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Research Article

Fuzzy Group Decision Making for Multiobjective Problems: Tradeoff between Consensus and Robustness

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Many decision making problems involve multiple decision makers and conflicting objectives. This paper refers to this kind of problems as group decision making for multiobjective problems (GDM-MOP). The task of GDM-MOP is to select final solution(s) from a set of nondominated solutions according to the decision makers' preferences. However, it is common that the preference could be imprecise. We study the GDM-MOP where preferences are expressed by fuzzy reference points, called as fuzzy GDMMOP (FGDM-MOP). This paper provides a decision support model to simultaneously consider two measures for FGDM-MOP: consensus measure and robustness measure. The former is used to reflect the acceptable degree of a solution by the decision making group, while the latter indicates a solution's ability to cope with any change on preferences. A multiobjective evolutionary approach is presented to solve the problem. Finally, a modified benchmark function is studied to illustrate the proposed approach.

1. Introduction

Many real-world decision problems involve multiple decision makers (DMs). For instance, decisions on a corporation's investment policy may need discussion and negotiation amongst the members of directorate, or alternative selection of a complex project needs the consultation of experts from different fields. Such decision problems are referred to as group decision making (GDM) problems. Many studies suggest that GDM is appropriate to be applied for arriving at a judgment based on the input and feedback of multiple individuals [1–3]. In the process of GDM, for each alternative or comparison of a pair of alternations, DMs need to provide their opinions or preferences on one or more evaluating criteria. Then the individual opinions are aggregated to form the collective one to select the final solution. However, in practice, it is very common that decisions need to be made with simultaneous consideration of multiple conflicting objectives; that is, DMs need to trade off between different objectives. Such problems are called as multiobjective problems (MOPs), and the term multiobjective optimization (MOO) refers to solving MOPs [4]. The results of MOPs are a set of mathematically equally good solutions, referred to as non-dominated set, among which any decision on the solutions cannot be made until the preference of decision maker(s) is provided. Usually, it is worthwhile to assist decision makers in refining obtained solutions. Thus, MOO methods are guided to move towards the preference of a decision maker. For instance, Abbass [5, 6] proposed an interactive evolutionary approach for MOPs with reference points. In the context of MOP, the task of GDM is to select the solutions most acceptable by the group among the nondominated set. We use group decision making for multi-objective problem (GDM-MOP) to indicate such a combination of group decision making and multi-objective problems.

The process of GDM-MOP is usually in the presence of uncertainty. On one hand, as the complexity of real-world decision problems and the limitation of DMs' background and expertise, it is difficult for a decision maker to precisely express his/her preference on each objective. On the other hand, the DMs' preferences might change over time according to various related factors, for example, the decision maker's attitude towards each objective or

the changes on external environment. In such a case, the group solution may change because of the perturbation on decision maker's preference. However, in MOP, it is expected that the selected solution(s) can stay in the non-dominated set in the presence of any potential change. In other words, the solutions should be robust against the perturbations on decision maker's preference. Abbass and Bender [7] addressed such kind of problems where tradeoff between the objectives to be optimized varies over time and proposed the concept of Pareto Operating Curve. For similar problems, two robustness measures were suggested in Bui et al. [8].

Although the imprecise preference of decision maker can take many other forms, in this research, we focus on the GDM-MOP where the preferences are given by fuzzy numbers. The problem is referred to as fuzzy group decision making for multi-objective problem (FGDM-MOP). To the best of our knowledge, there is no other literature that addressed such kind of problems. From the previous discussion, we may claim that in FGDM-MOP, two issues need to be addressed: the first is how acceptable is a solution by the decision group? The second is how robust is a solution against possible changes or perturbations? In the proposed approach, we employ *consensus* and *robustness* to indicate the degrees of the two above mentioned issues. In many cases, the consensus and robustness degrees are in conflict with each other. Thus, the FGDM-MOP itself can be considered as a multi-objective problem. This paper presents the model and procedure of FGDM-MOP and suggests consensus and robustness measures. A multiobjective evolutionary approach is proposed to solve the problem.

