Research Article

Path Transmissibility Analysis Considering Two Types of Correlations in Hydropower Stations

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A new vibration model is built by introducing the head-cover vibration transfer path based on a previous analysis of the vertical vibration model for hydropower station units and powerhouses. This research focuses on disturbance- and parameter-related transfer paths in a practical situation. In a complex situation, the application of the stochastic perturbation method is expanded using an algebra synthesis method the Hadamard product, and theoretical analyses, and numerical simulations of transfer paths in the new vibration model are carried out through the expanded perturbation method. The path transfer force, the path transmissibility, and the path disturbance ranges in the frequency domain are provided. The results indicate that the methods proposed in this study can efficiently reduce the disturbance range and can accurately analyze the transfer paths of hydraulic-source vertical vibration in hydropower stations.

1. Introduction

Hydraulic vibration is the main vibration source in hydropower station units and powerhouses. The vertical vibration in units is usually caused by hydraulic pressure fluctuations or other loads on the water turbine's flow passage components. Field and model tests have shown that there are three main vibration transfer paths running from the water turbine to the powerhouse [1]: (1) runner-shaft-bearing-fixed components (machine frame, head-cover-powerhouse); (2) flow pressure-spiral case-powerhouse; and (3) runner-runner negative pressure region-head cover-powerhouse. Previous studies concerning the vertical vibration produced by a hydraulic source mainly focus on path (1), while the effects of paths (2) and (3) are usually ignored [2]. However, as the scale and capacity of hydropower stations increase, the flow passage area of the head-cover system continually increases, with corresponding increases in the vibration of the head-cover system. Therefore, the influence of the head-cover system becomes more important in the hydraulic vibration transfer path, and ignoring path (3) will produce a larger error. Therefore, it is imperative that the contribution of path (3) should be analyzed; more specifically, the contribution of the vibration of the head-cover system of the hydropower station vertical vibration transfer should be analyzed. However, due to the presence of randomness, it is difficult to clearly and accurately describe the contribution of this transfer path to the structural vibration.

Theoretical analyses of the vibration transfer path can be classified as a stochastic structural system problem. At present, the Monte-Carlo numerical simulation method (MCSM) [3, 4] and the perturbation method [5–7] are the most popular analysis methods. MCSM is used less frequently because of the large amount of computation required when dealing with a large-scale structure. Conversely, the perturbation method is applied by many researchers in various fields. Collins and Thompson [8] initially employed the perturbation method to analyze stochastic dynamical systematic characteristics in 1969; the perturbation method was later employed in a static analysis by Hisada and Nakagiri [9] and in a dynamic analysis by Liu et al. [10]. Kronecker algebra was introduced to the expansion of the perturbation method by Vetter [11]. After several decades of development, relevant studies on perturbation theory were quite abundant. The primary methods included the L-P method [12], the multiple scale method [13, 14], the average method [15], the KBM
focused on the application of perturbation theory with two
types of correlation from the perspective of structure analysis.
Hydropower station units and powerhouses are large-scale structures. Most of the parameters’ disturbances should be
less than 10% of their mean values in such a large structure. Therefore, problems regarding hydropower stations can be
solved by the perturbation method. First, the head-cover system is introduced, which has the same vibration source
as a path (1); the elastic foundation constraint is selected; and a new vibration model is built on the basis of the
previous vertical vibration model. Furthermore, a method for solving the path transfer force is proposed using the
general method of dynamic analysis. Second, for the test signal, correlations between the two types of disturbances and
correlations between the parameters are considered based on the single disturbance vibration path analysis. By using
the coefficient of variation algebra synthesis method [39],
Kronecker algebra [11], and the Hadamard product [40],
gradient-sorting estimations of the transfer paths in the fre-
quency domain are carried out, and methods for determining
the transfer force and transmissibility and their disturbances
are proposed. Finally, the method described in this paper is
verified using the model of a large hydropower station.

