Risk Comparison of Improved Estimators in a Linear Regression Model with Multivariate \( t \) Errors under Balanced Loss Function

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Under a balanced loss function, we derive the explicit formulae of the risk of the Stein-rule (SR) estimator, the positive-part Stein-rule (PSR) estimator, the feasible minimum mean squared error (FMMSE) estimator, and the adjusted feasible minimum mean squared error (AFMMSE) estimator in a linear regression model with multivariate \( t \) errors. The results show that the PSR estimator dominates the SR estimator under the balanced loss and multivariate \( t \) errors. Also, our numerical results show that these estimators dominate the ordinary least squares (OLS) estimator when the weight of precision of estimation is larger than about half, and vice versa. Furthermore, the AFMMSE estimator dominates the PSR estimator in certain occasions.

1. Introduction

In the literature, many statisticians have studied the risk comparisons of various estimators in the linear model with normal errors and have generated substantial results. However, the assumption of normality restricts the range of possible applications. The multivariate \( t \) distributions are more realistic and accurate than multivariate normal distributions in modeling real-world data due to their heavy tails. Moreover, multivariate \( t \) distribution plays an important role in robust statistical inference. Therefore, various inference problems based on these distributions have been studied. The sampling performance of estimators is an important aspect among them.

Let us now consider a linear regression model

\[
y = X\beta + u, \tag{1}
\]

where \( y \) is an \( n \times 1 \) vector of observations on a dependent variable. \( X \) is an \( n \times k \) full rank matrix of observations. \( \beta \) is a \( k \times 1 \) vector of coefficients. We assume that \( u \) has a multivariate \( t \) distribution with the probability density function given by

\[
P(u | \alpha, \sigma) = \frac{g(\alpha)}{(\sigma^2)^{n/2}} \left( \frac{1}{(\alpha + u^t u/\sigma^2)^{(n+\alpha)/2}} \right), \tag{2}
\]

where \( g(\alpha) = \alpha^{\alpha/2} \Gamma((\alpha + n)/2)/\pi^{n/2} \Gamma(\alpha/2) \). It is well known that its mean vector and covariance matrix are given by

\[
E(u) = 0, \quad \text{for } \alpha > 1,
\]

\[
E(uu^t) = \frac{\alpha \sigma^2}{(\alpha - 2)} I_n, \quad \text{for } \alpha > 2. \tag{3}
\]

As is shown in Zellner [1], the multivariate \( t \) distribution can be viewed as a mixture of multivariate normal and inverted gamma distributions:

\[
P(u | \alpha, \sigma) = \int_0^\infty P_N(u | \tau) \cdot P_{IG}(\tau | \alpha, \sigma) d\tau, \tag{4}
\]
where
\[ P_N(u | \tau) = (2\pi\tau^2)^{-n/2} \exp\left(-\frac{uu'}{2\tau^2}\right), \]
\[ P_{IG}(\tau | \alpha, \sigma) = \frac{2(\alpha\sigma^2/2)^{\alpha/2}}{\Gamma(\alpha/2)} \cdot \tau^{-\alpha} \exp\left(-\alpha \cdot \frac{\sigma^2}{2\tau^2}\right). \]

The ordinary least squares (OLS) estimator of \( \beta \) is \( b = S^{-1}X' y \), where \( S = X'X \). Also, the Stein-rule (SR) estimator is
\[ b_{SR} = \left(1 - \frac{ae'e}{b'Sb}\right)b, \]
where \( e = y - Xb, v = n - k, \) and \( a \) is a constant such that \( 0 \leq a \leq 2(k-2)/(v+2) \). Under the mean squared error of prediction, Stein [2] and James and Stein [3] proved that the SR estimator dominates the OLS estimator when the numbers of explanatory variables are more than two and the MSE of the SR estimator is minimized if \( a = (k-2)/(v+2) \). Thus, we use this value of \( a \) hereafter. From then on, lots of improved estimators have been proposed. For example, Baranchik [4] proposed the positive-part Stein-rule (PSR) estimator defined as
\[ b_{PSR} = \max\left(0, 1 - \frac{ae'e}{b'Sb}\right)b. \]

