Research Article

Positive Periodic Solutions of a Periodic Discrete Competitive System Subject to Feedback Controls

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Species living in a fluctuating medium and human exploitation activities might result in the duration of continuous changes. Such changes can be well-approximated as feedback controls. In this contribution a periodic discrete competitive system subject to feedback controls is proposed. By using the methods of discrete inequality, fixed point theorem, and analysis techniques, a good understanding of the existence and global asymptotic stability of positive periodic solutions is obtained. Some numerical investigations are provided to verify our analytical results.

1. Introduction

Many mathematical models about bioecology play important roles for researchers to realize the interactions of ecological species. There has been increasing interest in studying the dynamical behaviors such as stability, permanence, and periodicity of competitive systems (see [1–5]). Recently, motivated by an autonomous competitive model of [6] in [7] we introduced a corresponding nonautonomous version

$$\begin{aligned} y_1'(t) &= y_1(t) [a_1(t) - b_1(t) y_1(t) - c_1(t) y_2(t) \\ &\quad - d_1(t) y_1^2(t)], \\ y_2'(t) &= y_2(t) [a_2(t) - b_2(t) y_2(t) - c_2(t) y_1(t) \\ &\quad - d_2(t) y_2^2(t)], \end{aligned} \quad (1)$$

where $y_1(t)$ and $y_2(t)$ can be interpreted as the density of two competing species at time t , respectively. $a_i(t)$ stand for the growth rates of species, $b_i(t)$ and $d_i(t)$ represent the effects of intraspecific competition, and $c_i(t)$ are the rates of inter-specific competition. Furthermore, we took the influence of almost periodic environment and impulsive perturbations into account and established sufficient conditions for the

uniformly asymptotic stability of a unique positive almost periodic solution for the above system (for details see [7]).

Note that ecosystems in the real world are often disturbed by outside continuous forces which can lead to changes in biological coefficients such as survival rates. In the language of control, we call the disturbance functions as feedback control variables. Many good results on this direction are deliberated (see [8–12]) and some similar works on the topic have been done (see [13, 14]). This paper is concerned with a discrete model and is a continuation of the work in [7]; in this contribution we search for certain schemes (such as harvesting procedure) to ensure the system coexists under appropriate conditions. For this reason, we consider a discrete version which corresponds to differential version (i.e., system (1)), meanwhile, replacing abrupt external perturbations (impulses) in [7] by continuous external perturbations (feedback control variables), a nonautonomous discrete controlled system can be described as follows:

$$\begin{aligned} y_1(k+1) &= y_1(k) \exp [a_1(k) - b_1(k) y_1(k) - c_1(k) y_2(k) \\ &\quad - d_1(k) y_1^2(k) - e_1(k) v_1(k)], \\ y_2(k+1) &= y_2(k) \exp [a_2(k) - b_2(k) y_2(k) - c_2(k) y_1(k) \\ &\quad - d_2(k) y_2^2(k) - e_2(k) v_2(k)], \end{aligned}$$

$$\begin{aligned}
\Delta v_1 &= v_1(k+1) - v_1(k) \\
&= h_1(k) - f_1(k) v_1(k) + g_1(k) y_1(k), \\
\Delta v_2 &= v_2(k+1) - v_2(k) \\
&= h_2(k) - f_2(k) v_2(k) + g_2(k) y_2(k), \quad k \in \mathbb{Z}^+.
\end{aligned} \tag{2}$$

Here $y_i(0) > 0$, $v_i(0) > 0$. $\Delta v_i(k)$ are the first-order forward difference operators, $y_i(k)$ stand for the densities of species y_i at the k th generation, $a_i(k)$ represent the natural growth rates of species y_i at the k th generation, $b_i(k)$ and $d_i(k)$ stand for the intraspecific effects of the k th generation of species y_i on own population, and $c_i(k)$ measure the interspecific effects of the k th generation of species y_j on species y_i . The coefficients $\{a_i(k)\}$, $\{b_i(k)\}$, $\{c_i(k)\}$, $\{d_i(k)\}$, $\{e_i(k)\}$, $\{f_i(k)\}$, $\{g_i(k)\}$, and $\{h_i(k)\}$ are T -periodic sequences with $0 < f_i(k) < 1$, $i, j = 1, 2$; $i \neq j$. $\mathbb{Z}^+$ is the set of nonnegative integers.

It is well known that the discrete models governed by difference equations may be more appropriate than the continuous ones when populations have a short life expectancy or nonoverlapping generations. Also, discrete models can provide efficient computational methods of continuous models for numerical simulations. A very important ecological problem associated with the study of multispecies population interaction in a periodic environment is the positive periodic solution which plays the role played by the equilibrium of the autonomous model. In this paper, we will discuss the above discrete system (2) and focus on the existence and stability of positive periodic solution of system (2). To the best of our knowledge, there are few published papers concerning system (2).

The rest structure of this paper is as follows. In the next section, we establish sufficient conditions for the existence of positive periodic solutions. In Section 3, we further discuss the global asymptotic stability of positive periodic solutions. In Section 4, we carry out an example and its numerical simulations to substantiate our theoretical results. To simplify the reading of paper, we give the proofs of lemmas and theorems in appendices.