The remaining sections of the paper are organized as follows. Section 2 briefly reviews the related work on group decision making and multi-objective problems. The mathematical model of FGDM-MOP is presented in Section 3. The measures of consensus and robustness are suggested in Section 4. Section 5 describes the evolutionary approach to solve FGDM-MOP. The experimental results with a modified benchmark function are reported in Section 6. Finally, conclusions and future works are discussed in Section 7.

2. Related Work

2.1. Multiobjective Optimization. Most real-world problems are characterized by multiple, noncommensurate, and often conflicting objectives [9]. The term multi-objective optimization problem (MOP) is used to refer to such kind of problem. In MOP, there are two or more conflicting objectives that needed to be optimized simultaneously. Formally, a MOP can be written as follows [4]:

$$\begin{aligned} \min \quad & f(x) = \left(f_1(x), f_2(x), \ldots, f_k(x)\right) \\ \text{s.t.} \quad & x \in X, \end{aligned} \tag{1}$$

where $X \subset \mathbb{R}^n$ is a feasible set of decision variables and $f : \mathbb{R}^n \to \mathbb{R}^k$. The n-dimensional space $\mathbb{R}^n$ is called a variable space, and the functions f_i $(i = 1, \ldots, k)$ are objective functions.

The solutions of MOP posed by (1) are called noninferior, non-dominated, efficient, or Pareto optimal solutions [4, 9–11]. This paper uses the naming convention of non-dominated or Pareto optimal solutions. A solution is called a Pareto optimal solution if there exists no other feasible solution which would decrease some criterion without causing a simultaneous increase in at least one other criterion [11]. To be more specific, we formally define the dominance and nondominated concepts as follows [4].

Definition 1. In (1), a vector $f(x)$, $x \in X$, is said to dominate another vector, $f(y)$, $y \in X$, if $f_i(x) \leq f_i(y)$ for all $i = 1, \ldots, k$, and the inequality is strict for at least one i.

Definition 2. In (1), a vector $f(x^*)$, $x^* \in X$, is non-dominated if there exists no other $x \in X$ such that $f(x)$ dominates $f(x^*)$.

A set which consists of all non-dominated solutions is called a non-dominated set. The topology of the set of non-dominated solutions in the objective space forms a curve that is known as the Pareto curve [7].

Evolutionary algorithms (EAs) have received considerable attention in dealing with MOPs during the last two decades as they are advantageous comparing with mathematical programming techniques. Currently, a wide variety of multi-objective evolutionary algorithms (MOEAs) are available in the literature. The representative MOEAs include SPEA [12], SPEA2 [13], PAES [14], and NSGA-II [15]. For a brief history of MOEAs, readers are referred to Coello [11].

From the definition of non-dominated solutions, one may notice that any choice among the set of non-dominated solutions could be difficult, unless we have additional information about the decision maker's preference structure [4]. Furthermore, in many cases, DMs might be interested in only some parts of the Pareto curve. Then, there is no need to visualize the whole Pareto curve, which is time intensive when the solution space is huge. Thus, it is important to utilize preference information during the optimization process. To perform this task, several preference-based multi-objective evolutionary algorithms are proposed and applied in recent years [4, 16–21]. However, most (if not all) existing literatures of preference-based multi-objective approaches neglect the following issues: (1) the preference is from a group of DMs, (2) the preference could be imprecise, and (3) the final solutions' ability to deal with any change on preference. This paper is aiming at simultaneously addressing the previous issues, which might be the first attempt in the field.

2.2. Group Decision Making. As its practical importance, group decision making has received great attention in many application areas, such as situation assessment [22], accident evaluation [23], emergency management [3], product development [24], and alternative selection [25]. The aim of group decision making is to find a group satisfactory solution which is most acceptable by the group individuals as a whole [26]. The procedure of GDM usually consists of two processes [27–31]: *consensus process* and *selection process*. The former is used to obtain the maximum degree of agreement between the set of DMs on the solution set of alternatives. Such

a process is defined as a dynamic and iterative group discussion process, coordinated by a moderator helping DMs bring their opinions closer [31]. The latter refers to obtain the solution set of alternatives according to the collective opinions of DMs.

