Research Article

Some Properties for an Integral Operator Defined by Generalized Hypergeometric Function

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We define a new general integral operator for meromorphic functions involving the generalized hypergeometric function. Furthermore, we study the characterization and other properties for this operator.

1. Introduction

Let Ω denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} a_k z^k, \quad (1)$$

which are analytic in the punctured open unit disk $\mathbb{U}^* = \{z : z \in \mathbb{C}, 0 < |z| < 1\} = \mathbb{U} \setminus \{0\}$.

We say that a function $f \in \Omega$ is meromorphic starlike of order η ($0 \leq \eta < 1$) and belongs to the class $\Omega^*(\eta)$, if it satisfies the following inequality:

$$-\Re \left(\frac{zf'(z)}{f(z)} \right) > \eta. \quad (2)$$

A function $f \in \Omega$ is a meromorphic convex function of order η ($0 \leq \eta < 1$), if f satisfies the following inequality:

$$-\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > \eta, \quad (3)$$

and we denote this class by $\Omega_k(\eta)$.

Also, let $\Omega_N(\xi)$ ($\xi > 1$, $z \in \mathbb{U}$) be the subclass of Ω , consisting of the functions f , which satisfy the inequality

$$-\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) < \xi \quad (\xi > 1, z \in \mathbb{U}), \quad (4)$$

where the subclass $\Omega_N(\xi)$ was introduced and studied by Wang et al. [1] and Nehari and Netanyahu [2].

A q -hypergeometric function is a power series in one complex variable z with power series coefficients which depend (apart from q) on r complex upper parameters α_i , β_j , ($i = 1, \dots, r$, $j = 1, \dots, s$, $\alpha_i \in \mathbb{C}$, $\beta_j \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$) as follows:

$$\begin{aligned} & \mathbb{V}(\alpha_1, \dots, \alpha_r; \beta_1, \dots, \beta_s, q, z) \\ &= \sum_{k=0}^{\infty} \frac{(\alpha_1, q)_k \cdots (\alpha_r, q)_k}{(q, q)_k (\beta_1, q)_k \cdots (\beta_s, q)_k} \left[(-1)^k q^{\binom{k}{2}} \right]^{1+s-r} z^k, \end{aligned} \quad (5)$$

with $\binom{k}{2} = k(k-1)/2$, where $q \neq 0$ when $r > s+1$, ($r, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$), and $(\alpha, q)_k$ is the Pochhammer symbol $(\alpha)_k$ defined by

$$\begin{aligned} & (\alpha, q)_k \\ &= \begin{cases} 1, & k = 0; \\ (1-\alpha)(1-\alpha q)(1-\alpha q^2) \cdots (1-\alpha q^{k-1}), & k \in \mathbb{N}. \end{cases} \end{aligned} \quad (6)$$

Tracing back the history of basic hypergeometric series defined by (5) has brought us to Heine dated in 1846. Therefore it is sometimes called Heine's series. For brief survey on q -hypergeometric functions, one may refer to [3–5] (see also [6–8]).

For $z \in \mathbb{U}$, $|q| < 1$, and $r = s + 1$, the basic hypergeometric function defined in (5) takes the form

$${}_r\Lambda_s(\alpha_1, \dots, \alpha_r; \beta_1, \dots, \beta_s, q, z) = \sum_{k=0}^{\infty} \frac{(\alpha_1; q)_k \cdots (\alpha_r; q)_k}{(q; q)_k (\beta_1; q)_k \cdots (\beta_s; q)_k} z^k, \quad (7)$$

which converges absolutely in the open unit disk $\mathbb{U}$.