2. Existence

In this section, we establish sufficient conditions for the existence of positive periodic solutions of system (2). To do this, we first give two preliminary lemmas. For convenience, given a bounded sequence $\{f(k)\}$, let f^U and f^L be defined as $f^U = \sup_{k \in \mathbb{Z}^+} f(k)$, $f^L = \inf_{k \in \mathbb{Z}^+} f(k)$, and

$$\begin{aligned}
M_i &= \frac{\exp(a_i^U - 1)}{b_i^L}, \quad N_i = \frac{h_i^U + g_i^U M_i}{f_i^L}, \\
m_i &= \frac{a_i^L - c_i^U M_j - d_i^U M_i^2 - e_i^U N_i}{b_i^U} \\
&\quad \times \exp(a_i^L - b_i^U M_i - c_i^U M_j - d_i^U M_i^2 - e_i^U N_i), \\
n_i &= \frac{(h_i^L + g_i^L m_i)}{f_i^U}, \quad i \neq j; \quad i, j = 1, 2.
\end{aligned} \tag{3}$$

Lemma 1. Every positive solution $\{y_1(k), y_2(k), v_1(k), v_2(k)\}$ of system (2) satisfies $\limsup_{k \rightarrow +\infty} y_i(k) \leq M_i$ and $\limsup_{k \rightarrow +\infty} v_i(k) \leq N_i$, $i = 1, 2$.

The proof of Lemma 1 is given in Appendix A.

Lemma 2. Assuming that

$$\begin{aligned}
a_1^L - c_1^U M_2 - d_1^U M_1^2 - e_1^U N_1 &> 0, \\
a_2^L - c_2^U M_1 - d_2^U M_2^2 - e_2^U N_2 &> 0
\end{aligned} \tag{4}$$

hold, then every positive solution $\{y_1(k), y_2(k), v_1(k), v_2(k)\}$ of system (2) satisfies $\liminf_{k \rightarrow +\infty} y_i(k) \geq m_i$ and $\liminf_{k \rightarrow +\infty} v_i(k) \geq n_i$, $i = 1, 2$.

The proof of Lemma 2 is given in Appendix B.

Theorem 3. If the inequalities in (4) hold, then system (2) has at least one strictly positive T -periodic solution.

The proof of Theorem 3 is given in Appendix C.

3. Global Asymptotic Stability

In this section, we further investigate the stability of positive periodic solutions of system (2). Denote

$$\begin{aligned}
\rho_1 &= \max \left\{ \left| 1 - b_1^L m_1 - 2d_1^L m_1^2 \right|, \left| 1 - b_1^U M_1 - 2d_1^U M_1^2 \right| \right\} \\
&\quad + c_1^U M_2 + e_1^U, \\
\rho_2 &= \max \left\{ \left| 1 - b_2^L m_2 - 2d_2^L m_2^2 \right|, \left| 1 - b_2^U M_2 - 2d_2^U M_2^2 \right| \right\} \\
&\quad + c_2^U M_1 + e_2^U, \\
\rho_3 &= 1 - f_1^L + g_1^U M_1, \\
\rho_4 &= 1 - f_2^L + g_2^U M_2.
\end{aligned} \tag{5}$$

Theorem 4. In addition to (4), further suppose that $\rho_i < 1$, $i = 1, 2, 3, 4$. Then the positive periodic solution of system (2) is globally asymptotically stable.

The proof of Theorem 4 is given in Appendix D.

4. An Example

In this section we give a numerical example and its corresponding simulations.

Example 1. Consider a discrete competitive system with feedback controls

$$\begin{aligned}
 y_1(k+1) &= y_1(k) \exp [0.98 + 0.003 \sin(k\pi) \\
 &\quad - (0.85 - 0.002 \cos(k\pi)) y_1(k) \\
 &\quad - (0.002 + 0.0001 \sin(k\pi)) y_2(k) \\
 &\quad - (0.004 - 0.0002 \cos(k\pi)) y_1^2(k) \\
 &\quad - (0.003 + 0.0001 \sin(k\pi)) v_1(k)], \\
 y_2(k+1) &= y_2(k) \exp [1.00 - 0.002 \sin(k\pi) \\
 &\quad - (0.92 + 0.001 \cos(k\pi)) y_2(k) \\
 &\quad - (0.003 + 0.0001 \cos(k\pi)) y_1(k) \\
 &\quad - (0.005 + 0.0003 \sin(k\pi)) y_2^2(k) \\
 &\quad - (0.002 + 0.0002 \sin(k\pi)) v_2(k)], \\
 \Delta v_1(k) &= 0.005 + 0.0002 \sin(k\pi) \\
 &\quad - (0.95 + 0.003 \cos(k\pi)) v_1(k) \\
 &\quad + (0.006 + 0.0001 \sin(k\pi)) y_1(k), \\
 \Delta v_2(k) &= 0.004 + 0.0001 \sin(k\pi) \\
 &\quad - (0.98 + 0.002 \sin(k\pi)) v_2(k) \\
 &\quad + (0.003 + 0.0002 \cos(k\pi)) y_2(k).
 \end{aligned} \tag{6}$$