Since consensus degree is employed to identify the agreement level amongst all DMs, it is preferable that the set of DMs can achieve the maximum consensus before applying the selection process [31]. Thus, how to obtain the maximum consensus for the given problem is a hot issue in GDM, and various approaches and methods have been developed in recent years. Hsu and Chen [32] presented a procedure for aggregating the individual opinions into a group consensus opinion. Herrera et al. [28] presented a consensus model in group decision making under linguistic assessments. Herrera et al. [33] proposed a model for the consensus reaching problem in heterogeneous group decision making. The model contains two types of linguistic consensus measures: linguistic consensus degrees and linguistic proximities to guide the consensus reaching process. A similar work was reported in Herrera-Viedma et al. [34], where a model of consensus support system was presented to assist the DMs in all phases of the consensus reaching process of group decision making problems with multigranular linguistic preference relations. Mata et al. [35] proposed an adaptive consensus support system model for the group decision making problems defined in multigranular linguistic contexts. Xu [36] developed an automatic approach to reaching consensus among group opinions for multiple attribute group decision making problems. It is worthwhile to note that in group decision making problems, besides consensus degree, some other measures might be used to indicate the final solution(s) from different aspects. Wu and Xu [37] studied individual consistency and group consensus in the process of group decision making with multiplicative preference relations and provided a decision support model to aid the group consensus process while keeping an acceptable individual consistency for each decision maker. In the present paper, we simultaneously consider consensus and robustness degrees of a solution for FGDM-MOP, providing the tradeoff between the solutions' acceptable degree among DMs and the ability to cope with any change on preferences.

3. Problem Formulation

The result of multi-objective optimization problem, shown as in (1), is a set of non-dominated solutions, denoted *ND*. The task of multi-objective decision making is to select a final solution among the non-dominated set.

It is usual that the selection process involves multiple DMs. Suppose the decision group consists of m DMs, denoted as D_j $(j = 1, 2, \ldots, m)$. The weight vector of DMs is represented as $\omega = (\omega_1, \omega_2, \ldots, \omega_m)$, satisfying $\sum_{j=1}^{m} \omega_j = 1$. DMs express their preference by reference points in objective space, denoted as $R_j = (r_{j1}, \ldots, r_{jk})$, where r_{ji} $(i = 1, \ldots, k)$ is the reference value of ith objective function given by the jth DM. The reference values given by DMs for each objective can be represented as triangular fuzzy numbers [38]; each value can be expressed as a triple $r_{ji} = (r_{ji}^{\text{lower}}, r_{ji}^{\text{most}}, r_{ji}^{\text{upper}})$ with its membership function defined as

$$\mu_A(r) = \begin{cases} \dfrac{\left(r - r_{ji}^{\text{lower}}\right)}{\left(r_{ji}^{\text{most}} - r_{ji}^{\text{lower}}\right)}, & r_{ji}^{\text{lower}} \le r \le \mathrm{r}_{ji}^{\text{most}}, \\ \dfrac{\left(r_{ji}^{\text{upper}} - r\right)}{\left(r_{ji}^{\text{upper}} - r_{ji}^{\text{most}}\right)}, & r_{ji}^{\text{most}} \le r \le r_{ji}^{\text{upper}}, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where r_{ji}^{most} is the most possible value of the fuzzy number A and r_{ji}^{lower} and r_{ji}^{upper} are, respectively, the lower and upper bounds, used to reflect the fuzziness of the DMs' preference.

Given the preferences of DMs, the consensus degree of each solution in non-dominated set is denoted as C_p $(p = 1, \ldots, N)$, where N is the number of non-dominated solutions. For each solution in non-dominated set, its ability to deal with the changes on DMs' preference is termed as *robustness* and is denoted as $\text{Rob}_p (p = 1, \ldots, N)$.

Thus, in FGDM-MOP, we need to find solutions which are both acceptable by the group and robust against fuzzy preferences of DMs. The decision making problem can be formally modeled as follows:

$$\begin{aligned} \text{obj.}: \quad & \max C(x) \\ & \max \text{Rob}(x). \end{aligned} \tag{3}$$

The calculation of *consensus* and *robustness* will be presented in the following section.

