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Research Article

Fare Design in Urban Transit Network considering Elastic Demand and Adverse Weather's Impact

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Due to the disturbance of unexpected effects and adverse weather conditions, transit supply and demand manifests many uncertainties. In this paper, we take account of these uncertainties and propose a transit fare structure design model including both ground and underground public transportation. Such transit fare design problem is described through bilevel programming, in which the upper level is the transportation authority's transit fare structure decision aiming to minimize the transit network's total travel and operation cost, while the lower level is a transit network assignment model considering supply and demand uncertainties that influence passengers' travel choice decisions. A heuristic algorithm is developed to solve the problem, and a numerical example is presented to illustrate the application. We get some important results: (1) a diversified fare structure considering uncertain weather's impact is quite necessary; (2) when the value of time is at a high level, metro fare should be higher than bus fare; (3) the optimal metro and bus fare should be close under an extremely adverse weather condition; (4) fare structure could be quite different with varied value of time.

1. Introduction

Transit service plays a vital role in urban transport. Nowadays, road traffic congestion, environmental contamination caused by motor vehicle exhaust emission, and other problems in transportation system emerging together with further urbanization and economic growth have been gradually impeding and dragging down cities' development. To deal with these problems and alleviate road capacity pressure, transit service should be greatly improved so that more and more residents are attracted to travel by public transport and much less private car mode to choose.

According to the investigation of the Bureau of Statistics in China, 65.5% of residents in Shanghai choose to commute by public transit. However, it takes them 50.4 minutes one way on average, and 80% of the interviewees' cost per day is no less than RMB 5 yuan. The degree of satisfaction in urban transit service is not that high, and residents have much great intention in traveling by private car.

Both time and monetary cost are main expenditure terms that people consider. Besides, the designation of transit line, location and number of transit stops, condition of vehicle devices such as air conditioners, transit service, conveniences for getting on and off ride, and others are all factors that could affect travelers' decisions in mode and route choice. We hope transit services could be safe, efficient, convenient, and with expected accuracy. But in reality, there is still a huge gap needed to be narrowed. In order to improve public transit services, operation management needs to be improved while the supply-demand contradiction should be coordinated.

Transit service demand is influenced by travelers' income level, travel cost (including transit fare), service level, comfort level, and car ownership [1–3]. Other travel modes, mainly private cars, electric motor cars, bicycles, and walk mode, can be regarded as the competitors of transit service [4]. In a word, transit demand can be affected to be stochastic according to many influencing factors and diversified competitive travel modes.

Recently, many researchers use schedule-based approach to formulate transit assignment problem. For example, Tong and Wong [5] developed a stochastic transit assignment

model using a dynamic schedule-based network and illustrated how it could be used to measure the performance of an urban metro system, while Poon et al. [6] focused on the route choice problems of travelers in a congested, dynamic, and schedule-based transit network. However, just as Tong et al. [7] pointed out, there are advantages in adopting a schedule-based approach when the transit vehicles are known to operate quite close to preannounced schedules and the vehicle speeds are not greatly affected by traffic conditions. Nuzzolo and Crisalli [8] also defined that the schedule-based approach requires explicit treatment for time-dependent segmentation of origin/destination matrix. Schedule-based approach can manage each vehicle in a more micro way, which is what frequency-based approach could not do; however, due to the lack of the basic requirement of the data and because the goal of this paper is not to deal with time-dependent analysis, we choose frequency-based approach which is often used for the strategic and long-range planning of the transit systems.

Uncertainty caused by unexpected accidents, not-informed events, traffic regulations violations, and adverse weather conditions such as dense fog, rainstorm, or snow also has great impact on transport network. Since unreliability has gradually become one of the major problems in transportation system [9], considering uncertainty is a necessity. However, the existing studies so far do not have much that covered the impact of the effects of uncertainty on urban transport. It is easy to find out that, in adverse weather conditions, the amount of outdoor activities is deduced, and most of unnecessary travels are cancelled. Transport demand would be changed greatly while the weather is not supportive. Therefore, demand elasticity has been an important factor that should be taken into consideration in transportation science research field.

Not only is the transit demand affected to be uncertain in adverse weather conditions, but also travel time is greatly influenced to be much unreliable. Variations and reliability of travel time have been studied for years from different perspectives, such as taking late arrival penalty into route cost [10] or considering travel time budget which depend upon travelers experience and risk aversion attitude [11]. Jackson and Jucker [12] and Abdel-Aty et al. [13] found that reliability is an important element that affects travelers' route choice decisions. Zhou and Chen [14] examined three different user equilibrium models, including traditional user equilibrium model, travel time reliability-based user equilibrium model, and α-reliable mean excess travel time user equilibrium model. They found that the latter two models are better in handling travelers' route choice decisions process under variable travel time. Siu and Lo [15] assumed both link capacity and demand are stochastic and divide the uncertain demand into two parts: regular travels by commuters and irregular trips by infrequent travelers. The two different groups have different travel behaviors, and the uncertain travel time is explained as a summation of expected travel time and travel time margin.

There are several means to deal with the uncertainties of transportation network, for example, to reduce the total travel cost through pricing designation or changing network capacity. Transit fare structure is one of the major factors that could help adjust transit demand and its assignment while improving residents' and transit operators' satisfaction and finally raising social welfare level. The optimization and design of transit fare structure have been studied over the years.

From the economic perspective, Mohring [16, 17] explained public transportation service under fixed demand by theories in microeconomics and Turvey and Mohring [18] discussed travel cost, frequency, unconstrained fare, and fare under the constraint of transit operators' revenue, but they did not give us a pricing model; we cannot quantify a reasonable fare value from their analysis of those factors' effect on each other. Cervero [19] considered social equity, discussed the difference between flat fare and differentiated fare structure through empirical studies of several districts' fare policy, and drew a conclusion that distance-based and time-based fare structure is better than flat fare while focusing on social equity, but flat fare has its strengths that public transportation authorities should take into account in many aspects.

