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Research Article

Mean-Square Exponential Stability Analysis of Stochastic Neural Networks with Time-Varying Delays via Fixed Point Method

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This work addresses the stability study for stochastic cellular neural networks with time-varying delays. By utilizing the new research technique of the fixed point theory, we find some new and concise sufficient conditions ensuring the existence and uniqueness as well as mean-square global exponential stability of the solution. The presented algebraic stability criteria are easily checked and do not require the differentiability of delays. The paper is finally ended with an example to show the effectiveness of the obtained results.

1. Introduction

Cellular neural networks (CNNs), firstly proposed by Chua and Yang in 1988 [1, 2], have become a research focus owing to their numerous successful applications in various fields such as optimization, linear and nonlinear programming, associative memory, pattern recognition, and computer vision. Taking into account the finite switching speed of amplifiers in the implementation of neural networks, we see that the time delays are inevitable and therefore a new important model, namely, delayed cellular neural networks (DCNNs), is put forward.

On the other hand, it is noteworthy that, besides delay effects, stochastic and impulsive as well as diffusion effects are also likely to exist in the neural networks. Up to now, there have been a mass of works [3–12] on the dynamic behaviors of complex CNNs such as impulsive delayed reaction-diffusion CNNs and stochastic delayed reaction-diffusion CNNs.

Referring to the current publications of complex CNNs, we note that Lyapunov theory is always the primary method for the stability analysis. However the unavoidable reality is that there also exist lots of difficulties in the application of corresponding results to specific problems. So it does seem that some new methods are needed to resolve those difficulties.

Encouragingly, the fixed point theory is successfully applied by Burton and other authors to investigate the stability of deterministic systems, followed by some valid conclusions presented; for example, see the monograph [13] and the papers [14–25]. Furthermore, this new idea is developed to discuss the stability of stochastic (delayed) differential equations, turning out to be effective for the stability analysis of dynamical systems with delays and stochastic effects; see [26–32]. Specifically, in [27–29], Luo used the fixed point theory to study the exponential stability of mild solutions for stochastic partial differential equations with bounded delays and with infinite delays. In [30, 31, 33–35], Sakthivel et al. used the fixed point theory to investigate the asymptotic stability in pth moment of mild solutions to nonlinear impulsive stochastic partial differential equations with bounded delays and with infinite delays. In [32], Luo used the fixed point theory to study the exponential stability of stochastic Volterra-Levin equations.

The motivation of this paper is discussing the feasibility of using the fixed point theory to tackle the stability research of complex CNNs and thereupon enlarging the applications of the fixed point theory as well as enriching the stability theory of complex CNNs. In detail, via Banach contraction mapping principle, studied in this paper is the mean-square global

exponential stability of stochastic delayed CNNs. Remarkably, Banach contraction mapping principle is far different from Lyapunov method. By establishing a new inequality, we first construct a proper Banach space and thereby investigate, in mean-square sense, the existence and uniqueness as well as global exponential stability of the solution simultaneously. The obtained results show that, in regard to the stability research of complex CNNs, the fixed point theory does work and has its own advantage; namely, it works with no need for Lyapunov functions. Some algebraic stability criteria are finally presented, which are easily checked and do not require the differentiability of delays, let alone the monotone decreasing behavior of delays.

2. Preliminaries

Let $\{\Omega, \mathcal{F}, P\}$ be a complete probability space equipped with some filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions; that is, the filtration is right continuous and $\mathcal{F}_0$ contains all P-null sets. Let $\{\omega(t), t \geq 0\}$ denote a standard Brownian motion defined on $\{\Omega, \mathcal{F}, P\}$. R^n stands for the n-dimensional Euclidean space and $\|\cdot\|$ represents the Euclidean norm. $\mathcal{N} \triangleq \{1, 2, \ldots, n\}$. $R_+ = [0, \infty)$. $C(X, Y)$ corresponds to the space of continuous mappings from the topological space X to the topological space Y.

Consider the following stochastic cellular neural network with time-varying delays:

$$\begin{aligned} dx_i(t) &= \left\{-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} c_{ij} g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right\} dt \\ &\quad + \sum_{j=1}^{n} \sigma_j\left(t, x_j(t), x_j\left(t-\tau_j(t)\right)\right) d\omega(t), \quad t \geq 0 \end{aligned} \tag{1}$$

$$x_i(s) = \varphi_i(s), \quad -\tau \leq s \leq 0, \tag{2}$$

where $i \in \mathcal{N}$ and n is the number of the neurons in the neural network. $x_i(t)$ corresponds to the state of the ith neuron at time t. $f_j(\cdot), g_j(\cdot) \in C(R, R)$; moreover, $f_j(x_j(t))$ is the activation function of the jth neuron at time t and $g_j(x_j(t-\tau_j(t)))$ is the activation function of the jth neuron at time $t-\tau_j(t)$. The constant b_{ij} represents the connection weight of the jth neuron on the ith neuron at time t and the constant $a_i > 0$ represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external inputs. The constant c_{ij} represents the connection strength of the jth neuron on the ith neuron at time $t-\tau_j(t)$, where $\tau_j(t)$ corresponds to the transmission delay along the axon of the jth neuron and satisfies $0 \leq \tau_j(t) \leq \tau$. $\sigma_j(t, x_j(t), x_j(t-\tau_j(t))) \in C(R_+ \times R \times R, R)$ denotes the diffusion coefficient. $\varphi(s) = (\varphi_1(s), \ldots, \varphi_n(s))^T \in R^n$ and $\varphi_i(s) \in C([-\tau, 0], R)$.

