Research Article

Passivity and Passification for Delay Fuzzy System Based on Delay Partitioning Approach

Xiangjie Liu,1 Dan Yue,1 and Xiuming Yao2

1 The State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China
2 Department of Automation, North China Electric Power University, Baoding 071003, China

Correspondence should be addressed to Xiuming Yao; xiumingyao@gmail.com

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A delay partitioning approach is introduced to solve problems of passivity and passification for continuous T-S fuzzy system with time delay. Our aim is to design a state feedback controller such that the resulting closed system is passive. By constructing a Lyapunov-Krasovskiifunctional, delay-dependent passivity/passification performance conditions are formulated in terms of linear matrix inequalities (LMIs). Finally, numerical examples are used to illustrate the effectiveness of the proposed approaches which can further reduce conservatism and become more obvious with partitioning getting thinner.

1. Introduction

The passivity concept was introduced by Willems [1] and developed further by Hill and Moylan [2]. Passivity of nonlinear systems has attracted great interest in the control area mainly because of the link between stability and passivity. The passivity theory provides a nice tool for analyzing the stability of systems and has found applications in diverse areas such as stability, signal processing, chaos control, and synchronization.

Since most physical systems in real world are nonlinear, researchers have been devoting their efforts to the control of nonlinear systems. Among the methods, the fuzzy control has been proven to be effective in dealing with the analysis of nonlinear systems, especially the T-S fuzzy control [3, 4]. It is denoted by a group of IF-THEN rules that the conventional linear system theory can be applied to the analysis of the class of nonlinear systems, and numerous nonlinear analysis problems have been studied based on this T-S fuzzy model, such as [5–7] reported the problem of stability analysis, and [8–10] investigated the $H_\infty$ control designs. References [11–13] mentioned the fault detection of the T-S fuzzy systems. $H_\infty$ model reduction is addressed in [14]. The fuzzy controller was carried out via Parallel Distributed Compensation (PDC) technique [15]. Based on the PDC technique, the fuzzy controller can also be designed to guarantee the passivity of T-S fuzzy systems.

On the other hand, the time delay exists naturally in various control systems. Time delays often degrade the system’s performance and even cause instability. Therefore, time delays have received great attention in recent years and many researchers have studied various analytical techniques and developed many synthesis methods for time-delay systems. For instance, model reduction is addressed in [16] and filtering problems are investigated in [17, 18]. So, the passivity and passification analysis of nonlinear systems with time delays is worth to be discussed and researched. To date, researches have gained many results in passivity control of T-S fuzzy systems; the passivity of delayed neural networks is considered in [19], passivity of fuzzy time-delay systems is investigated in [20] which adopt delay moom’s inequality, and passive controller design for T-S fuzzy systems is addressed in [21]. The passivity of uncertain fuzzy systems is considered in [22]. However, the above methods still have strong conservatism, and it is necessary for us to further study.

In this paper, we adopt a delay partitioning approach to study the passivity and passification of T-S fuzzy systems with time delay. Based on this idea, we can further reduce the
conservatism, and it becomes even less conservative when
the partitioning goes finer. The results of this paper are given
in terms of LMIs. The rest of the paper is organized as
follows. In Section 2, the problem to be studied is stated and
some preliminaries are presented. Passivity analysis results
are presented in Section 3. Based on the results obtained in
Section 3, we design the controller in Section 4. In Section 5,
numerical examples are given to demonstrate the effective-
ness of the theoretical results. Finally, conclusions are drawn
in Section 6.

Notations. Throughout the paper, $A^{-1}$ and $A^T$ denote
the inverse and transpose of a square matrix $A$. $R^n$ denotes
the $n$-dimensional Euclidean space and $\| \cdot \|$ refers to the
Euclidean vector norm. The notation $A > 0$ is used to define a
symmetric positive definite matrix and sym $(A)$ is defined as
$A + A^T$. Matrices are assumed to have compatible dimensions.

2. Problem Statement and Preliminaries

Consider the T-S fuzzy system with time delay has the
following form.

Plant Rule $i$:
If $Z_i(t)$ is $M_{i1}$ and ... and $Z_p(t)$ is $M_{ip}$ THEN
\[
\dot{x}(t) = A_i x(t) + A_{di} x(t-h) + B_i u(t) + B_{di} \omega(t),
\]
\[
y(t) = C_i x(t) + C_{di} x(t-h) + D_i \omega(t),
\]
\[
x(t) = \varphi(t), \quad t \in [-h,0],
\]
where $x(t) \in R^n$ is the state vector; $u(t)$ is the control input
vector; $h$ is a time delay; $\varphi(t)$ is the initial condition.