From the network equilibrium perspective, Lam et al. [20, 21] proposed user equilibrium assignment model considering congestion effect, overload delays, capacity constraint, and elastic frequency; based on these, Zhou and Lam [22] and Li et al. [23] studied fare structure design by bilevel programming, and line capacity constraint and transit service reliability in different market regimes are, respectively, further and deeper considerations.

Uncertainty in transit network leads to variations of in-vehicle travel time, waiting time, walking time, and other travel costs. The focus of most previous papers which consider uncertainty in transport network is the variability of travel time; some traditional studies considered mainly the mean travel time, and some scholars considered more the variance of travel time; the tradeoff between mean and variance of travel time depends upon travelers' risk aversion attitude. Thus, choice decision in transit network with uncertainty is much more complicated. The existing studies had not explicitly proposed an effective and reasonable fare structure considering adverse weather conditions and the uncertain demand under that situation. Li et al. [24] studied a network-based model which incorporated the unreliability of transit services and further discussed the effects of unreliability on optimal fares and the corresponding market. Little had they mentioned the specific fare structure designation features under adverse weather conditions and paid attention to different levels of impact of different weather. For example, while in a rain storm weather condition, for the higher reliability, travelers much more prefer to travel by rail than by bus. Thus, the demand assignment would be quite different from that of the normal weather condition. To adjust the uncertain demand under such conditions, transit fare is a powerful tool. Therefore, uncertain demand should be taken into account in models, especially under adverse weather conditions. Only considering the unreliability of travel time cannot exemplify enough true travel conditions of reality.

This paper tries to model passengers' choice decision behavior under adverse weather conditions and propose an optimal fare structure considering the elastic demand under

adverse weather conditions, where the decision variable is transit fare. Catching the phenomenon that travelers are more likely to travel by metro than by bus in adverse weather conditions, the main objective and contribution of this research is to specify an appropriate transit fare structure under adverse weather conditions, so that a more reasonable pricing of metro and bus can play an important role in transit network demand management. And at the same time the transit fare policy can be enriched and ticket family with more fare classes is amplified. In the optimization model, the upper level subprogramme determines the optimal transit fare in transit network, while the lower level subprogramme determines transit route choice and passenger flows with elastic demand. The system-wide objective functions are used to minimize the total travel cost and operation cost.

The remainder of this paper is organized as follows. In Section 2, some basic concepts and assumptions are described and explained. Section 3 presents the model formulation. Section 4 provides a numerical example to illustrate the application of the proposed model. The problem solving algorithm is presented in Section 5. Finally, conclusions and implications are given in Section 6.

2. Preliminary Basic Considerations

2.1. Network Representation and Basic Concepts. To better present the problem, we should first give some explanations on the basic concepts.

(1) *Transit Network*. A set of transit lines and a set of stops (stations).

(2) *Transit Line*. A set of vehicles that keep on running between two transit stops. It is always described by transit service frequency.

(3) *Transit Route*. A feasible path for passengers to travel between any given origin and destination (OD pair).

(4) *Transit Link (Line Section)*. The section of transit line between two consecutive stops.

We use the transit network tested by Nguyen and Dupius [25] as is shown in Figure 1. The transit network contains 13 transit nodes (transit stops). There are 19 line sections and four OD pairs in the network: (1, 2), (1, 3), (4, 2), and (4, 3). In this paper, we consider a transit network comprising metro and bus. Travelers need to walk from origin to transit station, from transit station to destination, and from transit station to station at transfer node. Sometimes, there are several transit lines running on the same transit route sections. These lines share some same stations and can be regarded as attractive lines.

2.2. Notations and Assumptions. The notations used throughout this paper are presented as follows:

$G(N, S)$: a transit network, with node (stop/station) set N and line section set S,

g_w: travel demand of OD pair w,

g_w^0: the potential demand of OD pair w,

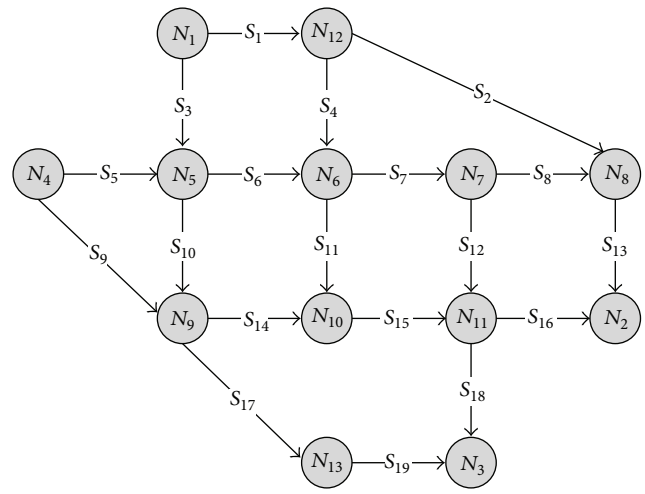

Figure 1: An example of transit network.

R_w: the set of feasible routes associated with the OD pair w,

W: set of all OD pairs,

A_s: set of attractive lines on transit link s,

π_w: parameter of demand sensitivity to travel disutility between OD pair w,

P_r^w: the probability of passenger amounts that choose transit route r between OD pair w,

t_s^0: in-vehicle travel time of transit s in the general BPR function,

u_r^w: expected travel disutility on transit route r between OD pair w,

$u_r^w(i)$: expected travel disutility on transit route r,

$N_l(i)$: number of vehicles on transit line l under weather category i,

P_i': the actual occurrence probability of weather category i,

h_r^w: passenger flow of route $r \in R_w$OD pair w,

v_s: passenger flow on transit link s,

v_s^l: passenger flow of transit line l while passing link s,

δ_{sr}: variable of 0-1 (it equals 1 if link s is on route r, and 0 otherwise),

C_s: capacity of transit link s,

η_s^l: probability of travelers choosing transit line l on transit link s,

fr_l: nominal frequency of transit line l,

$\mathrm{fr}_l(i)$: frequency of line l under weather category i,

k_l: vehicle capacity of transit line l,

T_r^w: in-vehicle travel time of transit route r between OD pair w,

T_{ws}: waiting time on transit link s,

T_{wks}: walking time on transit link s,

UC_s: the discomfort of transit link s,

UC_s^l: the discomfort of transit line l on transit link s,

tp_s: transit fare on transit link s,

tp_s^l: transit fare of transit line l on transit link s,

p_b: flat fare of bus lines,

p_u: flat fare of metro train lines,

$\mathbf{p}_b$: vector of flat fare of bus lines,

$\mathbf{p}_u$: vector of flat fare of metro train lines.

