Research Article

Determine the Inflow Performance Relationship of Water Producing Gas Well Using Multiobjective Optimization Method

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During the development of water drive gas reservoirs, the phenomena of gas escaping from water and water separating out from gas will change the seepage characteristics of formation fluid. Therefore, the traditional gas-water two-phase inflow performance relationship (IPR) models are not suitable for calculating the water producing gas well inflow performance relationship in water drive gas reservoirs. Based on the basic theory of fluid mechanics in porous medium, using the principle of mass conservation, and considering the process of dissolution and volatilization of gas and water formation, this paper establishes a new mathematical model of gas-water two-phase flow. Multiobjective optimization method is used to automatically match the sample well production data in water drive gas reservoirs and then we can achieve the sample well’s productivity equation, relative permeability curve, water influx intensity, and single well controlled reserves. In addition, the influence of different production gas water ratios (GWR) and gas-soluble water coefficients on absolute open flow rate (q_{AOF}) is discussed. This method remedied the limitation of well testing on site and was considered to be a new way to analyze the production behaviors in water producing gas well.

1. Introduction

Well productivity is one of primary concerns in field development and provides the basis for field development strategy [1]. Xiaoping and Birong [2] put forward a method to deduce the binomial productivity equation which could calculate the inflow performance relationship (IPR) curve of water producing gas well and presented the application of the IPR curve in determining gas and water production rate from water producing gas well. Zhu et al. [3] proposed three new formation evaluation parameters for low permeability gas reservoir. On the basis of the rate controlled mercury injection, nuclear magnetic resonance and physical simulation technologies. Wang et al. [4] analyzed gas and water phase relative permeability through cores with three different permeability leaves by the establishment of physical simulation experiment system and experimental process of water-gas mutual flooding.

Park et al. [5] proposed a fuzzy nonlinear programming approach to design production systems of gas fields. The synthetic optimization method could find a globally compromise solution and offer a new alternative with significant improvement over the existing conventional techniques. Cardoso [6] found that reduced-order model is well suited for reservoir simulation. Han et al. [7] presented a multi-objective evolutionary algorithm applied to history matching of water flooding projects, that is to search a feasible set of geological properties showing the reliable future performance. Cancelliere et al. [8] discussed benefits, limitations and drawbacks of assisted history matching, based on multi objective optimization and heuristic strategies. Attention was focused on the possibility offered by these methodologies of
obtaining a number of calibrated reservoir models. Shelkov et al. [9] described a comparison of single and multi-objective history matching of a medium-sized field in Western Siberia with nearly 100 wells and over 10 years of history. And they compared the performance of both single and multi objective versions of particle swarm optimization. Tan et al. studied the transient flow and two-phase flow behaviors in porous media [10–12]. Some of their research results can be used to solve the problem of inflow performance relationship of water producing gas well.

2. Gas–Water Two Phase IPR Equation

We assume the reservoir is homogeneous with uniform thickness, total compressibility of rock and fluid is low and constant. The water phase flow is isothermal and Darcy flow, and the gas phase flow is isothermal and non-Darcy flow at high velocity, ignoring the impact of gravity and capillary forces. No chemical reaction exists between gas and water phase. Fundamental filtration equations for the gas and water phase are defined as [13]

\[
\frac{dp}{dr} = \frac{\mu_g}{kk_{rg}} V_g + \beta_g \rho_g r^2,
\]

where, \( p \) is pressure, \( r \) is radial distance, \( \mu_g, \mu_w \) is gas and water viscosity, respectively, \( k \) is absolute permeability, \( k_{rg}, k_{rw} \) is gas and water phase relative permeability respectively, \( V_g, V_w \) is gas and water phase flow velocity, \( \beta_g \) is turbulence velocity coefficient, \( \rho_g \) is gas density.