Controller Rule $i$:
If $Z_i(t)$ is $M_{i1}$ and ... and $z_p(t)$ is $M_{ip}$ THEN
\[
u(t) = K_i x(t), \quad i = 1, \ldots, r,
\]
where $M_{ij}$ is the fuzzy set; $r$ is the number of IF-THEN
rules; $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]$ is the premise variables
vector; $K_i$, $i = 1, \ldots, r$ are constant matrices representing
state-feedback control gains. Let $\lambda_i(t)$ be the normalized
membership function of the fuzzy set
\[
\lambda_i(t) = \frac{\prod_{j=1}^{r} M_{ij}(z_j(t))}{\sum_{j=1}^{r} \prod_{j=1}^{r} M_{ij}(z_j(t))},
\]
where $M_{ij}(z_j(t))$ is the grade of membership function of $z_j(t)$
in $M_{ij}(t)$. It is assumed that $\prod_{j=1}^{r} M_{ij}(z_j(t)) \geq 0$, $i = 1, \ldots, r$
and $\sum_{j=1}^{r} \prod_{j=1}^{r} M_{ij}(z_j(t)) > 0$ for all $t$. Therefore, $\lambda_i(t) \geq 0$
and $\sum_{i=1}^{r} \lambda_i(t) = 1$ for all $t$. By applying (3) into (1), the fuzzy
system can be expressed as
\[
\dot{x}(t) = A(t)x(t) + A_d(t)x(t-h) + B(t)u(t) + B_1(t)\omega(t),
\]
\[
y(t) = C(t)x(t) + C_d(t)x(t-h) + D(t)\omega(t)
\]
with
\[
A(t) = \sum_{i=1}^{r} \lambda_i(t) A_i, \quad A_d(t) = \sum_{i=1}^{r} \lambda_i(t) A_{di},
\]
\[
B(t) = \sum_{i=1}^{r} \lambda_i(t) B_i, \quad B_1(t) = \sum_{i=1}^{r} \lambda_i(t) B_{di},
\]
\[
C(t) = \sum_{i=1}^{r} \lambda_i(t) C_i, \quad C_d(t) = \sum_{i=1}^{r} \lambda_i(t) C_{di},
\]
\[
D(t) = \sum_{i=1}^{r} \lambda_i(t) D_i.
\]

By applying (2) into (4), we can get the following closed-loop
system:
\[
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(z(t)) \lambda_j(z(t))
\]
\[
\times \left[ \left( A_i + B_i K_j \right) x(t) \right.
\]
\[
\left. + A_{di} x(t-h) + B_{di} \omega(t) \right],
\]
\[
y(t) = \sum_{i=1}^{r} \lambda_i(z(t)) \left[ C_i x(t) + C_{di} x(t-h) + D_i \omega(t) \right].
\]

Before formulating the main problem, we first give the
following definition.

Definition 1 (Li et al. [20]). The fuzzy system (1) is called
passive if there exists a scalar $\gamma \geq 0$ such that
\[
2 \int_{0}^{T} \omega(s)^T y(s) ds \geq -\gamma \int_{0}^{T} \omega^T(s) \omega(s) ds
\]
for all $T \geq 0$.

3. Passivity Analysis

The problems to be addressed in this paper can be expressed
as follows.

Problem 1 (passivity analysis). Given the feedback controller
gain matrices $K_i, i = 1, \ldots, r$ in (2), determine under what
conditions the closed-loop system (6) is passive for all $T \geq 0$
in the sense of Definition 1.

Problem 2 (passification). Determine the feedback controller
gain matrices, $K_i, i = 1, \ldots, r$ in (2), such that the closed-loop
system (6) is passive for all $T \geq 0$ in the sense of Definition 1.

In this section, we will present a sufficient condition in
terms of LMIs, under which the closed-loop system (6) is
passive.
**Theorem 2.** Given matrices $K_i$, an integer $m \geq 1$ and a scalar $h > 0$, if there exist symmetric positive definite matrices $P, Q_i$, $Z_i, R_i$ and matrices $S_{ij}, S_{2i}$ and scalar $\gamma > 0$, satisfying

\[
\Theta_{ijk} + \sigma W_\sigma^T W_\sigma S_{ij} + \frac{m}{h} Z_i < 0, \quad i, i, k = 1, \ldots, r \tag{8}
\]

\[
\Theta_{ijk} + \sigma W_\sigma^T W_\sigma S_{ij} + \frac{m}{h} Z_j < 0, \quad 1 \leq r < j \leq r, \quad i, k = 1, \ldots, r, \tag{9}
\]

\[
Z_i < R_i, \quad i, j = 1, \ldots, r, \tag{10}
\]