For the simplicity of descriptions, we assume the following.

(1) The travel demand between each OD pair is assumed to be influenced by transit service level and travel cost. Travelers may change their travel plans according to different travel information, such as the weather forecast. The OD demand then can be defined to be a function of the expected minimum utility under adverse weather conditions.

(2) It is assumed that the adverse weather conditions will decrease road capacity and increase the walking time and in-vehicle travel time and discomfort.

(3) There is only one transit service operator, that is, one transit service company. The fare structure is determined by the service operator under guide and control of the government.

(4) Travelers have the information of weather condition and road condition. Each of them has their own travel plan and will not change their route choice en-route.

(5) Besides, only single-class passengers are considered. This means that there is no difference in the importance of time value among passengers, and passengers who travel different distances would regard transit fare as flat and transit services as the same.

2.3. Stochastic Passenger Flow Distribution. As previously discussed, travelers' income levels, travel cost, service level, comfort level, and car ownership are the main factors that influence transit service demand. By taking into consideration adverse weather conditions, the uncertainty of transit demand is much greater. The OD demand will change according to different weather forecast, especially when there will be adverse weather conditions hindering some unessential outdoor activities. We assume the OD demand is elastic; when travel cost increases, the total demand decreases. According to random utility theory, the expected minimum travel disutility [26, 27] is

$$S_w = -\frac{1}{\theta} \ln \sum_{r \in R_w} \exp\left(-\theta u_r\right), \quad \forall w \in W, \tag{1}$$

where parameter $\theta > 0$ represents sensitivity of travelers' perception error on travel disutility. A greater θ value means the perception error is smaller. We assume the potential transit network demand is g_w^0. Then for the actual demand between OD pair w, in logit-based route choice transport network, we have

$$g_w = g_w^0 \exp\left(-\pi_w S_w\right), \tag{2}$$

where S_w is the expected minimum disutility of OD pair w. Thus the passenger flow on transit route r should be

$$\begin{aligned} h_r^w &= g_w P_r^w \\ g_w &= \sum_{r \in R_w} h_r^w, \quad w \in W, \end{aligned} \tag{3}$$

where P_r^w represents the probability of passenger amounts that choose transit route r:

$$P_r^w = \frac{\exp\left(-\theta u_r\right)}{\sum_{r \in R_w} \exp\left(-\theta u_r\right)}, \quad \forall r \in R_w,\ w \in W. \tag{4}$$

Then the passenger flow on transit link s should be

$$v_s = \sum_{w \in W} \sum_{r \in R_w} \delta_{sr} h_r^w. \tag{5}$$

3. Model Formulation

Travelers make their transit route choice decision by considering in-vehicle travel time, waiting time, walking time, in-vehicle crowding discomfort, transfer convenience, and transit fare. Thus, the disutility of transit route section (link) is a summation of these costs:

$$u_s = \alpha\left(T_s + T_{ws} + T_{wks}\right) + \alpha UC_s + tp_s, \tag{6}$$

where α is the monetary value of time. We assume the fare charged on the same transit link is a constant value.

3.1. Transit Route Disutility and Weather's Impact. (1) Walking time on both ends of origin and destination is usually neglected in most researches; however, as we consider the adverse weather conditions' impact, the walking time could be longer when there is a storm. So the walking time could be presented as

$$T_{wks} = \overline{T}_{wks} \varsigma(i), \tag{7}$$

where $\varsigma(i) \geq 1$ is a scaling function under weather condition i. $\overline{T}_{wks}$ is a constant value, representing the nominal walking time on each $s \in S$. Since besides origin and destination there is walking time cost at any transfer node, we define the walking time for each transit link.

(2) The in-vehicle travel time includes the time that travelers spend since boarding the bus or train till getting off. Since there is always different running speed which is affected by road or environment conditions and driver's driving behavior or any other uncertain factors, the in-vehicle time is a random variable. While considering adverse weather conditions' impact, it is a function of weather category I ($i = 1, 2, 3, 4, 5$) for bus. For metro train, since the in-vehicle travel time is much more reliable, it is rarely influenced by weather

condition. Thus the variation of travel time for metro is much smaller unless there is a need to slow down considering running safety when there is black storm or other severe adverse weather conditions. Follow the GBPR function in Lam et al. [28] and Sumalee et al. [4]:

$$T_s^l = g_{t_s^0}(i)\, t_s^0 + \beta_s \frac{1}{g_{C_s}(i)\, C_s}\left(v_s^l\right)^z, \quad \forall s \in S, \tag{8}$$

where $g_{t_s^0}(i)$ and $g_{C_s}(i)$ are, respectively, the scaling functions of free flow travel time and link capacity under weather condition i. β_s and z are the parameters in general BPR function. C_s is the link capacity. Consider

$$T_s = \sum_{l \in A_s} \eta_s^l T_s^l. \tag{9}$$

The in-vehicle travel time on transit link s is a weighted average of all T_s^l of attractive lines. η_s^l is the probability of travelers choosing transit line l:

$$\eta_s^l = \frac{\mathrm{fr}_l(i)}{\sum_{l \in A_s} \mathrm{fr}_l(i)}, \quad \forall l \in A_s,\ s \in S. \tag{10}$$