2. Analysis Model with the Introduction
of a Head-Cover System

For a hydroelectric generating unit, regardless of whether
it has a suspension or umbrella structure, the weight of its
rotating parts is successively transferred to the reinforced
concrete machine foundation through the thrust bearing,
frame (suspension units containing the stator frame), and
sole screw. The head-cover system is always fixed on the base
ring strengthening plate, and the head-cover system and the
strengthening plate are considered to be one part. Taking a
vertical vibration characteristic analysis of the umbrella unit
as an example, the model contains a shaft system, thrust
bearing, and lower bracket (see Figure 1). The heavy shaft can
be simplified as a massless elastic continuous beam, and then
its mass can be regarded as a mass attached to three nodes, $m_1$, $m_2$, and $m_3$. $m_1$ can be defined as the mass of the excitation
rotor and the shaft, the half shafting mass, which is measured
from the heavy shaft top to the rotor frame, and another mass
added on top of the heavy shaft; $m_2$ can be defined as the mass
of the central body of the rotor frame, the half mass of the
whole gate arm, and the half mass of the half shaft; and $m_3$
can be defined as the mass of the water, turbine runner, the
additional mass of water and the half shafting mass, which
is measured from the rotor frame to the hydraulic turbine.
The rotor gate arm can be simplified as a massless elastic
continuous rod, and then its mass can be assigned to the
runner margin and the central body of the rotor frame. $k_4$
can be defined as the sum of the vertical stiffness of the whole
gate arm, and $m_4$ can be defined as the lumped mass of the
runner margin. The outer end of the lower bracket is fixed to
the concrete foundation. The lower bracket gate arm can then
be simplified as a gravity-free beam if the coupling effect of
the foundation is ignored. $k_{52}$ can be defined as the vertical
The total mass matrix is obtained from the lumped mass:

\[ M = \text{diag} \{ m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8 \}. \]  

(3)

Generally, the characteristics of the vertical vibration source in the hydropower station are unique position, simple contact surface with structure, and few ingredients. Conversely, the characteristics of the lateral (radial) vibration are multipositions, strong nonlinear contact surface, and complex excitation. For simplifying the model and the calculation, the total structure of the unit’s powerhouse is considered as suffering vertical harmonic excitation. The vertical stiffness of the lower bracket gate arm, \( m_5 \), can be defined as the lumped mass of one end of the lower bracket, which is close to the heavy shaft, and the half mass of the gate arm; \( m_5 \) can be connected to \( m_3 \) by the thrust bearing, which is simplified by an equivalent stiffness \( k_{51} \).

With the introduction of the head-cover system, the vibration transfer path is as follows: first, the hydraulic vertical vibration is transferred to the head cover by the heavy shaft seal and guide bearing in the water turbine runner chamber; next, the vibration is transferred from the head cover to the spiral case base ring strengthening plate, which is connected to the outer end of the head cover; and finally, the vibration is transferred to the machine foundation by the wrapped concrete outside the spiral case. Ignoring the coupling effect, the control parts and other additional parts on the head cover can be regarded as the attached mass of the head-cover system. As the lumped mass is close to the heavy shaft, \( m_6 \) can be defined as the mass of the central body and the half mass of the whole head-cover system. The head-cover system can be simplified as a gravity-free beam, and \( k_{62} \) can be defined as the vertical stiffness. \( m_6 \) can be connected to \( m_3 \) by the sealing spring, which lies between the head-cover structure and the water turbine runner. The vertical stiffness of the connection can be simplified by an equivalent stiffness \( k_{61} \).

\( k_{51} \) is the series stiffness of the elastic oil tank stiffness (thrust bearing support system) and the oil film stiffness. \( k_{61} \) is the series stiffness of the sealing structure stiffness and the clearance water stiffness. These two parameters exhibit a linear relationship because the unit’s axial water thrust varies with the unit’s conditions. However, \( k_{51} \) and \( k_{61} \) are simplified as a single stochastic variable in this study.