A computation shows that

$$\begin{aligned}
 a_1^L &= 0.977, & a_1^U &= 0.983, & b_1^L &= 0.848, \\
 b_1^U &= 0.852, & c_1^L &= 0.0019, & c_1^U &= 0.0021, \\
 d_1^L &= 0.0038, & d_1^U &= 0.0042, & e_1^L &= 0.0029, \\
 e_1^U &= 0.0031, & f_1^L &= 0.947, & f_1^U &= 0.953, \\
 g_1^L &= 0.0059, & g_1^U &= 0.0061, & h_1^L &= 0.0048, \\
 h_1^U &= 0.0052, & a_2^L &= 0.998, & a_2^U &= 1.002, \\
 b_2^L &= 0.919, & b_2^U &= 0.921, & c_2^L &= 0.0029, \\
 c_2^U &= 0.0031, & d_2^L &= 0.0047, & d_2^U &= 0.0053, \\
 e_2^L &= 0.0018, & e_2^U &= 0.0022, & f_2^L &= 0.978, \\
 f_2^U &= 0.982, & g_2^L &= 0.0028, & g_2^U &= 0.0032, \\
 h_2^L &= 0.0039, & h_2^U &= 0.0041.
 \end{aligned} \tag{7}$$

Furthermore, it follows from (3) that

$$\begin{aligned}
 M_1 &\approx 1.1594, & M_2 &\approx 1.0903, \\
 N_1 &\approx 0.0130, & N_2 &\approx 0.0078, \\
 m_1 &\approx 1.1162, & m_2 &\approx 1.0557, \\
 n_1 &\approx 0.0119, & n_2 &\approx 0.0070, \\
 a_1^L - c_1^U M_2 - d_1^U M_1^2 - e_1^U N_1 &\approx 0.9690 > 0, \\
 a_2^L - c_2^U M_1 - d_2^U M_2^2 - e_2^U N_2 &\approx 0.9881 > 0.
 \end{aligned} \tag{8}$$

Obviously, the assumptions in (4) are satisfied, and moreover, one has

$$\begin{aligned}
 \rho_1 &= \max \left\{ \left| 1 - b_1^L m_1 - 2d_1^L m_1^2 \right|, \left| 1 - b_1^U M_1 - 2d_1^U M_1^2 \right| \right\} \\
 &\quad + c_1^U M_2 + e_1^U \approx 0.0494 < 1, \\
 \rho_2 &= \max \left\{ \left| 1 - b_2^L m_2 - 2d_2^L m_2^2 \right|, \left| 1 - b_2^U M_2 - 2d_2^U M_2^2 \right| \right\} \\
 &\quad + c_2^U M_1 + e_2^U \approx 0.0251 < 1, \\
 \rho_3 &= 1 - f_1^L + g_1^U M_1 \approx 0.0601 < 1, \\
 \rho_4 &= 1 - f_2^L + g_2^U M_2 \approx 0.0255 < 1,
 \end{aligned} \tag{9}$$

so the assumptions of Theorem 4 are also satisfied. Thus, there exists a globally asymptotically stable positive periodic solution of system (6). Figure 1 shows that system (6) exists a positive periodic solution $(y_1^*(k), y_2^*(k), v_1^*(k), v_2^*(k))$, and the two-dimensional phase portrait of periodic system (6) is displayed in Figure 2. From Figure 3, we can see that any positive solution $(y_1(k), y_2(k), v_1(k), v_2(k))$ tends to the periodic solution $(y_1^*(k), y_2^*(k), v_1^*(k), v_2^*(k))$, and the two-dimensional phase portrait reflects the fact in Figure 4.

Appendices

A. Proof of Lemma 1

We consider the following Cases I and II to show that

$$\limsup_{k \rightarrow +\infty} y_i(k) \leq M_i, \quad i = 1, 2. \tag{A.1}$$

Case I. Suppose that there is a $k_0 \in \mathbb{Z}^+$ such that $y_1(k_0) \leq y_1(k_0 + 1)$, it follows from the first equation of system (2) that

$$\begin{aligned}
 &y_1(k_0 + 1) \\
 &= y_1(k_0) \exp [a_1(k_0) - b_1(k_0) y_1(k_0) - c_1(k_0) y_2(k_0) \\
 &\quad - d_1(k_0) y_1^2(k_0) - e_1(k_0) v_1(k_0)] \\
 &\leq y_1(k_0) \exp [a_1^U - b_1^L y_1(k_0)] \\
 &\leq \frac{\exp(a_1^U - 1)}{b_1^L} = M_1.
 \end{aligned} \tag{A.2}$$

FIGURE 1: Positive periodic solution of system (6). (a)–(d) Time-series of $y_1^*(k)$, $y_2^*(k)$, $v_1^*(k)$, and $v_2^*(k)$ with initial values $y_1^*(0) = 1.1560$, $y_2^*(0) = 1.0720$, $v_1^*(0) = 0.0128$, and $v_2^*(0) = 0.0078$ for $k \in [0, 100]$, respectively.

Here we use the fact that $\max_{x \in \mathbb{R}^+} [x \exp(a - bx)] = [\exp(a - 1)]/b$ for $a, b > 0$ and $\mathbb{R}^+$ is the set of all positive real numbers. Thus $y_1(k_0) \leq y_1(k_0 + 1) \leq M_1$.