(3) The waiting time at transit station depends upon the distribution of passenger arrival and the arrival frequency of transit vehicle on transit link s [20, 29], and waiting time is assumed to follow

$$T_{ws}(i) = \frac{1}{\sum_{l \in A_s} \mathrm{fr}_l(i)}. \tag{11}$$

(4) Crowding discomfort in transit vehicles is gradually becoming an important factor in making transit route choice decisions. It is generally more comfortable in metro trains than in bus because of the steadier running. According to previous studies [30], crowding cost function is a product of in-vehicle travel time and average crowding cost per unit time which is an increasing function of passenger number in the vehicle:

$$UC(n, \tau) = g(n)\, \tau, \tag{12}$$

where n represents the amount of passengers and τ represents the in-vehicle travel time, with $g(n) \geq 0$ and $g(0) = 0$.

In this paper we assume the crowding cost function is linear:

$$UC_s^l = \left(\beta_0 + \beta_1\left(v_s^l - k_l\right)\right) T_s^l, \quad \forall l \in A_s,\ s \in S, \tag{13}$$

where β_0 is a parameter which represents the discomfort cost while transit vehicle is vacant and β_1 is a coefficient that represents the discomfort cost related to the amounts of passengers with limited vehicle capacity per unit time. $\beta_1 = 0$, if $v_s^l < k_l$; otherwise, $\beta_1 > 0$.

The transit link in-vehicle discomfort cost is a weighted average of discomfort cost of attractive lines:

$$UC_s = \sum_{l \in A_s} \eta_s^l UC_s^l, \quad \forall s \in S. \tag{14}$$

(5) Transit fare can be flat, distanced based, zone based, or time/peak/period based. For simplicity, we assume the fare structure for bus is flat. Thus we have $tp_s = tp_s^l$. For bus lines, $tp_s^l = p_b$, and for metro lines we have $tp_s^l = p_u$.

Table 1: Rainfall category adopted in the weather forecast.

Rainfall categories (i)	Expected hourly average of rainfall intensity (ρ_i) (mm/h)
No rain/light rain ($i = 1$)	5
Normal rain ($i = 2$)	20
Amber rain storm ($i = 3$)	30
Red rainstorm ($i = 4$)	50
Black rainstorm ($i = 5$)	70

Source: Sumalee et al. (2011) [4], Lam et al. (2008) [28].

3.2. Modeling Adverse Weather Conditions' Impact. We assume that travelers get weather forecast information from all sources, and the information is given by chances of different weather conditions. We now present Table 1 showing different weather conditions and the probabilities, as in Sumalee et al. [4] and Lam et al. [28].

Rainfall categories are generally concluded into five categories. The expected hourly average of rainfall intensity of each category is presented in Table 1. The forecasted probability is not the actual occurrence probability. Travelers may have a perception of the accurate probability through their knowledge and experience in the past. One means to calculate the updated probability is to apply Bayes' Theorem, as by Lam et al. [28].

According to Bayes' Theorem, the actual occurrence probability of weather category i should be

$$p_i' = \Pr\left[\frac{i}{\check{p}}\right] = \frac{\widehat{p}_{i/\check{p}_i}}{\sum_{i=1}^5 \check{p}_i \widehat{p}_{i/\check{p}}} \check{p}_i. \tag{15}$$

And $\check{p}_i$ is the probability of weather category i being forecasted. $\widehat{p}_{i/\check{p}_i}$ is the conditional probability that weather i happens while the weather forecast information is given.

The summation of p_i' under all weather categories equals 1. If we consider all weather categories, then the generalized disutility of transit route would be $\sum_{i=1}^5 p_i' \cdot u_r^w(i)$.

3.3. User Equilibrium Model under Adverse Conditions. Transit route disutility is a summation of transit link disutility:

$$u_r^w = \sum_{s \in S} \delta_{sr} u_s, \quad \forall r \in R_w,\ w \in W. \tag{16}$$

According to Wardrop's user equilibrium (UE) principle, travelers choose the route which has the minimum disutility. It will gradually come to an equilibrium state while the utilities of all used routes are equal, and no one could take less than the disutility to travel on either route. Thus, the rule of which route to be followed by passengers who make transit route choice decision can be described as follows:

$$\begin{gathered} h_r^{w^*} > 0 \quad u_r^{w^*} = \widehat{u_r^w}, \\ h_r^{w^*} = 0 \quad u_r^{w^*} > \widehat{u_r^w}, \\ \forall r \in R_w,\ w \in W, \end{gathered} \tag{17}$$

where $h_r^{w^*}$ denotes the passenger flow on transit route r under user equilibrium (UE) condition, $u_r^{w^*}$ denotes the utility on transit route r under UE condition, and the route flows and resultant network demand should satisfy

$$\begin{aligned} h_r^w &\geq 0, \quad \forall w \in W, \ \forall r \in R_w \\ g_w &\geq 0, \quad \forall r \in R_w. \end{aligned} \tag{18}$$

Since the passenger flow is decided by the cost or disutility on transit route, the disutility is affected by passenger flow. Thus, the above equilibrium functions and equations ((1)–(5)) can be described as a fixed point problem:

$$\mathbf{h}^* = \mathbf{F}(\mathbf{h}^*), \quad \forall \mathbf{h}^* \in \Omega, \tag{19}$$

where $\mathbf{F}(\mathbf{h}) = (g_w P_r^w$, for all $r \in R_w, \forall w \in W)$, $\Omega = \{\mathbf{h} \mid \sum_{r \in R_w} h_r^w = g_w$, for all $w \in W\}$.

According to the theorem of existence of solution of Brower's fixed point theorem, if the feasible set Ω is a bounded closed convex set and the function is a continuous function in set Ω, then there would be at least one solution to this problem. So the above fixed point problem would have at least one solution.

