Research Article

Adaptive Sliding Mode Controller Design for Projective Synchronization of Different Chaotic Systems with Uncertain Terms and External Bounded Disturbances

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Synchronization is very useful in many science and engineering areas. In practical application, it is general that there are unknown parameters, uncertain terms, and bounded external disturbances in the response system. In this paper, an adaptive sliding mode controller is proposed to realize the projective synchronization of two different dynamical systems with fully unknown parameters, uncertain terms, and bounded external disturbances. Based on the Lyapunov stability theory, it is proven that the proposed control scheme can make two different systems (driving system and response system) be globally asymptotically synchronized. The adaptive global projective synchronization of the Lorenz system and the Lü system is taken as an illustrative example to show the effectiveness of this proposed control method.

1. Introduction

The cooperative behavior of coupled nonlinear oscillators is of interest in connection with a wide variety of different phenomena in physics, engineering, biology, and economics. For example, systems of coupled nonlinear oscillators may be used to explain how different sectors of the economy adjust their individual commodity cycles relative to one another through the exchange of goods and capital units or via aggregate signals in the form of varying interest rates or raw materials prices. As part of the cooperative behaviors, the synchronization plays very important role in many applications. Synchronization occurs when oscillatory (or repetitive) systems via some kind of interaction adjust their behaviors relative to one another so as to attain a state where they work in unison [1], since the individual oscillators display chaotic dynamics in many cases and it is very important to analyze the synchronization of chaotic systems.

Synchronization of two coupled chaotic systems has attracted much attention for both theoretical studies and practical applications since the synchronization of the different chaotic systems was observed by Pecora et al. in 1997 [2]. Many investigations have been devoted to synchronization due to its potential application in many fields recently, such as security communication [3, 4], information processing [5, 6], and biological systems [7]. Many different synchronization strategies of different chaotic systems have been developed. Beside the generalized synchronization [8, 9], there are some other types of synchronization which are also very interesting and useful, like phase synchronization [10, 11], antiphase synchronization [12, 13], complete synchronization [14, 15], and lag synchronization [16, 17]. In 1999, a new synchronization observed by Mainieri and Rehacek in partially linear chaotic systems is called projective synchronization [18]. The dynamical behavior of projective synchronization refers to that two identical or different systems which are synchronized up to a constant scaling factor.

So far, great progress has been made in the research of projective synchronization among all types of synchronization because of its adjustable proportionality between the synchronized dynamical states [19, 20]. Xu et al. designed a control scheme to manipulate the scaling factor onto any
desired value [21]. Wen realized full-state projective synchroniza-
tion by using an observer-based control [22]. Sudheer and Sabir designed a coupling function for unidirec-
tional coupling in identical and mismatched oscillators to
realize function projective synchronization through open-
plus-closed-loop coupling method [23]. In addition, Peng
and Jiang realized generalized projective synchronization 
of a class of fractional-order chaotic systems via a scalar 
transmitted signal [24]. When projective synchronization is
applied to secure communications, the binary sequences can
be extended to N-ary sequences in digital communication
for achieving higher speed [25], which would exert a great
influence on communication.

To achieve projective synchronization of identical or
donidentical chaotic systems with different initial conditions,
many effective control methods have been proposed, such as
linear and nonlinear feedback control [26], adaptive control
[27], impulsive control [28], and sliding mode control [29].
With these control schemes, many dynamical systems, such as
the Lorenz system, the Chen system, the Rössler system,
and some other chaotic or hyperchaotic systems [30, 31],
with known or unknown parameters, are all synchronized
by various control methods based on Lyapunov stability
theory. Sliding mode control is a variable structure control
algorithm as it alters the dynamics of a nonlinear system by
the application of a high-frequency switching control. In view
of the simple structure, good robustness, and high reliability,
sliding mode control is widely applied to motion control
in engineering, especially used in determinacy system with
an accurate mathematical model [32]. When the dynamical
system has been precisely modeled, this control method is
an effective way to realize the projective synchronization
of various dynamical systems. Mou et al. and Yang et al. pro-
vided an adaptive sliding mode control method to achieve the
generalized synchronization of integral-order and fractional-
order chaotic dynamics with fully unknown parameters [33, 34]. However, the existing synchronizations were realized for
only identical structure chaotic systems, and these methods
cannot be directly applied to the systems with uncertain
terms and disturbances which are the most cases in the
applications.