Corresponding to the function ${}_r\Lambda_s(\alpha_1, \dots, \alpha_r; \beta_1, \dots, \beta_s, q, z)$, let

$$\begin{aligned} {}_r\Psi_s(\alpha_1, \dots, \alpha_r; \beta_1, \dots, \beta_s, q, z) &= \frac{1}{z} \\ {}_r\Lambda_s(\alpha_1, \dots, \alpha_r; \beta_1, \dots, \beta_s, q, z) &= \frac{1}{z} + \sum_{k=1}^{\infty} \frac{(\alpha_1; q)_{k+1} \cdots (\alpha_r; q)_{k+1}}{(q; q)_{k+1} (\beta_1; q)_{k+1} \cdots (\beta_s; q)_{k+1}} z^k. \end{aligned} \quad (8)$$

Analogous to the differential operator defined in [9] which involves the q -hypergeometric functions on the normalized analytic functions, we define the following differential operator $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] : \Omega \rightarrow \Omega$ on the space of meromorphic functions in the class Ω by

$$\begin{aligned} \mathcal{N}_{r,s}^0[a_i, b_j; q] f(z) &= f(z) * {}_r\Psi_s(a_i, b_j; q; z) \\ \mathcal{N}_{r,s}^1[a_i, b_j; q] f(z) &= (1 - \lambda) f(z) * {}_r\Psi_s(a_i, b_j; q; z) \\ &\quad + \lambda z D_q(f(z) * {}_r\Psi_s(a_i, b_j; q; z)) \\ &\quad \vdots \\ \mathcal{N}_{r,s}^n[a_i, b_j; q] f(z) &= \mathcal{N}_{r,s}^1(\mathcal{N}_{r,s}^{n-1}(f(z))). \end{aligned} \quad (9)$$

If $f \in \Omega$, then from (9) we may deduce that

$$\mathcal{N}_{r,s}^n[a_i, b_j; q] f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} [1 + (k-1)\lambda]^n \Upsilon_k a_k z^k, \quad (10)$$

where

$$\Upsilon_k = \frac{(\alpha_1; q)_{k+1} \cdots (\alpha_r; q)_{k+1}}{(q; q)_{k+1} (\beta_1; q)_{k+1} \cdots (\beta_s; q)_{k+1}}, \quad (11)$$

$\lambda \geq 0$, and $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

It should be remarked that the operator given by (10) is a generalization of many other operators considered earlier; for example, consider the following.

- (i) For $\lambda = 1$, $r = 1$, $s = 0$, $\alpha_1 = q$, $q \rightarrow 1$ the operator $\mathcal{N}_{1,0}[q, q] f(z) = I^n f(z)$ was defined by Frasin and Darus [10] and studied by El-Ashwah and Aouf [11].
- (ii) For $n = 0$, $\alpha_i = q^{\alpha_i}$, $\beta_j = q^{\beta_j}$, $\alpha_i > 0$, $\beta_j > 0$, ($i = 1, \dots, r$; $j = 1, \dots, s$, $r = s + 1$), $q \rightarrow 1$ the operator $\mathcal{N}_{r,s}[q^{\alpha_i}, q^{\beta_j}, q] f(z) = \mathcal{H}_{r,s}[\alpha_i, \beta_j] f(z)$ was imposed by Liu and Srivastava [12].

(iii) For $n = 0$, $r = 2$, $s = 1$, $\alpha_2 = q$, $q \rightarrow 1$, the operator $\mathcal{N}_{2,1}[\alpha_1, q, \beta_1, q] f(z) = \mathcal{N}[\alpha_1; \beta_1] f(z)$ was introduced and studied by Liu and Srivastava [13]. Further, we note in passing that this operator $\mathcal{N}[\alpha_1; \beta_1] f(z)$ is closely related to the Carlson-Shaffer operator $\mathcal{N}[\alpha_1; \beta_1] f(z)$ defined on the space of analytic univalent functions in $\mathbb{U}$.

(iv) For $n = 0$, $r = 1$, $s = 0$, $\alpha_1 = \lambda + 1$, $q \rightarrow 1$, the operator $\mathcal{N}_{1,0}[\lambda + 1, q] f(z) = \mathcal{D}^\lambda f(z) = (1/z(1-z)^{\lambda+1}) * f(z)(\lambda > -1)$, where $\mathcal{D}^\lambda$ is the differential operator, was introduced by Ganigi and Uralegaddi [14], and then it was generalized by Yang [15].

Now, making use of the differential operator $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]$ given by (10), we introduce the following integral operator in the class Ω .