Farebrother [5] proposed the feasible minimum mean squared error (FMMSE) estimator which is
\[ b_{FMMSE} = \left(\frac{b'Sb}{b'Sb + e'e/v}\right)b. \]

Further, Ohtani [6] extended the FMMSE estimator to the adjusted feasible minimum mean squared error (AFMMSE) estimator by adjusting the degrees of the freedom of the component of the FMMSE estimator. The AFMMSE estimator is
\[ b_{AFMMSE} = \left(\frac{b'Sb/k}{b'Sb/k + e'e/v}\right)b. \]

Some results related to the comparisons of these estimators have been established. For example, Giles [7] considered the pretest estimator for linear restrictions. Namba [8] studied the PMSE performance of the biased estimators in a regression model when relevant regressors are omitted. Namba and Ohtani [9] gave the risk comparison of the Stein-rule estimator under the Pitman nearness criterion. There is a common characteristic in their studies. That is, the used loss functions were the quadratic function and its variants. However, in regression analysis, we are often interested in using an estimator which has high precision of estimation and high goodness of fit of model. In this situation, Zellner [10] proposed a balanced loss function which takes account of both precision of estimation and goodness of fit. Balanced loss function is a more comprehensive and reasonable standard than quadratic loss and residual sum of squares. Much work has been done about the balanced loss risk comparisons of improved estimators in the normal linear model. Some examples are Giles et al. [11], Ohtani et al. [12], Ohtani [13], and so on. Their results show that SR estimator is not admissible and is dominated by PSR estimator. However, do the conclusions still hold under multivariate \( t \) errors and balanced loss function? And, do these estimators still dominate the OLS estimator? It is interesting to discuss them under multivariate \( t \) distributions and balanced loss function. Thus, we will give the explicit formulae for the balanced loss risk of these estimators and compare their sampling performance by theoretical and numerical analysis. In the next section, the explicit formulae of balanced loss risk of these estimators are derived. In Section 3, we compare the risk performance by numerical evaluations. The proofs of main results are given in Section 4.

2. Balanced Loss Function and Risk

In order to discuss the performance of considered estimators, we consider the balanced loss function as
\[ L(\tilde{\beta}, \beta) = \theta (y - X\tilde{\beta})' (y - X\tilde{\beta}) + (1 - \theta) \times (X\tilde{\beta} - X\beta)' (X\tilde{\beta} - X\beta), \]
where \( \theta \) is a scalar such that \( 0 \leq \theta \leq 1 \), and \( \tilde{\beta} \) is any estimator of \( \beta \). The corresponding risk function is \( R(\tilde{\beta}) = E[L(\tilde{\beta}, \beta)] \). Since \( u \) has a multivariate \( t \) distribution which can be viewed as the mixture of multivariate normal and inverted gamma distribution, we have
\[ R(\tilde{\beta}) = E[L(\tilde{\beta}, \beta)] = E, E \left[ L(\tilde{\beta}, \beta) \mid \tau \right]. \]

If the null hypothesis is \( H_0: \beta = 0 \) and the alternative is \( H_1: \beta \neq 0 \), then the test statistic for \( H_0 \) is \( F = (b'Sb/k) / (e'e/v) \). In the same way as that of Namba [8], we consider the general pretest estimator as
\[ \tilde{\beta} = I(F \geq c) \left(1 + \frac{e'e}{b'Sb}\right)^{\omega} b, \]
where \( I(A) \) is an indicator function such that \( I(A) = 1 \) if an event \( A \) occurs and \( I(A) = 0 \) otherwise. \( c \) is the critical value of the pretest, and \( \omega \) is an arbitrary integer. The term \( \tilde{\beta} \) reduces to the SR estimator when \( c = 0, \gamma = -a, \) and \( \omega = 1 \), and it reduces to the PSR estimator when \( c = av/k, \gamma = -a, \) and \( \omega = 1 \). Furthermore, \( \tilde{\beta} \) reduces to the FMMSE estimator when \( c = 0, \gamma = 1/v, \) and \( \omega = -1 \), and it reduces to the AFMMSE estimator when \( c = 0, \gamma = k/v, \) and \( \omega = -1 \), respectively.