In the process of examining the unit’s vertical vibration, the machine foundation pier can be regarded as an elastic foundation. The structure can be treated as a single node, \( m_7 \), which ranges from the machine foundation pier to the concrete floor in the turbine layer. The structure can be treated as a single node, \( m_8 \), which ranges from the turbine layer to the foundation. The stiffness of this structure is replaced by the equivalent stiffness, which is based on the strengthening plate, spiral structure, and concrete structure. Thus, the hydropower station base is treated by dividing it into two nodes at the connection of the head-cover system and the hydropower station base, where \( k_{62} \) is only connected to \( m_8 \).

For the entire model, the form and meaning of the dampness matrix are similar to those of the stiffness matrix. Assuming that the system is linear, the differential equation of the vibration is found from the Lagrange equation:

\[ M\ddot{u} + C\dot{u} + Ku = F(t). \]  

(1)

By merging these dynamical balance equations for the shaft, the rotor, the lower bracket, the head-cover system, and the machine foundation pier, an equation with 8 degrees of freedom and 26 parameters can be obtained, and the total stiffness matrix can be written as

\[ K = \begin{pmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_3 + k_5 + k_6 & -k_3 & -k_4 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_4 & 0 & k_4 & 0 & 0 & 0 & 0 \\ 0 & -k_5 & 0 & 0 & k_5 & 0 & 0 & 0 \\ 0 & 0 & -k_6 & 0 & 0 & k_6 + k_7 & -k_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_6 & 0 & -k_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_6 + k_7 & k_6 + k_7 + k_8 \end{pmatrix}. \]  

(2)
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harmonic excitation is located on the water turbine runner, and the response of the total structure is in a steady state. In addition, the initial phase remains constant throughout the whole process. Setting the steady-state response as $u(t) = \mathcal{U} e^{i(\omega t + \varphi)}$ and $U_i = \mathcal{U} e^{i\varphi_i}$ yields

$$u_i(t) = U_i e^{i\omega t}, \quad \dot{u}_i(t) = i\omega U_i e^{i\omega t}, \quad (4a)$$

$$\ddot{u}_i(t) = -\omega^2 U_i e^{i\omega t}, \quad (4b)$$

$$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}^T, \quad (4c)$$

$$F(t) = \{0, 0, F_0 e^{i\omega t}, 0, 0, 0, 0\}^T. \quad (4c)$$

Substituting (4a)–(4c) into (1) gives the dynamical balance equation of the hydropower station's vertical vibration with the head-cover system in the frequency domain:

$$(-\omega^2 M + i\omega C + K) U = F(t). \quad (5)$$

The response vector $U$ at each node of the structure was obtained by numerical calculations. $F_{zhou}$ and $F_{ding}$ denote the forces that are transferred to the machine foundation pier from the water turbine by the shaft system and the head-cover system, respectively, and can be written as

$$F_{zhou} = k_{zhou} (u_h - u_7) + c_{zhou} (\dot{u}_h - \dot{u}_7), \quad (6a)$$

$$F_{ding} = k_{ding} (u_h - u_7) + c_{ding} (\dot{u}_h - \dot{u}_7), \quad (6b)$$

In (6a)–(6b), $k_{zhou}$ and $c_{zhou}$ denote the path stiffness and the path dampness of the vibration path through the shaft system, respectively. Similarly, $k_{ding}$ and $c_{ding}$ denote the path stiffness and the path dampness through the head-cover system, respectively. When calculating the path stiffness and dampness of the shaft system, $m_1$ and $m_4$ are treated as dynamic vibration absorbers with dampness.

3. **Vibration Path Sorting Considering Two Types of Correlations**

In the analysis of a hydropower station, parameters such as the stiffness, mass, and dampness are attributed to the multiplicative disturbances arising from the material properties, manufacturing technology, and other factors. These parameters, obtained by measurement, are attributed to the additive disturbances due to the testing noise and environmental noise. Furthermore, because the testing method and environment are the same for every parameter, the two disturbances of each parameter are interrelated. These parameters are also interrelated by means of continuous structure discretization.

