Hindawi
Journal of Applied Mathematics
Volume 2018, Article ID 9701268, 14 pages
https://doi.org/10.1155/2018/9701268

Research Article

Study of Two-Sided Similarity Methods Using a Radiation "Switch on" Imploding Shock in a Magnetic Field

J. R. A. J. NiCastro

Systems Science Research Group, P.O. Box 4051 Walnut Creek CA., 94596, USA

Correspondence should be addressed to J. R. A. J. NiCastro; jrajnic@earthlink.net

Received 4 December 2017; Revised 29 March 2018; Accepted 28 May 2018; Published 24 June 2018

Academic Editor: Syed Abdul Mohiuddine

This paper explores aspects of two-sided similarity modeling using cylindrical geometry for radiating shock waves embedded in a medium with a magnetic field. Two-sided similarity solution techniques may be used to link states influenced by long range near instantaneous fields that continually modify the pre- and postshock zones. Emergent radiation scaling relations are immediately available from consistent homologies. For both small angle and large angle measurements, an approximate analytic technique in the vicinity of luminous fronts together with the high symmetry implications delineated in Lemma provides direct access to the homology parameters. The parameters obtained using this process can augment the constraint relations and contribute to establishing relevant similarity homologies.

1. Introduction

In nonlinear materials, strong compression or shock phenomenon may result from an initially sufficiently strong impulse, mechanical or otherwise, propagating into the fluid. Similarity techniques have been used to study these problems involving radiation effects. Evidently there has been an interest in radiation driven cylindrical shocks in plasma systems. [1] Vishwakarma and Pandey have studied the propagation of shock waves under the action of monochromatic radiation in nonideal gas using a van deWaals model. [2] Nath and Sahu have studied propagation in an axisymmetric system involving a magnetic field and monochromatic radiation. Further studies have found solutions to spherical plasma systems. [3] NiCastro, in an early work using similarity analysis, studied the possibility of initiating nuclear effects in an electromagnetically driven quasispherical convergent geometry dominated by thermal radiation. [4] Singh has found a solution for the self-similar flow behind a radiative spherical shock wave. [5] Singh and Vishwakarma have studied propagation of spherical waves with radiation heat flux in a dusty gas, a mixture of gas, and small particles and include the effect of an exponential density gradient. [6] D. Mihalas and B. W. Mihalas have prepared a summary bibliography of work on radiation hydrodynamics that spans a history of work and deals with a diversity of problems. [7] Nath, Dutta, and Pathak have obtained an exact solution for propagating shocks including magnetic field, radiative heat flux, and gravitating effects.

When one departs from purely fluid mechanics variables to include radiation (photons and neutrons) and electromagnetic field effects, pre- and postshock regions are coupled by these fields which have long range near instantaneous effects. Both regions will inevitably be modified by these fields during the shock development. Similarity techniques applied to the pre- and postshock regions, and two-sided similarity seems to suggest itself in approaching these problems. More generally, if problems require that the domain be divided by contact surfaces into "n" regions with "p" regions in a constant state, then "n-p" similarity models may be warranted. Some form of conservation equations would be expected to be preserved across the "n-1" contact surfaces.

Similarity equations are based on a symmetrization of the original physics equations and as such imply a strong underlying assumption about the nature of the process. There is no a priori justification for these symmetry assumptions. Assumptions must be justified through an iterative process involving experimental observation and if necessary homology redesign. If the symmetry properties discussed in Lemma 1 are valid, an analytic process, that is an extension of a perspective of [8] Auluck and Tandon, can be used to address this issue using "emergent radiation observables" in the vicinity of a luminous front. The objective of this paper is to develop this process.

Additionally, radiation phenomenon while not initially dominant may become so as the shock onset goes through its development phase and becomes a developed shock front. The initial conditions are masked by what is essentially a radiation switch on (RSO) phenomenon. A radiative "precursor" and a "postcursor" detach the radiating shock from information (initial conditions) about its formation. Indeed it may easily be the case that many different initial conditions lead to the same or indistinguishably similar RSO results! While it is convenient to use assigned values on boundaries to solve numerical problems, the results are academic if the boundary values do not evolve reflecting the energy released in the pre- and postshock zones (fissions and fusion events or even exothermic chemical explosives). Also it would be suspect to define the solution by initial conditions when the dominant phenomenology determining the homology does not develop until the shock is formed. Consider the example case of a tamper plate (contact surface) moving into a latent explosive material. A compression wave may build into a shock and a detonation front is formed. The fluid and thermodynamic parameters at the plate and the movement of the tamper must reflect the evolutionary release of energy occurring behind the detonation front.