One claims that $y_1(k) \leq M_1$ for all $k \geq k_0$. Otherwise, if there is a $l_0 > k_0$ such that $y_1(l_0) > M_1$, then $l_0 \geq k_0 + 2$. Set

$$\bar{l}_0 = \min \{l_0 : l_0 \geq k_0 + 2, y_1(l_0) > M_1\}; \quad (\text{A.3})$$

namely, $y_1(\bar{l}_0) > M_1$ and $\bar{l}_0 \geq k_0 + 2$; then $y_1(\bar{l}_0) > M_1 \geq y_1(\bar{l}_0 - 1)$. From the above discussion one obtains that $y_1(\bar{l}_0) \leq M_1$, which is a contradiction. Hence, $y_1(k) \leq M_1$ for all $k \geq k_0$; then $\limsup_{k \rightarrow +\infty} y_1(k) \leq M_1$. *Case I* is complete.

Case II. Assume that $y_1(k) > y_1(k + 1)$ for $k \in \mathbb{Z}^+$. $\lim_{k \rightarrow +\infty} y_1(k)$ especially exists, denoted by $\bar{y}_1$. One claims

that $\bar{y}_1 \leq a_1^U/b_1^L$. Otherwise, if $\bar{y}_1 > a_1^U/b_1^L$, then it follows from the first equation of system (2) that

$$\lim_{k \rightarrow +\infty} \begin{bmatrix} a_1(k) - b_1(k) y_1(k) - c_1(k) y_2(k) \\ -d_1(k) y_1^2(k) - e_1(k) v_1(k) \end{bmatrix} = 0, \quad (\text{A.4})$$

which is a contradiction since

$$\lim_{k \rightarrow +\infty} \begin{bmatrix} a_1(k) - b_1(k) y_1(k) - c_1(k) y_2(k) \\ -d_1(k) y_1^2(k) - e_1(k) v_1(k) \end{bmatrix}$$

FIGURE 2: Two-dimensional phase portrait. (a) Phase portrait of $y_1^*(k)$ and $y_2^*(k)$ with initial values $y_1^*(0) = 1.1560$ and $y_2^*(0) = 1.0720$ for $k \in [0, 100]$, respectively. (b) Phase portrait of $v_1^*(k)$ and $v_2^*(k)$ with initial values $v_1^*(0) = 0.0128$ and $v_2^*(0) = 0.0078$ for $k \in [0, 100]$, respectively.

$$\begin{aligned} &\leq \lim_{k \rightarrow +\infty} [a_1(k) - b_1(k) y_1(k)] \\ &\leq a_1^U - b_1^L \bar{y}_1 < 0. \end{aligned} \quad (\text{A.5})$$

This proves the claim. By $\bar{y}_1 \leq a_1^U/b_1^L$ and the fact that $\min_{x \in \mathbb{R}^+} [\exp(x-1)/x] = 1$, one can show that

$$\bar{y}_1 \leq \frac{a_1^U}{b_1^L} \leq \left(\frac{a_1^U}{b_1^L} \right) \frac{\exp(a_1^U - 1)}{a_1^U} = M_1. \quad (\text{A.6})$$

Hence $\limsup_{k \rightarrow +\infty} y_1(k) \leq M_1$. We can prove that $\limsup_{k \rightarrow +\infty} y_2(k) \leq M_2$ in the same way.

In the following, we prove that $\limsup_{k \rightarrow +\infty} v_1(k) \leq N_1$. For any $\varepsilon > 0$, there exists an integer $m_0 \in \mathbb{Z}^+$ such that $y_1(k) \leq M_1 + \varepsilon$ for all $k \geq m_0$. We have from the third equation of system (2) that

$$\begin{aligned} v_1(k) &= \prod_{i=0}^{k-1} (1 - f_1(i)) \left[v_1(0) + \sum_{i=0}^{k-1} \frac{h_1(i) + g_1(i) y_1(i)}{\prod_{j=0}^i (1 - f_1(j))} \right] \\ &\leq (1 - f_1^L)^k (v_1(0) + \mu_1) \\ &\quad + [h_1^U + g_1^U (M_1 + \varepsilon)] \sum_{i=m_0}^{k-1} \prod_{j=i+1}^{k-1} (1 - f_1(j)) \\ &\leq (1 - f_1^L)^k (v_1(0) + \mu_1) \\ &\quad + [h_1^U + g_1^U (M_1 + \varepsilon)] \sum_{i=m_0}^{k-1} (1 - f_1^L)^{k-i-1}, \end{aligned} \quad (\text{A.7})$$

where $\mu_1 = \sum_{i=0}^{m_0-1} ((h_1(i) + g_1(i) y_1(i)) / (\prod_{j=0}^i (1 - f_1(j))))$. Since $0 < f_1^L < 1$, we can choose a constant $\theta > 0$ such that $1 - f_1^L = e^{-\theta}$. By using the Stolz theorem, one has, as $k \rightarrow +\infty$,

$$\sum_{i=m_0}^{k-1} (1 - f_1^L)^{k-i-1} = \frac{\sum_{i=m_0}^{k-1} e^{\theta(i+1)}}{e^{\theta k}} \rightarrow \frac{1}{1 - e^{-\theta}} = \frac{1}{f_1^L}. \quad (\text{A.8})$$

So $\limsup_{k \rightarrow +\infty} v_1(k) \leq (h_1^U + g_1^U (M_1 + \varepsilon)) / f_1^L$. Since ε is arbitrary, we obtain that $\limsup_{k \rightarrow +\infty} v_1(k) \leq N_1$.