Throughout this paper, we always assume that $f_j(0) = g_j(0) = \sigma_j(t, 0, 0) = 0$ $(j \in \mathcal{N})$. Denote by $\mathbf{x}(t) \triangleq \mathbf{x}(t; s, \varphi) = (x_1(t; s, \varphi_1), \ldots, x_n(t; s, \varphi_n))^T \in R^n$ the solution of (1) with the initial condition (2).

Definition 1. Equation (1) is said to be globally exponentially stable in mean square if, for any $\varphi(s) \in C([-\tau, 0], R^n)$, there exists a pair of positive constants λ and M such that

$$\mathbb{E}\left\{\|\mathbf{x}(t;\ s, \varphi)\|^2\right\} \leq M e^{-\lambda t}, \quad \forall t \geq 0. \tag{3}$$

Lemma 2. *Assume that $\mathbf{q}(t), \mathbf{h}(t) \in \mathcal{U}^2([-\tau, \infty), R^n)$, where $\mathcal{U}^2([-\tau, \infty), R^n)$ denotes the space consisting of functions $\boldsymbol{\chi}(t) : [-\tau, \infty) \to R^n$ satisfying $\mathbb{E}\{\int_0^t \|\boldsymbol{\chi}(s)\|^2 ds\} < \infty$; then*

$$\begin{aligned} &\sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau} \left|q_i(t) + h_i(t)\right|^2\right\}} \\ &\quad \leq \sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau} q_i^2(t)\right\}} + \sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau} h_i^2(t)\right\}} \end{aligned} \tag{4}$$

holds for $\mathbf{q}(t) = (q_1(t), \ldots, q_n(t))^T$ and $\mathbf{h}(t) = (h_1(t), \ldots, h_n(t))^T$.

Proof. As

$$\begin{aligned} \left\{\mathbb{E}\left(\sup_{t\geq -\tau}\left(2\left|q_i h_i\right|\right)\right)\right\}^2 &\leq 4\mathbb{E}\left(\sup_{t\geq -\tau}\left(q_i^2 h_i^2\right)\right) \\ &\leq 4\mathbb{E}\left(\sup_{t\geq -\tau}\left(q_i^2\right) \sup_{t\geq -\tau}\left(h_i^2\right)\right) \\ &= 4\mathbb{E}\left(\sup_{t\geq -\tau}\left(q_i^2\right)\right)\left(\mathbb{E}\sup_{t\geq -\tau}\left(h_i^2\right)\right), \end{aligned} \tag{5}$$

we derive $\mathbb{E}(\sup_{t\geq -\tau}(2q_i h_i)) \leq \mathbb{E}(\sup_{t\geq -\tau}(2|q_i h_i|)) \leq 2\sqrt{\mathbb{E}(\sup_{t\geq -\tau}(q_i^2))}\sqrt{\mathbb{E}(\sup_{t\geq -\tau}(h_i^2))}$. Hence, it is easy to see

$$\begin{aligned} &\sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau}\left[\left|q_i(t) + h_i(t)\right|^2\right]\right\}} \\ &\quad \leq \sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau}\left[q_i^2(t)\right]\right\}} + \sum_{i=1}^{n} \sqrt{\mathbb{E}\left\{\sup_{t\geq -\tau}\left[h_i^2(t)\right]\right\}}. \end{aligned} \tag{6}$$

□

Lemma 3 (see [36]). *Assume that $\alpha(t) \in \mathcal{U}^2(R_+, R)$, where $\mathcal{U}^2(R_+, R)$ denotes the space consisting of functions $\chi(t) : R_+ \to R$ satisfying $\mathbb{E}\{\int_0^t |\chi(s)|^2 ds\} < \infty$, $\beta \geq 2$; then*

$$\begin{aligned} &\mathbb{E}\left\{\left|\int_0^t \alpha(s)\, d\omega(s)\right|^\beta\right\} \\ &\quad \leq \left[\frac{\beta(\beta-1)}{2}\right]^{\beta/2} t^{((\beta/2)-1)} \int_0^t \mathbb{E}\left\{|\alpha(s)|^\beta\right\} ds. \end{aligned} \tag{7}$$

Remark 4. In Lemma 3, it is derived from letting $\beta = 2$ that

$$\mathbb{E}\left\{\left|\int_0^t \alpha(s)\, d\omega(s)\right|^2\right\} \leq \int_0^t \mathbb{E}\left\{|\alpha(s)|^2\right\} ds. \tag{8}$$

The consideration of this paper is based on the following fixed point theorem.

Theorem 5 (see [37]). *Let Υ be a contraction operator on a complete metric space Θ; then there exists a unique point $\zeta \in \Theta$ for which $\Upsilon(\zeta) = \zeta$.*

3. Main Results

In this section, we discuss, by means of the contraction mapping principle stated in Theorem 5, the existence and uniqueness as well as global exponential stability of the solution to (1)-(2) in mean-square sense. Before proceeding, we introduce some assumptions as follows.