3.4. Bilevel Transit Fare Design Model. Bilevel programming is usually applied to model the decision process when the upper level decision can influence the lower level decision, and the lower level decision making also has effect back on the upper level. It is quite adequate here to model the fare design decision making in mainland China because of the intervention of the government and authorities in transport policies.

The upper level program in this paper is designed to minimize the total travel cost and variable operation cost; the lower level program is a user equilibrium model considering the elastic demand and adverse weather conditions. For the reason that the fixed operation cost is basically steady, we consider only the variable operation cost part. The cost of time is converted into monetary unit and we measure the travel cost by monetary cost.

$$\begin{aligned} \min \quad & Z((\mathbf{p}_b, \mathbf{p}_u), g(\mathbf{p}_b, \mathbf{p}_u), v(\mathbf{p}_b, \mathbf{p}_u)) \\ & = \sum_{s \in A_s} u_s [v_s(\mathbf{p}_b, \mathbf{p}_u), g_s] \cdot v_s(\mathbf{p}_b, \mathbf{p}_u) \\ & \quad + \sum_l E(C_l(i) \cdot N_l(i)) \\ \text{subject to} \quad & v_s(\mathbf{p}_b, \mathbf{p}_u) = \sum v_s^l(\mathbf{p}_b, \mathbf{p}_u) \\ & p_b^{\min} \leq p_b \leq p_b^{\max} \\ & p_u^{\min} \leq p_u \leq p_u^{\max}, \end{aligned} \tag{20}$$

where $C_l(i)$ represents operation cost per vehicle increase of transit line l under weather condition i. Since the frequency is the number of transit vehicles on line l divided by the cycle journey time, tThus $N_l(i) = \mathrm{fr}_l(i) \cdot CT_l$, and

$$E(C_l(i) \cdot N_l(i)) = \sum_{i=1}^{5} p_i' \cdot C_l(i) \cdot N_l(i). \tag{21}$$

The lower level program is the user equilibrium model under adverse weather conditions. We had previously proposed the model as a fixed point problem.

4. Solution Algorithm

Bilevel programming problems are generally not convex, and it is quite difficult to get an optimal solution. There are several solution algorithms that can be applied to solve this problem. We adopt a heuristic algorithm in this paper.

For the upper level model, we use simulating annealing algorithm; and for the lower level model, the method of successive average (MSA) is applied to solve the equilibrium assignment defined by the fixed point model. It has been proved to be very effective in many researches [23, 31].

4.1. Simulating Annealing Algorithm

Step 1. initialization. Give an initial point $\mathbf{x}^0 = (p_b^0, p_u^0) \in \Omega$, initial temperature T_0, and parameters $\xi (0 < \xi < 1)$, M, and T_s; set $n = 1$, $k = 1$, and $j = 1$.

Step 2. inside iteration. Calculate the objective function when $\mathbf{x} = \mathbf{x}^0$. Then for each k at a given temperature, give a random increment $\Delta\mathbf{x}$ for a current solution $\mathbf{x}^k$ according to a given rule, and a new $\mathbf{y} = \mathbf{x} + \Delta\mathbf{x}$ solution is generated. If the new solution is better than the previous optimal solution, that is, $\Delta Z = Z(\mathbf{y}) - Z(\mathbf{x}) < 0$, then $\mathbf{y}$ is accepted by the probability of 1, or it is accepted according to the Metropolis rule. If one inside iteration ended, then $j = j + 1$.

Step 3 (Metropolis rule). If $\Delta Z > 0$, $\exp(-\Delta Z)/T^{(n)} > \mathrm{random}[0, 1]$, then let $\mathbf{x}^{(k^{(n)}+1)} = \mathbf{y}$ and $k = k + 1$, back to Step 2; otherwise, let $\mathbf{x}^{(k^{(n)}+1)} = \mathbf{x}^{(k^{(n)})}$, and $k = k + 1$, back to Step 2.

Step 4 (outside iteration). Decrease the temperature and keep optimizing. $T^{(n+1)} = T^{(n)} \cdot \xi$, if j equals the given M value; let $n = n + 1$.

Step 5 (stop rule). If $T^{(n)} < T_s$, iteration is terminated, and output the optimal $\mathbf{x}^* = (p_b^*, p_u^*)$; otherwise, go back to Step 4.

4.2. Method of Successive Averages

Step 6 (initialization). Define a tolerance to end the algorithm, which can be defined as $\varepsilon = 0.1$, $j = 1$. Initialize the value of u_r^w.

Table 2: OD pairs and transit routes of the example transit network.

OD pair	g_w^0	Route	Transit link
(1, 2)	1000	1	S_1-S_2-S_{13}
		2	S_3-S_6-S_7-S_8-S_{13}
		3	S_3-S_6-S_7-S_{12}-S_{16}
		4	S_3-S_6-S_{11}-S_{15}-S_{16}
		5	S_3-S_{10}-S_{14}-S_{15}-S_{16}
		6	S_1-S_4-S_7-S_8-S_{13}
		7	S_1-S_4-S_7-S_{12}-S_{16}
		8	S_1-S_4-S_{11}-S_{15}-S_{16}
(1, 3)	1100	9	S_3-S_{10}-S_{17}-S_{19}
		10	S_3-S_6-S_7-S_{12}-S_{18}
		11	S_3-S_6-S_{11}-S_{15}-S_{18}
		12	S_3-S_{10}-S_{14}-S_{15}-S_{18}
		13	S_1-S_4-S_7-S_{12}-S_{18}
		14	S_1-S_4-S_{11}-S_{15}-S_{18}
(4, 2)	1000	15	S_9-S_{14}-S_{15}-S_{16}
		16	S_5-S_6-S_7-S_8-S_{13}
		17	S_5-S_6-S_7-S_{12}-S_{16}
		18	S_5-S_6-S_{11}-S_{15}-S_{16}
		19	S_5-S_{10}-S_{14}-S_{15}-S_{16}
(4, 3)	1200	20	S_9-S_{17}-S_{19}
		21	S_9-S_{14}-S_{15}-S_{18}
		22	S_5-S_{10}-S_{17}-S_{19}
		23	S_5-S_6-S_7-S_{12}-S_{18}
		24	S_5-S_6-S_{11}-S_{15}-S_{18}
		25	S_5-S_{10}-S_{14}-S_{15}-S_{18}

Step 7 (computation). For all $r \in R_w$, $\forall w \in W$, for all $s \in S$, compute the assignment of transit route flow $h_r^{w(j)}$ and the transit line section flow $v_s^{(j)}$.