where

\[
\Omega_{ijk} = W_{p}^T \hat{P} W_{p} + \frac{1}{m} W_{r}^T R_{i} W_{r} + W_{q_1}^T Q_{i} W_{q_1} - W_{q_2}^T Q_{i} W_{q_2} + \text{sym} \left(S_{ij} W_{skj}\right),
\]

\[
\Theta_{ijk} = \Omega_{ijk} - \left(W_{1} C_{j}^T W_{2} + W_{2} C_{j}^T W_{1}^T\right)
\]

\[
+ W_{2} D_{j}^T W_{2} + W_{2} C_{j}^T W_{3} + W_{3} C_{j}^T W_{2} + \gamma W_{2}^T W_{2},
\]

\[
W_{p} = \begin{pmatrix}
I_n & O_{n,mn} & O_{n,n} & O_{n,n} \\
O_{n,n} & I_n & O_{n,n} & O_{n,n} \\
o_{n,n} & O_{n,n} & O_{n,n} & O_{n,n}
\end{pmatrix},
\]

\[
\hat{P} = \begin{pmatrix}
o_{n,n} & P & O_{n,n} & O_{n,n} \\
P & O_{n,n} & O_{n,n} & O_{n,n} \\
o_{n,n} & O_{n,n} & O_{n,n} & O_{n,n}
\end{pmatrix},
\]

\[
W_{\sigma} = \begin{pmatrix}
I_n & O_{n,mn+2n} \\
o_{n,n} & O_{n,n} & O_{n,n} \\
o_{n,n} & O_{n,n} & O_{n,n}
\end{pmatrix},
\]

\[
W_{r} = \begin{pmatrix}
o_{n,mn+1} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n}
\end{pmatrix},
\]

\[
W_{q_1} = \begin{pmatrix}
o_{n,mn+1} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n} \\
o_{n,n} & O_{n,n} & O_{n,n}
\end{pmatrix},
\]

\[
W_{q_2} = \begin{pmatrix}
o_{n,mn} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n}
\end{pmatrix},
\]

\[
S_{ij} = \begin{pmatrix}
o_{n,n} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n} \\
o_{n,n} & I_{n} & O_{n,n}
\end{pmatrix},
\]

\[
W_{q_{jk}} = \begin{pmatrix}
I_{n} & -I_{n} & O_{n,n+mn} \\
A_{j} + B_{j} K_{k} & O_{n,mn-n} & A_{j} + B_{j} \times I_{n} \\
-I_{n} & -B_{i} & O_{n,n+mn}
\end{pmatrix},
\]

\[
W_{1} = \begin{pmatrix}
o_{n,mn+2n} & I_{n} \\
o_{n,n} & I_{n} \\
o_{n,n} & I_{n}
\end{pmatrix},
\]

\[
W_{2} = \begin{pmatrix}
o_{n,mn+2n} & I_{n} \\
o_{n,n} & I_{n} \\
o_{n,n} & I_{n}
\end{pmatrix},
\]

\[
W_{3} = \begin{pmatrix}
o_{n,mn} & I_{n} \\
o_{n,n} & I_{n} \\
o_{n,n} & I_{n}
\end{pmatrix},
\]

\[
\Pi_{1} = 2 \xi(t) S_{1}(t)
\]

\[
x(t) - x(t) - \int_{\tau/h}^{t} \dot{x}(\alpha) d\alpha = 0,
\]

\[
\Pi_{2} = 2 \xi(t) S_{2}(t) \left[ A(t) + B(t) K(t) \right] x(t)
\]

\[
+ A_d(t) x(t-h) + B_1(t) \omega(t) - \dot{x(t)} = 0,
\]

\[
\Pi_{3} = \frac{h}{m} \xi(t) S_{1}(t) Z^{-1}(t) S_{1}^T(t) \xi(t)
\]

\[
- \int_{\tau/h}^{t} \xi(t) S_{1}(t) Z^{-1}(t) S_{1}^T(t) \xi(t) d\alpha = 0.
\]

Proof. Choose a Lyapunov-Krasovskii functional as $V(t) = V_1(t) + V_2(t) + V_3(t)$ with

\[
V_1(t) = x^T(t) P x(t),
\]

\[
V_2(t) = \int_{t-h/m}^{t} \dot{x}(\alpha) R(\alpha) \dot{x}(\alpha) d\alpha d\beta,
\]

\[
V_3(t) = \int_{t-h/m}^{t} y^T(\alpha) Q(\alpha) y(\alpha) d\alpha,
\]

where

\[
y(t) = \begin{bmatrix} x^T(t) & x^T(t-h) & \ldots & x^T(t-(m-1)h) \end{bmatrix}^T,
\]