Under the boundary condition of steady radial state flow, the integral of (1) can be written as follows [14]

\[
\int_{p_{w}}^{p} \frac{kk_{rg} \mu_g}{B_g} dp = \int_{r_w}^{r} \frac{q_g}{2\pi rh} dr + \frac{q_g^2}{4\pi^2 h^2} \int_{r_w}^{r} \frac{\beta_g \rho_g k k_{rg} B_g}{\mu_g r^2} dr,
\]

\[
\int_{p_{w}}^{p} \frac{kk_{rw} \mu_w}{B_w} dp = \int_{r_w}^{r} \frac{q_w}{2\pi rh} dr,
\]

where \( p_R \) is reservoir pressure, \( p_{w} \) is bottom hole flowing pressure, \( r, r_w \) is external boundary and wellbore radius respectively, \( B_g, B_w \) is gas and water volume factor, respectively, \( q_g, q_w \) is gas and water production rate, respectively, \( h \) is reservoir thickness.

Fevang and Whitson [15] defined the gas and water phase pseudo pressure in two phase filtration. The equations are as follows

\[
\Delta m(p)_g = \int_{p_{w}}^{p} \left( \frac{k_{rw} - R_{sgw}}{B_w \mu_g} + \frac{k_{rg}}{B_g \mu_g} \right) dp,
\]

\[
\Delta m(p)_w = \int_{p_{w}}^{p} \left( \frac{k_{rw}}{B_w \mu_w} + \frac{k_{rg} - R_{sgw}}{B_g \mu_g} \right) dp,
\]

where, \( R_{sgw} \) is solution gas water ratio and \( R_{sgw} \) is solution water gas ratio.

By combing (2) and (3), the gas and water phase pseudo pressure can be expressed

\[
\Delta m(p)_g = q_g \left( \frac{1}{R_{pgw}} \int_{r_w}^{r} \frac{R_{sgw}}{2\pi rh} dr + \frac{1}{2\pi rh} \right) + \frac{q_g^2}{4\pi^2 h^2} \int_{r_w}^{r} \frac{\beta_g \rho_g k k_{rg} B_g}{\mu_g r^2} dr,
\]

\[
\Delta m(p)_w = q_w \left( \frac{r_w}{2\pi rh} \int_{r_w}^{r} \frac{R_{sgw}}{2\pi rh} dr + R_{pgw} \int_{r_w}^{r} \frac{R_{sgw}}{2\pi rh} dr \right) + \frac{q_w^2 R_{pgw}^2}{4\pi^2 h^2} \int_{r_w}^{r} \frac{\beta_g \rho_g k k_{rg} B_g}{\mu_g r^2} dr,
\]

where \( R_{pgw} \) is production gas water ratio.

In order to simplify the expressions of gas and water phase pseudo pressure, we define four parameters

\[
A = \int_{r_w}^{r} \frac{1}{2\pi rh} dr,
\]

\[
B = \frac{1}{4\pi^2 h^2} \int_{r_w}^{r} \frac{\beta_g \rho_g k k_{rg} B_g}{\mu_g r^2} dr,
\]

\[
C_g = \int_{r_w}^{r} \frac{R_{sgw}}{2\pi rh} dr,
\]

\[
C_w = \int_{r_w}^{r} \frac{R_{sgw}}{2\pi rh} dr.
\]

By combing (4), we obtain the gas-water two phase IPR equation (6).
This equation is determined by the four parameters $A, B, C_g, C_w$, where $A$ is laminar coefficient, $B$ is turbulence coefficient, $C_g$ is gas soluble coefficient, representing dissolved gas within gas well control range, $C_w$ is water soluble coefficient, representing dissolved formation water in gas within the well control range.

### 3. Gas-Water Two Phase Comprehensive Model and the Solution

#### 3.1. Gas and Water Relative Permeability

Gas and water phase relative permeability $k_{rg}, k_{rw}$ are needed in order to calculate the gas-water two phase IPR equation. $k_{rg}, k_{rw}$ is a function of water saturation $S_w$, the empirical equations are represented asfollows [15]

$$
k_{rg} = (1 - (S_w - S_{wi}))^2 \left(1 - (S_w - S_{wi})^{(5-D)/(3-D)}\right),
$$

$$
k_{rw} = (S_w - S_{wi})^{(11-3D)/(3-D)},
$$

(7)

where, $S_w$ is water saturation in reservoir, $S_{wi}$ is initial water saturation, $D$ is relative permeability index.