These comments suggest in many cases the shock may have to be considered as a standalone event. Complex problems may require dividing the solution domain into more than two regions partitioned by appropriate contact surfaces. In these cases models should be built from the conservation laws and shock jump conditions meshed with measurements and observations. Radiation observables at luminous fronts may play an important role in this process.

Historically the analysis of various aspects of the nature of a shock front has undergone considerable discussion in the literature. It has been suggested by [9] Germain that, to some degree, there appears to have been two avenues of shock wave studies.

In the first case, the continuum physics equations are used to describe the asymptotic character of the shock transition which is represented as a discontinuity across which the conservation laws must apply. Rankine and Hugoniot (R-H) produced the classic result formulating the shock jump conditions from the conservation laws in the shock moving frame which are summarized in [10] Friedrich and Courant. Implosions studied by [11] Guderly and explosions studied by [12] Taylor have used similarity techniques in the strong shock limit. Similarity solutions in the strong shock limit of a magneto hydrodynamic shock have also received some attention by [13] Cole and Greifinger and [14] Greenspan. It would appear that these early works and others might have been motivated by the study of large weapons systems. The "Strong Shock" model assumption, acceptable in modeling strong explosions, belies the issue of coupling the two regions using any two-sided similarity methods.

In the second case, attention is concentrated on the shock transition. Important are the effects of radiation and the dissipative mean free paths, among others, on the character of the transition. Numerous research papers have developed these issues. Some work by [15] Heaslet and Baldwin and [16] Clarke suggests that radiation may smooth the shock structure under certain flow conditions. [17] Mitchner and Vinokur suggest the presence of a magnetic field might to some extent countervail this smoothing effects. [18] Imshennik has discussed shock wave structure in the case the radiation energy and pressure are significant and may exceed the kinetic pressure and energy. These conditions are likely to prevail in stars. One might expect they could also prevail in the early stages of a fusion explosion.

2. Problem

The predicate for using two-sided similarity techniques is that the pre- and postshock regions are tied together by some form of radiation exchange. In this paper, we develop the discussion with thermal radiation but neutron transport and electromagnetic effects are also issues. For example, a neutron "switch on" wave is likely to be accompanied by significant thermal radiation. A neutron "switch on" wave may be characterized as a wave initiated in a medium that has the potentiality of a self-supporting process of fission or fusion or both leading to the emission of neutron radiation. When the wave steepens into a shock or compression wave and either the number of MFP or the "ρx"product of the fluid reaches a critical value, the process may become self-sustaining or may simply be a dissipating pulse. The neutron and thermal radiation fronts are likely to coexist and require new physical considerations. These adjunct processes distinguish these problems from purely mechanical shock problems.

In this paper we use imploding radiating shock in a magnetic field as a vehicle to study some aspects of two-sided similarity problem formulations. The same ideas used in this model can be used in more complex problems, if the solution space is divided by "(n-1)" contact surfaces into "n" zones and "p" zones are a constant states.

Assume a cylindrical column of gas is permeated by a background magnetic field. The cylinder implodes. Conventional explosives would be one of several means of accomplishing this. In laboratory plasmas, external pulsed fields initiate snowplow and shock wave processes. A compression wave is initiated and may build into an isothermal shock. The gas is ionized in front of and behind the shock front by the radiation cooling of the shock zone. The resulting high conductivity can lead to what is referred to as "frozen in magnetic field." The fluid, radiation, and magnetic field sensitively depend on the homologies and the initial values of the dependent variables. It is of importance to exploit the high symmetry characteristics, to be discussed in Lemma 1, of a presumed similarity solution to obtain measurements that would validate or guide the values of homology parameters.

3. Basic Model Equations

The descriptive equations can be tailored depending on the specific engineering problem. In this discussion we will use basic conservation laws. In the text by [19] Sutton and

Sherman, these equations are sometimes referred to as the MHD approximation. It is frequently useful to gain insight into the nature of the physics with a less detailed model, especially if there are measurements available to guide the model development. In cylindrical coordinates the equations take the form

$$r\partial_t \rho + \partial_r (r\rho v) = 0 \quad (1)$$

$$\rho (\partial_t v + v\partial_r v) + \partial_r P - jxB = 0 \quad (2)$$

$$\rho (\partial_t E + v\partial_r E) = -P\left(\partial_r v + \frac{v}{r}\right) - \left(\partial_r F + \frac{F}{r}\right) \quad (3)$$

$$\frac{-1}{r\partial_r (r\partial_r B)} = \mu\sigma \left[-\partial_t B - B\left(\partial_r v + \frac{v}{r}\right) - v\partial_r B\right] \quad (4)$$

Equations (1)-(4) are the continuity, momentum, and energy and magnetic diffusion, respectively. The symbols ρ, v, P, B, j, F, and E are the density, velocity, kinetic pressure, magnetic field, current, radiation flux, and internal energy. In general, the radiation flux may be of any kind, photons or neutrons, for example. Also, the pressure may generally include hydrodynamic, magnetic, and radiation contributions. We have excluded Ohmic heating, assuming the conductivity will be sufficiently large in the region of interest to justify this. It is unlikely this will be the case in all phases of an experiment or phenomenon under study. If thermal gradients are large, then (4) needs to be reassessed.