(A1) There exist nonnegative constants l_j such that, for any $\eta, \upsilon \in R$,

$$\left|f_j(\eta) - f_j(\upsilon)\right| \le l_j |\eta - \upsilon|, \quad j \in \mathcal{N}. \tag{9}$$

(A2) There exist nonnegative constants k_j such that, for any $\eta, \upsilon \in R$,

$$\left|g_j(\eta) - g_j(\upsilon)\right| \le k_j |\eta - \upsilon|, \quad j \in \mathcal{N}. \tag{10}$$

(A3) There exist nonnegative constants ξ_j and ζ_j such that, for any $\eta_1, \eta_2, \upsilon_1, \upsilon_2 \in R$,

$$\left|\sigma_j(t, \eta_1, \upsilon_1) - \sigma_j(t, \eta_2, \upsilon_2)\right| \le \xi_j |\eta_1 - \eta_2| + \zeta_j |\upsilon_1 - \upsilon_2|, \quad j \in \mathcal{N}. \tag{11}$$

Let $\mathcal{H} = \mathcal{H}_1 \times \cdots \times \mathcal{H}_n$, and let $\mathcal{H}_i$ $(i \in \mathcal{N})$ be the space consisting of $\mathcal{F}$-adapted processes $\phi_i(t, \omega) : [-\tau, \infty) \times \Omega \to R$ which satisfy, for fixed $\omega \in \Omega$, the following:

(1) $\phi_i(t, \omega)$ is continuous in mean-square;

(2) $\phi_i(s, \omega) = \varphi_i(s)$ on $s \in [-\tau, 0]$;

(3) $\lim_{t\to\infty} e^{\alpha t}\mathbb{E}\{\phi_i^2(t, \omega)\} = 0$, where α is a positive constant satisfying $\alpha < \min_{i\in\mathcal{N}}\{2a_i\}$;

here $\varphi_i(s)$ is defined as shown in (2). From Lemma 2, we equip $\mathcal{H}$ with the norm $\|\overline{\phi}\| = \sum_{i=1}^n \sqrt{\mathbb{E}\{\sup_{t\in[-\tau,\infty)}\phi_i^2(t)\}}$, where $\overline{\phi}(t) = (\phi_1(t), \ldots, \phi_n(t)) \in \mathcal{H}$; thereby $\mathcal{H}$ is also a complete metric space.

Theorem 6. *Assume that conditions (A1)–(A3) hold. If there exist constants $0 < \varepsilon, \eta < 1$ such that $\sum_{i=1}^n \sqrt{\lambda_i} < 1$, where*

$$\lambda_i = \frac{n}{a_i^2(1-\varepsilon)(1-\eta)} \max_{j\in\mathcal{N}}\left\{b_{ij}^2 l_j^2\right\} + \frac{n}{a_i\varepsilon}\max_{j\in\mathcal{N}}\left\{\xi_j^2 + \zeta_j^2\right\} + \frac{n}{a_i^2(1-\varepsilon)\eta}\max_{j\in\mathcal{N}}\left\{c_{ij}^2 k_j^2\right\}, \tag{12}$$

then (1) *is globally exponentially stable in mean square.*

Proof. By Ito formula, we compute the differential of $e^{a_i t}x_i(t)$ along the solution of (1)-(2):

$$\begin{aligned}
de^{a_i t}x_i(t) &= e^{a_i t}dx_i(t) + a_i x_i(t)e^{a_i t}dt \\
&= e^{a_i t}\left\{-a_i x_i(t) + \sum_{j=1}^n b_{ij}f_j\left(x_j(t)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right\}dt \\
&\quad + e^{a_i t}\sum_{j=1}^n \sigma_j\left(t, x_j(t), x_j\left(t-\tau_j(t)\right)\right)d\omega(t) + a_i x_i(t)e^{a_i t}dt \\
&= e^{a_i t}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(t)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right\}dt \\
&\quad + e^{a_i t}\sum_{j=1}^n \sigma_j\left(t, x_j(t), x_j\left(t-\tau_j(t)\right)\right)d\omega(t), \\
&\qquad t > 0, \quad i \in \mathcal{N}
\end{aligned} \tag{13}$$

which yields after integrating from 0 to $t > 0$

$$\begin{aligned}
x_i(t) &= \varphi_i(0)e^{-a_i t} \\
&\quad + e^{-a_i t}\int_0^t e^{a_i s}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(s)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(s-\tau_j(s)\right)\right)\right\}ds \\
&\quad + e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right)d\omega(s), \\
&\qquad t > 0, \quad i \in \mathcal{N}.
\end{aligned} \tag{14}$$

Note $x_i(0) = \varphi_i(0)$ in (14); hence we define the following operator π acting on $\mathcal{H}$, for $\overline{\mathbf{x}}(t) = (x_1(t), \ldots, x_n(t)) \in \mathcal{H}$,

$$\pi(\overline{\mathbf{x}})(t) = \left(\pi(x_1)(t), \ldots, \pi(x_n)(t)\right), \tag{15}$$

where $\pi(x_i)(t) : [-\tau, \infty) \to R$ obeys the following rules:

$$\begin{aligned}
\pi(x_i)(t) &= \varphi_i(0)e^{-a_i t} \\
&\quad + e^{-a_i t}\int_0^t e^{a_i s}\left\{\sum_{j=1}^n b_{ij}f_j\left(x_j(s)\right) + \sum_{j=1}^n c_{ij}g_j\left(x_j\left(s-\tau_j(s)\right)\right)\right\}ds \\
&\quad + e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right)d\omega(s)
\end{aligned} \tag{16}$$

on $t \ge 0$ and $\pi(x_i)(s) = \varphi_i(s)$ on $s \in [-\tau, 0]$, $i \in \mathcal{N}$.