4. Consensus and Robustness Measures

Usually, in group decision making, consensus is employed to measure the closeness among DMs' opinions [31]. For each final selected solution, it is expected to be as close to the collective opinion as possible. The term of consensus employed in this paper is slightly different from other literatures. In Herrera et al. [28], Herrera-Viedma et al. [29] consensus degree is used to reflect the closeness of the DMs' preferences on a set of alternatives. While in our problem, the DMs' preferences are expressed as reference points in the objective space. Thus, we use consensus to reflect not only the closeness among DMs' preferences, but also the distance of each solution in the Pareto curve to the collective preference. Since the preferences of DMs are fuzzy, the robustness is employed to measure each solution's ability of dealing with any change on the preferences.

4.1. Consensus Measure. Once the reference points are given, the consensus degree for each solution is calculated as follows.

(1) Distance between the solution and each reference point.

For each solution x, the objective values are $(f_1(x), \ldots, f_k(x))$, where k is the number of objectives. Since

the reference value on each objective is expressed by fuzzy number, the relative distance between the solution and reference point on each objective is given by a fuzzy distance. For the reference value on the ith objective given by decision maker D_j, that is, $r_{ji} = (r_{ji}^{\text{lower}}, r_{ji}^{\text{most}}, r_{ji}^{\text{upper}})$, we employ the concept of vertex method [39] to calculate the distance, given as follows:

$$d_{ji}(x) = \frac{\sqrt{(1/3)\left(f_i(x) - r_{ji}^{\text{lower}}\right)^2 + (1/3)\left(f_i(x) - r_{ji}^{\text{most}}\right)^2 + (1/3)\left(f_i(x) - r_{ji}^{\text{upper}}\right)^2}}{f_i(x)^{\max} - f_i(x)^{\min}}, \tag{4}$$

where $j = 1, \ldots, m$, $i = 1, \ldots, k$, $f_i(x)^{\max}$, and $f_i(x)^{\min}$, respectively, represent maximum and minimum values of the ith objective function in a given set.

By aggregating the distance measures on all objectives, the distance between the solution and the reference point given by decision maker D_j can be obtained and given as follows:

$$d_j(x) = \phi\left(d_{ji}(x), i = 1, \ldots, k\right), \tag{5}$$

where ϕ is aggregation function, which is arithmetic mean in this research. However, different aggregation operators could be used. For example, if a weight vector is used in objective functions, the aggregation operator could be replaced with weighted aggregation.

(2) Aggregation of distances to obtain the consensus degree.

The distance $d_j(x)$ indicates how far the solution is from the individual decision maker's preference. In order to obtain the consensus measure among the group, the distances from all DMs' reference points need to be considered. In our case, we use additive weight aggregation operator to calculate the consensus degree, given as follows:

$$\text{cd}(x) = \sum_{j=1}^{m} \omega_j d_j(x), \tag{6}$$

where m is the number of decision makers.

4.2. Robustness Measure. In order to evaluate the effect of perturbations in the objective space of MOP, Bui et al. [8] defined the *preference robustness* as follows: *preference robustness* of a *non-dominated solution is defined as the minimum transition costs in decision space when the solution is perturbed in objective space.* Preference robustness is specifically tailored for MOP and indicates the closeness of other non-dominated solutions in the decision space.

Following the principle of Bui et al. [8], the term of *robustness* used in this paper refers to the preference robustness. Clearly, for each solution, given a neighborhood radius δ in the n-dimensional decision space, if there are more neighbors and smaller distances, the solution is expected to be more capable of moving to another non-dominated solution with a less transition cost. It is worthwhile to note that the transition cost varies depending on problem domain [8]. Thus, we modify the preference robustness measure and approximate it as follows:

$$\text{rd}(x) = \frac{1}{|ND_\delta| + \epsilon} + \frac{\sum_{y \in ND_\delta} d(x, y)}{|ND_\delta| + \epsilon}, \tag{7}$$

where ND_δ represents the set of all non-dominated solutions in the area of neighborhood with radius δ of the non-dominated solution x, $|ND_\delta|$ indicates the number of non-dominated solutions in the neighborhood, and ϵ is a small number used to avoid potential singularities in the denominator, which is set to $1E$-06. $d(x, y)$ is the relative distance between the solution x and its neighbor y, given as follows:

$$d(x, y) = \frac{\sum_{q=1}^{n} \left(\left|x_q - y_q\right| / \left(x_q^{\max} - x_q^{\min}\right)\right)}{n}, \tag{8}$$

where $x_q^{\max}$ and $x_q^{\min}$, respectively, indicate the maximum and minimum values of the qth decision variable in the non-dominated set. It can be seen from (7) that the calculation of robustness $\text{rd}(x)$ consists of two parts: the first part reflects the number of neighbors in the neighborhood, while the second part indicates the average distance of the non-dominated solution from its neighbors.