Repeating a similar argument, we can prove that $\limsup_{k \rightarrow +\infty} v_2(k) \leq N_2$. The proof of Lemma 1 is complete.

B. Proof of Lemma 2

In view of Lemma 1, we obtain that there exist $\varepsilon > 0$ and $k^* \in \mathbb{Z}^+$ such that, for $k \geq k^*$,

$$y_i(k) \leq M_i + \varepsilon, \quad v_i(k) \leq N_i + \varepsilon, \quad i = 1, 2. \quad (\text{B.1})$$

The first inequality of (4) implies that

$$a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon) > 0. \quad (\text{B.2})$$

Similar to Lemma 1, we consider the following Cases I and II to show that

$$\liminf_{k \rightarrow +\infty} y_1(k) \geq m_1. \quad (\text{B.3})$$

FIGURE 3: Global asymptotic stability. (a)–(d) Time-series of $y_1^*(k)$, $y_2^*(k)$, $v_1^*(k)$, and $v_2^*(k)$ with initial values $y_1^*(0) = 1.1560$, $y_2^*(0) = 1.0720$, $v_1^*(0) = 0.0128$, and $v_2^*(0) = 0.0078$ and $y_1(k)$, $y_2(k)$, $v_1(k)$, and $v_2(k)$ with initial values $y_1(0) = 1.1300$, $y_2(0) = 1.0850$, $v_1(0) = 0.0123$, and $v_2(0) = 0.0071$ for $k \in [0, 100]$, respectively.

Case I. Suppose that there is a nonnegative integer $k_0 \geq k^*$ such that $y_1(k_0 + 1) \leq y_1(k_0)$; we have from the first equation of system (2) that

$$\begin{aligned}
 y_1(k_0 + 1) &= y_1(k_0) \exp [a_1(k_0) - b_1(k_0)y_1(k_0) \\
 &\quad - c_1(k_0)y_2(k_0) - d_1(k_0)y_1^2(k_0) \\
 &\quad - e_1(k_0)v_1(k_0)] \\
 &\geq y_1(k_0) \exp [a_1^L - b_1^U y_1(k_0) - c_1^U (M_2 + \varepsilon) \\
 &\quad - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)],
 \end{aligned} \tag{B.4}$$

and hence

$$\begin{aligned}
 a_1^L - b_1^U y_1(k_0) - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon) \\
 \leq 0.
 \end{aligned} \tag{B.5}$$

Note that (B.2) and ε is sufficiently small; one has from (B.5) that

$$\begin{aligned}
 y_1(k_0) &\geq \frac{a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)}{b_1^U} \\
 &> 0.
 \end{aligned} \tag{B.6}$$

FIGURE 4: Two-dimensional phase portrait. (a) Phase portrait of $y_1^*(k)$ and $y_2^*(k)$ with initial values $y_1^*(0) = 1.1560$ and $y_2^*(0) = 1.0720$ and $y_1(k)$ and $y_2(k)$ with initial values $y_1(0) = 1.1300$ and $y_2(0) = 1.0850$ for $k \in [0, 100]$, respectively. (b) Phase portrait of $v_1^*(k)$ and $v_2^*(k)$ with initial values $v_1^*(0) = 0.0128$ and $v_2^*(0) = 0.0078$ and $v_1(k)$ and $v_2(k)$ with initial values $v_1(0) = 0.0123$ and $v_2(0) = 0.0071$ for $k \in [0, 100]$, respectively.

Furthermore, it follows from (B.4) and (B.6) that

$$\begin{aligned}
 y_1(k_0 + 1) &= y_1(k_0) \exp \left[a_1(k_0) - b_1(k_0) y_1(k_0) - c_1(k_0) y_2(k_0) \right. \\
 &\quad \left. - d_1(k_0) y_1^2(k_0) - e_1(k_0) v_1(k_0) \right] \\
 &\geq \frac{a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)}{b_1^U} \\
 &\quad \times \exp \left[a_1^L - b_1^U (M_1 + \varepsilon) - c_1^U (M_2 + \varepsilon) \right. \\
 &\quad \left. - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon) \right] \\
 &> 0.
 \end{aligned} \tag{B.7}$$

Hence $y_1(k_0) \geq y_1(k_0 + 1) \geq y_1^\varepsilon$, where

$$\begin{aligned}
 y_1^\varepsilon &= \frac{a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)}{b_1^U} \\
 &\quad \times \exp \left[a_1^L - b_1^U (M_1 + \varepsilon) - c_1^U (M_2 + \varepsilon) \right. \\
 &\quad \left. - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon) \right].
 \end{aligned} \tag{B.8}$$

Now, we claim that $y_1(k) \geq y_1^\varepsilon$ for all $k \geq k_0$. Otherwise, if there exists a $q_0 > k_0$ such that $y_1(q_0) < y_1^\varepsilon$, then $q_0 \geq k_0 + 2$. Set

$$\bar{q}_0 = \min \{q_0 : q_0 \geq k_0 + 2, y_1(q_0) < y_1^\varepsilon\}; \tag{B.9}$$

then $y_1(\bar{q}_0 - 1) > y_1(\bar{q}_0)$. The above argument produces that $y_1(\bar{q}_0) \geq y_1^\varepsilon$, which is a contradiction. Thus $y_1(k) \geq y_1^\varepsilon$ for all $k \geq k_0$. Since ε is sufficiently small, this reduces to

$$\begin{aligned}
 y_1(k) &\geq \frac{a_1^L - c_1^U M_2 - d_1^U M_1^2 - e_1^U N_1}{b_1^U} \\
 &\quad \times \exp \left[a_1^L - b_1^U M_1 - c_1^U M_2 - d_1^U M_1^2 - e_1^U N_1 \right] = m_1.
 \end{aligned} \tag{B.10}$$

Hence, $\liminf_{k \rightarrow +\infty} y_1(k) \geq m_1$. This verifies *Case I*.

