nabla\cdot u_0=0$ in the sense of distributions. Then there exist a constant $T>0$ and a unique strong solution (u,w) of the 3D micropolar fluid equations* (1) *and* (2) *such that*

$$u\in BC\left([0,T);L^p\left(\mathbf{R}^3\right)\right), \quad t^{1/2}\nabla u\in BC\left([0,T);L^p\left(\mathbf{R}^3\right)\right). \tag{12}$$

Theorem 4. *Suppose $T>0$ and $(u_0,w_0)\in L^2(\mathbf{R}^3)\cap L^4(\mathbf{R}^3)$ and $\nabla\cdot u_0$ in the sense of distributions and assume (u,w) is a weak solution of the 3D micropolar fluid equations* (1) *and* (2) *on $(0,T]$. If one of the partial derivatives of the pressure, say, $\partial_3\pi$, satisfies*

$$\partial_3\pi \in L^p\left(0,T;L^q\left(\mathbf{R}^3\right)\right), \quad \frac{2}{p}+\frac{3}{q}\le 2, \quad 3<q<\infty,\ 1<p<\infty, \tag{13}$$

then (u,w) is regular on $[0,T]$.

3. Proof of Theorem 4

We first establish some fundamental estimates between the pressure and the velocity of the micropolar fluid equations (1) and (2). Taking the operator div to both sides of the first equation of (1) and noting the fact of the divergence-free velocity, one shows that

$$-\Delta\pi = \sum_{i,j=1}^{3} \frac{\partial^2}{\partial x_i \partial x_j}\left(u_i u_j\right), \tag{14}$$

together with Calderon-Zygmund inequality, implies that for any $1 < p < \infty$

$$\begin{aligned}\|\pi\|_{L^p} &= \left\|(-\Delta)^{-1}\sum_{i,j=1}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\left(u_i u_j\right)\right\|_{L^p} \\ &\le C\|u\|_{L^{2p}}^2.\end{aligned} \tag{15}$$

Similarly, acting the operator ∇ div on both sides of the second equation of (1), we have

$$\begin{aligned}\nabla\pi &= (-\Delta)^{-1}\sum_{i,j=1}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\left(\nabla\left(u_i u_j\right)\right), \\ \|\nabla\pi\|_{L^p} &= \left\|(-\Delta)^{-1}\sum_{i,j=1}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\left(\nabla\left(u_i u_j\right)\right)\right\|_{L^p} \\ &\le C\||u|\nabla u\|_{L^p}.\end{aligned} \tag{16}$$

In order to prove the main result, we also need some auxiliary estimates of the weak solutions of the micropolar fluid equations (1). Thanks to the divergence-free velocity fields and application to the integration by parts, we have the following estimates:

$$\int_{\mathbf{R}^3}(u\cdot\nabla u)\cdot u|u|^2dx = 0, \qquad \int_{\mathbf{R}^3}(u\cdot\nabla w)\cdot w|w|^2dx = 0. \tag{17}$$

In particular, by direct computation, we also have

$$\begin{aligned}&(\nu+\kappa)\int_{\mathbf{R}^3}(-\Delta u)\,u|u|^2dx \\ &\quad = (\nu+\kappa)\int_{\mathbf{R}^3}(\nabla u)\cdot\left(\nabla\left(u|u|^2\right)\right)dx \\ &\quad = (\nu+\kappa)\int_{\mathbf{R}^3}|\nabla u|^2|u|^2dx + \frac{\nu+\kappa}{2}\int_{\mathbf{R}^3}\left|\nabla|u|^2\right|^2dx, \\ &\gamma\int_{\mathbf{R}^3}(-\Delta w)\,w|w|^2dx \\ &\quad = \gamma\int_{\mathbf{R}^3}(\nabla w)\cdot\left(\nabla\left(w|w|^2\right)\right)dx \\ &\quad = \gamma\int_{\mathbf{R}^3}|\nabla w|^2|w|^2dx + \frac{\gamma}{2}\int_{\mathbf{R}^3}\left|\nabla|w|^2\right|^2dx.\end{aligned} \tag{18}$$

We now begin to prove Theorem 4.