Now we will, by applying the contraction mapping principle, prove the existence and uniqueness as well as global exponential stability of solution to (1)-(2) in mean-square sense. The subsequent proof can be divided into two steps.

Step 1. We need to prove $\pi(\mathscr{H}) \subset \mathscr{H}$. To testify $\pi(\mathscr{H}) \subset \mathscr{H}$, it is necessary to show the mean-square continuity of $\pi(x_i)(t)$ on $[-\tau, \infty)$ and $\lim_{t\to\infty} e^{\alpha t}\mathbb{E}\{|\pi(x_i)(t)|^2\} = 0$ for $i \in \mathscr{N}$. First, in light of the expression of $\pi(x_i)(t)$, we have, for a fixed time $t_1 > 0$,

$$\pi(x_i)(t_1 + r) - \pi(x_i)(t_1) = I_1 + I_2 + I_3 + I_4, \tag{17}$$

where

$$\begin{aligned}
I_1 &= \varphi_i(0)\, e^{-a_i(t_1+r)} - \varphi_i(0)\, e^{-a_i t_1},\\
I_2 &= e^{-a_i(t_1+r)} \int_0^{t_1+r} e^{a_i s} \sum_{j=1}^n b_{ij} f_j\left(x_j(s)\right) ds\\
&\quad - e^{-a_i t_1} \int_0^{t_1} e^{a_i s} \sum_{j=1}^n b_{ij} f_j\left(x_j(s)\right) ds,\\
I_3 &= e^{-a_i(t_1+r)} \int_0^{t_1+r} e^{a_i s} \sum_{j=1}^n c_{ij} g_j\left(x_j\left(s-\tau_j(s)\right)\right) ds\\
&\quad - e^{-a_i t_1} \int_0^{t_1} e^{a_i s} \sum_{j=1}^n c_{ij} g_j\left(x_j\left(s-\tau_j(s)\right)\right) ds,\\
I_4 &= e^{-a_i(t_1+r)} \int_0^{t_1+r} e^{a_i s} \sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right) d\omega(s)\\
&\quad - e^{-a_i t_1} \int_0^{t_1} e^{a_i s} \sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right) d\omega(s).
\end{aligned} \tag{18}$$

It is obvious that $\mathbb{E}\{|\pi(x_i)(t_1 + r) - \pi(x_i)(t_1)|^2\} \le 4\sum_{i=1}^4 \mathbb{E}\{I_i^2\}$ holds. In addition, it is easy to see that $\lim_{r\to 0}\mathbb{E}\{I_i^2\} = 0$ for $i = 1, 2, 3$. Furthermore, by Lemma 3, we get

$$\begin{aligned}
&\mathbb{E}\left\{I_4^2\right\}\\
&\quad \le 2e^{-2a_i(t_1+r)}\\
&\qquad \times \int_0^{t_1+r} e^{2a_i s}\mathbb{E}\left[\left|\sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right)\right|^2\right] ds\\
&\qquad + 2e^{-2a_i t_1}\\
&\qquad \times \int_0^{t_1} e^{2a_i s}\mathbb{E}\\
&\qquad\quad \times \left[\left|\sum_{j=1}^n \sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right)\right|^2\right] ds \longrightarrow 0,\\
&\qquad\qquad \text{as } r \longrightarrow 0.
\end{aligned} \tag{19}$$

Thus, we know $\lim_{r\to 0}\mathbb{E}\{|\pi(x_i)(t_1 + r) - \pi(x_i)(t_1)|^2\} = 0$ for $i \in \mathscr{N}$, which means $\pi(x_i)(t)$ is continuous in mean square on $(0, \infty)$. Moreover, owing to $\varphi_i(s) \in C([-\tau, 0], R)$, we conclude $\pi(x_i)(t)$ is indeed continuous in mean square on $[-\tau, \infty)$ for $i \in \mathscr{N}$.

Next, we will prove $e^{\alpha t}\mathbb{E}\{|\pi(x_i)(t)|^2\} \to 0$ as $t \to \infty$ for $i \in \mathscr{N}$. It is derived from (A1) that

$$\begin{aligned}
P_1 &= e^{\alpha t}\mathbb{E}\left\{\left|e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^n b_{ij} f_j\left(x_j(s)\right) ds\right|^2\right\}\\
&\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|b_{ij} f_j\left(x_j(v)\right)\right| dv\right)^2\\
&\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|b_{ij} l_j x_j(v)\right| dv\right)^2\\
&\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|b_{ij} l_j\right| \sup_{v\in[0,s]}\left|x_j(v)\right| dv\right)^2\\
&= \left(e^{-a_i s}\sum_{j=1}^n \left|b_{ij} l_j\right| \sup_{v\in[0,s]}\left|x_j(v)\right| \int_0^s e^{a_i v} dv\right)^2\\
&\le e^{-2a_i s} n \sum_{j=1}^n \left|b_{ij} l_j\right|^2 \sup_{v\in[0,s]}\left|x_j(v)\right|^2 \frac{e^{2a_i s}}{a_i^2}\\
&= \frac{n}{a_i^2}\sum_{j=1}^n \left|b_{ij} l_j\right|^2 \sup_{v\in[0,s]}\left|x_j(v)\right|^2.
\end{aligned} \tag{20}$$