Case II. We assume that $y_1(k+1) \geq y_1(k)$ for all large $k \geq k^*$. In this case, $\lim_{k \rightarrow +\infty} y_1(k)$ exists, denoted by y_{1*} . One claims that $y_{1*} \geq (a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)) / b_1^U$; otherwise, assume that $y_{1*} < (a_1^L - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon)) / b_1^U$. It follows from the first equation of system (2) that

$$\begin{aligned}
 \lim_{k \rightarrow +\infty} \left[a_1(k) - b_1(k) y_1(k) - c_1(k) y_2(k) \right. \\
 \left. - d_1(k) y_1^2(k) - e_1(k) v_1(k) \right] &= 0;
 \end{aligned} \tag{B.11}$$

however,

$$\begin{aligned}
 \lim_{k \rightarrow +\infty} \left[a_1(k) - b_1(k) y_1(k) - c_1(k) y_2(k) \right. \\
 \left. - d_1(k) y_1^2(k) - e_1(k) v_1(k) \right] \\
 \geq \lim_{k \rightarrow +\infty} \left[a_1(k) - b_1(k) y_1(k) - c_1(k) (M_2 + \varepsilon) \right. \\
 \left. - d_1(k) (M_1 + \varepsilon)^2 - e_1(k) (N_1 + \varepsilon) \right]
 \end{aligned}$$

$$\begin{aligned} &\geq a_1^L - b_1^U y_{1*} - c_1^U (M_2 + \varepsilon) - d_1^U (M_1 + \varepsilon)^2 - e_1^U (N_1 + \varepsilon) \\ &> 0, \end{aligned} \quad (\text{B.12})$$

which is a contradiction. This fact implies that $\liminf_{k \rightarrow +\infty} y_1(k) \geq y_1^\varepsilon$. Since ε can be sufficiently small, one yields $\liminf_{k \rightarrow +\infty} y_1(k) \geq m_1$, where m_1 is defined as (B.10).

Similarly, one can derive that $\liminf_{k \rightarrow +\infty} y_2(k) \geq m_2$.

In the following, we prove that $\liminf_{k \rightarrow +\infty} v_1(k) \geq n_1$. For any $\varepsilon > 0$, there exists an integer $p_0 \in \mathbb{Z}^+$ such that $y_1(k) \geq m_1 - \varepsilon$ for $k \geq p_0$. We have from the third equation of system (2) that

$$\begin{aligned} v_1(k) &= \prod_{i=0}^{k-1} (1 - f_1(i)) \left[v_1(0) + \sum_{i=0}^{k-1} \frac{h_1(i) + g_1(i) y_1(i)}{\prod_{j=0}^i (1 - f_1(j))} \right] \\ &\geq (1 - f_1^U)^k (v_1(0) + \mu_2) \\ &\quad + [h_1^L + g_1^L (m_1 - \varepsilon)] \sum_{i=p_0}^{k-1} \prod_{j=i+1}^{k-1} (1 - f_1(j)) \\ &\geq (1 - f_1^U)^k (v_1(0) + \mu_2) \\ &\quad + [h_1^L + g_1^L (m_1 - \varepsilon)] \sum_{i=p_0}^{k-1} (1 - f_1^U)^{k-i-1}, \end{aligned} \quad (\text{B.13})$$

where $\mu_2 = \sum_{i=0}^{p_0-1} ((h_1(i) + g_1(i) y_1(i)) / (\prod_{j=0}^i (1 - f_1(j))))$. Since $0 < f_1^U < 1$, we can choose a constant $\eta > 0$ such that $1 - f_1^U = e^{-\eta}$; then by Stolz's theorem one yields

$$\sum_{i=p_0}^{k-1} (1 - f_1^U)^{k-i-1} = \frac{\sum_{i=p_0}^{k-1} e^{\eta(i+1)}}{e^{\eta k}} \rightarrow \frac{1}{1 - e^{-\eta}} = \frac{1}{f_1^U} \quad (\text{B.14})$$

as $k \rightarrow +\infty$.

Then $\liminf_{k \rightarrow +\infty} v_1(k) \geq (h_1^L + g_1^L (m_1 - \varepsilon)) / f_1^U$. Since ε can be sufficiently small, we have

$$\liminf_{k \rightarrow +\infty} v_1(k) \geq n_1 = \frac{(h_1^L + g_1^L m_1)}{f_1^U}. \quad (\text{B.15})$$

Analogously,

$$\liminf_{k \rightarrow +\infty} v_2(k) \geq n_2 = \frac{(h_2^L + g_2^L m_2)}{f_2^U}. \quad (\text{B.16})$$

The proof of Lemma 2 is complete.