To derive the formulae of \( R(\tilde{\beta}) \), we first compute \( E[I(\tilde{\beta}, \beta) \mid \tau] \), assuming that \( \tau \) is given. If we denote \( u_1 = b'Sb/\tau^2, \) \( u_2 = e'e/\tau^2, \) then \( u_1 \sim \chi^2_\lambda(\lambda_1), \) and \( u_2 \sim \chi^2_{\lambda-k} \) for given \( \tau \), where \( \lambda_1 = b'Sb/\tau^2, \) \( \chi^2_\lambda(\lambda) \) is the noncentral chi-square distribution with \( f \) degrees of freedom and noncentrality
parameter $\lambda$. Thus, using $u_1$ and $u_2$, we define the functions $H(p, q, \gamma, \psi)$ and $J(p, q, \gamma, \psi)$ as

$$H(p, q, \gamma, \psi) = E \left[ I \left( \frac{\nu^T u_1}{\nu} \geq \psi \right) \left( \frac{u_1 + \nu^T u_2}{u_1} \right)^p u_1^\tau | \tau \right],$$

$$J(p, q, \gamma, \psi) = E \left[ I \left( \frac{\nu^T u_1}{\nu} \geq \psi \right) \left( \frac{u_1 + \nu^T u_2}{u_1} \right)^p u_1^\tau | \tau \right],$$

where $p, q$ are arbitrary integers. By direct computation, we have

$$E \left[ L(\hat{\beta}, \beta) | \tau \right] = \theta \left( \beta^T S \beta + m \tau^2 - 2 \tau^2 H(\omega, 1, \gamma, \psi) + \tau^2 J(\omega, 1, \gamma, \psi) \right) + (1 - \theta) \left( \tau^2 H(\omega, 1, \gamma, \psi) - 2 \tau^2 J(\omega, 0, \gamma, \psi) + \beta^T S \beta \right).$$

(14)

In the following, we first give one lemma in order to obtain the explicit formulae of risk.

**Lemma 1.** The explicit formulae of $H(p, q, \gamma, \psi)$ and $J(p, q, \gamma, \psi)$ are

$$H(p, q, \gamma, \psi) = \sum_{i=0}^{\infty} w_i (\lambda_1) G_i(p, q, \gamma, \psi),$$

$$J(p, q, \gamma, \psi) = \lambda_1 \sum_{i=0}^{\infty} w_i (\lambda_1) G_{i+1}(p, q, \gamma, \psi),$$

where $G_i(p, q, \gamma, \psi) = \frac{2 \Gamma(n/2 + i + q)}{\Gamma(k/2 + i \Gamma(\nu/2)^i) (1 - \nu)^{y/2 - 1}} (1 - \nu)^{y/2 - 1} \Gamma(\nu/2)^i (1 - \nu)^{y/2 - 1} \Gamma(\nu/2)^i dt,$

$$w_i(\lambda) = \exp(-\lambda/2)(\lambda/2)^{i!},$$

and $c^* = kc/(kc + v)$.

By this lemma and (11) and (14), we have the following theorem.

**Theorem 2.** Under model (1) and loss function (10), the risk of the general pretest estimator $\hat{\beta}$ is

$$R(\hat{\beta}) = \beta^T S \beta + n \theta \sigma^2 \frac{\alpha}{\alpha - 2} + \frac{\sigma^2 \alpha^{\alpha/2}}{2} \Gamma(\alpha/2)$$

$$\times \sum_{i=0}^{\infty} G_i(2\omega, 1, \nu, \psi) \left( \Gamma(\alpha/2 + i - 1) \right)^{\theta_i} \left( \theta_1 + \alpha \right)^{\alpha^{\alpha/2 + 1}}$$