So, $\sup_{s\in[-\tau,t]} P_1 \le (n/a_i^2)\sum_{j=1}^n |b_{ij} l_j|^2 \sup_{s\in[-\tau,t]}|x_j(s)|^2$, which leads to

$$\begin{aligned}
&e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]} P_1\right)\\
&\quad \le \frac{n}{a_i^2}\sum_{j=1}^n \left|b_{ij} l_j\right|^2 e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]}\left|x_j(s)\right|^2\right) \longrightarrow 0, \quad \text{as } t \longrightarrow \infty.
\end{aligned} \tag{21}$$

In addition, from (A2), we deduce that

$$\begin{aligned}
P_2 &\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|c_{ij} g_j\left(x_j\left(v-\tau_j(v)\right)\right)\right| dv\right)^2\\
&\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|c_{ij} k_j x_j\left(v-\tau_j(v)\right)\right| dv\right)^2\\
&\le \left(e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^n \left|c_{ij} k_j\right| \sup_{u\in[-\tau,s]}\left|x_j(u)\right| dv\right)^2
\end{aligned}$$

$$
\begin{aligned}
&= \left(e^{-a_i s}\sum_{j=1}^{n}\left|c_{ij}k_j\right|\sup_{u\in[-\tau,s]}\left|x_j(u)\right|\int_0^s e^{a_i v}dv\right)^2 \\
&\le ne^{-2a_i s}\sum_{j=1}^{n}\left|c_{ij}k_j\right|^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2\frac{e^{2a_i s}}{a_i^2} \\
&= \frac{n}{a_i^2}\sum_{j=1}^{n}\left|c_{ij}k_j\right|^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2.
\end{aligned}
\tag{22}
$$

Therefore, $\sup_{s\in[-\tau,t]}P_2 = (n/a_i^2)\sum_{j=1}^{n}|c_{ij}k_j|^2\sup_{s\in[-\tau,t]}|x_j(s)|^2$, which leads to

$$
\begin{aligned}
&e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]}P_2\right) \\
&\quad\le \frac{n}{a_i^2}\sum_{j=1}^{n}\left|c_{ij}k_j\right|^2 e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]}\left|x_j(s)\right|^2\right)\longrightarrow 0, \quad \text{as } t\longrightarrow\infty.
\end{aligned}
\tag{23}
$$

Moreover, from (A3) and Lemma 3, we get

$$
\begin{aligned}
P_3 &= \left|e^{-a_i s}\int_0^s e^{a_i v}\sum_{j=1}^{n}\sigma_j\left(v, x_j(v), x_j\left(v-\tau_j(v)\right)\right)d\omega(v)^2\right|^2 \\
&\le e^{-2a_i s}\int_0^s ne^{2a_i v}\sum_{j=1}^{n}\left|\sigma_j\left(v, x_j(v), x_j\left(v-\tau_j(v)\right)\right)\right|^2 dv \\
&\le e^{-2a_i s}\int_0^s ne^{2a_i v} \\
&\qquad\times\sum_{j=1}^{n}\left(\xi_j\left|x_j(v)\right|+\zeta_j\left|x_j\left(v-\tau_j(v)\right)\right|\right)^2 dv \\
&\le e^{-2a_i s}\int_0^s 2ne^{2a_i v} \\
&\qquad\times\sum_{j=1}^{n}\left(\xi_j^2\left|x_j(v)\right|^2+\zeta_j^2\left|x_j\left(v-\tau_j(v)\right)\right|^2\right)dv \\
&\le e^{-2a_i s}\int_0^s 2ne^{2a_i v}\sum_{j=1}^{n}\left(\xi_j^2\sup_{v\in[0,s]}\left|x_j(v)\right|^2\right)dv \\
&\quad+e^{-2a_i s}\int_0^s 2ne^{2a_i v}\sum_{j=1}^{n}\left(\zeta_j^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2\right)dv \\
&= e^{-2a_i s}\sum_{j=1}^{n}\left(\xi_j^2\sup_{v\in[0,s]}\left|x_j(v)\right|^2\right)\int_0^s 2ne^{2a_i v}dv \\
&\quad+e^{-2a_i s}\sum_{j=1}^{n}\left(\zeta_j^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2\right)\int_0^s 2ne^{2a_i v}dv \\
&\le 2ne^{-2a_i s}\sum_{j=1}^{n}\left(\xi_j^2\sup_{v\in[0,s]}\left|x_j(v)\right|^2\right)\frac{e^{2a_i s}}{2a_i} \\
&\quad+2ne^{-2a_i s}\sum_{j=1}^{n}\left(\zeta_j^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2\right)\frac{e^{2a_i s}}{2a_i} \\
&= \frac{n}{a_i}\sum_{j=1}^{n}\left(\xi_j^2\sup_{v\in[0,s]}\left|x_j(v)\right|^2\right) \\
&\quad+\frac{n}{a_i}\sum_{j=1}^{n}\left(\zeta_j^2\sup_{u\in[-\tau,s]}\left|x_j(u)\right|^2\right).
\end{aligned}
\tag{24}
$$

So,

$$
\begin{aligned}
&e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]}P_3\right) \\
&\quad\le \frac{n}{a_i}\sum_{j=1}^{n}\left(\left(\xi_j^2+\zeta_j^2\right)e^{\alpha t}\mathbb{E}\left(\sup_{s\in[-\tau,t]}\left|x_j(s)\right|^2\right)\right)\longrightarrow 0, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{as } t\longrightarrow\infty.
\end{aligned}
\tag{25}
$$

It then follows from (21)–(25) that $e^{\alpha t}\mathbb{E}\{|\pi(x_i)(t)|^2\}\to 0$ as $t\to\infty$ for $i\in\mathcal{N}$. We therefore conclude $\pi(\mathcal{H})\subset\mathcal{H}$.