Step 8 (update and move). Compute the cost of transit route and get the transit route disutility u_r^w.

Step 9. Descending Direction and Step Searching. Update u_r^w, and get an accessory solution $g_r^{w(j)}$; the new passenger flow of transit route r is $h_r^{w(j+1)} = h_r^{w(j)} + (1/j)(g_r^{w(j)} - h_r^{w(j)})$.

Step 10 (check convergence). If $\|h_r^{w(j+1)} - h_r^{w(j)}\| \le \varepsilon$, then end; otherwise, return to Step 7, $j = j + 1$.

5. Numerical Example

We use the transit network in Figure 1 for the numerical example. Transit routes of each OD pair, the relationship between transit line and transit link, and t_s^0 and C_s of each transit link are shown in Tables 2, 3, and 4, respectively.

We set the operation cost for metro line and bus line as 180 and 20 yuan per vehicle, respectively, under weather category 1. $\theta = 0.4$, $\pi_w = 0.01$, $\beta_0 = 0.5$; if $v_s^l < k_l$, $\beta_1 = 0$; otherwise, $\beta_1 = 0.1$; z is 4 according to the BPR function; $p_b^{\min}$ is 1 and $p_b^{\max}$ is 10, while $p_u^{\min}$ is 1 and $p_u^{\max}$ is 10. For weather forecast information of different categories, as in Sumalee et al. [4], we show it in Table 5 and the probabilities in bold face are the most likely weather condition in each weather category scenario.

Table 3: The relationship between transit line and transit link.

Transit line	Transit link
Bus Line 1	S_1
Bus Line 2	S_1-S_4
Bus Line 3	S_2
Metro Line 4	S_6-S_{11}
Metro Line 5	S_7-S_{12}
Bus Line 6	S_3
Bus Line 7	S_3-S_6
Bus Line 8	S_5
Bus Line 9	S_5-S_6
Bus Line 10	S_5-S_{10}
Bus Line 11	S_9
Bus Line 12	S_9-S_{14}
Bus Line 13	S_{13}
Bus Line 14	S_{16}
Bus Line 15	S_{18}
Bus Line 16	S_{17}-S_{19}
Bus Line 17	S_4-S_7
Bus Line 18	S_{15}-S_{18}
Bus Line 19	S_{15}-S_{16}
Bus Line 20	S_{12}-S_{18}
Bus Line 21	S_8
Bus Line 22	S_8-S_{13}
Bus Line 23	S_{11}-S_{15}
Bus Line 24	S_3-S_{10}
Bus Line 25	S_{12}-S_{16}
Metro Line 26	S_2
Metro Line 27	S_{18}
Metro Line 28	S_{16}
Metro Line 29	S_5-S_{10}
Metro Line 30	S_{14}-S_{15}

Under categories 1 and 2, the bus line and the metro line are set to be at the nominal frequency. The frequency reduces by one and reduces by two under categories 3 and 4, respectively. All bus frequencies are supposed to be 2 vehicles per hour under category 5. Under categories 4 and 5, the frequency of metro train is supposed to reduce by one and reduces by two, respectively. The variable operation cost would become 1.1, 1.2, 1.3, and 1.5 times the nominal cost under categories 2, 3, 4, and 5, respectively. The scaling factor of walking time is $\varsigma(i) = 2(1 + \rho_i/100)$.

The parameters for the GBPR functions are

$$g_{t_s^0}(i) = g_{C_s}(i) = \exp\left(0.05 \times \frac{\rho_i}{100}\right), \quad \forall s \in S. \tag{22}$$

$\beta_s = t_s^0/(C_s)^2$, and $u_r^w = E(u_r^w(i))$.

From Table 6, we find out that the total cost increases, while the total flows of each OD pair and the whole network decrease as the value of time increases. This is because the

Table 4: t_s^0 and C_s of each transit link.

Transit link	t_s^0 (minutes)		C_s (pass/hour)
	Bus	Metro	
S_1	12		1000
S_2	80	40	1100
S_3	12		1200
S_4	12		1100
S_5	12	6	1050
S_6	12	6	1500
S_7	12	6	1000
S_8	12		1080
S_9	30		1200
S_{10}	12	6	1150
S_{11}	12	6	1300
S_{12}	12	6	1200
S_{13}	12		1130
S_{14}	12	6	1300
S_{15}	12	6	1500
S_{16}	12	6	1200
S_{17}	30		1100
S_{18}	12	6	1250
S_{19}	30		1150

Table 5: Scenarios for different weather forecast information.