The time-derivative of $V(t)$ along the trajectory of the system in (6) is given by

\[
\dot{V}_1 = x^T(t) P \dot{x}(t) + x^T(t) P \dot{x}(t),
\]

\[
\dot{V}_2 = \frac{h}{m} x^T(t) R(t) \dot{x}(t) - \int_{t-h/m}^{t} x^T(\alpha) R(\alpha) \dot{x}(\alpha) d\alpha
\]

\[
\dot{V}_3 = y^T(t) Q(t) y(t) - y^T(t) Q(t-h) y(t-h) y(t-h).
\]

Define

\[
\xi(t) = \begin{bmatrix} y^T(t) & x^T(t-h) & \ldots & \omega(t) \end{bmatrix}^T,
\]

\[
\omega(t) = [\omega_1(t) \ldots \omega_n(t)]^T
\]

and according to the Newton-Leibniz formula and the system in (6), we have

\[
\Pi_1 = 2 \xi(t) S_1(t)
\]

\[
\times \begin{bmatrix} x(t) - x(t-h) \dot{x}(\alpha) d\alpha \end{bmatrix} = 0,
\]

\[
\Pi_2 = 2 \xi(t) S_2(t) \left[ A(t) + B(t) K(t) \right] x(t)
\]

\[
+ A_d(t) x(t-h) + B_1(t) \omega(t) - \dot{x(t)} = 0,
\]

\[
\Pi_3 = \frac{h}{m} \xi(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t)
\]

\[
- \int_{t-h/m}^{t} \xi(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t) d\alpha = 0.
\]

Then the fuzzy system (1) is passive in the sense of Definition 1 for the time delay $0 \leq \tau \leq h$. 
Therefore
\[
\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \Pi_1 + \Pi_2 + \Pi_3
\]
\[
= x^T(t)Px(t) + x^T(t)P\dot{x}(t) + \frac{h}{m}x^T(t)R(t)\dot{x}(t)
\]
\[
- \int_{t-h/m}^t x^T(\alpha)R(\alpha)\dot{x}(\alpha) d\alpha + y^T(t)Q(t)y(t)
\]
\[
- y^T\left(t - \frac{h}{m}\right)Q\left(t - \frac{h}{m}\right)y\left(t - \frac{h}{m}\right) + 2\xi^T(t)S_1(t)
\]
\[
\times \left[ x(t) - x\left(t - \frac{h}{m}\right) - \int_{t-h/m}^t \dot{x}(\alpha) d\alpha \right]
\]
\[
+ 2\xi^T(t)S_2(t)
\]
\[
\times \left[ (A(t) + B(t)K(t))x(t) + A_d(t)x(t-h)
+ B_1(t)\omega(t) - \dot{x}(t) \right]
\]
\[
+ \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)
\]
\[
- \int_{t-h/m}^t \xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) d\alpha
\]
\[
= \Lambda(t) + \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)
\]
\[
- \int_{t-h/m}^t \xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) d\alpha
\]
\[
- 2\xi^T(t)S_1(t)\int_{t-h/m}^t \dot{x}(\alpha) d\alpha
\]
\[
- \int_{t-h/m}^t \xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) d\alpha.
\]

where
\[
\Lambda(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \frac{h}{m}x^T(t)R(t)\dot{x}(t)
\]
\[
+ y^T(t)Q(t)y(t) - y^T\left(t - \frac{h}{m}\right)Q\left(t - \frac{h}{m}\right)y\left(t - \frac{h}{m}\right)
\]
\[
+ 2\xi^T(t)S_1(t)
\]
\[
\times \left[ (A(t) + B(t)K(t))x(t) + A_d(t)x(t-h) + B_1(t)\omega(t) - \dot{x}(t) \right]
\]
\[
+ \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)
\]
\[
- \int_{t-h/m}^t \xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) d\alpha.
\]

If
\[
Z(t) < R(\alpha).
\]
Then
\[
Z^{-1}(t) > R^{-1}(\alpha),
\]
\[
- \int_{t-h/m}^t \xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) d\alpha
\]
\[
< - \int_{t-h/m}^t \xi^T(t)S_1(t)R^{-1}(\alpha)S_1^T(t)\xi(t) d\alpha.
\]

So, we can obtain
\[
\dot{V}(t) < \Lambda(t) + \frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)
\]
\[
- \int_{t-h/m}^t \left( x^T(\alpha)R(\alpha) + \xi^T(t)S_1(t) \right) R^{-1}(\alpha)
\]
\[
\times \left( R(\alpha)\dot{x}(\alpha) + S_1^T(t)\xi(t) \right) d\alpha,
\]

(20)

Besides
\[
\dot{x}(t) = (O_{n,nn}+n, I_n, O_{n,nn}), \\
x(t) = (I_n, O_{n,nn+2n}), \\
y(t) = (I_{nm}, O_{nm,3n}), \\
y\left(t - \frac{h}{m}\right) = (O_{nm,n}, I_{nm}, O_{mm,2n}).
\]