Taking the inner product of the first equation of (1) with $u|u|^2$ and the second equation of (1) with $w|w|^2$, respectively, it follows that

$$\begin{aligned}&\frac{1}{4}\frac{d}{dt}\int_{\mathbf{R}^3}|u|^4dx + (\nu+\kappa) \\ &\quad\times\int_{\mathbf{R}^3}||u|\nabla u|^2dx + \frac{\nu+\kappa}{2}\int_{\mathbf{R}^3}|\nabla|u||^2dx \\ &\quad = 2\kappa\int_{\mathbf{R}^3}(\nabla\times w)\cdot u|u|^2dx - \int_{\mathbf{R}^3}u\cdot\nabla\pi|u|^2dx, \\ &\frac{1}{4}\frac{d}{dt}\int_{\mathbf{R}^3}|w|^4dx + \gamma\int_{\mathbf{R}^3}||w|\nabla w|^2dx \\ &\quad + \frac{\gamma}{2}\int_{\mathbf{R}^3}\left|\nabla|w|^2\right|^2dx + \frac{(\alpha+\beta)}{2}\int_{\mathbf{R}^3}|\nabla\cdot w|^2|w|^2dx \\ &\quad \le 2\kappa\int_{\mathbf{R}^3}(\nabla\times u)\cdot w|w|^2dx - 4\kappa\int_{\mathbf{R}^3}|w|^4dx.\end{aligned} \tag{19}$$

Applying Hölder inequality, Young inequality, and integration by parts, one shows that

$$\begin{aligned}&2\kappa\int_{\mathbf{R}^3}(\nabla\times w)\cdot u|u|^2dx \\ &\quad + 2\kappa\int_{\mathbf{R}^3}(\nabla\times u)\cdot w|w|^2dx - 4\kappa\int_{\mathbf{R}^3}|w|^4dx \\ &\quad \le 2\kappa\|w\|_{L^4}\||u|\nabla u\|_{L^2}\|u\|_{L^4} \\ &\quad + 2\kappa\|u\|_{L^4}\||w|\nabla w\|_{L^2}\|w\|_{L^4} - 4\kappa\|w\|_{L^4}^4 \\ &\quad \le C\left(\|u\|_{L^4}^4 + \|w\|_{L^4}^4\right) + \frac{\nu+\kappa}{2}\||u|\nabla u\|_{L^2}^2 + \frac{\gamma}{2}\||w|\nabla w\|_{L^2}^2.\end{aligned} \tag{20}$$

Summing up (19) and (20), we have

$$\begin{aligned}&\frac{1}{4}\frac{d}{dt}\left(\int_{\mathbf{R}^3}|u|^4dx + \int_{\mathbf{R}^3}|w|^4dx\right) \\ &\quad + \frac{\nu+\kappa}{2}\int_{\mathbf{R}^3}||u|\nabla u|^2dx + \frac{\gamma}{2}\int_{\mathbf{R}^3}||w|\nabla w|^2dx \\ &\quad \le -\int_{\mathbf{R}^3}u\cdot\nabla\pi|u|^2dx.\end{aligned} \tag{21}$$

Now we estimate the right hand side of (21). Employing Hölder inequality and Young inequality firstly yields

$$\begin{aligned}\left|-\int_{\mathbf{R}^3}u\cdot\nabla\pi|u|^2dx\right| &\le C\|\pi\|_{L^4}\|u\|_{L^4}\left\|\nabla|u|^2\right\|_{L^2} \\ &\le C\|\pi\|_{L^4}^2\|u\|_{L^4}^2 + \frac{\nu+\kappa}{4}\||u|\nabla u\|_{L^2}^2.\end{aligned} \tag{22}$$

Applying interpolation inequality and Lemma 1 together with (14) and (16), it follows that

$$\begin{aligned}\|\pi\|_{L^4}^2\|u\|_{L^4}^2 &\le C\|\pi\|_{L^\rho}^{2(1-\theta)}\|\pi\|_{L^2}^{2\theta}\|u\|_{L^4}^2\\ &\le C\|\pi\|_{L^\rho}^{2(1-\theta)}\|u\|_{L^4}^{4\theta+2}\\ &\le C\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/3}\|\nabla_h\pi\|_{L^{4/3}}^{(4(1-\theta))/3}\|u\|_{L^4}^{4\theta+2}\\ &\le C\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/3}\|\nabla\pi\|_{L^{4/3}}^{(4(1-\theta))/3}\|u\|_{L^4}^{4\theta+2}\\ &\le C\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/3}\||u|\nabla\pi\|_{L^{4/3}}^{(4(1-\theta))/3}\|u\|_{L^4}^{4\theta+2},\end{aligned} \tag{23}$$

where we have used interpolation inequality

$$\frac{1-\theta}{\rho}+\frac{\theta}{2}=\frac{1}{4}\quad(0\le\theta\le1), \tag{24}$$

$$1+\frac{3}{\rho}=\frac{1}{q}+\frac{2}{4/3} \tag{25}$$

(let $\gamma=\rho$, $\mu=q$, and $\lambda=4/3$ in Lemma 1).