In view of its great potential value in secure communica-
tion, the projective synchronization is increasingly becoming
a very important research topic in the fields of synchro-
nization and control of chaos. Since the system structure of
the projective synchronization belongs to driving-response
type (master-slave), the driving system and the response
system synchronize up proportionally to a constant scaling
factor by using a proper control scheme. Therefore, it is
desirable to achieve the projective synchronization of non-
identical chaotic systems with unknown parameters, uncer-
tain terms, and bounded external disturbances via adaptive
sliding mode controller. This paper mainly focuses on the
projective synchronization of nonidentical structure chaotic
systems with fully unknown parameters, uncertain terms,
and external bounded disturbances. Based on the Lyapunov
stability theory, a kind of adaptive sliding mode controller
is designed to achieve projective synchronization of two
nonidentical structure chaotic systems. Here, the parameters
of the response system are fully unknown, and there are
uncertain terms and bounded external disturbances in the
response system. Moreover, the complete projective synchro-
nization and the antiphase projective synchronization can be
realized by adjusting the scaling factor.

The remainder of this paper is organized as follows.
Section 2 presents the general theory of projective synchro-
nization. In Section 3, an adaptive sliding mode controller
is proposed and the robust stability is analyzed. Synchro-
nization analysis of the Lorenz system and the Lu system is
given in Section 4, and finally, Sections 5 and 6 provide the
numerical simulations and concluding remarks of this study,
respectively.

2. General Theory of
Projective Synchronization

The main problem discussed in this paper is projective synchro-
nization, which will be implemented by driving-response
scheme. The driving system can be described by the nonlinear
differential equation

\[ \dot{x} = f(x, \phi, t), \]  

where \( y \in \mathbb{R}^n \) denotes the observable state variables
of the response system (2), \( \theta \in \mathbb{R}^{m+1} \) represents an \((m+1)\)-
dimensional column vector, \( f(\cdot) \in \mathbb{R}^m \) is an \( l \)-dimensional parameter vector, and \( f(\cdot) \in \mathbb{R}^n \) is a continuous or non-
continues nonlinear function vector.

The response system can be expressed as

\[ \dot{y} = g(y, \theta, t) + u(t), \]  

where \( y \in \mathbb{R}^n \) denotes \( y \in \mathbb{R}^n \) denotes the observable state variables
of the response system (2), \( \theta \in \mathbb{R}^{m+1} \) represents an \((m+1)\)-
dimensional column vector, \( g(\cdot) \in \mathbb{R}^m \) is a function
vector similar with \( f(\cdot) \), and \( u(t) \in \mathbb{R}^l \) is an \( n \)-dimensional
control input vector which can be used to realize projective
synchronization of the systems (1) and (2).

When the tracking error is defined as \( e_i = y_i - \kappa x_i \) \( (i = 
1, 2, \ldots, n) \), the synchronization error differential equation
can be obtained as

\[ \dot{e} = \dot{y} - \kappa \dot{x} = g(y, \theta, t) - \kappa f(x, \phi, t) + u(t). \]  

Here, the parameter \( \kappa \) is a scaling factor, which can adjust the
synchronization proportion of the systems (1) and (2).

In order to achieve the synchronization of the driving
system (1) and the response system (2), the control input \( u(t) \)
needs to satisfy

\[ \lim_{t \to \infty} \| e(t) \| \to 0. \]  

Using a proper control scheme, this kind of synchronization
is called projective synchronization if \( \lim_{t \to \infty} \| e(t) \| \to 0 \) by
varying the scaling factor \( \kappa \).