Generally, P=P(ρ,S). Since F and ΔF≠0, ΔS≠0. We will use the ideal gas equation of state functional form together with an internal energy proportional to the temperature. So P=RρT and E=c_vT serve in this analysis. This is called the polytropic model. We view these equations and the specific heat ratio as an heuristic. If S=constant, the pressure would be proportional to the density to a constant power γ, the specific heat ratio. Also, the ideal gas form of the constitutive equation of state does not always represent the best modeling of high temperature processes.

In some analysis, different values of γ are used to capture a range of behavior due to different degrees of ionization, collision frequencies, and magnetic field strength. In a "frozen in field" B/ρ is constant. One might be inclined to discuss a $\gamma_\perp$ and $\gamma_\parallel$ for the fluid due to the field in a collisionless or weakly collisional gas, as might occur in some laboratory or an astrophysical problem. This might be a fruitful modeling approach in such cases, but in this analysis, we are assuming a collision dominated fluid. Additionally, we will not consider contributions from the thermal conductivity or viscosity. It is important to model the detailed constitutive properties of the fluid, if one does a complete computer calculation.

These basic model equations are usually augmented by Faraday and Ampere equation (5) and generalized Ohm's Law. In this example the magnetic field is axial, the bulk fluid velocity is radial, and the induction electric field and consequent currents are azimuthal. The displacement field term in Ampere equation is ignored.

$$\nabla XE = -\partial_t B \nabla XB = \mu jj = \sigma (E + vXB) \quad (5)$$

Because of the geometry of the B field and its gradient, jxB in (2) can be represented as in

$$j \times B = \left(\nabla \times \frac{B}{\mu}\right) \times B = -\frac{\partial_r B^2}{(2\mu)} \quad (6)$$

We note that the diffusion equation (4) for the magnetic field is derived by using the generalized Ohm's Law in Ampere's equation (5), then by taking the curl of (5).While the high conductivity approximation is acceptable in a high temperature zone, ignoring gradients in the conductivity is suspect if there are strong thermal gradients. In this case (4) would have to be reassessed. Faraday's equation is necessary if the relative gradient in conductivity cannot be ignored and if flux slippage is something to be modeled. Ignoring the tensor character of the conductivity, some researchers have used a constant value or the Spitzer Harm (SH) power law approximation to capture aspects of the temperature dependence of conductivity. $\sigma_c=\chi T^{\zeta}$; $\chi \approx 1.5\text{x}10^{-2}/\ln\Omega$; ζ=3/2. Note that depending on the structure of the plasma, $\ln\Lambda \approx$6-23. This covers a significant range effect.

Within a given problem it is unlikely that just one radiation functional structure will represent the entire domain of the problem. This fact was noted by F.C. Auluck and J.N. Tandon, hereafter referred to as AT, and their perspective will form the basis of the approximate calculations to follow. The Flux "F," (3), is radiation energy transfer internal to the plasma and is likely due to many different processes. The mean free path is of significance, for example, in determining whether the plasma is optically thick, thin, or something intermediate. There are a number of recognized expressions for emergent radiation loss mechanisms that would constitute the "external observables" of the experiment or phenomenon depending on the properties of the plasma.

By "external observables" the signature a measuring instrument external to the phenomenon would record is meant. The measurements might be a superposition of the functions in (7). The "external observables" might have to be decomposed into individual contributions from different sources before using (45). This might require deconvolving the frequency distribution of the measured radiation signature.