(23)

Then
\[
\Lambda(t) = \xi^T(t)\Omega(t)\xi(t),
\]
\[
\Omega(t) = W_p^TPW_p + \frac{h}{m}W_r^TR(t)W_r + W_{q1}^TQ(t)W_{q1}
\]
\[
- W_{q2}^TQ\left(t - \frac{h}{m}\right)W_{q2} + \text{sym}(S(t)W_S(t)),
\]
\[
\hat{P} = \begin{pmatrix} O_{n,n} & O_{nn} \\ O_{nn} & O_{nn} \end{pmatrix}, \\
S(t) = \begin{bmatrix} S_1(t) & S_2(t) \end{bmatrix}
\]
\[
W_p = \begin{pmatrix} I_n & O_{n,nn} & O_{n,n} & O_{n,n} \\ O_{n,n} & I_n & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & I_n & O_{n,n} \end{pmatrix},
\]
\[
W_r = (O_{n,nn+1}, I_n, O_{n,n}), \\
W_{q1} = (I_{nm}, O_{mm,3n}),
\]
\[
W_{q2} = (O_{nm,n}, I_{nm}, O_{mm,2n}),
\]
\[
W_S(t) = \begin{bmatrix} A(t) + B(t)K(t) & O_{nm,n} & A_d(t) - I_n & B_1(t) \end{bmatrix}.
\]

(24)
Besides
\[ 2\omega^T(t) y(t) + \gamma \omega^T(t) \omega(t) \]
\[ = x^T(t) C_i^T \omega(t) + \omega^T(t) C_i x(t) \]
\[ + \omega^T(t) \left( D_i^T + D_i \right) \omega(t) + \omega^T(t) C_{di} x(t - h) \]
\[ + x^T(t - h) C_j^T \omega(t) + \gamma \omega^T(t) \omega(t) \]
\[ = \xi^T(t) \left[ W_1 C_i^T W_2 + W_2^T C_i^T W_1^T + W_2^T \left( D_i^T + D_i \right) W_2 \right. \]
\[ + W_2^T C_{di} W_3 + W_3^T C_{di}^T W_2 + \gamma W_2^T W_2 \] \]
\[ \left. \right] \xi(t), \]
\[ (25) \]
where
\[ W_1 = \left( I_n \right), \quad W_2 = \left( O_{nn,n}, I_n \right), \quad W_3 = \left( O_{n,nn}, I_n, O_{n,n} \right). \]

So, we have
\[ \dot{V}(t) - 2 \omega^T(t) y(t) - \lambda \omega^T(t) \omega(t) \]
\[ \leq \xi^T(t) \Theta(t) \xi(t) + \frac{h}{m} \xi^T(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t) \]
\[ - \int_{t-h/m}^{t} \left( \dot{x}^T(\alpha) R(\alpha) + \xi^T(t) S_1(t) \right) R^{-1}(\alpha) \]
\[ \times \left( R(\alpha) \dot{x}(\alpha) + S_1^T(t) \xi(t) \right) d\alpha \]
\[ (27) \]
with
\[ \Theta(t) = \Omega(t) - \left( W_1 C_i^T W_2 + W_2^T C_i^T W_1^T \right. \]
\[ + W_2^T \left( D_i^T + D_i \right) W_2 \]
\[ \left. + W_2^T C_{di} W_3 + W_3^T C_{di}^T W_2 + \gamma W_2^T W_2 \right). \]
\[ (28) \]
If
\[ \Theta(t) + \sigma W_\sigma^T W_\sigma + \frac{h}{m} S_1(t) Z^{-1}(t) S_1^T(t) < 0 \]
\[ (29) \]
then
\[ \Theta(t) + \frac{h}{m} S_1(t) Z^{-1}(t) S_1^T(t) < -\sigma W_\sigma^T W_\sigma, \]
\[ \dot{V}(t) - 2 \omega^T(t) y(t) - \gamma \omega^T(t) \omega(t) \]
\[ < \xi^T(t) \Theta(t) \xi(t) + \frac{h}{m} \xi^T(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t) \]
\[ < -\xi^T(t) \sigma W_\sigma^T W_\sigma \xi(t) - \sigma \|x(t)\|^2 < 0. \]
\[ (30) \]
We can obtain
\[ \dot{V}(t) < 2 y^T(t) \omega(t) + \gamma \omega^T(t) \omega(t). \]
\[ (31) \]
It follows by integrating (31) with respect to \( t \) over the time period \( 0 \sim T \) that (7) holds, and hence the delayed fuzzy system (1) is passive in the sense of Definition 1. Our next objective is to convert the inequalities in (18) and (29) to some finite LMIs, then (29) can be rewritten as
\[ \eta_{ijlk} = \eta_{ik} + \left( \eta_{ijk<l} + \eta_{ij<l,k} \right) < 0, \]
\[ \eta_{ijlk} = \sum_{l=1}^{r} \sum_{j=1}^{r} \lambda_i \left( \theta(t) \right) \sum_{k=1}^{r} \lambda_j \left( \theta(t) \right) \]
\[ \times \left[ \Omega_{ijlk} - \left( W_1 C_j^T W_2 + W_2^T C_j^T W_1^T \right. \right. \]
\[ \left. \left. + W_2^T (D_j + D_j^T) W_2 \right. \right. \]
\[ \left. + W_2^T C_{dj} W_3 + W_3^T C_{dj}^T W_2 + \gamma W_2^T W_2 \right) \]
\[ + \sigma W_\sigma^T W_\sigma \]
\[ + \frac{h}{m} S_1 Z_i^{-1} S_1^T \right]. \]
\[ (32) \]
Equation (18) can be rewritten as
\[ Z_i < R_f \]
\[ (33) \]
From the Schur complement, we can get the inequalities (8), (9), and (10), and the proof is completed. \( \square \)