5. Evolutionary Approach for GDM-MOO

It is clear that for both consensus and robustness measures, the smaller $\text{cd}(x)$ and $\text{rd}(x)$, the better the consensus and robustness a solution has. Thus, the decision making problem presented in (3) can be replaced as follows:

$$\begin{aligned} \text{obj.:} \quad & \min \text{cd}(x) \\ & \min \text{rd}(x). \end{aligned} \tag{9}$$

We employ one of classical multi-objective evolutionary algorithms, NSGA-II [15], as the optimizer for multi-objective optimization problem. NSGA-II is an elitism-based multiobjective evolutionary algorithm, whose main feature is an elitism-preserving operation. In NSGA-II, the parent population and offspring are combined and sorted in order to generate a population for the next generation. A non-dominated sorting mechanism is performed to classify the combined population into different ranks of nondomination. A crowding-distance assignment is employed to ensure that diversity is maintained among non-dominated solutions.

Table 1: The fuzzy reference points given by DMs for M-BINH.

Decision maker (D_j)	r_{j1}	r_{j2}
D_1	(36.0, 40.0, 44.6)	(8.2, 10.0, 13.3)
D_2	(26.9, 30.0, 32.1)	(13.0, 15.0, 19.5)
D_3	(16.6, 20.0, 25.6)	(3.9, 5.0, 8.5)
D_4	(38.1, 45.0, 48.8)	(12.4, 15.0, 17.3)
D_5	(18.3, 20.0, 21.1)	(17.8, 20.0, 22.4)

In the original NSGA-II, the mechanism of dominance ranking is used to classify the population into a number of layers, such that the first layer is the non-dominated set in the population and with the rank value of 1. Similarly, in the second layer, the solutions are non-dominated in the population with the first layer removed and with the rank value of 2. The sorting procedure is continued until all solutions in the population are classified into a layer and assigned a rank value. In order to perform the decision making task and incorporate the DMs' preference information into the optimization process, we slightly modify this ranking procedure of NSGA-II.

The final solutions of problem in (9) is a subset of the solutions of original multi-objective optimization problem in (3). In other words, some of the solutions with the first non-dominated rank will be excluded because they are dominated for (9). Thus, after each generation, we modify the non-dominated rank of the obtained solutions as follows:

$$\mathrm{Ind}_p^{\mathrm{rank}} = \begin{cases} \mathrm{Ind}_p^{\mathrm{rank}}, & \text{if } \mathrm{Ind}_p^{\mathrm{rank}} = 1 \wedge \mathrm{Ind}_p \\ & \text{is nondominated for (9)} \\ \mathrm{Ind}_p^{\mathrm{rank}} + 1, & \text{else,} \end{cases} \tag{10}$$

where Ind_p is the pth individual solution in the population and $\mathrm{Ind}_p^{\mathrm{rank}}$ indicates the non-dominated rank of the solution Ind_p.

6. Experimental Results

We illustrate the proposed approach using and modifying a well-known benchmark function. We investigated the problem introduced by Binh and Korn [40] with two objectives, denoted as BINH and given as follows:

$$\begin{aligned} &\min f_1(x_1, x_2) = x_1^2 + x_2^2, \\ &\min f_2(x_1, x_2) = (x_1 - 5)^2 + (x_2 - 5)^2, \end{aligned} \tag{11}$$

where $-5 \le x_1, x_2 \le 10$.