Equations (7) are a listing of some these radiation processes along with their dependence on thermodynamic and fluid variables. It is likely that a measurement would contain a superposition of the various sources indicated in (7). In order to condense the presentation, modeling details are condensed in the scaling coefficients. The values of the coefficients depend on the particular model calculation and plasma composition. For example, for Bremsstrahlung, for a pure deuterium plasma ρ^2 would be proportional to n_d^2 and for a mixed deuterium-tritium plasma it would be proportional to $n_d n_t$. For Cyclotron radiation ρ is proportional to the electron number density. In both cases T is proportional to T_e the electron temperature. For a Black body, T is the equilibrium temperature.

$$\text{Bremsstrahlung:} \quad P = P_{brs}\rho^2\sqrt{T} watts/cm^3$$

$$\text{Cyclotron:} \quad P = P_{cys}\rho T B^2 watts/cm^3$$

$$\begin{aligned}
&\text{Black Body:} \quad P \\
&\qquad = P_{bbs} 5.75\text{x}10^{-5} T^4 ergscm^{-2} sec^{-1} \left(K^0\right)^{-4} \\
&\text{Radio Frequency:} \quad P = P_{rfs} E \times H = P_{rfs} v \times \frac{B^2}{\mu_0} wattscm^{-2}
\end{aligned} \tag{7}$$

RF emissions, which are the result of the bulk compression (or movement) of any "embedded" axial magnetic field and the azimuthal induction field at the shock wave interface, are included. RF emission is present in a complete treatment of the energy equation and might serve as a useful diagnostic signature.

Black Body radiation flux diffusion is characterized by the gradient of the radiative energy and is a model considered in many papers. It is not the focus in this paper, but we will indicate the transformation constraints for that radiative model. The frequency averaged mass absorption or Rosseland opacity is κ. The mean free path, μ=1/$\rho\kappa$, is frequently modeled as a product of a power of the density times a separate power of the temperature. This is not done but recognize it allows modeling flexibility and will change the homology representation of the fluid's opacity. It may be possible to model the pre- and postshock material with different mean free path functions, when using the more general representation of the mean free path, as long as the homologies cancel across the shock. The quantity ac/4 is the Stephan Boltzmann constant σ_R. This is summarized in

$$\begin{aligned}
&U^R = aT^4; \\
&F = -\left(\frac{4\sigma_R}{3\langle\kappa\rangle\rho}\right)\nabla T^4; \quad \sigma_R = \frac{ac}{4}
\end{aligned} \tag{8}$$

$$\begin{aligned}
&r^{-1}\partial_r rF \\
&\quad = -\left(\frac{4\sigma_R}{3\langle\kappa\rangle}\right)\left[\rho^{-1}\partial_{rr}T^4 + \left((r\rho)^{-1} - \rho^{-2}\partial_r\rho\right)\partial_r T^4\right]
\end{aligned} \tag{9}$$

It is unlikely that the electron and ion kinetic temperatures and the radiation field are in thermal equilibrium in most cases. A shape factor equal to the ratio of the systems dimensions to the radiation MFP has been used in modeling radiation diffusion to compensate for this property. In addition to Bremsstrahlung, a significant contribution of the radiation in the presence of a magnetic field would likely come from Cyclotron effects. Unless one is analyzing extreme radiation environments, radiation pressure and energy density can usually be ignored in the energy and momentum equations. In those cases a more complete description of the radiative process is required.

4. Solution

4.1. The Overall Approach. To use (45), a solution (model) on both sides of the vicinity of the shock front needs to be obtained. This calculation is an example of the iterative process of calculation and measurement to help define relevant homologies consistent with measured radiation spectral structure. The solution involves solving or constructing a preshock state and joining the solution across the shock front to the postshock state using the shock jump conditions. We choose to illustrate the technique using analytic approximation but a detailed computer calculation is equally acceptable. However the neighborhood of the luminous front is the only region of interest to perform this analysis. The homologies of (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f) reveal important aspects of the flow, if narrow angle observations about a luminous front, shock front, or contact surface can be made. The approximate analytic or computer solutions about the shock front can be used when the experimental observations about a line of constant η require more than a very narrow subtended observation angle.

To obtain the analytic approximation, this paper extends the suggestion and perspective of AT. In their particular calculation, they work under the assumption that the details of the radiation field are difficult to postulate a priori. This is well suited to the process that involves incorporating experimental data of the luminous front to determine the character of the radiation instead of stipulating it as an assumption. This analysis asserts that if the radiation loss is not a dominant mechanism, the homology should not determine the dynamics. Their analysis centers on the jump equations. We expand their point of view by developing simple approximate two-sided similarity representations in the vicinity of the shock front. The complex dependence on homology parameters, which is frequently lost with detailed discreet computer calculations, can be retained. Then as an operational alternative to postulating the homology of the radiation, the homologies of the external emergent radiation functions can be evaluated and compared with observation ((44a)-(46)). There are at least three approaches to consider.