From (7), we obtain

$$
\frac{k_{rg}}{k_{rw}} = \frac{(1 - (S_w - S_{wi}))^2 \left(1 - (S_w - S_{wi})^{(5-D)/(3-D)}\right)}{(S_w - S_{wi})^{(11-3D)/(3-D)}}.
$$

(8)

Using the ratio of gas and water production, a method aimed to obtain the ratio of gas and water phase relative permeability is proposed by Jokhio and Tiab [16].

$$
\frac{k_{rg}}{k_{rw}} = \left(\frac{B_g\mu_g}{B_w\mu_w}\right)\left(\frac{R_{pgw} - R_{swg}}{1 - R_{pgw}R_{swg}}\right).
$$

(9)

The parameters $\mu_g, \mu_w, B_g, B_w, R_{pgw}, R_{swg}$ in (9) are functions of pressure $p$. Therefore, the ratio of gas and water phase relative permeability $k_{rg}/k_{rw}$ can be obtained by means of $p$ and $R_{pgw}$. Then $S_w$ can be calculated. At last, $k_{rg}, k_{rw}$ can be obtained.

By Combining (8) and (9), we obtain:

$$
\frac{1 - (S_w - S_{wi})^{(5-D)/(3-D)}}{(S_w - S_{wi})^{(11-3D)/(3-D)}} = \left(\frac{B_g\mu_g}{B_w\mu_w}\right)\left(\frac{R_{pgw} - R_{swg}}{1 - R_{pgw}R_{swg}}\right).
$$

(10)

From (7) and (10), the equations for calculating $k_{rg}, k_{rw}$ are defined as follows

$$
k_{rg} = (1 - (S_w - S_{wi}))^2 \left(1 - (S_w - S_{wi})^{(5-D)/(3-D)}\right),
$$

$$
k_{rw} = (S_w - S_{wi})^{(11-3D)/(3-D)},
$$

$$
(1 - (S_w - S_{wi}))^2 \left(1 - (S_w - S_{wi})^{(5-D)/(3-D)}\right)
$$

$$
= \left(\frac{B_g\mu_g}{B_w\mu_w}\right)\left(\frac{R_{pgw} - R_{swg}}{1 - R_{pgw}R_{swg}}\right).
$$

(11)

Based on the material balance equation in the water drive gas reservoir, the relationships between average formation...
pressure and geologic reserve, cumulative gas production and water invasion intensity can be obtained in (12) [17]
\[
P = \frac{P_i}{z_i} \left( 1 - \frac{\left( G_p/G \right)}{\left( 1 - \frac{G_p/G}{G_R} \right)^R} \right),
\]
where, \( p, p_i \) is current and initial reservoir pressure, respectively, \( z, z_i \) is gas deviation factor under current and initial reservoir pressure, respectively, \( G_p, G \) is cumulative gas production and dynamic reserves, respectively, \( R \) is water invasion coefficient.