In BINH, the solutions evenly spread in both decision and objective spaces. In order to illustrate the proposed approach clearer, we modified the original BINH as a many-to-one problem. Similar with in Bui et al. [8], the function's domain is divided into different parts. Each part uses a different resolution for forming intervals that are mapped to a single point in objective space. The modified BINH is denoted as M-BINH and given as follows:

$$\begin{gathered} x_p^c = \frac{x_p^{\max} + x_p^{\min}}{2}, \\ r_{1\text{-}p} = 0.2, \\ r_{2\text{-}p} = \frac{x_p}{x_p^{\max}}, \\ x_p = \begin{cases} x_p, & \text{if } x_p \le 1 \\ x_p^{\min} + \mathrm{floor}\left(\dfrac{x_p - x_p^{\min}}{r_{1\text{-}p}}\right) r_{1\text{-}p}, & \text{if } x_p < x_p^c \\ x_p^c + \mathrm{floor}\left(\dfrac{x_p - x_p^c}{r_{2\text{-}p}}\right) r_{2\text{-}p}, & \text{else,} \end{cases} \\ p = 1, 2, \\ \min f_1(x_1, x_2) = x_1^2 + x_2^2, \\ \min f_2(x_1, x_2) = (x_1 - 5)^2 + (x_2 - 5)^2, \end{gathered} \tag{12}$$

where $0 \le x_1, x_2 \le 5$, $x_p^{\min}$ and $x_p^{\max}$ $(p = 1, 2)$, respectively, indicate the lower and upper bounds of x-domain. $r_{1\text{-}p}$ and $r_{2\text{-}p}$ are resolutions for the different parts of the domain of x_p.

It is supposed that there are five DMs, whose weight coefficients are $(0.12, 0.20, 0.18, 0.22, 0.28)$. The fuzzy reference points given by each decision maker are presented in Table 1.

6.1. Parameter Setting. The population size was set to 100, the crossover rate was 0.95, and the mutation rate was $1/n$, where n is the number of decision variables. The distribution indices for crossover and mutation operators were, respectively, set to $\eta_c = 20$ and $\eta_m = 10$. The evolution process was terminated after 400 generations. In the calculation of robustness, the radius of neighborhood δ was set to 0.08. Furthermore, in order to alleviate the effect of algorithm's stochastic nature, each experiment is run repeatedly for 10 times with different random seeds.

6.2. Results Analysis. At first, we report the obtained non-dominated solutions of M-BINH in both decision and objective spaces, respectively, presented in Figures 1 and 2. From the figures, one can see that the non-dominated solutions are discontinuous and each part with different resolution. In particular, in the domain $x_1, x_2 \in [0, 1]$, the non-dominated solutions are dense, continuous, and evenly spread in both decision and objective spaces. While for $x_1, x_2 \in (1, 5]$, the non-dominated solutions are divided into several parts with different densities.

With reference points, we can obtain the non-dominated solutions of interest to DMs, shown as in Figure 3. In the figure, the most preferable values of the fuzzy reference points given by DMs are represented as black stars, while the non-dominated solutions obtained with preferences are indicated by blue triangles. It can be seen that with DMs' preference, the

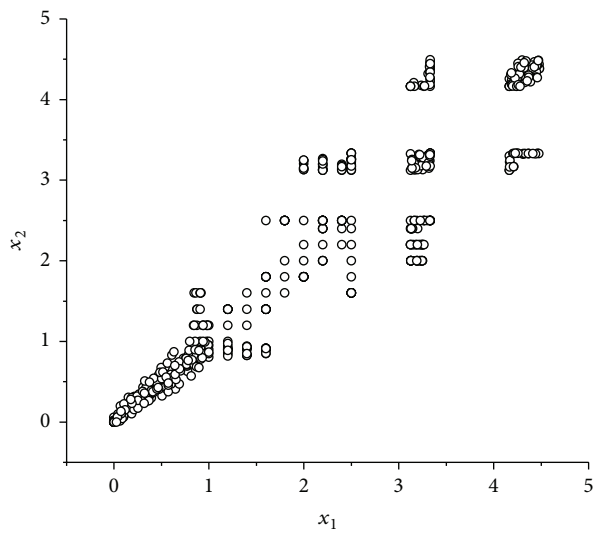

Figure 1: The obtained nondominated solutions of M-BINH problem, projected in decision space.