C. Proof of Theorem 3

It follows from Lemmas 1 and 2 that

$$\Theta = [m_1, M_1] \times [m_2, M_2] \times [n_1, N_1] \times [n_2, N_2] \quad (\text{C.1})$$

is an invariant set of system (2). Define the continuous mapping Π on Θ by

$$\begin{aligned} &\Pi \{y_1^*(0), y_2^*(0), v_1^*(0), v_2^*(0)\} \\ &= \Pi \{y_1^*(T), y_2^*(T), v_1^*(T), v_2^*(T)\}, \end{aligned} \quad (\text{C.2})$$

for $\{y_1^*(0), y_2^*(0), v_1^*(0), v_2^*(0)\} \in \Theta$. Clearly, Π depends continuously on $\{y_1^*(0), y_2^*(0), v_1^*(0), v_2^*(0)\}$; then Π is continuous and maps the compact set Θ into itself. Hence, Π has a fixed point $\{\tilde{y}_1, \tilde{y}_2, \tilde{v}_1, \tilde{v}_2\}$. It is easy to see that the solution $\{y_1^*(k), y_2^*(k), v_1^*(k), v_2^*(k)\}$ which passes through $\{\tilde{y}_1, \tilde{y}_2, \tilde{v}_1, \tilde{v}_2\}$ is a T -periodic solution of system (2). The proof of Theorem 3 is complete.

D. Proof of Theorem 4

Let $\{y_1^*(k), y_2^*(k), v_1^*(k), v_2^*(k)\}$ be positive periodic solution and let $\{y_1(k), y_2(k), v_1(k), v_2(k)\}$ be any positive solution of system (2), respectively. Let us make the change of variables as follows:

$$\begin{aligned} x_1(k) &= \ln y_1(k) - \ln y_1^*(k), \\ x_2(k) &= \ln y_2(k) - \ln y_2^*(k), \end{aligned} \quad (\text{D.1})$$

$$u_1(k) = v_1(k) - v_1^*(k), \quad u_2(k) = v_2(k) - v_2^*(k).$$

By the mean-value theorem, we have from the first equation of system (2) that

$$\begin{aligned} x_1(k+1) &= \ln y_1(k+1) - \ln y_1^*(k+1) \\ &= \ln y_1(k) - \ln y_1^*(k) - b_1(k) (y_1(k) - y_1^*(k)) \\ &\quad - c_1(k) (y_2(k) - y_2^*(k)) \\ &\quad - d_1(k) (y_1^2(k) - y_1^{*2}(k)) \\ &\quad - e_1(k) (v_1(k) - v_1^*(k)) \\ &= \ln y_1(k) - \ln y_1^*(k) - b_1(k) \exp(\theta_1(k)) \\ &\quad \times (\ln y_1(k) - \ln y_1^*(k)) - c_1(k) \exp(\theta_2(k)) \\ &\quad \cdot (\ln y_2(k) - \ln y_2^*(k)) - 2d_1(k) \exp(2\theta_3(k)) \\ &\quad \times (\ln y_1(k) - \ln y_1^*(k)) \\ &\quad - e_1(k) (v_1(k) - v_1^*(k)) \\ &= [1 - b_1(k) \exp(\theta_1(k)) \\ &\quad - 2d_1(k) \exp(2\theta_3(k))] x_1(k) \\ &\quad - c_1(k) \exp(\theta_2(k)) x_2(k) - e_1(k) u_1(k). \end{aligned} \quad (\text{D.2})$$

Similarly, we can arrive at the following desired results:

$$\begin{aligned} x_2(k+1) &= [1 - b_2(k) \exp(\theta_4(k)) \\ &\quad - 2d_2(k) \exp(2\theta_6(k))] x_2(k) \\ &\quad - c_2(k) \exp(\theta_5(k)) x_1(k) - e_2(k) u_2(k), \\ u_1(k+1) &= (1 - f_1(k)) u_1(k) + g_1(k) \exp(\theta_7(k)) x_1(k), \\ u_2(k+1) &= (1 - f_2(k)) u_2(k) + g_2(k) \exp(\theta_8(k)) x_2(k), \end{aligned} \quad (D.3)$$

where $\theta_1(k)$, $\theta_3(k)$, $\theta_5(k)$, and $\theta_7(k)$ lie between $\ln y_1^*(k)$ and $\ln y_1(k)$ and $\theta_2(k)$, $\theta_4(k)$, $\theta_6(k)$, and $\theta_8(k)$ lie between $\ln y_2^*(k)$ and $\ln y_2(k)$.