Probability	Scenario				
	1	2	3	4	5
$\breve{p}_1$	**80.0**	8.8	4.5	3.5	4.0
$\breve{p}_2$	8.0	**75.0**	10.5	7.0	8.0
$\breve{p}_3$	6.0	8.8	**70.0**	12.3	12.0
$\breve{p}_4$	4.0	5.0	10.5	**65.0**	16.0
$\breve{p}_5$	2.0	2.5	4.5	12.3	**60.0**
$\widehat{p}_{1/\breve{p}}$	**90.0**	7.0	4.5	4.0	5.0
$\widehat{p}_{2/\breve{p}}$	4.0	**80.0**	10.5	8.0	10.0
$\widehat{p}_{3/\breve{p}}$	3.0	7.0	**70.0**	14.0	15.0
$\widehat{p}_{4/\breve{p}}$	2.0	4.0	10.5	**60.0**	20.0
$\widehat{p}_{5/\breve{p}}$	1.0	2.0	4.5	14.0	**50.0**

actual travel demand of OD pair is a downsloping exponential function, and as the expected travel disutility increases, the actual travel demand of OD pair decreases. Due to the fare structure having nonlinear influence on the flows and the cost, both bus fare and metro fare structures are not monotonically increasing or decreasing. Since travel by bus takes more time, in order to balance the flows of each route, the metro fare should be much larger than the bus fare when the value of time is high. Specifically, the changes of transit route flows of each OD pair are shown in Table 7.

From Table 7, the flows of transit route 6 (S_5-S_{10}-S_{14}-S_{15}-S_{18}) between OD pair (4,3) increase as the value of time changes from 10 to 15, 15 to 20, 20 to 25, and 25 to 30. However the flows of transit route 5 (S_5- S_6- S_{11}- S_{15}- S_{18}) increase only

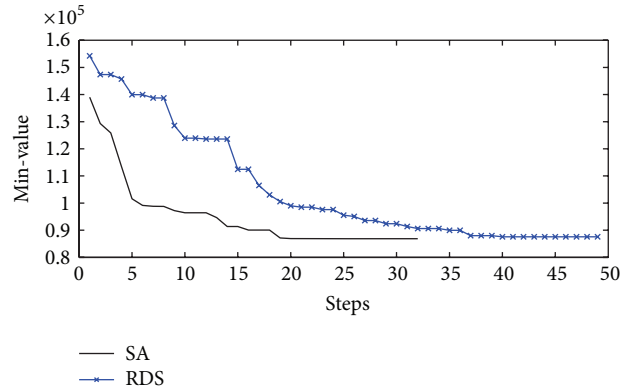

Figure 2: The convergence plot of SA algorithm and RDS algorithm.

when the value of time changes from 15 to 20. Although these two routes seem similar to 2 bus lines and 3 metro lines, S_{10}-S_{14} affects route 6 less than S_6- S_{11} affects route 5, because the flows of route 1 and 2 decrease which shared S_{10}-S_{14} and the flows of route 4 increase or decrease very little which shared S_6. We can also see from Table 7 that the flow of the longest transit route between each OD pair (e.g., route 1 between DO pair (1, 2), route 2 between OD pair (4, 2)) always decrease as the value of time increases. For non-metro line transit route, such as route 1 between OD pair (4, 3), the flow also decreases with the value of time increases.

Table 8 shows the results when considering different scenarios described in Table 5 when the value of time is 5 yuan. We can find out that, in scenarios 1 and 2, the metro fare should be larger than bus fare, while in scenario 3 the bus fare is larger than metro fare and in scenario 4 the bus fare and the metro fare are mostly equal. The flows of each OD pair and the whole network decrease as the expected hourly average of rainfall intensity increases. Because the worse the weather is, the larger the travel disutility is, which results in less travel demand. However, the volume of disutility increment is larger than the demand decrease, which brings about the total cost increase as the weather becomes worse. Table 9 shows the changes of transit route flows under each scenario with value of time at 5 yuan.

From Table 9, we can see that for route 3 and route 5 between OD pair (1, 2) and (4, 2), and route 4 between OD pair (4, 3), the flow increases as the weather becomes worse. It is because these routes contain more metro lines by at least 2/3, and we know that the metro line is affected little by weather condition. Reversely, for route 6 between OD pair (1, 2), route 1 and 5 between OD pair (1, 3), route 2 between OD pair (4, 2), and route 1 and 2 between OD pair (4, 3), the flow decreases as the weather get worse. Such kind of results is due to weather conditions affect bus line largely.

Taking the data in Tables 6 and 8 into account, we see that the total travel cost and the operation cost would grow much higher when the weather conditions change frequently and the value of time varies among individual largely. Thus, diversified fare structure design is of great necessity.

Figure 2 shows the convergence of SA algorithm and random direction search (RDS) algorithm when calculating

Table 6: Transit fare structure and transit route flows considering scenario 1 with different value of time.

α	Bus fare	Metro fare	OD pair flows ($\times 10^3$)					Total cost ($\times 10^5$)
			(1, 2)	(1, 3)	(4, 2)	(4, 3)	Total	
5	1.0291	1.9560	855.2	908.8	878.6	990.5	3633.1	0.86888
8	1.5812	1.8726	808.4	859.7	832.5	938.3	3438.9	1.01018
10	1.7040	1.2593	759.6	810.0	782.7	881.6	3233.9	1.15290
15	1.2972	2.1066	680.5	719.9	703.0	781.9	2885.3	1.35909
20	1.1082	2.5958	611.8	643.9	633.9	698.0	2587.6	1.50215
25	1.3387	2.8702	548.7	580.8	571.2	630.5	2331.2	1.59487
30	1.0812	2.6616	504.0	538.1	525.8	584.3	2152.2	1.64499

Table 7: The changes of transit route flows considering scenario 1 with different value of time.