Definition 1. Let $n \in \mathbb{N}$, $i \in \{1, 2, \dots, n\}$, $\delta_i > 0$. We define the integral operator $I_{\delta_1, \dots, \delta_n} : \Omega^n \rightarrow \Omega$ by

$$\begin{aligned} I_{\delta_1, \dots, \delta_n}(z) &= \frac{1}{z^2} \int_0^z \left(-u^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_1)'(u) \right)^{\delta_1} \\ &\quad \cdots \left(-u^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_n)'(u) \right)^{\delta_n} du. \end{aligned} \quad (12)$$

Remark 2. Note that for $n = 0$, $r = 1$, $s = 0$, $\alpha_1 = q$, we obtain the integral operator defined by Mohammed and Darus [16]; see also [17–19].

With the aid of differential operator $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f(z)$ given by (10), we define a new subclass of functions in Ω as follows.

Definition 3. Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega_k(\alpha_i, \beta_j; q, \eta)$ if, and only if, f satisfies

$$-\Re \left\{ 1 + \frac{z (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f)'(z)} \right\} > \eta, \quad (13)$$

where $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f$ is defined in (10) and $0 \leq \eta < 1$.

Definition 4. Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega_{r,s}\mathcal{H}(\alpha_i, \beta_j, q, \eta, b)$ if, and only if, f satisfies

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{z (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f)'(z)} + 2 \right) \right\} > \eta, \quad (14)$$

where $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f$ is defined in (10) and $b \in \mathbb{C} \setminus \{0\}$, $0 \leq \eta < 1$.

Definition 5. Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega_{r,s}\mathcal{KU}(\alpha_i, \beta_j; q, \alpha, \eta, b)$ if, and only if, f satisfies

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)'(z)} + 2 \right) \right\} > \alpha \left| \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)'(z)} + 2 \right) \right| + \eta, \quad (15)$$

where $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f$ is defined in (10) and $\alpha \geq 0, -1 \leq \eta < 1, \alpha + \eta \geq 0, b \in \mathbb{C} \setminus \{0\}$.

Definition 6. Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega_{r,s}\mathcal{KH}(\alpha_i, \beta_j, q, \alpha, b)$ if, and only if, f satisfies

$$\left| 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)'(z)} + 2 \right) - 2\alpha(\sqrt{2}-1) \right| < \Re \left\{ \sqrt{2} \left(1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f)'(z)} + 2 \right) \right) \right\} + 2\alpha(\sqrt{2}-1), \quad (16)$$

where $\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q]f$ is defined in (10) and $\alpha > 0, b \in \mathbb{C} \setminus \{0\}$.

Remark 7. Putting $n = 0, r = 1, s = 0$ and $\alpha_1 = q$ in Definition 3, we obtain the result corresponding to the class mentioned in (3).

For $n = 0, r = 1, s = 0$ and $\alpha_1 = q$ in Definitions 4, 5, and 6, we obtain $\Omega\mathcal{K}_b(\eta), \Omega\mathcal{KU}(\alpha, \eta, b)$ and $\Omega\mathcal{KH}(\alpha, b)$ the classes of meromorphic functions, introduced and studied by Mohammed and Darus [20].

2. Preliminary Definitions

We begin by recalling each of the following definitions of subclasses of meromorphic functions $\Omega\mathcal{T}_1(\eta, b), \Omega\mathcal{T}_2(\alpha, \eta, b)$ and $\Omega\mathcal{T}_3(\alpha, b)$ which will be required in our investigation.

Definition 8 (see [20]). Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega\mathcal{T}_1(\eta, b)$ if, and only if, f satisfies

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) \right\} > \eta, \quad (17)$$

where $b \in \mathbb{C} \setminus \{0\}, 0 \leq \eta < 1$.

Definition 9 (see [20]). Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega\mathcal{T}_2(\alpha, \eta, b)$ if, and only if, f satisfies

$$\Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) \right\} > \alpha \left| \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) \right| + \eta, \quad (18)$$

where $\alpha \geq 0, -1 \leq \eta < 1, \alpha + \eta \geq 0, b \in \mathbb{C} \setminus \{0\}$.

Definition 10 (see [20]). Let a function $f \in \Omega$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Omega\mathcal{T}_3(\alpha, b)$ if, and only if, f satisfies

$$\left| 1 - \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) - 2\alpha(\sqrt{2}-1) \right| < \Re \left\{ \sqrt{2} \left(1 - \frac{1}{b} \left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1 \right) \right) \right\} + 2\alpha(\sqrt{2}-1), \quad (19)$$

where $\alpha > 0, b \in \mathbb{C} \setminus \{0\}$.