3.2. Gas-Water Two Phase Comprehensive Model. By Combining (6), (10) and (12), the gas-water two phase comprehensive model can be expressed
\[
q_g = \left( -\left( A + \frac{1}{R_{ppw}} C_g \right) \right)
+ \sqrt{\left( A + \frac{1}{R_{ppw}} C_g \right)^2 + 4B \cdot \Delta m(p)_g} \end{equation}
(2B)^{-1},
\]
\[
q_w = \left( -\left( A + R_{ppw} C_w \right) \right)
+ \sqrt{\left( A + R_{ppw} C_w \right)^2 + 4BR^2_{ppw} \cdot \Delta m(p)_w} \end{equation}
\times (2BR^2_{ppw})^{-1},
\]
\[
\Delta m(p)_g = \int_{p_c}^{p_e} \left( \frac{k_{rg}}{B_{gw} \mu_g} - \frac{k_{rg}}{B_{gw} \mu_g} \right) dp,
\]
\[
\Delta m(p)_w = \int_{p_c}^{p_e} \left( \frac{k_{rg}}{B_{gw} \mu_g} + \frac{k_{rg}}{B_{gw} \mu_g} \right) dp,
\]
\[
k_{rg} = \left( 1 - (S_w - S_{uw}) \right)^2 \left( 1 - (S_w - S_{uw})^{1-D}/(1-D) \right),
\]
\[
k_{rw} = \left( S_w - S_{uw} \right)^{(1-3D)/(3-D)}
\]
\[
1 - (S_w - S_{uw})^2 \left( 1 - (S_w - S_{uw})^{1-D}/(1-D) \right)
\]
\[
\left( S_w - S_{uw} \right)^{(1-3D)/(3-D)}
\]
\[
= \left( \frac{B_{gw} \mu_g}{B_{gw} \mu_g} \right) \frac{R_{ppw} - R_{awg}}{1 - R_{ppw} R_{awg}}.
\]
\[
\frac{P}{Z} = \frac{p}{z_i} \left( 1 - \frac{\left( G_p/G \right)}{\left( 1 - \frac{G_p/G}{G_R} \right)^R} \right).
\]
(13)

3.3. The Solution of Gas-Water Two Phase Comprehensive Model. As (13) shows above, Laminar coefficient \( A \), turbulence coefficient \( B \), gas soluble coefficient \( C_g \), water soluble coefficient \( C_w \), relative permeability index \( D \), water invasion intensity \( R \) and single well controlled reserve \( G \) are needed to solve. An automatic fitting multi-objective optimization method is given in this paper to solve the problem in the complicated percolation model mentioned above. The essence of this method is seeking the best fitting between theoretical value and measured value. The solution is defined as follows
\[
E = \sum_{i=1}^{n} \left( q_g \left( A, B, C_g, C_w, D, R, G \right) - q_g \right)^2
\]
+ \sum_{i=1}^{n} \left( q_w \left( A, B, C_g, C_w, D, R, G \right) - q_w \right)^2,
\]
where, \( q_g(A, B, C_g, C_w, D, R, G) \) and \( q_w(A, B, C_g, C_w, D, R, G) \) is theoretical gas and water production rate, respectively, \( q_g \) and \( q_w \) are actual gas and water production rate, respectively, \( E \) is expressed as the target function to be fitted. The proper parameters can be obtained to minimum the target function by means of the automatic fitting method. The flow chart for plotting type curves is shown in Figure 1.

### Table 1: The basic parameters of an actual well.

<table>
<thead>
<tr>
<th>Well depth (m)</th>
<th>The relative density of gas</th>
<th>The relative density of water</th>
<th>Formation pressure (MPa)</th>
<th>Formation temperature (°C)</th>
<th>Initial water saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2994</td>
<td>0.78</td>
<td>1.02</td>
<td>30.08</td>
<td>78</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table 2: The target parameter in an actual well.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C_g</th>
<th>C_w</th>
<th>D</th>
<th>R</th>
<th>G (10^4 m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.41 × 10^{-5}</td>
<td>5.23 × 10^{10}</td>
<td>0.12</td>
<td>4.21 × 10^{-6}</td>
<td>−1.97</td>
<td>5.37</td>
<td>3.59</td>
</tr>
</tbody>
</table>

4. Case Analysis

4.1. Calculating of the Target Parameter. During the process of acquiring the parameters like laminar coefficient \( A \), turbulence coefficient \( B \), gas soluble coefficient \( C_g \), water soluble coefficient \( C_w \), relative permeability index \( D \), water invasion intensity \( R \) and single well controlled reserve \( G \), the theoretical gas and water production can be obtained based on (13). By fitting the practical gas and water production on the basis of the automatically fitting method in (14), the result shown in Figure 2 can be acquired. It is clearly that the theoretical results verified with the practical ones which indicates the reliability of the results. The basic parameters of an actual well are shown in Table 1.