Step 2. We need to prove π is contractive. For $\bar{\mathbf{x}} = (x_1(t),\ldots,x_n(t))\in\mathcal{H}$ and $\bar{\mathbf{y}} = (y_1(t),\ldots,y_n(t))\in\mathcal{H}$, we know that, for $0<\varepsilon,\eta<1$,

$$
\begin{aligned}
&\left|\pi(x_i)(t)-\pi(y_i)(t)\right|^2 \\
&\quad= \left(Q_1+Q_2+Q_3\right)^2 \\
&\quad\le \frac{1}{(1-\varepsilon)(1-\eta)}Q_1^2+\frac{1}{(1-\varepsilon)\eta}Q_2^2+\frac{1}{\varepsilon}Q_3^2,
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
Q_1 &= e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^{n}\left\{b_{ij}\left(f_j\left(x_j(s)\right)-f_j\left(y_j(s)\right)\right)\right\}ds, \\
Q_2 &= e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^{n}\left\{c_{ij}\left(g_j\left(x_j\left(s-\tau_j(s)\right)\right)\right.\right. \\
&\qquad\qquad\left.\left.-g_j\left(y_j\left(s-\tau_j(s)\right)\right)\right)\right\}ds, \\
Q_3 &= e^{-a_i t}\int_0^t e^{a_i s}\sum_{j=1}^{n}\left\{\sigma_j\left(s, x_j(s), x_j\left(s-\tau_j(s)\right)\right)\right. \\
&\qquad\qquad\left.-\sigma_j\left(s, y_j(s), y_j\left(s-\tau_j(s)\right)\right)\right\}d\omega(s).
\end{aligned}
\tag{27}
$$

Note

$$\begin{aligned}
Q_1^2 &= e^{-2a_it}\left|\int_0^t e^{a_is}\sum_{j=1}^n\left\{b_{ij}\left(f_j\left(x_j(s)\right)-f_j\left(y_j(s)\right)\right)\right\}ds\right|^2\\
&\le e^{-2a_it}\left(\int_0^t e^{a_is}\sum_{j=1}^n\left|b_{ij}\left(f_j\left(x_j(s)\right)-f_j\left(y_j(s)\right)\right)\right|ds\right)^2\\
&\le e^{-2a_it}\left(\int_0^t e^{a_is}\sum_{j=1}^n\left|b_{ij}l_j\right|\left|x_j(s)-y_j(s)\right|ds\right)^2\\
&\le e^{-2a_it}\left(\left(\sum_{j=1}^n\left|b_{ij}l_j\right|\sup_{s\in[0,t]}\left|x_j(s)-y_j(s)\right|\right)\int_0^t e^{a_is}ds\right)^2\\
&\le e^{-2a_it}\left(\left(\sum_{j=1}^n\left|b_{ij}l_j\right|\sup_{s\in[0,t]}\left|x_j(s)-y_j(s)\right|\right)\frac{e^{a_it}}{a_i}\right)^2\\
&\le \frac{n}{a_i^2}\sum_{j=1}^n\left|b_{ij}l_j\right|^2\sup_{s\in[0,t]}\left|x_j(s)-y_j(s)\right|^2
\end{aligned}\tag{28}$$

which implies

$$\mathbb{E}\left\{\sup_{t\ge-\tau}Q_1^2\right\}\le\frac{n}{a_i^2}\left(\sum_{j=1}^n\left|b_{ij}l_j\right|^2\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right),$$

$$\begin{aligned}
Q_2^2 &= e^{-2a_it}\left|\int_0^t e^{a_is}\sum_{j=1}^n\left[c_{ij}\left(g_j\left(x_j\left(s-\tau_j(s)\right)\right)\right.\right.\right.\\
&\qquad\left.\left.\left.-g_j\left(y_j\left(s-\tau_j(s)\right)\right)\right)\right]ds\right|^2\\
&\le e^{-2a_it}\left(\int_0^t e^{a_is}\sum_{j=1}^n\left|c_{ij}k_j\right|\left|x_j\left(s-\tau_j(s)\right)\right.\right.\\
&\qquad\left.\left.-y_j\left(s-\tau_j(s)\right)\right|ds\right)^2\\
&\le e^{-2a_it}\left(\sum_{j=1}^n\left|c_{ij}k_j\right|\sup_{v\in[-\tau,t]}\left|x_j(v)-y_j(v)\right|\int_0^t e^{a_is}ds\right)^2\\
&\le \frac{n}{a_i^2}\sum_{j=1}^n\left|c_{ij}k_j\right|^2\sup_{v\in[-\tau,t]}\left|x_j(v)-y_j(v)\right|^2
\end{aligned}\tag{29}$$

which implies

$$\mathbb{E}\left\{\sup_{t\ge-\tau}Q_2^2\right\}\le\frac{n}{a_i^2}\left(\sum_{j=1}^n\left|c_{ij}k_j\right|^2\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right).\tag{30}$$