3. Main Results

In this section, we study some properties for the integral operator $l_{\delta_1, \dots, \delta_n}(z)$ defined by (12) of the subclasses given by Definitions 3, 4, 5, and 6.

Theorem 11. Let $\delta_i > 0$, for $i \in \{1, 2, \dots, n\}$ and $f_i \in \Omega$. If

$$\sum_{i=1}^n \delta_i (1 - \eta_i) < 2, \quad (0 \leq \eta_i < 1) \quad (20)$$

and $f_i \in \Omega_k(\alpha_i, \beta_j, q, \eta_i)$, then $l_{\delta_1, \dots, \delta_n}(z)$ is in the class $\Omega_N(\xi)$, $\xi > 1$.

Proof. By differentiating the equality given by (12), we get

$$\begin{aligned} & z^2 l'_{\delta_1, \dots, \delta_n}(z) + 2z l_{\delta_1, \dots, \delta_n}(z) \\ &= \left(-z^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_1)'(z) \right)^{\delta_1} \\ & \quad \dots \left(-z^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_n)'(z) \right)^{\delta_n}. \end{aligned} \quad (21)$$

Again, differentiating (21), we deduce that

$$\begin{aligned} & z^2 l''_{\delta_1, \dots, \delta_n}(z) + 4z l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z) \\ &= \sum_{i=1}^n \delta_i \left(\frac{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + \frac{2}{z} \right), \end{aligned} \quad (22)$$

$$\begin{aligned} & \left[\left(-z^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_1)'(z) \right)^{\delta_1} \right. \\ & \quad \left. \dots \left(-z^2 (\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_n)'(z) \right)^{\delta_n} \right]. \end{aligned} \quad (23)$$

By using (21) and (23), we have

$$\frac{z^2 l''_{\delta_1, \dots, \delta_n}(z) + 4z l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)}{z^2 l'_{\delta_1, \dots, \delta_n}(z) + 2z l_{\delta_1, \dots, \delta_n}(z)} = \sum_{i=1}^n \delta_i \left(\frac{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + \frac{2}{z} \right). \quad (24)$$

Multiplying (24) by z yields that

$$\frac{z^2 l''_{\delta_1, \dots, \delta_n}(z) + 4z l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} = \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right), \quad (25)$$

or, equivalently, that

$$\frac{z(z l''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{z l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 = \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right). \quad (26)$$

Thus, we get

$$\begin{aligned} & - \left(\frac{z l''_{\delta_1, \dots, \delta_n}(z)}{l'_{\delta_1, \dots, \delta_n}(z)} + 1 \right) - 2 \\ & = \left(- \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \\ & \quad \times \left(1 + \frac{2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z)} \right). \end{aligned} \quad (27)$$

That is,

$$\begin{aligned} & - \left(\frac{z l''_{\delta_1, \dots, \delta_n}(z)}{l'_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \\ & = \left(\frac{2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z)} \right) \\ & \quad \times \left(- \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \\ & \quad + \sum_{i=1}^n \delta_i \left(- \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 1 \right) \right) \\ & \quad + 3 - \sum_{i=1}^n \delta_i. \end{aligned} \quad (28)$$

Taking the real parts of both terms of the last expression and using the fact that $\operatorname{Re}(z) \leq |z|$ for all z , we impose

$$\begin{aligned} & - \Re \left(\frac{z l''_{\delta_1, \dots, \delta_n}(z)}{l'_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \\ & = \Re \left\{ \left(\frac{2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z)} \right) \right. \\ & \quad \times \left(- \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \Big\} \\ & \quad + \sum_{i=1}^n \delta_i \Re \left\{ - \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 1 \right) \right\} \\ & \quad + 3 - \sum_{i=1}^n \delta_i \\ & \leq \left| \frac{2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z)} \right| \\ & \quad \times \left(- \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \\ & \quad + \sum_{i=1}^n \delta_i \Re \left\{ - \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 1 \right) \right\} \\ & \quad + 3 - \sum_{i=1}^n \delta_i. \end{aligned} \quad (29)$$