Given the factors mentioned above, the mass, stiffness, and dampness are described as a random vector $\mathbf{a}$ with $n$ random variables, where every random variable involves two types of disturbances. When the variable disturbance is lower than 15% of the mean value, the stochastic variable can be expressed as

$$a_i = a_i^1 a_i^2 + a_i^3. \quad (7)$$

In (7), $a_i$ denotes the $i$th element in the random vector $\mathbf{a}$. $a_i^1$ and $a_i^2$ denote the multiplicative and additive disturbance of the random variable $a_i$, respectively. Assuming that their mean values are 1 and 0, respectively, $a_i^d$ denotes the deterministic component of $a_i$, which represents the mean value after multiple samplings.

According to the coefficient of variation algebra synthesis method [40], calculating the mathematical expectation and variance of (7) yields

$$E_{a_i} = E \left[ a_i^1 a_i^d + a_i^2 \right] = E (a_i^1 a_i^d) + E (a_i^2) = a_i^d, \quad \sigma_{a_i}^2 = \operatorname{Var} (a_i) = E \left[ (a_i - E_{a_i})^2 \right]$$

$$= E \left[ \left( (a_i^1 - 1) a_i^d + a_i^2 \right)^2 \right] = (a_i^d)^2 \sigma_{a_i^1}^2 + \sigma_{a_i^2}^2 + 2 a_i^d \operatorname{Cov} (a_i^1, a_i^2),$$

where

$$\operatorname{Cov} (a_i^1, a_i^2) = \rho_{a_i^1, a_i^2} \sigma_{a_i^1} \sigma_{a_i^2}, \quad (9)$$

where $\rho_{a_i^1, a_i^2}$ is the correlation coefficient of the multiplicative and additive disturbances contained in the parameter $a$. If these parameters are interrelated, then

$$\operatorname{Cov} (a_i, a_j) = \rho_{a_i, a_j} \sigma_{a_i} \sigma_{a_j}, \quad (10)$$

where $\rho_{a_i, a_j}$ is the correlation coefficient of the random variables $a_i$ and $a_j$. Generally, if the random variables follow a normal distribution, their linear transformations and multiplication will also follow a normal distribution. For example, if the vector $\mathbf{a}$ follows a normal distribution, then the function $F_i(\mathbf{a})$ will follow a normal distribution. Using the Taylor expansion, expanding the transfer force $F_i$ at the mean value $F_i^d$ yields

$$F_i = F_i^d + \frac{\partial F_i^d}{\partial \mathbf{a}} (\mathbf{a} - \mathbf{a}^d) + O (\mathbf{a}^2)$$

$$= F_i^d + \frac{\partial F_i^d}{\partial \mathbf{a}} \mathbf{a}^p + O (\mathbf{a}^2), \quad (11)$$

where

$$\mathbf{a}^p = \mathbf{a} - \mathbf{a}^d = (a_1^1 - 1) \mathbf{a}^d + \mathbf{a}^3. \quad (12)$$

Neglecting components of the second order and above, the disturbance of the transfer force is

$$F_i^p = \sum_{k=1}^{m} \frac{\partial F_i^d}{\partial a_k} a_k^p. \quad (13)$$
In (13), \( \partial F_i^d/\partial a_k \) is the partial derivative of \( F_i \) with respect to the random variable \( a_k \), which is also the first-order sensitivity of \( a_k \). The covariance of \( F_i^d \) and \( F_j^d \) is

\[
\text{Cov}(F_i, F_j) = E \left[ F_i^d F_j^d \right]
= E \left[ \left( \sum_{k=1}^{m} \frac{\partial F_i^d}{\partial a_k} a_k^p \right) \left( \sum_{l=1}^{m} \frac{\partial F_j^d}{\partial a_l} a_l^q \right) \right].
\]  

(14)

That is,

\[
\text{Cov}(F_i, F_j) = \sum_{k=1}^{m} \sum_{l=1}^{m} \frac{\partial F_i^d}{\partial a_k} \frac{\partial F_j^d}{\partial a_l} E(a_k^p a_l^q) - E(a_k^p) E(a_l^q) \text{Cov}(a_k, a_l).
\]

(15)