4. Controller Design

In this section, fuzzy state feedback controllers will be designed based on the result developed in the previous section.

**Theorem 3.** Given an integer \( m > 1 \) and scalars \( h > 0, \tau_1, \tau_2, \ldots, \tau_{(m+3)} \), there exists a fuzzy state feedback controller such that the closed-loop system (4) is passive if there exist symmetric
positive definite matrices $P_i, Q_i, Z_i, R_i$, and matrices $X, S_{1i}, M_i$ and scalars $\gamma > 0, \sigma > 0$, satisfying

$$\begin{bmatrix}
    \phi_{ji} & S_{ij} & W_0^TX^T & W_2^TX^T & W_2^TX^T \\
    * & -m_i Z_i & 0 & 0 & 0 \\
    * & * & -\sigma^{-1}I_n & 0 & 0 \\
    * & * & * & D_i^{-T} + D_i^{-1} & 0 \\
    * & * & * & * & \gamma^{-1}I_n \\
\end{bmatrix} < 0 \quad (34)
$$

for $i, l = 1, \ldots, r$.

$$\begin{bmatrix}
    \phi_{ji} & S_{ij} & W_0^TX^T & W_2^TX^T & W_2^TX^T \\
    * & m_i Z_i & 0 & 0 & 0 \\
    * & * & -\sigma^{-1}I_n & 0 & 0 \\
    * & * & * & D_i^{-T} + D_i^{-1} & 0 \\
    * & * & * & * & \gamma^{-1}I_n \\
\end{bmatrix} > 0 \quad (35)
$$

where

$$\begin{bmatrix}
    \psi_{ji} & S_{ij} & W_0^TX^T & W_2^TX^T & W_2^TX^T \\
\end{bmatrix} = \begin{bmatrix}
    \psi_{ji} & S_{ij} & W_0^TX^T & W_2^TX^T & W_2^TX^T \\
    * & m_i Z_i & 0 & 0 & 0 \\
    * & * & -\sigma^{-1}I_n & 0 & 0 \\
    * & * & * & D_i^{-T} + D_i^{-1} & 0 \\
    * & * & * & * & \gamma^{-1}I_n \\
\end{bmatrix} \quad (36)
$$

If the above conditions are feasible, the gains of the controller are given by

$$K_i = M_iX_i^{-1}, \quad i = 1, \ldots, r. \quad (38)$$

Proof. Assume that $X$ is invertible, define $S = X^{-T}$, and

$$G_1 = \text{diag} \left\{ S, \ldots, S, I_n, S, S, S \right\} \in \mathbb{R}^{(m+3n) \times (m+3n)},$$

$$G_2 = \text{diag} \left\{ S, \ldots, S, I_n \right\} \in \mathbb{R}^{(m+3n) \times (m+3n)}, \quad (39)$$

$$G_3 = \text{diag} \{ S, \ldots, S \} \in \mathbb{R}^{mn \times mn}.$$ 

Premultiplying and postmultiplying (34) with $G_1$ and $G_1^T$, then we obtain

$$\begin{bmatrix}
    \Delta & G_2 \bar{S}_{ij} S_i^T & G_2 W_0^TX_iS_i^T & G_2 W_2^TX_iS_i^T & G_3 W_2^TX_iS_i^T \\
    * & \frac{m}{h} S_i Z_i S_i^T & 0 & 0 & 0 \\
    * & * & -\sigma^{-1}SS^T & 0 & 0 \\
    * & * & * & S \left(D_i^{-T} + D_i^{-1}\right) S_i^T & 0 \\
    * & * & * & * & \gamma^{-1}SS^T \\
\end{bmatrix} < 0, \quad (40)
$$