Setting

$$\begin{aligned} \xi & = \left| \frac{2l_{\delta_1, \dots, \delta_n}(z)}{z l'_{\delta_1, \dots, \delta_n}(z)} \right| \\ & \quad \times \left(- \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \\ & \quad + \sum_{i=1}^n \delta_i \Re \left(- \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 1 \right) \right) \\ & \quad + 3 - \sum_{i=1}^n \delta_i, \end{aligned} \quad (30)$$

and since

$$\left| \frac{2l_{\delta_1, \dots, \delta_n}(z)}{zl'_{\delta_1, \dots, \delta_n}(z)} \times \left(-\sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) + 1 \right) \right| > 0, \quad f_i \in \Omega_k(\alpha_i, \beta_j, q, \eta_i), \quad (31)$$

we conclude that

$$\xi > \sum_{i=1}^n \delta_i (\eta_i - 1) + 3. \quad (32)$$

Hence, by (20), we have $\xi > 1$, so that $l_{\delta_1, \dots, \delta_n}(z) \in \Omega_N(\xi)$, which evidently completes the proof of Theorem 11. $\square$

Theorem 12. Let $\delta_i > 0$, for $i \in \{1, 2, \dots, n\}$, $f_i \in \Omega_{r,s}\mathcal{K}(\alpha_i, \beta_j, q, \eta_i, b)$ ($0 \leq \eta_i < 1$) and $b \in \mathbb{C} \setminus \{0\}$. If

$$0 < \sum_{i=1}^n \delta_i (1 - \eta_i) \leq 1, \quad (33)$$

then $l_{\delta_1, \dots, \delta_n}(z)$ is in the class $\Sigma\mathcal{G}_1(\gamma, b)$, where $\gamma = 1 - \sum_{i=1}^n \delta_i (1 - \eta_i)$.

Proof. We know from the proof of Theorem 11 that

$$\begin{aligned} & \frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \\ &= \sum_{i=1}^n \delta_i \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right), \end{aligned} \quad (34)$$

which is equivalent to

$$\begin{aligned} & 1 - \frac{1}{b} \left\{ \frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right\} \\ &= 1 - \sum_{i=1}^n \delta_i \\ &+ \sum_{i=1}^n \delta_i \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\}. \end{aligned} \quad (35)$$

Taking the real part of both terms of (35), we get

$$\begin{aligned} & \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right\} \\ &= \sum_{i=1}^n \delta_i \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\ &+ 1 - \sum_{i=1}^n \delta_i. \end{aligned} \quad (36)$$

Now, since $f_i \in \Omega_{r,s}\mathcal{K}(\alpha_i, \beta_j, q, \eta_i, b)$, we have

$$\begin{aligned} & \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right\} \\ &> \sum_{i=1}^n \delta_i \eta_i + 1 - \sum_{i=1}^n \delta_i = 1 - \sum_{i=1}^n \delta_i (1 - \eta_i). \end{aligned} \quad (37)$$

Letting

$$\gamma = 1 - \sum_{i=1}^n \delta_i (1 - \eta_i), \quad (38)$$

and in view of (33), it follows that $0 \leq \gamma < 1$, so that $l_{\delta_1, \dots, \delta_n}(z) \in \Omega\mathcal{G}_1(\gamma, b)$. This completes the proof of Theorem 12. $\square$

Theorem 13. Let $\delta_i > 0$, for $i \in \{1, 2, \dots, n\}$, $f_i \in \Omega_{r,s}\mathcal{K}\mathcal{U}(\alpha_i, \beta_j, q, \alpha, \eta, b)$ ($\alpha \geq 0, -1 \leq \eta < 1, \alpha + \eta \geq 0$) and $b \in \mathbb{C} \setminus \{0\}$. If

$$\sum_{i=1}^n \delta_i - 1 \leq 0, \quad (39)$$

then $l_{\delta_1, \dots, \delta_n}(z)$ is in the class $\Omega\mathcal{G}_2(\alpha, \eta, b)$.