The parameters like laminar coefficient \( A \), turbulence coefficient \( B \), gas soluble coefficient \( C_g \), water soluble coefficient \( C_w \), relative permeability index \( D \), water invasion intensity \( R \) and single well controlled reserve \( G \) are shown in Table 2.

The value of water soluble coefficient in this table is very small which suggests that the content of the formation water dissolving into the natural gas is little. And the energy of the
formation water in the well controlled range is weak when the water invasion intensity is greater than 4. The gas and water phase relative permeability curve in different water saturations is obtained. The results are shown in Figure 3.

4.2. The IPR Curves of Water Producing Gas Well. The IPR curves in different production gas water ratios (GWR) are expressed in Figure 4. It is noted that the IPR curves expressed with a left offset when production gas water ratio decreases. Because when the GWR decreases, the water saturation in formation increases, and the flow resistance increases too. It becomes harder to flow in formation when the GWR decreases. The absolute open flow rates \( q_{AOF} \) of water producing gas well in different production gas water ratios can be shown in Table 3. From the curves, when the production gas water ratios is 1, 0.5, 0.2 and 0.1, respectively, the absolute open flow rates reduced 32.76, 31.71, 28.79, and 24.61 \( \times 10^4 \) m\(^3\)/d accordingly. The absolute open flow rates is reducing by 3.21%, 12.12% and 24.88% when compared to the one whose production gas water ratio is 0.1. It is shown that water invasion will greatly reduce the gas production capacity of water producing gas well.

The IPR curves in different gas soluble coefficient are shown in Figure 5. It can be concluded that the IPR curves expressed with a left offset when the gas soluble coefficient increases. Gas soluble coefficient represents the solubility of gas in water, which lead to a bigger flow resistance. The absolute open flow rates \( q_{AOF} \) of water producing gas well in different gas soluble coefficient can be shown in Table 4. From the curves, it is clearly that when gas soluble coefficient is 0.1, 0.5, 1 and 2, respectively, absolute open flow rates is 32.94, 29.57, 25.91 and 20.22 \( \times 10^4 \) m\(^3\)/d, respectively. The absolute open flow rates are reduced by 10.23%, 21.34% and 38.62% when compared to the one whose gas soluble coefficient is 0.1. It is shown that the more gas dissolved in the formation water, the more liquid phase will exist in the formation fluid. It will increase the gas flowing resistance and result in the greater productivity impairment.

<table>
<thead>
<tr>
<th>GWR ( (10^4 \text{m}^3/\text{m}^3) )</th>
<th>( q_{AOF} ) ( (10^4 \text{m}^3/\text{d}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.76</td>
</tr>
<tr>
<td>0.5</td>
<td>31.71</td>
</tr>
<tr>
<td>0.2</td>
<td>28.79</td>
</tr>
<tr>
<td>0.1</td>
<td>24.61</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>GWR ( (10^4 \text{m}^3/\text{m}^3) )</th>
<th>( q_{AOF} ) ( (10^4 \text{m}^3/\text{d}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>32.94</td>
</tr>
<tr>
<td>0.5</td>
<td>29.57</td>
</tr>
<tr>
<td>1</td>
<td>25.91</td>
</tr>
<tr>
<td>2</td>
<td>20.22</td>
</tr>
</tbody>
</table>
5. Conclusions

Based on the basic theory of fluid mechanics in porous medium, taking the solution and volatilization of gas and water into consideration, a gas-water two phase IPR equation was established. Combining with gas-water two phase IPR equation, relative permeability equation and material balance equation in the water drive gas reservoir, we deduced the gas-water two phase comprehensive model, which is influenced by laminar coefficient, turbulence coefficient, gas soluble coefficient, water soluble coefficient, relative permeability index, water invasion intensity and single well controlled reserve. The influences of different production gas water ratios and gas soluble coefficients on the absolute open flow rate were discussed. The method proposed in this paper provided a new theoretical method for single well analysis of productivity and inflow performance, and got rid of the limitation that the well productivity can be only determined according to field well test.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References


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