Projective synchronization includes several kinds of
synchronization. Complete synchronization and antiphase
synchronization are two special cases corresponding with
scaling factor \( \kappa = 1 \) and \( \kappa = -1 \), respectively.
In view of the structure of the systems (1) and (2), the projective synchronization can be divided into identical structure projective synchronization and nonidentical structure projective synchronization. In this paper, we focus on the projective synchronization of two nonidentical structure systems, which means that the driving system (1) and the response system (2) are not same, and the identical structure projective synchronization can be looked as a specific case of nonidentical structure projective synchronization.

3. Adaptive SMC Design and Robust Stability Analysis

In the most of the practical applications, the parameters of the response system are not known, and there are uncertain terms and external bounded disturbances in the response system. Therefore, it is necessary to consider this practical matter. In the following sections, this kind of response systems will be applied to the design and analysis of synchronization. This kind of response system can be described by the linear parameterization structure

$$\dot{y} = g(y, \theta, t) + \Delta g(y, \theta, t) + d(t) + u(t),$$

(5)

where parameter vector $\theta \in \mathbb{R}^{m+1}$ is unknown, $m$ is the number of unknown parameters, $\Delta g(y, \theta, t)$ denotes uncertain and bounded function vector, and $d(t)$ is the unknown and bounded external disturbances, such as DC signal, AC signal, any kind of bounded noise signals, and chaotic signal and control input vector. Hence, $\Delta g(y, \theta, t)$ and $d(t)$ should meet the following conditions:

$$\|d(t)\| \leq D(t) \in L_{\infty},$$

(6)

$$\|\Delta g(y, \theta, t)\| \leq H(y, t) \in L_{\infty},$$

where $D(t)$ and $H(y, t)$ are two known upon bound of external disturbance and uncertain function, respectively, which are always set to be constants without loss of generality. In general, $g(y, \theta, t)$ can be described by the linear parameterization structure

$$g(y, \theta, t) = G(y, t) \theta,$$

(7)

where the elements of matrix $G(y, t) \in \mathbb{R}^{n \times (m+1)}$ and parameter vector $\theta_i$ $(i = 1, 2, \ldots, n)$ can be one-order, high-order, or constant terms. Therefore, the response system (5) can be rewritten as

$$\dot{y} = G(y, t) \theta + \Delta g(y, \theta, t) + d(t) + u(t).$$

(8)

Using the definition of tracking error, one can further obtain the synchronization error differential vector

$$\dot{e} = \dot{y} - \kappa \dot{x} = G(y, t) \theta + \Delta g(y, \theta, t) + d(t) - \kappa f(x, \phi, t) + u(t),$$

(9)

where $e = [e_1, e_2, \ldots, e_n]^T$.

Definition 1. For any initial conditions of the systems (1) and (8), the zero solution of the error system (9) is globally stable if the motion trajectory of the error dynamical system (9) satisfies $|e_i| \to 0$ $(i = 1, 2, \ldots, n)$ as $t \to +\infty$. The driving-response systems (1) and (8) are globally projective synchronization when the scaling factor $\kappa \neq 0$. Specially, for $\kappa = 1$, it is called global complete synchronization, and it is called globally antiphase synchronization for $\kappa = -1$.

We construct an $n$-dimensional sliding mode surface $s = [s_1, s_2, \ldots, s_n]^T$ with $s_i = \mu_i e_i$ $(i = 1, 2, \ldots, n)$ and adopt a constant-velocity reaching law as a control scheme to realize globally projective synchronization of two nonidentical structure systems. Here, an $n$-dimensional matrix of gain adjustment $\Gamma$ can be used to denote the parameters $\mu_i$ $(i = 1, 2, \ldots, n)$, that is, $\Gamma = \text{diag}(\mu_1, \mu_2, \ldots, \mu_n)$. Further, the $n$-dimensional sliding mode surface $s$ can be represented as $s = \Gamma e$.