Proof. We want to show that

$$\begin{aligned} & \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right\} \\ &- \alpha \left| \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right| - \eta \\ &> 0. \end{aligned} \quad (40)$$

We find from (26) and (36) that

$$\begin{aligned}
 & \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right\} \\
 & - \alpha \left| \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right| - \eta \\
 & = 1 - \sum_{i=1}^n \delta_i \\
 & + \sum_{i=1}^n \delta_i \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\
 & - \alpha \left| \sum_{i=1}^n \delta_i \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right| - \eta.
 \end{aligned} \tag{41}$$

Since $f_i \in \Omega_{r,s} \mathcal{K}\mathcal{U}(\alpha_i, \beta_j, q, \alpha, \delta, b)$, it follows from Definition 5 that

$$\begin{aligned}
 & \Re \left\{ 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right\} \\
 & - \alpha \left| \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right| - \eta \\
 & > 1 - \sum_{i=1}^n \delta_i \\
 & + \sum_{i=1}^n \delta_i \left\{ \alpha \left| \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right| + \eta \right\} \\
 & - \alpha \sum_{i=1}^n \delta_i \left| \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right| - \eta \\
 & = 1 - \sum_{i=1}^n \delta_i + \sum_{i=1}^n \delta_i \eta - \eta.
 \end{aligned} \tag{42}$$

From hypothesis (39), we note that

$$(1 - \eta) \left(1 - \sum_{i=1}^n \delta_i \right) \geq 0. \tag{43}$$

Hence $l_{\delta_1, \dots, \delta_n} \in \Omega \mathcal{G}_2(\alpha, \eta, b)$. Therefore, we complete the proof of Theorem 13. $\square$

Theorem 14. Let $\delta_i > 0$ for $i \in \{1, 2, \dots, n\}$, $f_i \in \Omega_{r,s} \mathcal{K}\mathcal{U}\mathcal{H}(\alpha_i, \beta_j, q; \alpha, b)$ ($\alpha > 0$ and $b \in \mathbb{C} \setminus \{0\}$). If

$$\sum_{i=1}^n \delta_i - 1 \leq 0, \tag{44}$$

then $l_{\delta_1, \dots, \delta_n}(z)$ is in the class $\Omega \mathcal{G}_3(\alpha, b)$.

Proof. Our aim is to prove that

$$\begin{aligned}
 & \Re \left\{ \sqrt{2} \left(1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right) \right\} \\
 & + 2\alpha(\sqrt{2} - 1) \\
 & - \left| 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right| \\
 & - 2\alpha(\sqrt{2} - 1) \Big| \\
 & > 0.
 \end{aligned} \tag{45}$$

With the aid of (26), (36) can be written as follows:

$$\begin{aligned}
 & \Re \left\{ \sqrt{2} \left(1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right) \right\} \\
 & + 2\alpha(\sqrt{2} - 1) \\
 & - \left| 1 - \frac{1}{b} \left(\frac{z(zl''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{zl'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right| \\
 & - 2\alpha(\sqrt{2} - 1) \Big| \\
 & = \Re \left\{ \sqrt{2} \left[1 - \sum_{i=1}^n \delta_i \frac{1}{b} \right. \right. \\
 & \quad \times \left. \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right] \Big\} \\
 & + 2\alpha(\sqrt{2} - 1) \\
 & - \left| 1 - \sum_{i=1}^n \delta_i \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right| \\
 & - 2\alpha(\sqrt{2} - 1) \Big| \\
 & = \sqrt{2} - \sqrt{2} \sum_{i=1}^n \delta_i \Re \left\{ \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n[\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\
 & + 2\alpha(\sqrt{2} - 1)
 \end{aligned}$$