$$+ \frac{\sigma^2 \alpha^{\alpha/2}}{\Gamma(\alpha/2)} \left( \sum_{i=0}^{\infty} G_i(\omega, 1, \gamma, \psi) \left( \Gamma(\alpha/2 + i - 1) \right)^{\theta_i} \left( \theta_1 + \alpha \right)^{\alpha^{\alpha/2 + 1}} \right)$$

$$- 2 (1 - \theta) \sigma^2 \frac{\alpha^{\alpha/2}}{\Gamma(\alpha/2)}$$

$$\times \sum_{i=0}^{\infty} G_{i+1}(\omega, 0, \gamma, \psi) \left( \Gamma(\alpha/2 + i) \right)^{\theta_{i+1}} \left( \theta_1 + \alpha \right)^{\alpha^{\alpha/2 + 1}}.$$
Table 1: Relative risk of the SR, PSR, FMMSE, and AFMMSE estimators for \( n = 20, k = 5 \) and \( \alpha = 3 \).

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
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<tr>
<td>SR</td>
<td>0</td>
<td>0.4705</td>
<td>0.8234</td>
<td>0.9999</td>
<td>1.1058</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.5015</td>
<td>0.8338</td>
<td>0.9999</td>
<td>1.0996</td>
</tr>
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<td></td>
<td>2</td>
<td>0.5262</td>
<td>0.8420</td>
<td>1.0000</td>
<td>1.0947</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.5640</td>
<td>0.8546</td>
<td>1.0000</td>
<td>1.0872</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.5923</td>
<td>0.8641</td>
<td>1.0000</td>
<td>1.0815</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.6148</td>
<td>0.8716</td>
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<td>1.0700</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.6333</td>
<td>0.8778</td>
<td>1.0000</td>
<td>1.0733</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.6686</td>
<td>0.8895</td>
<td>1.0000</td>
<td>1.0663</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.6942</td>
<td>0.8980</td>
<td>1.0000</td>
<td>1.0611</td>
</tr>
</tbody>
</table>

This indicates that the dominance results of the FMMSE and AFMMSE estimators over the OLS estimator do not hold necessarily under the balanced loss function. It is easy to see that the risk of the AFMMSE estimator is much smaller than the risks of the SR and PSR estimators if \( \theta < 0.5 \). However, the AFMMSE estimator does not dominate the FMMSE estimator under the balanced loss function when \( \theta \geq 0.75 \).

Table 2: Relative risk of the SR, PSR, FMMSE, and AFMMSE estimators for \( n = 20, k = 5 \) and \( \alpha = 20 \).

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>0</td>
<td>0.4705</td>
<td>0.8234</td>
<td>0.9999</td>
<td>1.1058</td>
</tr>
<tr>
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<td>1</td>
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<td>1.0892</td>
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<tr>
<td></td>
<td>2</td>
<td>0.6176</td>
<td>0.8725</td>
<td>1.0000</td>
<td>1.0765</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.7074</td>
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<td>1.0000</td>
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<td>0.7662</td>
<td>0.9221</td>
<td>1.0000</td>
<td>1.0468</td>
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<tr>
<td></td>
<td>8</td>
<td>0.8068</td>
<td>0.9356</td>
<td>1.0000</td>
<td>1.0386</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td>0.9608</td>
<td>1.0000</td>
<td>1.0235</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.9087</td>
<td>0.9696</td>
<td>1.0000</td>
<td>1.0183</td>
</tr>
</tbody>
</table>

In sum, our results show that when the loss function and error terms are extended from the usual quadratic loss function and normal distribution to balanced loss function and multivariate t distribution, the dominance of the PSR estimator over the SR estimator is robust. However, the dominance of these estimators over the OLS estimator is not robust.
4. Proof of Main Results

Proof of Lemma 1. For given \( \tau, u_1 \sim \chi_2^2(\lambda_1) \) and \( u_2 \sim \chi_2^2(\lambda_2) \); meanwhile, \( u_1 \) and \( u_2 \) are mutually independent. Therefore, we have
\[
H(p, q, \gamma, c) = \mathbb{E} \left[ I \left( \frac{\nu u_1}{u_2} \geq c \right) \left( \frac{u_1 + \nu u_2}{u_1} \right)^p \right] \\
= \sum_{i=0}^{\infty} w_i(\lambda_1) \frac{(1/2)^{n/2+i}}{\Gamma(k/2 + i + 1)} \times \int_0^\infty (u_1 + \nu u_2)^p u_1^{k/2+i-q-1} u_2^{n/2-1} \exp \left( -\frac{u_1 + \nu u_2}{2} \right) du_1 du_2, \\
\tag{17}
\]
where \( R \) is the region such that \((\nu/k)(u_1/u_2) \geq c\).