Equation (16) shows that \( \text{Cov}(F_i, F_j) \) of the transfer force \( F_i \) and \( F_j \) can be expressed by \( \text{Cov}(a_k, a_l) \) of the random structure parameter:

\[
\text{Cov}(F_i, F_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial F_i^d}{\partial a_i} \frac{\partial F_j^d}{\partial a_j} \rho_{a_i a_j} \sigma_{a_i} \sigma_{a_j}.
\]

(16)

If \( i = j \),

\[
\sigma_i^2 = \text{Cov}(F_i, F_i) = \sum_{k=1}^{m} \frac{\partial F_i^d}{\partial a_k} \frac{\partial F_i^d}{\partial a_k} \rho_{a_k a_k} \sigma_{a_k}^2 \sigma_{a_k}.
\]

(17)

According to Kronecker algebra [11], the corresponding stochastic analysis theory and Hadamard product [41] yield

\[
\sigma_i^2 = \left| \frac{\partial F_i^d}{\partial a^t} \right|^2 \left( \rho \ast (a^t) \right)^2,
\]

(18)

where the subscript [2] denotes the Kronecker power; that is, \( a^2 = a \ast a; \) if \( a \) is of order \( n \times 1 \), then \( a^2 \) is of order \( n^2 \times 1 \). \( \rho \) is the stacking vector of the matrix of correlation coefficients, which is also of order \( n^2 \times 1 \). The symbol \( \ast \) denotes the Hadamard product. Therefore, \( \sigma_i^2 \) is of order \( 1 \times 1 \).

The sensitivity matrix \( [\partial F_i^d(a)/\partial a^t] \) of the transfer force of each parameter is

\[
\left| \frac{\partial F_i^d}{\partial a^t} \right| = \left| \begin{array}{c} \frac{\partial F_i^d}{\partial a_1} \\ \vdots \\ \frac{\partial F_i^d}{\partial a_m} \end{array} \right|.
\]

(19)

Substituting (19) into (18), the variance of the transfer force \( F_i \) for each path can be calculated. Equation (11) only involves the first-order Taylor expansion. A higher-order Taylor expansion will improve the accuracy, but it involves complicated mathematical calculations. Equation (18) shows that the transfer force variance can be directly obtained from the random variables’ numerical characteristics. The calculations can thus be simplified because the sample is not included in the mathematical operations. The correlations between parameters are merely modified in (18) and do not increase the number of calculations required. The transmissibility is defined as the ratio of the amplitudes between the transfer force and the vibration source excitation force:

\[
\beta_i = \left| \frac{F_i}{F_0} \right|.
\]

(20)

Neglecting the disturbance of the excitation force \( F_0 \), based on the random variable algebra synthesis method, the expectation, variance, and transfer coefficient of the transmissibility can be written as

\[
E_\beta = E \left[ \beta(a) \right] = \frac{E(F_i)}{E(F_0)},
\]

(21a)

\[
\sigma_\beta^2 = \frac{E[F_i^2]}{E[F_0^2]} \left| \frac{\text{Var}[F_i]}{|E(F_i)|^2} \right|,
\]

(21b)

\[
\theta = \frac{E_\beta}{\sigma_\beta}.
\]

(21c)

The result of a random variable that follows a normal distribution divided by a constant also follows a normal distribution. The transfer coefficient is the transfer efficiency of the excitation force. The gradient sorting of the vibration path transmissibility in the frequency domain can be obtained by changing the frequency of the excitation force.

A solution of the vibration transfer path for the two types of disturbances and correlations is proposed based on the above methods. The path transmissibility and its probabilistic characteristic are provided. In this study, the solution only involves the first-order sensitivity of the random parameter and the probabilistic characteristic of the random variable. Additionally, the introduction of the two types of correlations does not involve excessive calculations. Therefore, the calculation accuracy is improved for practical problems.