$$\begin{aligned}
& - \left| 1 - \sum_{i=1}^n \delta_i \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \\
& \quad \left. - 2\alpha(\sqrt{2} - 1) \right| \\
& = \sqrt{2} + \sqrt{2} \sum_{i=1}^n \delta_i \Re \\
& \quad \times \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\
& \quad - \sqrt{2} \sum_{i=1}^n \delta_i + 2\alpha(\sqrt{2} - 1) \\
& \quad - \left| 1 + \sum_{i=1}^n \delta_i \left[1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \right. \\
& \quad \quad \left. \left. - 2\alpha(\sqrt{2} - 1) \right] \right| \\
& \quad - \sum_{i=1}^n \delta_i + 2\alpha(\sqrt{2} - 1) \sum_{i=1}^n \delta_i - 2\alpha(\sqrt{2} - 1) \Bigg| \\
& = \sqrt{2} \left(1 - \sum_{i=1}^n \delta_i \right) + 2\alpha(\sqrt{2} - 1) \\
& \quad + \sqrt{2} \sum_{i=1}^n \delta_i \Re \\
& \quad \times \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\
& \quad - \left| [1 - 2\alpha(\sqrt{2} - 1)] \left(1 - \sum_{i=1}^n \delta_i \right) \right. \\
& \quad \quad \left. + \sum_{i=1}^n \delta_i \left[1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \right. \\
& \quad \quad \left. \left. - 2\alpha(\sqrt{2} - 1) \right] \right| \\
& \geq \sqrt{2} \left(1 - \sum_{i=1}^n \delta_i \right) + 2\alpha(\sqrt{2} - 1) \\
& \quad + \sqrt{2} \sum_{i=1}^n \delta_i \Re
\end{aligned}$$

$$\begin{aligned}
& \times \left\{ 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right\} \\
& - \sum_{i=1}^n \delta_i \left| 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \\
& \quad \left. - 2\alpha(\sqrt{2} - 1) \right| \\
& - |1 - 2\alpha(\sqrt{2} - 1)| \left(1 - \sum_{i=1}^n \delta_i \right) \\
& = \sum_{i=1}^n \delta_i \left\{ \Re e \sqrt{2} \right. \\
& \quad \times \left[1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \\
& \quad \quad \left. + 2\alpha(\sqrt{2} - 1) \right. \\
& \quad \quad \left. - \left| 1 - \frac{1}{b} \left(\frac{z(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)''(z)}{(\mathcal{N}_{r,s}^n [\alpha_i, \beta_j; q] f_i)'(z)} + 2 \right) \right. \right. \\
& \quad \quad \left. \left. - 2\alpha(\sqrt{2} - 1) \right| \right] + \sqrt{2} \left(1 - \sum_{i=1}^n \delta_i \right) \\
& \quad + 2\alpha(\sqrt{2} - 1) - 2\alpha(\sqrt{2} - 1) \sum_{i=1}^n \delta_i \\
& \quad - |1 - 2\alpha(\sqrt{2} - 1)| \left(1 - \sum_{i=1}^n \delta_i \right) \Bigg\} \\
& \quad + \sqrt{2} \sum_{i=1}^n \delta_i \Re
\end{aligned} \tag{46}$$

Since $f_i \in \Omega_{r,s} \mathcal{H}(\alpha, \beta_j, q, \alpha, b)$, it follows from Definition 6 and by a simple calculation that

$$\begin{aligned}
& \Re \left\{ \sqrt{2} \left(1 - \frac{1}{b} \left(\frac{z(z'l''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{z'l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right) \right\} \\
& \quad + 2\alpha(\sqrt{2} - 1) \\
& \quad - \left| 1 - \frac{1}{b} \left(\frac{z(z'l''_{\delta_1, \dots, \delta_n}(z) + 3l'_{\delta_1, \dots, \delta_n}(z))}{z'l'_{\delta_1, \dots, \delta_n}(z) + 2l_{\delta_1, \dots, \delta_n}(z)} + 1 \right) \right. \\
& \quad \quad \left. - 2\alpha(\sqrt{2} - 1) \right|
\end{aligned}$$

$$\begin{aligned}
&> \left[\sqrt{2} + 2\alpha(\sqrt{2} - 1) - |1 - 2\alpha(\sqrt{2} - 1)| \right] \left(1 - \sum_{i=1}^n \delta_i \right) \\
&> \left(1 - \sum_{i=1}^n \delta_i \right) \min \{ (\sqrt{2} - 1)(1 + 4\alpha), \sqrt{2} + 1 \} \\
&\geq 0,
\end{aligned} \tag{47}$$

which means that $l_{\delta_1, \dots, \delta_n} \in \Omega\mathcal{E}_3(\alpha, b)$, and we obtain the proof of the theorem. $\square$

Conflict of Interests

The authors declare that they have no competing interests.

Authors' Contribution

Ibtisam Aldawish and Maslina Darus read and approved the final paper.

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