α	OD pair	Changes of transit route flow							
		Route 1	Route 2	Route 3	Route 4	Route 5	Route 6	Route 7	Route 8
10 − 5	1, 2	−83.77	−24.71	**13.06**	**4.83**	**9.41**	−23.59	**7.58**	**1.54**
15 − 10		−88.15	**2.16**	−5.75	−5.49	−0.56	**9.94**	**4.95**	**3.85**
20 − 15		−69.49	−4.27	**1.27**	−1.50	**2.88**	−1.90	**3.70**	**0.65**
25 − 20		−40.84	−13.53	**7.18**	**2.43**	**7.28**	−18.20	−2.37	−5.10
30 − 25		−20.39	−12.62	**4.43**	**2.27**	**6.72**	−14.73	−5.20	−5.14
10 − 5	1, 3	−19.40	−8.83	−21.15	−13.03	−13.70	−22.68		
15 − 10		−46.07	−18.61	−17.00	−9.23	**0.84**	**0.01**		
20 − 15		−34.90	−9.54	−12.10	−6.53	−4.94	−8.01		
25 − 20		−14.70	−3.73	−7.79	−2.98	−16.35	−17.59		
30 − 25		−2.13	−4.73	−5.25	−1.44	−15.62	−13.53		
10 − 5	4, 2	−32.67	−60.33	**4.85**	−7.59	−0.14			
15 − 10		−29.98	−4.29	−18.91	−16.86	−9.69			
20 − 15		−33.00	−13.90	−7.47	−10.02	−4.74			
25 − 20		−33.46	−25.34	**0.13**	−4.50	**0.49**			
30 − 25		−32.19	−18.70	**1.46**	−0.45	**4.44**			
10 − 5	4, 3	−43.31	−39.25	−11.70	**0.72**	−11.28	−4.08		
15 − 10		−65.90	−2.56	−35.87	−0.80	−1.64	**7.06**		
20 − 15		−56.79	−14.39	−33.05	**7.81**	**2.14**	**10.34**		
25 − 20		−23.91	−35.53	−16.51	**4.84**	−1.75	**5.32**		
30 − 25		−2.96	−42.11	−2.55	−0.15	−2.07	**3.67**		

Note: the number of the first column, for example, 10 − 5, means the transit route flow with value of time 10 minus the transit route flow with value of time 5.

the minimum value of the total cost at the up-level in scenario with the value of time at 5 yuan. We can see that the SA algorithm converges from about 20 steps, and RDS algorithm converges after 40 steps. SA algorithm is better than RDS algorithm in convergence step and accuracy when solving the urban transit network fare design problem formulated in this paper.

6. Conclusions and Implications

In this paper we propose a model for designing a reasonable fare structure under elastic demand and adverse weather conditions' impact. The upper level model aims to minimize the total network cost including travelers' travel cost and transit operators' variable operation cost; the lower level model describes the user equilibrium considering the adverse weather conditions. We designed a heuristic algorithm which combines the method of successive averages (MSA) and simulated annealing algorithm to solve this bilevel problem. Through a numerical example, we illustrated the validity and effectiveness of the algorithm. We demonstrate by the numerical example that a diversified transit fare structure considering the improvement of transit service under adverse weather conditions is quite necessary.

We get several important conclusions and implications that would be applicable and quite practical: (a) different kinds of weather categories will result in very different total cost and fare structure; thus when designing a single fare

Table 8: Transit fare structure and transit route flows considering different scenarios ($\alpha = 5$).

Scenarios	Bus fare	Metro fare	OD pair flows					Total cost ($\times 10^5$)
			(1, 2)	(1, 3)	(4, 2)	(4, 3)	Total	
1	1.0291	1.9560	855.2	908.8	878.6	990.5	3633.1	0.86888
2	1.2297	1.7873	846.6	902.1	872.6	985.6	3607.0	0.89634
3	1.6547	1.1386	827.6	881.0	856.9	965.6	3531.1	0.95199
4	1.4429	1.4467	791.7	832.5	832.2	925.2	3381.6	1.05387
5	1.0882	1.1114	712.0	750.6	776.8	853.5	3100.9	1.23013

Table 9: The changes of transit route flows considering different scenarios ($\alpha = 5$).

Scenario	OD pair	Transit route flows changes							
		Route 1	Route 2	Route 3	Route 4	Route 5	Route 6	Route 7	Route 8
2 − 1	1, 2	−10.61	−5.97	**4.72**	2.35	**2.32**	−5.90	**3.23**	**1.30**
3 − 2		**23.16**	−20.96	**14.40**	−1.95	**10.49**	−28.13	−3.04	−13.01
4 − 3		−10.51	−14.93	**2.05**	−6.19	**4.13**	−9.66	**2.45**	−3.28
5 − 4		−40.43	−24.17	**19.32**	−1.83	**17.05**	−22	−6.66	−12.97
2 − 1	1, 3	−4.32	**2.83**	−1.57	−1.61	**0.77**	−2.77		
3 − 2		−12.91	**24.01**	−8.46	**16.71**	−10.38	−30.07		
4 − 3		−15.82	−4.55	−18.08	**1.06**	−0.78	−10.37		
5 − 4		−32.96	**14.22**	−16.98	**11.58**	−25.65	−32.06		
2 − 1	4, 2	−1.19	−14.10	**5.83**	**1.76**	**1.72**			
3 − 2		−30.53	−35.74	**29.43**	−0.96	**22.03**			
4 − 3		−37.55	−20.74	**16.41**	−2.50	**19.75**			
5 − 4		−62.42	−48.84	**33.32**	−6.58	**29.11**			
2 − 1	4, 3	−7.14	−2.63	−2.08	**4.92**	**1.00**	**0.96**		
3 − 2		−50.77	−25.03	−4.98	**33.26**	**2.15**	**25.41**		
4 − 3		−39.07	−35.90	−4.98	**18.60**	−1.04	**21.98**		
5 − 4		−56.12	−60.75	−28.03	**40.85**	−3.23	−56.12		

Note: the number of the first column, for example, 2 − 1, means the transit route flows for scenario 2 minus the transit route flows for scenario 1.

structure all kinds of weather categories should be taken into account; (b) when the value of time is very high, bus fare should be lower than metro fare; (c) if under an extremely adverse weather condition, such as red storm and black storm, then the value of metro fare and bus fare should be close; (d) if the value of time becomes larger, the fare structure should be greatly different from that under a lower value of time condition.

Conflict of Interests

The authors declare that they do not have any commercial or associative interest that represents a conflict of interests in connection with the work submitted.

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