Theorem 2. For the system (9), if the control input is designed as

$$u(t) = -G(y, t) \bar{\theta} + \kappa f(x, \phi, t) - \rho(t) \text{sgn}(s),$$

(10)

the zero solution of the error system (9) is globally stable when $\rho(t) = H(y, t) + D(t) + \eta$ $(\eta > 0)$. Here, $\bar{\theta}$ is the estimated vector of the unknown parameter $\theta$ of the system (8), $\rho(t)$ denotes the constant-velocity of the trajectory inclining to the switch surface $s = 0$, and the expression $\text{sgn}(s)$ is a sign function vector. In other words, the systems (1) and (8) can asymptotically achieve projective synchronization by using the control law (10) when $\rho(t) = H(y, t) + D(t) + \eta$ $(\eta > 0)$.

Proof. Construct the positive Lyapunov function

$$V(s, \theta) = \frac{1}{2} s^T s + \frac{1}{2} \bar{\theta}^T \bar{\theta},$$

(11)

where $\bar{\theta} = \theta - \hat{\theta}$ is the estimation error vector of the parameter vector. Then, one can obtain $\dot{\bar{\theta}} = -\bar{\theta}$ as $\theta$ is a constant parameter vector for a certain response system. Taking time derivative of (11) along the solution of error dynamics (9) and control input (10),

$$\dot{V}(\cdot) = s^T \dot{s} + \bar{\theta}^T \dot{\bar{\theta}}$$

$$= s^T \Gamma \dot{e} + \bar{\theta}^T \dot{\bar{\theta}}$$

$$= s^T \Gamma \left[ G(y, t) \theta + \Delta g(y, \theta, t) + d(t) - \kappa f(x, \phi, t) + u(t) \right] + \bar{\theta}^T \dot{\bar{\theta}}$$

$$= s^T \Gamma \left[ G(y, t) \hat{\theta} + \Delta g(y, \theta, t) + d(t) - \rho(t) \text{sgn}(s) \right] + \bar{\theta}^T \dot{\bar{\theta}}$$
\[ s^T \Gamma [\Delta g(y, \theta, t) + d(t) - \rho(t) \text{sgn}(s)] \\
+ \theta^T G(y, t) \Gamma s - \theta^T \Theta \]
\[ = \begin{cases} 
  s^T \Gamma [\Delta g(y, \theta, t) + d(t) - (H(y, t) + D(t) + \eta) v] \\
  + \theta^T (G(y, t) \Gamma s - \theta) & s_i \geq 0, \ i = 1, 2, \ldots, n \\
  s^T \Gamma [\Delta g(y, \theta, t) + d(t) + (H(y, t) + D(t) + \eta) v] \\
  + \theta^T (G(y, t) \Gamma s - \theta) & s_i < 0, \ i = 1, 2, \ldots, n
\end{cases} \\
\leq -\eta |s^T \Gamma| + \theta^T (G(y, t) \Gamma s - \theta), \quad (12)
\]
where \( v \) is an \( n \)-dimensional vector with all entries equal to one, namely, \( v = [1, 1, \ldots, 1]^T \). If the parameter identification update law is as follows:
\[ \hat{\Theta} = G(y, t) \Gamma s = G(y, t) \Gamma \Theta e. \quad (13) \]
One can further obtain
\[ \dot{V}(\cdot) \leq -\eta |s^T \Gamma| = -\eta \left( \sum_{i=1}^{n} |\mu_i s_i| \right) \leq 0. \quad (14) \]
Then,
\[ V(\cdot) \geq 0, \]
\[ \dot{V}(\cdot) \leq 0 \rightarrow s_i \in L_{\infty}, \]
\[ \rightarrow e_i \in L_{\infty} \quad \text{expression (14)} \]
\[ \rightarrow \hat{\Theta} \in L_{\infty}, \quad (15) \]
\[ V(\cdot) \geq 0, \]
\[ \dot{V}(\cdot) \leq 0 \rightarrow \hat{\Theta} \in L_{\infty} \]
\[ \rightarrow (\hat{\Theta} = \Theta) \text{ expression (10)} \]
\[ \rightarrow e_i \in L_{\infty}, \quad (16) \]
\[ \dot{e}_i = \begin{cases} 
  g_i^e \dot{y}_i & e_i = 0 \ \\
  e_i & e_i \neq 0
\end{cases}, \quad (17) \]
\[ x_i, y_i, s_i, \hat{\Theta}, \rho \rightarrow u_i \in L_{\infty}, \quad (18) \]
In view of this, we define a new function \( f(\cdot) = \sum_{i=1}^{n} k_i e_i^2 \), and the following equation (19) can be determined by derivative operation:
\[ \dot{f}(\cdot) = 2 \sum_{i=1}^{n} k_i e_i^2 \in L_{\infty}. \quad (19) \]
As can be seen from (15) to (18), the estimated parameters, error variables, and state variables are bounded. Based on (15), (16), (19), and the Barbalat lemma, \( |e_i| \rightarrow 0 \ (i = 1, 2, \ldots, n) \) as \( t \rightarrow +\infty \), and the error dynamical system (9) is asymptotically stable at the origin. Therefore, this projective synchronization theorem was proved.  