By the inequalities in (5), we will now select a constant ε small enough such that

$$\begin{aligned} \rho_1^\varepsilon &= \max \left\{ \left| 1 - b_1^L(m_1 - \varepsilon) - 2d_1^L(m_1 - \varepsilon)^2 \right|, \right. \\ &\quad \left. \left| 1 - b_1^U(M_1 + \varepsilon) - 2d_1^U(M_1 + \varepsilon)^2 \right| \right\} \\ &\quad + c_1^U(M_2 + \varepsilon) + e_1^U < 1, \\ \rho_2^\varepsilon &= \max \left\{ \left| 1 - b_2^L(m_2 - \varepsilon) - 2d_2^L(m_2 - \varepsilon)^2 \right|, \right. \\ &\quad \left. \left| 1 - b_2^U(M_2 + \varepsilon) - 2d_2^U(M_2 + \varepsilon)^2 \right| \right\} \\ &\quad + c_2^U(M_1 + \varepsilon) + e_2^U < 1, \\ \rho_3^\varepsilon &= 1 - f_1^L + g_1^U(M_1 + \varepsilon) < 1, \\ \rho_4^\varepsilon &= 1 - f_2^L + g_2^U(M_2 + \varepsilon) < 1. \end{aligned} \quad (D.4)$$

From Lemmas 1 and 2, we know that there exists a $\hat{k} \in \mathbb{Z}^+$ such that, for $k \geq \hat{k}$,

$$\begin{aligned} 0 &< m_1 - \varepsilon \leq y_1^*(k) \leq M_1 + \varepsilon, \\ 0 &< m_1 - \varepsilon \leq y_1(k) \leq M_1 + \varepsilon, \\ 0 &< m_2 - \varepsilon \leq y_2^*(k) \leq M_2 + \varepsilon, \\ 0 &< m_2 - \varepsilon \leq y_2(k) \leq M_2 + \varepsilon, \\ 0 &< n_1 - \varepsilon \leq v_1^*(k) \leq N_1 + \varepsilon, \\ 0 &< n_1 - \varepsilon \leq v_1(k) \leq N_1 + \varepsilon, \\ 0 &< n_2 - \varepsilon \leq v_2^*(k) \leq N_2 + \varepsilon, \\ 0 &< n_2 - \varepsilon \leq v_2(k) \leq N_2 + \varepsilon. \end{aligned} \quad (D.5)$$

It is easy to see that $\exp(\theta_1(k))$, $\exp(\theta_3(k))$, $\exp(\theta_5(k))$, and $\exp(\theta_7(k))$ are between $y_1^*(k)$ and $y_1(k)$. Meanwhile, $\exp(\theta_2(k))$, $\exp(\theta_4(k))$, $\exp(\theta_6(k))$, and $\exp(\theta_8(k))$ are

between $y_2^*(k)$ and $y_2(k)$. It follows from the equation of (D.2) that

$$\begin{aligned} |x_1(k+1)| &= |1 - b_1(k) \exp(\theta_1(k)) - 2d_1(k) \exp(2\theta_3(k))| \\ &\quad \cdot |x_1(k)| + |c_1(k) \exp(\theta_2(k))| \cdot |x_2(k)| \\ &\quad + |e_1(k)| \cdot |u_1(k)| \\ &\leq \max \left\{ \left| 1 - b_1^L(m_1 - \varepsilon) - 2d_1^L(m_1 - \varepsilon)^2 \right|, \right. \\ &\quad \left. \left| 1 - b_1^U(M_1 + \varepsilon) - 2d_1^U(M_1 + \varepsilon)^2 \right| \right\} \\ &\quad \times |x_1(k)| + c_1^U(M_2 + \varepsilon) |x_2(k)| + e_1^U |u_1(k)| \\ &\leq \rho_1^\varepsilon \max \{|x_1(k)|, |x_2(k)|, |u_1(k)|, |u_2(k)|\}. \end{aligned} \quad (D.6)$$

Analogously,

$$\begin{aligned} |x_2(k+1)| &\leq \rho_2^\varepsilon \max \{|x_1(k)|, |x_2(k)|, |u_1(k)|, |u_2(k)|\}, \\ |u_1(k+1)| &\leq \rho_3^\varepsilon \max \{|x_1(k)|, |x_2(k)|, |u_1(k)|, |u_2(k)|\}, \\ |u_2(k+1)| &\leq \rho_4^\varepsilon \max \{|x_1(k)|, |x_2(k)|, |u_1(k)|, |u_2(k)|\}. \end{aligned} \quad (D.7)$$

Let $\rho^\varepsilon = \max\{\rho_1^\varepsilon, \rho_2^\varepsilon, \rho_3^\varepsilon, \rho_4^\varepsilon\}$; then $\rho^\varepsilon < 1$. Hence, when $k \geq \hat{k}$ we have

$$\begin{aligned} &\max \{|x_1(k+1)|, |x_2(k+1)|, |u_1(k+1)|, |u_2(k+1)|\} \\ &\leq \rho^\varepsilon \max \{|x_1(k)|, |x_2(k)|, |u_1(k)|, |u_2(k)|\} \\ &\leq (\rho^\varepsilon)^{k-\hat{k}} \max \{|x_1(\hat{k})|, |x_2(\hat{k})|, |u_1(\hat{k})|, |u_2(\hat{k})|\}, \end{aligned} \quad (D.8)$$

which implies

$$\begin{aligned} \lim_{k \rightarrow +\infty} |y_1(k) - y_1^*(k)| &= 0, & \lim_{k \rightarrow +\infty} |y_2(k) - y_2^*(k)| &= 0, \\ \lim_{k \rightarrow +\infty} |v_1(k) - v_1^*(k)| &= 0, & \lim_{k \rightarrow +\infty} |v_2(k) - v_2^*(k)| &= 0. \end{aligned} \quad (D.9)$$

So the positive periodic solution $\{y_1^*(k), y_2^*(k), v_1^*(k), v_2^*(k)\}$ of system (2) is globally asymptotically stable. The proof of Theorem 4 is complete.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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