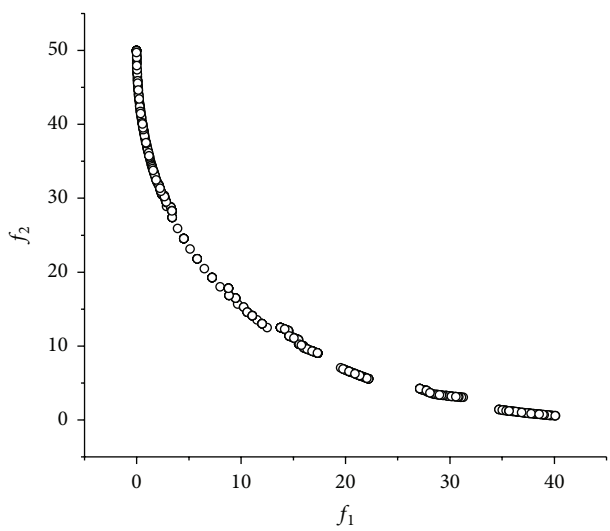

Figure 2: The obtained non-dominated solutions of M-BINH problem, projected in objective space.

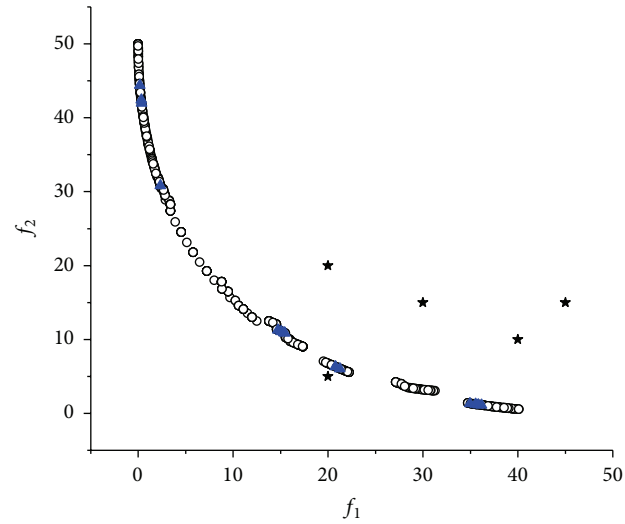

Figure 3: The obtained non-dominated solutions of M-BINH problem with reference points, projected in objective space.

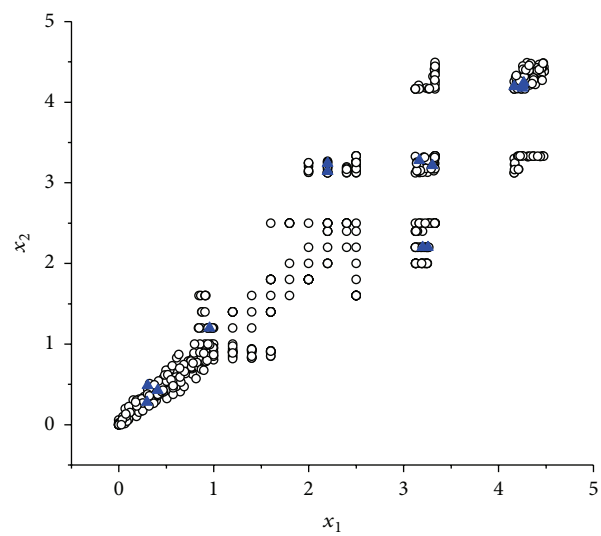

Figure 4: The obtained non-dominated solutions of M-BINH problem with reference points, projected in decision space.

obtained solutions are a subset of the non-dominated solutions obtained without any preference. One may notice that some solutions far from the reference points are included, for example, the solutions with lower values on objective f_1 and higher values on f_2. This is because these solutions provide higher values on robustness measures. Since the robustness of a solution is measured in decision space, we also depicted the projection of the non-dominated solutions obtained with reference points on decision space, shown as in Figure 4. Similarly, the solutions obtained with reference points are denoted as blue triangles. From Figure 4, one can see that the final selected non-dominated solutions are the ones which are surrounded by other non-dominated solutions. Thus, these non-dominated solutions can easily move to another position in the non-dominated set in the presence of perturbation on decision space or DMs' preference.