4. Projective Synchronization of the Lorenz System and the Lü System

In this section, the projective synchronization of the Lorenz system and the Lü system is used as an example to validate the proposed adaptive control technique in Section 3, where the Lorenz system is regarded as the driving system, and another with control inputs \( u(t) \), unknown system parameters \( \theta \), uncertain terms \( \Delta g(y, \theta, t), \) and bounded external disturbances \( d(t) \) acts as the response system.

The Lorenz system [35] is given by
\[ \begin{align*}
  \dot{x}_1 &= a_1 (x_2 - x_1), \\
  \dot{x}_2 &= b_1 y_1 - x_1 x_3, \\
  \dot{x}_3 &= x_1 x_2 - c_1 x_3,
\end{align*} \quad (20) \]
which has a typical butterfly chaotic attractor and captures lots of features of chaotic systems when the parameters \( a_1 = 10, b_1 = 28, \) and \( c_1 = 8/3 \). The Lü system [36] is a typical chaos anticontrol model, which is a chaotic system between the Lorenz system and the Chen system in the unified Lorenz system family. The nonlinear differential equations of the Lü system are
\[ \begin{align*}
  \dot{y}_1 &= a_2 (y_2 - y_1), \\
  \dot{y}_2 &= b_2 y_3 - y_1 y_3, \\
  \dot{y}_3 &= y_1 y_2 - c_2 y_3.
\end{align*} \quad (21) \]
There exists a chaotic attractor in the Lü system when the parameters \( a_2 = 30, b_2 = 22, \) and \( c_2 = 3. \) In view of (8), the controlled Lü system (21) can be rearranged as
\[ y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \]
\[ = G(y, t) \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} \Delta g_1 (y, \theta, t) \\ \Delta g_2 (y, \theta, t) \\ \Delta g_3 (y, \theta, t) \end{bmatrix} \]
\[ + \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad (22) \]
where
\[ G(y, t) = \begin{bmatrix} y_2 - y_1 & 0 & 0 \\ 0 & y_3 & -y_1 y_3 \\ 0 & 0 & -y_3 \end{bmatrix}, \quad (23) \]
\[ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \quad (24) \]
Then, one can obtain the following error differential equations when the system (20) and (22) are the driven system and the response system, respectively:

\[
\dot{\epsilon} = \begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \end{bmatrix} = G(y, t) \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} + \begin{bmatrix} \Delta g_1(y, \theta, t) \\ \Delta g_2(y, \theta, t) \\ \Delta g_3(y, \theta, t) \end{bmatrix} + \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{bmatrix} - \begin{bmatrix} \rho(t) \\ \rho(t) \\ \rho(t) \end{bmatrix} \begin{bmatrix} \text{sgn}(s_1) \\ \text{sgn}(s_2) \\ \text{sgn}(s_3) \end{bmatrix},
\]

where

\[
\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}, \quad \tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \\ \tilde{\theta}_4 \end{bmatrix}, \quad \Delta \theta = \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta \theta_3 \\ \Delta \theta_4 \end{bmatrix},
\]

and the responsesystem, respectively:

\[
\dot{\epsilon} = \begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \end{bmatrix} = G(y, t) \tilde{\theta} + \begin{bmatrix} \Delta g_1(y, \theta, t) \\ \Delta g_2(y, \theta, t) \\ \Delta g_3(y, \theta, t) \end{bmatrix} + \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{bmatrix} - \begin{bmatrix} \rho(t) \\ \rho(t) \\ \rho(t) \end{bmatrix} \begin{bmatrix} \text{sgn}(s_1) \\ \text{sgn}(s_2) \\ \text{sgn}(s_3) \end{bmatrix},
\]

hence, one can obtain

Consequently,

\[
V(s, \tilde{\theta}) = \frac{1}{2} s^T s + \frac{1}{2} \tilde{\theta}^T \tilde{\theta}.
\]

Taking time derivative of (33) along the solution of error dynamics (25) and the control input (28),

\[
\dot{V}(\cdot) = s^T \dot{s} + \tilde{\theta}^T \dot{\tilde{\theta}} = s^T \Gamma \dot{\epsilon} + \tilde{\theta}^T \dot{\tilde{\theta}}
\]

Construct the following positive Lyapunov function:

\[
\dot{V}(\cdot) = s^T \dot{s} + \tilde{\theta}^T \dot{\tilde{\theta}} = s^T \Gamma \dot{\epsilon} + \tilde{\theta}^T \dot{\tilde{\theta}}
\]

where \( \bar{\theta} = [\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3, \bar{\theta}_4]^T \) is the online estimation for unknown parameter vector \( \theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T \) of the system (22), that is,

\[
\bar{\theta} = \begin{bmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \\ \bar{\theta}_4 \end{bmatrix} = \begin{bmatrix} \bar{a}_2 \\ \bar{b}_2 \\ \bar{c}_2 \\ 1 \end{bmatrix},
\]
The last part of this section is the proof of the projective synchronization of the Lorenz system and the Lü system using the control law (28) and the parameter update law (35). Hence, one can conclude that the sliding mode controller (28). In view of (15) to (19), one can obtain

\[ e_1, e_2, e_3, y_1, y_2, y_3, \hat{a}_2, \hat{b}_2, \hat{c}_2, \tilde{a}_2, \tilde{b}_2, \tilde{c}_2, u_1, u_2, u_3, \hat{e}_1, \hat{e}_2, \hat{e}_3, \tilde{y}_1, \tilde{y}_2, \tilde{y}_3 \in \mathcal{L}_\infty. \]

(37)

The proof of the projective synchronization of the Lorenz system and the Lü system is thus completed.

5. Numerical Simulations

In this section, numerical simulations for the projective synchronization of the systems (20) and (22) are given to demonstrate the effectiveness and feasibility of the proposed controllers (28) and the online parameter update law (35). All the differential equations are solved by using the fourth-order Runge-Kutta method, and step size is set to 0.001.

In the simulations, parameters \( a_1, b_1, c_1 \) and the initial conditions of the Lorenz system are chosen as 10, 8/3, 28,
Figure 1: Time series and synchronization error of trajectories of the Lorenz system and the Lü system: (a) $x_1(y_1)$ versus $t$; (b) $e_1$ versus $t$.