where

$$\Delta = G_2 \bar{S}_{ij} G_1^T, \quad G_2 \bar{S}_{ij} G_1^T = W_p^T \begin{bmatrix}
    O_{n,n} & SP_i S_i^T & O_{n,n} \\
    O_{n,n} & O_{n,n} & O_{n,n} \\
    O_{n,n} & O_{n,n} & O_{n,n} \\
\end{bmatrix} W_p + \frac{h}{m} W_0^T SR_i S_i^T W_r \\
+ \frac{m}{h} W_0^T SR_i S_i^T W_r \\
+ \frac{m}{h} W_0^T SR_i S_i^T W_r \\
+ \text{sym} \left( G_2 V_i W_0 S_i G_2^T \right) - W_i C_i^{-T} W_2 \quad (41)
$$

We defining

$$p = SP_i S_i^T, \quad r_1 = SR_i S_i^T, \quad q_1 = G_3 Q_i G_3^T,$$

$$S_{1i} = G_2 S_i S_i^T, \quad Z_i = S_i Z_i S_i^T, \quad (42)$$

$$S_{2i} = \left[ \tau_1 S_i^T \tau_2 S_i^T \ldots \tau_{(m+2)} S_i^T \tau_{(m+3)} I_n \right]^T.$$
Then
\[
G_2 V_i W S_i^T G_2^T = G_2 S_i^T \left[ I_n - I_n O_{n,mn} \right] G_2^T + G_2 U \left[ A_i X + B_i M_i A_{di} X - X B_{1i} \right] G_2^T = G_2 S_i \left[ I_n - I_n O_{n,mn} \right]
\]
\[
+ S_{2i} \left[ A_i + B_i K_k O_{n,mn} A_{di} - I_n B_{1i} \right]
\]
\[
= \begin{bmatrix} I_n - I_n O_{n,mn} \\ A_i + B_i K_k O_{n,mn} A_{di} - I_n B_{1i} \end{bmatrix}
\]

(43)

then
\[
G_2 q_{ii} G_2^T = W_p^T \begin{bmatrix} O_{n,n} & P \\ P & O_{n,n} \end{bmatrix} W_p + \frac{h}{m} W_r^T R_i W_r
\]
\[
+ W_{q1}^T Q_1 W_{q1} - W_{q2}^T Q_2 W_{q2} + \text{sym} \left( G_2 V_i W S_i^T G_2^T \right)
\]
\[
- W_i C_{i1} W_1 - W_i C_{i2} W_2
\]
\[
- W_i C_{di} W_3.
\]

(44)

Thus, by the Schur complement, we can obtain (40) is equivalent to (8). Pre- and postmultiplying (35) with \( G_1 \) and \( G_1^T \), we obtain (9), pre- and postmultiplying (36) with \( S \) and \( S^T \), we obtain (10). The proof is completed.

\[ \square \]

5. Numerical Example

Without delay and uncertainty, Example 1 designs different passive controllers by applying the theorem of our paper and the literature [22], respectively. We can compare the region of feasible solution.

**Example 1.** Consider a fuzzy system of the following form.

Plant Rules:

Rule 1: IF \( x_1(t) \) is \( M_1 \), THEN

\[
\dot{x}(t) = A_1 x(t) + B_1 u(t) + B_{11} \omega(t), \quad y(t) = C_1 x(t) + D_1 \omega(t).
\]

(45)

Rule 2: IF \( x_1(t) \) is \( M_2 \), THEN

\[
\dot{x}(t) = A_2 x(t) + B_2 u(t) + B_{12} \omega(t), \quad y(t) = C_2 x(t) + D_2 \omega(t).
\]

(46)

We can obtain Figure 1 by applying our method and obtain Figure 2 by applying method in [22]. Symbol “*” shows that

\[
A_1 = \begin{bmatrix} a & -0.02 & 1 \\ 1 & 0 & 5 \\ 1 & 2 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.225 & -0.02 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \\ b \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad a \in [-2,3], \quad b \in [6,10].
\]

(47)
feasible solution exists at that point; symbol "o" shows that feasible solution does not exist at that point.

It is obvious that Theorem 3 can relax the conditions in [22], and our method further reduced the conservatism.

Example 2 (Li et al. [20]). Consider a delayed fuzzy system of the following form.