That is to say, we may check q as

$$q=\frac{4(1-\theta)}{1-4\theta}. \tag{26}$$

With the aid of Hölder inequality and Young inequality, $\|\pi\|_{L^4}^2\|u\|_{L^4}^2$ is now further estimated as

$$\begin{aligned}\|\pi\|_{L^4}^2\|u\|_{L^4}^2 &\le C\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/3}\|\nabla u\|_{L^2}^{(4(1-\theta))/3}\right)\\ &\quad\times\left(\|u\|_{L^4}^2\|u\|_{L^4}^{(4+8\theta)/3}\right)\\ &\le C\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/(1+2\theta)}+\|\nabla u\|_{L^2}^2\right)\\ &\quad\times\left(\|u\|_{L^4}^4+\|u\|_{L^4}^{(8+16\theta)/3}\right).\end{aligned} \tag{27}$$

We choose

$$0<\theta\le\frac{1}{4}; \tag{28}$$

that is,

$$\frac{8+16\theta}{3}\le4. \tag{29}$$

Thus

$$\|u\|_{L^4}^{(8+16\theta)/3}\le C\left(\|u\|_{L^4}^4+1\right) \tag{30}$$

which together with (27) gives

$$\begin{aligned}&\|\pi\|_{L^4}^2\|u\|_{L^4}^2\\ &\quad\le C\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/(1+2\theta)}+\|\nabla u\|_{L^2}^2\right)\left(\|u\|_{L^4}^4+1\right).\end{aligned} \tag{31}$$

Collecting (21), (22), and (31), we derive

$$\begin{aligned}&\frac{d}{dt}\left(\int_{\mathbf{R}^3}|u|^4dx+\int_{\mathbf{R}^3}|w|^4dx\right)\\ &\quad+\frac{\nu+\kappa}{2}\int_{\mathbf{R}^3}||u|\nabla u|^2dx+\frac{\gamma}{2}\int_{\mathbf{R}^3}||w|\nabla w|^2dx\\ &\quad\le C\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/(1+2\theta)}+\|\nabla u\|_{L^2}^2\right)\left(\|u\|_{L^4}^4+1\right)\end{aligned} \tag{32}$$

or

$$\begin{aligned}&\frac{d}{dt}\left(\|u\|_{L^4}^4+\|w\|_{L^4}^4+1\right)\\ &\quad\le C\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/(1+2\theta)}+\|\nabla u\|_{L^2}^2\right)\\ &\quad\times\left(\|u\|_{L^4}^4+\|w\|_{L^4}^4+1\right).\end{aligned} \tag{33}$$

Taking Gronwall inequality into consideration yields

$$\begin{aligned}&\operatorname{ess\,sup}_{0<t<T}\left(\|u(t)\|_{L^4}^4+\|w(t)\|_{L^4}^4\right)\\ &\quad\le\left(\|u_0\|_{L^4}^4+\|w_0\|_{L^4}^4+1\right)\\ &\quad\times\exp\left\{C\int_0^T\left(\|\partial_3\pi\|_{L^q}^{(2(1-\theta))/(1+2\theta)}+\|\nabla u\|_{L^2}^2\right)d\tau\right\}.\end{aligned} \tag{34}$$

By the definitions of the weak solutions, we have

$$\int_0^T\|\nabla u(\tau)\|_{L^2}^2d\tau\le C\left(\|u_0\|_{L^2},\|w_0\|_{L^2}\right). \tag{35}$$

Letting $p=(2(1-\theta))/(1+2\theta)$, we could obtain

$$\frac{2}{p}+\frac{3}{q}=2-\frac{1-4\theta}{4(1-\theta)}\le2, \tag{36}$$

which exactly satisfies the growth condition (13) in Theorem 4; therefore, we derive the uniform bounds of (u,w)

$$\operatorname{ess\,sup}_{0<t<T}\left(\|u(t)\|_{L^4}^4+\|w(t)\|_{L^4}^4\right)\le C, \tag{37}$$

together with Lemma 3; we now complete the proof of Theorem 4.

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