Figure 2: Time series and synchronization error of trajectories of the Lorenz system and the Lü system: (a) $x_2(y_2)$ versus $t$; (b) $e_2$ versus $t$.

Figure 3: Time series and synchronization error of trajectories of the Lorenz system and the Lü system: (a) $x_3(y_3)$ versus $t$; (b) $e_3$ versus $t$. 
and $x_0 = (2, -2, -5)$, respectively. Parameters $a_2$, $b_2$, $c_2$ and the initial conditions of the Lü system are set to 35, 7, 2.92, and $y_0 = (0, 1, -2)$, respectively, and other parameters are

$$
\begin{align*}
\begin{bmatrix}
\Delta g_1(y, \theta, t) \\
\Delta g_2(y, \theta, t) \\
\Delta g_3(y, \theta, t)
\end{bmatrix} &=
\begin{bmatrix}
0.2 \sin(t) \\
0.5 \sin(3t) \\
0.3 \sin(2t)
\end{bmatrix}, \\
\begin{bmatrix}
d_1(t) \\
d_2(t) \\
d_3(t)
\end{bmatrix} &=
\begin{bmatrix}
0.5 \sin(5t) \\
0.3 \sin(2t) \\
0.1 \sin(4t)
\end{bmatrix}, \\
\Gamma &=
\begin{bmatrix}
20 & 0 & 0 \\
0 & 20 & 0 \\
0 & 0 & 20
\end{bmatrix}, \\
\eta &= 9, \quad \kappa = 1, \quad \rho(t) = 10.
\end{align*}
$$

(38)

Figures 1(a), 2(a), and 3(a) display the synchronized state trajectories between the Lorenz system and the Lü system. From the evolution of the state variables, it can be found that the different trajectories from the initial values $x_0$ and $y_0$ are gradually close to each other and will overlap ultimately. Figures 1(b), 2(b), and 3(b) display that the errors converge to zero as time increases, which implies that the projective synchronization has been realized. Figure 4 shows that the relation curve is a line with a slope of $45^\circ$, which illustrates that the trajectories generated from the Lü system will equal that from the Lorenz system.

Figure 5 is the time series of control inputs $u_1$, $u_2$, and $u_3$, whose main role is to ensure that the response of the Lü system is the same as that of the Lorenz system with the lapse of time. Figure 6 is the projection of the completely synchronized attractors of the Lorenz system and the Lü system on phase plane. The trajectories of the driving system and the response system from different conditions will converge to an identical attractor. Figure 7 depicts the time
series of the estimated parameter, which are frozen at $a_2 = 35$, $b_2 = 7$, and $c_2 = 2.92$, respectively.

When the scaling factor $\kappa = -1.2$, using the control of inputs $u_1$, $u_2$, and $u_3$ and the parameter update law (35), the antphase projection of synchronized attractors on $x_1 - x_3$ ($y_1 - y_3$) plane and synchronization behavior of the Lorenz system and the Lü system are shown in Figures 8 and 9, respectively. It is obvious that the antphase projective synchronization in proportion is obtained by varying the scaling factor $\kappa$.

6. Conclusions

In this paper, a projective synchronization theorem was proposed based on an adaptive sliding mode control algorithm when the parameters of the response system are unknown and there are uncertain terms and external bounded disturbances in the response system. As an application of this projective synchronization theorem, the projective synchronization of the Lorenz system and the Lü system with uncertain terms and external bounded disturbances was also analyzed. The globally projective synchronization was achieved by a sliding mode controller based on the adaptive technique. The numerical simulations demonstrated the effectiveness and feasibility of the proposed technique. For the proposed method, there are potential applications in higher speed secure digital communication. In the future, we will try to extend the proposed method to the consensus of multiantgent systems such as the networked chaotic systems and complex networks.

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