4. Example Analysis

The main structure of the umbrella unit in a large hydropower station is shown in Figure 2, and the simplified model of transfer path is shown in Figure 1. In this example, the effect of the spiral case and substructure is ignored [41, 42] because their effect is far less than that of the upper structure. The excitation is assumed to be a simple harmonic excitation. The mean values of the random parameters can be obtained from the hydropower station design diagrams. These mean values are \( m_1 = 8.28 \times 10^4 \), \( m_2 = 1.042 \times 10^6 \), \( m_3 = 3.29 \times 10^5 \), \( m_4 = 9 \times 10^5 \), \( m_5 = 1.2 \times 10^7 \), \( m_6 = 1.15 \times 10^7 \), \( m_7 = 1.39 \times 10^7 \) and \( m_8 = 8.92 \times 10^7 \), in units of kg; \( k_1 = 7.26 \times 10^{10} \), \( k_2 = 5.72 \times 10^{10} \), \( k_3 = 2.32 \times 10^{10} \), \( k_4 = 2.20 \times 10^{12} \), \( k_5 = 9.41 \times 10^9 \), \( k_6 = 1.73 \times 10^8 \), \( k_7 = 1.73 \times 10^8 \), \( k_8 = 7.70 \times 10^8 \), and \( k_9 = 4.26 \times 10^8 \), where the unit of the stiffness \( k \) is N/m; and \( c_1 = 5.48 \times 10^5 \), \( c_2 = 4.11 \times 10^6 \), \( c_3 = 1.02 \times 10^8 \), \( c_4 = 2.57 \times 10^6 \), \( c_5 = 7.51 \times 10^6 \), \( c_6 = 2.23 \times 10^6 \), \( c_7 = 9.99 \times 10^5 \), \( c_8 = 1.64 \times 10^6 \), and \( c_9 = 9.74 \times 10^5 \), where the unit of the dampness \( c \) is N \cdot s/m. Each parameter comprises...
two disturbances that follow a normal distribution. The multiplicative disturbance is related to the difficulty of obtaining the parameters. The multiplicative variance coefficients of $k_{51}$ (including the vertical stiffness of the thrust bearing) and $k_{61}$ (the sealing equivalent vertical stiffness between the head cover and runner) are set to 0.10. The equivalent bending rigidity of $k_{62}$ multiplicative variance coefficient is set to 0.075 because there are many uncertain factors in the control components and other attached components on the head cover. The multiplicative random variance coefficients of the other parameters are set at 0.05. The additive disturbance is related to the measurement range. The standard deviations of the additive random variable with respect to mass and stiffness are set at $10^4$ and $10^5$, respectively, according to each parameter's mean value. The variance coefficient of damping is similar to that of the mass and stiffness. In this example, the parameters’ disturbances are generated by a function in the MATLAB software, and the sample size is 10000 for each disturbance. All errors are determined to be smaller than 0.1% by comparing the variances and mean values of the generated samples with the corresponding set values. Therefore, the generated sample variances and mean values are used in the study. The correlation function is determined by the generated function. The partial derivative is calculated by the software Mathematica. The coefficient correlation of the generated sample is 0.0329.

Figure 3 displays the transmissibility variances of the head-cover transfer path obtained using different methods. Method 1 does not involve the two types of correlations. The Monte-Carlo simulation is not used in method 1 (due to the inclusion of 26 random variables and because the calculation time is too long, this method only focuses on how each group of variables changes with time, that is, the number of calculations is equal to the sample size). Method 2 (following the method developed by W. J. Vetter in 1973) only involves the correlation with respect to one parameter, which involves the correlations between the disturbances of the parameters. Method 3 (similar to method 2) involves the correlations between parameters instead of the correlations between the disturbances of the parameters. Method 4 involves both types of correlations. The results for method 4 are obtained from (18). Figure 3 displays the characteristic curve of the head-cover system transmissibility variances in the frequency band of 0–1000 rad/s. Figure 4 presents the optimal curve of the two types of correlations for the transmissibility variance range as the correlation coefficient changes. This figure contains two curves: the disturbance curve, which describes the mean values of the results of method 2 subtracted from method 1 and the results of method 4 subtracted from method 3, and the parameter curve, which is similar to the disturbance curve.