Plant Rules:
Rule 1: IF $x_1(t)$ is $M_1$, THEN
\[
\dot{x}(t) = A_1 x(t) + A_{d1} x(t-h) + B_1 u(t) + B_{11} \omega(t),
\]
\[
y(t) = C_1 x(t) + C_{d1} x(t-h) + D_1 \omega(t).
\]
(48)

Rule 2: IF $x_1(t)$ is $M_2$, THEN
\[
\dot{x}(t) = A_2 x(t) + A_{d2} x(t-h) + B_2 u(t) + B_{12} \omega(t),
\]
\[
y(t) = C_2 x(t) + C_{d2} x(t-h) + D_2 \omega(t).
\]
(49)

with
\[
A_1 = \begin{bmatrix} -1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -0.5 \end{bmatrix},
\]
\[
A_{d1} = \begin{bmatrix} 0.1 & 0.6 \\ 0 & -0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix},
\]
\[
B_1 = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
\]
\[
C_1 = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix},
\]
\[
C_{d2} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}
\]
(50)

we let $h = 7.5$, $m = 2$, $r_1 = 10$, $r_2 = 0$, $r_3 = 5$, $r_4 = 10$, $r_5 = 10$ and we can obtain
\[
P = \begin{bmatrix} 29.0804 & 1.8737 \\ 1.8737 & 3.7272 \end{bmatrix}, \quad X = \begin{bmatrix} 1.8020 & 0.0328 \\ 0.1385 & 0.4057 \end{bmatrix},
\]
\[
\gamma = 237.5809.
\]
(51)

Table 1: Allowable maximum time delay $h$.

<table>
<thead>
<tr>
<th>Methods</th>
<th>The upper bound of $h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[21]</td>
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<tr>
<td>Theorem 3, $m = 1$</td>
<td>2</td>
</tr>
<tr>
<td>Theorem 3, $m = 4$</td>
<td>3.16</td>
</tr>
<tr>
<td>Theorem 3, $m = 5$</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Rule 2: IF $x_2(t)$ is $M_2$, THEN
\[
\dot{x}(t) = A_2 x(t) + A_{d2} x(t-h) + B_2 u(t) + B_{12} \omega(t),
\]
\[
y(t) = C_2 x(t) + D_2 \omega(t).
\]
(53)

with
\[
A_1 = \begin{bmatrix} -0.1125 & -0.02 \\ 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.1125 & -1.527 \\ 1 \end{bmatrix},
\]
\[
A_{d1} = \begin{bmatrix} -0.0125 & -0.005 \\ 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.0125 & -0.23 \\ 0 \end{bmatrix},
\]
\[
B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{11} = B_{12} = I_2,
\]
\[
C_1 = C_2 = D_1 = D_2 = I_2.
\]
(54)

The membership function is
\[
M_{11}(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25}, \quad M_{21}(x_2(t)) = \frac{x_2^2(t)}{2.25}.
\]
(55)

The initial state is
\[
x(0) = [-1 - 1.2]^T, \quad \omega(t) = \begin{bmatrix} \sin(3t) & \cos(3t) \end{bmatrix}^T.
\]
(56)

First, we should find the allowable maximum time delay $h$ which let the fuzzy system passive. Table 1 shows the maximum time delay obtained by [21] and our method.

It clearly shows that the method in our paper can get larger upper bound than before. It also shows that conservatism is further reduced when $m$ increases.

Then we let $h = 1.5$, $m = 4$, and we can obtain
\[
P = \begin{bmatrix} 9.4980 & 9.1547 \\ 9.1547 & 8.8849 \end{bmatrix}, \quad X = \begin{bmatrix} 0.6351 & 0.4997 \\ 0.5807 & 0.4826 \end{bmatrix}.
\]
(57)

The fuzzy controller gains by our method are given by
\[
K_1 = [150.7358 \quad -218.3595], \quad K_2 = [150.9502 \quad -219.0519].
\]
(58)

Figure 3 shows the state response $x_1(t)$ of the closed-loop system with the controller gains in (58), and Figure 4 shows the state response $x_2(t)$ of the closed-loop system.

Example 3. Consider a delayed fuzzy system of the following form.

Plant Rules:
Rule 1: IF $x_1(t)$ is $M_1$, THEN
\[
\dot{x}(t) = A_1 x(t) + A_{d1} x(t-h) + B_1 u(t) + B_{11} \omega(t),
\]
\[
y(t) = C_1 x(t) + D_1 \omega(t).
\]
(52)
6. Conclusion

This paper has adopted the delay partitioning approach to analyse the passivity and passification of delay fuzzy system based on T-S model. The theorems given in this paper are all in terms of LMIs. Examples have illustrated the effectiveness of our results. The method in our paper has further reduced the conservatism and the effect has been more apparent when \( m \) increases. In addition, the results obtained in this paper also can be extended to the fuzzy system with time-varying delay and uncertainties.

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