As indicated earlier, for each solution, consensus and robustness measures are in conflict with each other to a certain degree. In other words, in the final selected non-dominated set, the values of consensus and robustness of all solutions form another non-dominated set in consensus-robustness space. Figure 5 shows such a confliction between consensus and robustness. In order to clearly present the mapping among decision space, objective space, and consensus-robustness space, Table 2 lists the decision variables, objective functions, and consensus and robustness measures of the final selected non-dominated set.

Table 2: The decision variables, objective functions, and consensus and robustness measures of the final selected nondominated solutions for M-BINH.

Index	x_1	x_2	f_1	f_2	cd	rd
1	3.2576	2.2000	15.4516	10.8761	0.2412	0.1538
2	3.2006	2.2000	15.0836	11.0780	0.2449	0.1511
3	3.1648	3.2792	20.7687	6.3294	0.2112	0.1577
4	0.3035	0.4851	0.3274	42.4414	0.6620	0.1122
5	0.2967	0.2782	0.1654	44.4161	0.6840	0.1069
6	2.2000	3.2415	15.3476	10.9322	0.2399	0.1539
7	2.2000	3.1431	14.7190	11.2881	0.2461	0.1498
8	4.2535	4.1775	35.5441	1.2337	0.2687	0.1329
9	4.1674	4.1938	34.9552	1.3432	0.2652	0.1484
10	4.2658	4.2414	36.1870	1.1144	0.2726	0.1146
11	4.2581	4.2069	35.8298	1.1794	0.2704	0.1225
12	0.9549	1.2000	2.35185	30.8028	0.5242	0.1133
13	0.4089	0.4281	0.3505	41.9797	0.6487	0.1125
14	3.3027	3.2169	21.2564	6.0602	0.2105	0.2155

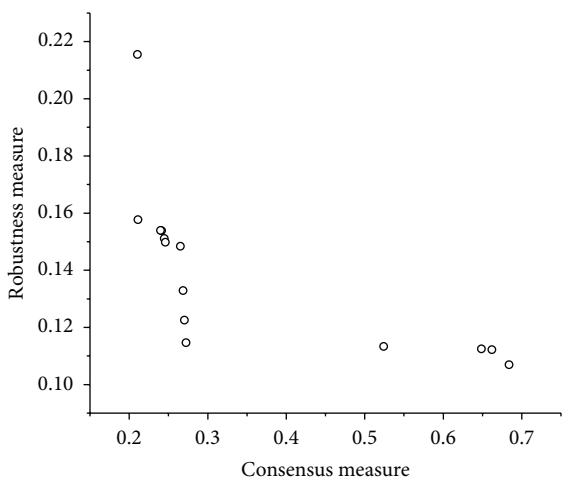

Figure 5: The consensus and robustness measures of the obtained non-dominated solutions of M-BINH problem with reference points.

7. Conclusion

In many real-world decision problems, a set of decision makers need to trade off between multiple conflicting objectives then provide their preferences to select the final solution(s). This paper studies such an amalgamation of group decision making (GDM) and multi-objective problems (MOP), referred to as GDM-MOP. As the limitation of the DMs' background and/or knowledge structure, it is usually hard for them to give precise preference information. Thus, the GDM-MOP with fuzzy preference is addressed in this research, called as fuzzy GDM-MOP (FGDM-MOP). In the context of GDM, consensus is used to indicate the distance of a solution from the group preference. In the presence of uncertain preference, robustness is suggested to measure the ability of coping with change on preference. Thus, in the process of FGDM-MOP, it is preferable to pursue both great consensus and robustness degrees. This paper firstly describes the mathematical model of FGDM-MOP. Then, a new multi-objective model is introduced for FGDM-MOP, where the measures of consensus and robustness are simultaneously considered. To obtain the refined non-dominated set of new established multi-objective model, one of the classical multi-objective evolutionary algorithms—NSGA-II—is employed and modified to solve the problem. However, how to identify the final solution with the consideration of consensus and robustness highly depends on the decision makers' preferences on different measures. We leave this new decision making problem to our future research. We examine a benchmark function to illustrate the concept and procedure of the proposed approach. In our future study, some real cases will be addressed through the proposed approach.

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