Figures 3 and 4 show the following results: (1) compared to the other methods, the variance range for the method that contains two types of correlations is the smallest. This finding indicates that this method significantly decreases the disturbance range under theoretical calculations. The path transmissibility variance decreases by 72.51%. (2) In optimizing the method considering correlations between the disturbances and variances, the range merely decreases by 0.644%. This small decrease may have arisen because the difference between the additive and multiplicative disturbances is large, and the optimizing function is weak. (3) The optimization effect is significant for the method considering the correlations between parameters. The variance range
decreases by 75.71% when the correlation coefficient is 0.0085. Thus, neglecting this type of correlation leads to an inaccurate analysis. (4) As the correlation coefficient increases, the optimization of the transmissibility variance is more efficient, and the effect on the disturbance range is more significant. This result demonstrates that the relationship between the optimization range and the correlation coefficients for the parameters is linear and that the relationship between the optimization range and the correlation coefficients for the disturbances is proportional. However, the latter relationship does exhibit some fluctuations.

Figure 5 compares the characteristic curve for each path transmissibility, $\beta_i$, to the excitation frequency. This curve shows the ratio between the forces that are transmitted to the machine pier by the corresponding paths and the excitation force of the water turbine hydraulic source. This ratio is equivalent to the amplification coefficient. The curve indicates two results: (1) resonance occurs when the frequency of the excitation force is the same as the natural frequency. The transmissibility reaches its maximum value for each path at this frequency. This result indicates that the path has a large effect on the disturbance range. (2) Considering the whole process of vertical vibration transfer, the transmissibility of the head cover is significantly smaller than that of the shafting system. Throughout the entire frequency range, the mean value of the path transmissibility ratio is 32.37. This value indicates that the vibration effect of the head-cover system can be neglected in accurate calculations.

Figure 6 shows that the characteristic curve of the transmissibility variance varies with the excitation frequency for the two types of correlations. This curve reflects the diversion of the transmissibility and indicates the following findings. (1) The path transmissibility variance of the shaft system is greater than that of the head-cover system due to the large number of components and the randomness of these components’ parameters. Thus, the variance of a structure with a high level of parameter randomness is large in the transfer path. (2) The variance maximum occurs at the natural frequency. This frequency is determined by the structural parameter, which accounts for the parameter contribution of every transfer path, enabling the transfer force to be obtained from the expectation and variance of the transmissibility.

Figure 7 shows that the characteristic curve of the transfer coefficients varies with the excitation frequency for the two types of correlations. This curve reflects the transfer efficiency of the excitation force. The figure shows that the transfer efficiency of the head-cover system is more efficient than that of the shafting system. Therefore, more attention should be given to the assigned vibration proportion of the shaft and head-cover system in a design.
5. Conclusions

(1) The multivibration source, multipath vibration model, is improved with respect to the coupling effect between hydropower station units and powerhouses by the introduction of a head-cover system. The scalar expression of each vibration transfer path is provided in this study.

(2) A perturbation analysis of the transfer path is carried out by using a test signal. A complex situation with two types of correlations is considered. Kronecker algebra, the Hadamard product, and probabilistic statistics are employed to develop an analysis method for the transfer path. In the solution process, excessive additional computations are not required because only the numerical characteristics of the random variables are used. The application of the stochastic perturbation method is expanded, and a method for analyzing the vibration transfer path is developed. The analysis method efficiently decreases the disturbance range of the path's contribution. The disturbance ranges of the path transmissibility and the contribution rate are efficiently reduced by considering the two types of correlations. This reduction is significant for the optimization function with respect to the results.

(3) The simulation results indicate that when the effect of the spiral case and substructure is ignored, the influence on the disturbance range continually increases as the correlation coefficient increases in the hydropower station model. The optimization function of the correlations between parameters is too important to be neglected when calculating the disturbance range.

(4) In conclusion, the analysis of the proposed model shows that the effect of the water turbine head cover is not evident in the vertical vibration transfer, but its transfer efficiency is significant.

The analysis of vibration transfer paths of hydropower station units and powerhouses is complicated. Based on the complex disturbances and parameters, the contribution of each transfer path can be calculated by analyzing the sensitivity of the transfer force and the transmissibility of the vibration model in the frequency domain. This study provides a reference for future comprehensive research on the transfer paths of hydropower station units and powerhouses.

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