Making use of the change of variables, \( v_1 = u_1/u_2, v_2 = u_2 \), the integral in (17) reduces to
\[
\int_{k/c}^{\infty} \int_{0}^{v_1^{k/2+i-q-1} (v_1 + \gamma)^p (1 + v_1)^{-(n/2+i-q)}} \exp \left( -\frac{v_1 (1 + v_1)}{2} \right) dv_2 dv_1. \\
\tag{18}
\]
Again, making use of the change of variables, \( z = v_2(1 + v_1)/2, v_1 = v_1 \), the integral in (18) becomes
\[
2^{n/2+i} t^{(n/2 + i + q)} \\
\times \int_{k/c}^{\infty} v_1^{k/2+i-q-1} (v_1 + \gamma)^p (1 + v_1)^{-(n/2+i-q)}} \exp \left( -\frac{v_2 (1 + v_1)}{2} \right) dv_2 dv_1. \\
\tag{19}
\]
Further, making use of the change of variable, \( t = v_1/(1 + v_1) \), the integral in (19) reduces to
\[
\int_{k/c/(k+c)}^{1} t^{k/2+i-q-1} (1 - t)^{k/2+i-q-1} (1 - t)^{p-n/2+i-q} dt. \\
\tag{20}
\]
By (17)–(20), we have
\[
H(p, q, \gamma, c) = \sum_{i=0}^{\infty} w_i(\lambda_1) G_i(p, q, \gamma, c). \\
\tag{21}
\]

Next, we derive the formula for \( J(p, q, \gamma, c) \). Noting that \( \partial \lambda_1/\partial \beta = 2\beta b/r^2 \) and differentiating \( H(p, q, \gamma, c) \) with respect to \( \beta \), we have
\[
\frac{\partial H(p, q, \gamma, c)}{\partial \beta} = \sum_{i=0}^{\infty} \frac{\partial w_i(\lambda_1)}{\partial \beta} G_i(p, q, \gamma, c) \\
= -\frac{2b}{r^2} H(p, q, \gamma, c) + \frac{2b}{r^2} \sum_{i=0}^{\infty} w_i(\lambda_1) G_{i+1}(p, q, \gamma, c). \\
\tag{22}
\]

Since \( u_1 = b' S b/r^2 \) and \( b \sim N(\beta, r^2(X'X)^{-1}) \), \( H(p, q, \gamma, c) \) can be expressed as
\[
H(p, q, \gamma, c) = \int_{R} \left( \frac{u_1 + \nu u_2}{u_1} \right)^p u_1^q f_1(b) f_2(u_2) du_2 db, \\
\tag{23}
\]
where \( f_2(u_2) \) is the density function of \( u_2 \) and
\[
f_1(b) = \frac{1}{(2\pi)^{k/2} r^2 (X'X)^{1/2}} \exp \left[ -\frac{(b - \beta)' X' X (b - \beta)}{2r^2} \right]. \\
\tag{24}
\]
Differentiating (23) with respect to \( \beta \), we have
\[
\frac{\partial H(p, q, \gamma, c)}{\partial \beta} = \int_{R} \left( \frac{u_1 + \nu u_2}{u_1} \right)^p u_1^q f_1(b) f_2(u_2) \frac{S b - S \beta}{r^2} du_2 db \\
= \int_{R} \left( \frac{u_1 + \nu u_2}{u_1} \right)^p u_1^q f_1(b) f_2(u_2) S b \frac{d u_2}{r^2} db \\
- \frac{S \beta}{r^2} J(p, q, \gamma, c), \\
\tag{25}
\]
which together with (22) yields that
\[
\int_{R} \left( \frac{u_1 + \nu u_2}{u_1} \right)^p u_1^q f_1(b) f_2(u_2) \frac{S b}{r^2} du_2 db \\
= \frac{S \beta}{r^2} \sum_{i=0}^{\infty} w_i(\lambda_1) G_{i+1}(p, q, \gamma, c). \\
\tag{26}
\]