In addition,

$$\begin{aligned}
Q_3^2 &= e^{-2a_it}\left|\int_0^t e^{a_is}\sum_{j=1}^n\left\{\sigma_j\left(s,x_j(s),x_j\left(s-\tau_j(s)\right)\right)\right.\right.\\
&\qquad\left.\left.-\sigma_j\left(s,y_j(s),y_j\left(s-\tau_j(s)\right)\right)\right\}d\omega(s)\right|^2\\
&\le e^{-2a_it}\int_0^t e^{2a_is}\left|\sum_{j=1}^n\left\{\sigma_j\left(s,x_j(s),x_j\left(s-\tau_j(s)\right)\right)\right.\right.\\
&\qquad\left.\left.-\sigma_j\left(s,y_j(s),y_j\left(s-\tau_j(s)\right)\right)\right\}\right|^2ds\\
&\le ne^{-2a_it}\int_0^t e^{2a_is}\sum_{j=1}^n\left\{\left|\sigma_j\left(s,x_j(s),x_j\left(s-\tau_j(s)\right)\right)\right.\right.\\
&\qquad\left.\left.-\sigma_j\left(s,y_j(s),y_j\left(s-\tau_j(s)\right)\right)\right|^2\right\}ds\\
&\le ne^{-2a_it}\int_0^t e^{2a_is}\sum_{j=1}^n\left\{\left[\xi_j\left|x_j(s)-y_j(s)\right|\right.\right.\\
&\qquad+\zeta_j\left|x_j\left(s-\tau_j(s)\right)\right.\\
&\qquad\left.\left.\left.-y_j\left(s-\tau_j(s)\right)\right|\right]^2\right\}ds\\
&\le 2ne^{-2a_it}\int_0^t e^{2a_is}\sum_{j=1}^n\left\{\xi_j^2\left|x_j(s)-y_j(s)\right|^2\right.\\
&\qquad+\zeta_j^2\left|x_j\left(s-\tau_j(s)\right)\right.\\
&\qquad\left.\left.-y_j\left(s-\tau_j(s)\right)\right|^2\right\}ds\\
&\le 2ne^{-2a_it}\int_0^t e^{2a_is}\sum_{j=1}^n\left\{\xi_j^2\sup_{s\in[-\tau,t]}\left|x_j(s)-y_j(s)\right|^2\right.\\
&\qquad\left.+\zeta_j^2\sup_{s\in[-\tau,t]}\left|x_j(s)-y_j(s)\right|^2\right\}ds\\
&= 2ne^{-2a_it}\sum_{j=1}^n\left\{\left(\xi_j^2+\zeta_j^2\right)\sup_{s\in[-\tau,t]}\left|x_j(s)-y_j(s)\right|^2\right\}\\
&\qquad\times\int_0^t e^{2a_is}ds\\
&\le \frac{n}{a_i}\sum_{j=1}^n\left\{\left(\xi_j^2+\zeta_j^2\right)\sup_{s\in[-\tau,t]}\left|x_j(s)-y_j(s)\right|^2\right\}
\end{aligned}\tag{31}$$

which implies

$$\begin{aligned}
&\mathbb{E}\left\{\sup_{t\ge-\tau}Q_3^2\right\}\\
&\quad\le\frac{n}{a_i}\left(\sum_{j=1}^n\left(\xi_j^2+\zeta_j^2\right)\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right).
\end{aligned}\tag{32}$$

Therefore,

$$\begin{aligned}
&\mathbb{E}\left\{\sup_{t\geq-\tau}\left\{\left|\pi\left(x_i\right)(t)-\pi\left(y_i\right)(t)\right|^2\right\}\right\} \\
&\quad\leq \frac{1}{(1-\varepsilon)(1-\eta)}\mathbb{E}\left\{\sup_{t\geq-\tau}Q_1^2\right\}+\frac{1}{(1-\varepsilon)\eta}\mathbb{E}\left\{\sup_{t\geq-\tau}Q_2^2\right\} \\
&\qquad+\frac{1}{\varepsilon}\mathbb{E}\left\{\sup_{t\geq-\tau}Q_3^2\right\} \\
&\quad\leq \frac{n}{a_i^2(1-\varepsilon)(1-\eta)} \\
&\qquad\times\left(\sum_{j=1}^{n}\left|b_{ij}l_j\right|^2\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right) \\
&\qquad+\frac{n}{a_i^2(1-\varepsilon)\eta}\left(\sum_{j=1}^{n}\left|c_{ij}k_j\right|^2\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right) \\
&\qquad+\frac{n}{\varepsilon a_i}\left(\sum_{j=1}^{n}\left(\xi_j^2+\zeta_j^2\right)\mathbb{E}\left(\sup_{s\in[-\tau,\infty]}\left|x_j(s)-y_j(s)\right|^2\right)\right) \\
&\quad\leq \left\{\begin{array}{l}\dfrac{n}{a_i^2(1-\varepsilon)(1-\eta)}\max\limits_{j\in\mathcal{N}}\left\{b_{ij}^2l_j^2\right\}+\dfrac{n}{a_i\varepsilon}\max\limits_{j\in\mathcal{N}}\left\{\xi_j^2+\zeta_j^2\right\} \\ +\dfrac{n}{a_i^2(1-\varepsilon)\eta}\max\limits_{j\in\mathcal{N}}\left\{c_{ij}^2k_j^2\right\}\end{array}\right\} \\
&\qquad\times\sum_{j=1}^{n}\mathbb{E}\left(\sup_{s\in[-\tau,\infty)}\left|x_j(s)-y_j(s)\right|^2\right) \\
&\quad\triangleq \lambda_i\sum_{j=1}^{n}\mathbb{E}\left(\sup_{s\in[-\tau,\infty)}\left|x_j(s)-y_j(s)\right|^2\right),
\end{aligned} \tag{33}$$

which results in

$$\begin{aligned}
&\sum_{i=1}^{n}\sqrt{\mathbb{E}\left\{\sup_{t\in[-\tau,\infty)}\left|\pi\left(x_i\right)(t)-\pi\left(y_i\right)(t)\right|^2\right\}} \\
&\quad\leq\left(\sum_{i=1}^{n}\sqrt{\lambda_i}\right)\sqrt{\sum_{j=1}^{n}\mathbb{E}\left(\sup_{s\in[-\tau,\infty)}\left|x_j(s)-y_j(s)\right|^2\right)} \\
&\quad\leq\left(\sum_{i=1}^{n}\sqrt{\lambda_i}\right)\sum_{j=1}^{n}\sqrt{\mathbb{E}\left(\sup_{s\in[-\tau,\infty)}\left|x_j(s)-y_j(s)\right|^2\right)}.
\end{aligned} \tag{34}$$

As $\sum_{i=1}^{n}\sqrt{\lambda_i}<1$, π is a contraction mapping and hence there exists a unique fixed point $\overline{\mathbf{x}}(\cdot)$ of π in $\mathscr{H}$ which means $\overline{\mathbf{x}}^T(\cdot)$ is the solution of (1)-(2) and $e^{\alpha t}\mathbb{E}\{\|\overline{\mathbf{x}}^T(\cdot)\|^2\}\to 0$ as $t\to\infty$. This completes the proof. □

Remark 7. The main idea of this proof is based on the fixed point theory rather than Lyapunov method. By using Banach contraction mapping principle with no need for Lyapunov functions, we simultaneously explore the existence and uniqueness as well as global exponential stability of solution to (1)–(14) in mean-square sense, whereas Lyapunov method fails to do this.

Lemma 8. *Assume that conditions (A1)–(A3) hold. If* $\sum_{i=1}^{n}\sqrt{\lambda_i}<1$, *where* $\lambda_i=(3n/a_i^2)\max_{j\in\mathcal{N}}\{b_{ij}^2l_j^2\}+(3n/a_i)\max_{j\in\mathcal{N}}\{\xi_j^2+\zeta_j^2\}+(3n/a_i^2)\max_{j\in\mathcal{N}}\{c_{ij}^2k_j^2\}$, *then* (1) *is globally exponentially stable in mean square.*

Proof. Lemma 8 is the direct corollary of Theorem 6 by choosing $\varepsilon=1/3$ and $\eta=1/2$. □

Remark 9. The obtained algebraic stability criteria are easily checked and do not require even the differentiability of delays, let alone the monotone decreasing behavior of delays which is necessary in some relevant works.

4. Example

Consider the following two-dimensional stochastic cellular neural network with time-varying delays:

$$\begin{aligned}
&dx_i(t) \\
&\quad=\left\{-a_ix_i(t)+\sum_{j=1}^{2}b_{ij}f_j\left(x_j(t)\right)+\sum_{j=1}^{2}c_{ij}g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right\}dt \\
&\qquad+\sum_{j=1}^{2}\sigma_j\left(t,x_j(t),x_j\left(t-\tau_j(t)\right)\right)d\omega(t),\quad t\geq 0
\end{aligned} \tag{35}$$

with the initial conditions $x_1(s)=\cos(s)$, $x_2(s)=\sin(s)$ on $-\tau\leq s\leq 0$, where $a_1=a_2=9$, $b_{ij}=0$, $c_{11}=0$, $c_{12}=-1/7$, $c_{21}=0$, $c_{22}=1/7$, $f_j(s)=g_j(s)=(|s+1|-|s-1|)/4$, $\sigma_j(t,x_j(t),x_j(t-\tau_j(t)))=\arctan(0.5x_j(t-\tau_j(t)))$. It is easy to see that $k_j=1/2$ and $\zeta_j=1/2$. We compute, for $i=1,2$,

$$\lambda_i=\frac{3n}{a_i}\max_{j\in\mathcal{N}}\left\{\zeta_j^2\right\}+\frac{3n}{a_i^2}\max_{j\in\mathcal{N}}\left\{c_{ij}^2k_j^2\right\}=\frac{1}{6}+\frac{1}{54\times 49}, \tag{36}$$

which yields $2\sqrt{\lambda_i}<1$. From Lemma 8, we conclude this two-dimensional stochastic cellular neural network with time-varying delays is mean-square globally exponentially stable.

5. Conclusions

The main contribution of this work is confirming the feasibility of utilizing the fixed point theory to address the stability research of complex CNNs and thereby enlarging the applications of the fixed point theory as well as enriching the stability theory of complex CNNs. Specifically, by Banach contraction mapping principle with no need for Lyapunov functions, we complete the proof of the existence and uniqueness as well as global exponential stability of solution to stochastic delayed neural networks simultaneously, whereas Lyapunov method cannot do this. The derived algebraic stability criteria are novel and easily checked and do not

require the differentiability of delays. As we all know, the fixed point theory has various forms, for example, Krasnosleskii's fixed point theorem. Considering many mathematical models can be transformed into a linear part and other nonlinear parts, our future work is trying to explore the application of Krasnosleskii's fixed point theorem to the stability analysis of complex CNNs.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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