Multiplying \( \beta' \) from the left of the above, we have
\[
J(p, q, \gamma, c) = \lambda_1 \sum_{i=0}^{\infty} w_i(\lambda_1) G_{i+1}(p, q, \gamma, c). \\
\tag{27}
\]

This completes the proof of this lemma. \( \square \)

Proof of Theorem 2. By Lemma 1, we have
\[
E_\tau \left[ \tau^2 H(p, q, \gamma, c) \right] \\
= \sum_{i=0}^{\infty} G_i(p, q, \gamma, c) \int_0^\infty \tau^2 W_i(\tau) P_{X\tau}(\tau | \alpha, \sigma) d\tau \\
= \sum_{i=0}^{\infty} G_i(p, q, \gamma, c) \frac{\alpha^{i/2} \eta_i^2}{i!} \frac{(\lambda/2)^{1-\alpha/2-i}}{\Gamma(\alpha/2)} \\
\times \int_0^\infty \tau^{-(\alpha/2-i)} d\tau \exp \left( -\frac{\eta_i + \alpha \sigma^2}{2\tau^2} \right) d\tau, \\
\tag{28}
\]
where $\eta_1 = \beta'S\beta$. Making use of the change of a variable, $t_1 = (\eta_1 + \alpha \sigma^2)/2\tau_2$, (28) becomes

$$
\frac{\sigma^2}{2} \sum_{i=0}^{\infty} G_i (p, q, y, c) \frac{\alpha^{i/2}}{i!} \frac{\Gamma(\alpha/2)}{\Gamma(\alpha/2)} 
\times \Gamma\left(\frac{\alpha}{2} + i - 1\right) \frac{\eta_1}{\sigma^2} + \alpha^{i/2} \tau_2^{-1}. (28)
$$

Taking $\theta_1 = \eta_1/\sigma^2$, (29) becomes

$$
\frac{\sigma^2}{2} \sum_{i=0}^{\infty} G_i (p, q, y, c) \frac{\alpha^{i/2}}{i!} \frac{\Gamma(\alpha/2 + i - 1)}{(\theta_1 + \alpha)^{\alpha/2+i}},
$$

which together with (28) and (29) yields

$$
E_r \left[ r^2 H (p, q, y, c) \right] = \frac{\sigma^2}{2} \frac{\alpha^{i/2}}{\Gamma(\alpha/2)} 
\times \sum_{i=0}^{\infty} G_i (p, q, y, c) \frac{\Gamma(\alpha/2 + i - 1)}{(\theta_1 + \alpha)^{\alpha/2+i+1}}.
$$

In a similar way, we have

$$
E_r \left[ y^2 f (p, q, y, c) \right] = \frac{\sigma^2}{2} \frac{\alpha^{i/2}}{\Gamma(\alpha/2)} 
\times \sum_{i=0}^{\infty} G_i (p, q, y, c) \frac{\Gamma(\alpha/2 + i)}{(\theta_1 + \alpha)^{\alpha/2+i+1}}.
$$

From (35), when $\gamma = -a, \omega = 1$, a condition for $R(\beta)$ to be monotonically decreasing is

$$
-a + (1 + a) \frac{kc}{kc + v} \leq 0. (36)
$$

Thus, $R(\beta)$ is monotonically decreasing on $c \in [0, av/k]$ if $\gamma = -a, \omega = 1$. Since $\beta$ becomes the SR estimator when $\gamma = -a, \omega = 1$, and $c = 0$ it reduces to the PSR estimator when $\gamma = -a, \omega = 1$, and $c = av/k$, the PSR estimator dominates the SR estimator. This completes the proof. \hfill \Box

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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