(1) Assume a strong shock condition where the shock assumes limiting values and the preshock state is not strongly influential in the process. This eliminates the need for structuring the preshock state explicitly but has limited applicability. It is well known that there are substantial photon and neutron precursors that strongly influence the preshock regions in nuclear weapons effects into which the blast wave eventually propagates. Nonetheless the strong blast approach produces useful insights into the mechanical nature of the blast waves and has been successfully utilized in a number of past studies.

(2) The relevant equation can be solved with boundary point conditions that are physically reasonable, such as bounded physical variable values at a well-defined contact surface (piston) for the functions and if necessary their first derivatives. Assigning initial values at select points to obtain solutions is sometimes a necessity but it is in reality at best a convenience since discreet contact surfaces seldom exist in real situations for all the dependent variables. The solutions are more useful if the initial conditions are tied to experimental measurements. There are various techniques for measuring the plasma properties, as described by [20] Griem. Temperature and densities are among them.

(3) The preshock state being strongly influenced by near instantaneous long range field effects can be postulated (calculated), be a prepared laboratory experimental state,

or be the observed result of a natural process. The state can be mathematically structured enabling wide modeling applicability. The strong blast assumption is a limiting case.

5. The Transformation

Introduce the homology representation for the dependent and independent variables equations (10a)-(10b). One of the first applications of this was by [21] Boltzman in his treatment of the heat diffusion problem. Before WWII, [22] Weizacker studied similarity solutions presumable as part of the fission implosion program. While (10c) is a composite variable, the quantities, ρ_λ, T_λ, B_λ, F_λ, r_λ, and t_λ are, respectively, the scale density, temperature and magnetic field and radiation flux, scale length, and time.

$$\rho = r^k \rho_0 \varrho(\eta), \qquad v = \left(\frac{r}{t}\right) v(\eta), \tag{10a}$$

$$T = \left(\frac{r^m}{t^n}\right) T_0 T(\eta), \qquad B = \left(\frac{r^w}{t^l}\right) B_0 B(\eta), \qquad F = \left(\frac{r^a}{t^b}\right) F_0 F(\eta) \tag{10b}$$

$$\frac{B(r,t)}{\rho(r,t)} = \left(\frac{r^{w-k}}{t^l}\right)\left(\frac{B_0 B(\eta)}{\rho_0 \varrho(\eta)}\right) \tag{10c}$$

$$\eta = \frac{r}{t^\delta}, \qquad \partial_r \eta = \left(\frac{\eta}{r}\right), \qquad \partial_t \eta = -\frac{\delta\eta}{t}, \tag{10d}$$

$$\rho_0 = \frac{\rho_\lambda}{r_\lambda^k}; \qquad T_0 = T_\lambda\left(\frac{t_\lambda^n}{r_\lambda^m}\right); \qquad B_0 = \left(\frac{t_\lambda^l}{r_\lambda^w}\right) B_\lambda; \qquad F_0 = \left(\frac{t_\lambda^b}{r_\lambda^a}\right) F_\lambda \tag{11}$$

The functional structures of (10a)-(11) are not unique and have been used by many authors. The similarity transformation, while a simple mathematical process, implies a strong underlying symmetry assumption about the dependent variables. Equations (12a) are the similarity structures and the scaling relations of the composite emergent radiation functions in (7). The terms in brackets are the respective homologies.

$$\begin{aligned}
&\text{Bremsstrahlung:} \quad P = P_{brs}\left[\frac{r^{(2k+m/2)}}{t^{(n/2)}}\right]\varrho^2(\eta)\sqrt{T(\eta)}\,watts/cm^3\\
&\text{Cyclotron:} \quad P = P_{cys}\left[\frac{r^{(k+m+2w)}}{t^{(n+2l)}}\right]\varrho(\eta)\,T(\eta)\,B(\eta)^2\,watts/cm^3\\
&\text{Black Body:} \quad P = P_{bbs}\left[\frac{r^{4m}}{t^{4n}}\right]5.75\text{x}10^{-5}T(\eta)^4\,ergscm^{-2}sec^{-1}\left(K^0\right)^{-4}\\
&\text{Radio Frequency} \quad P = P_{rfs}E\times H = \overline{P_{rfs}}\left[r^{(1+2w)}/t^{(1+2l)}\right]v(\eta)\times\frac{B(\eta)^2}{\varsigma_0}wattscm^{-2}
\end{aligned} \tag{12a}$$

The approximate analysis in subsequent sections provides a representation for the dependent invariants in (12a), (12b), and (12c). Equations 12(a) are the radiation observables, among others, that can be used to test the similarity hypothesis.

Lemma 1. *If the problem is self-similar and if any set of functions such as $B(\eta)$,$T(\eta)$,$v(\eta)$, and $\rho(\eta)$ are dependent invariants, then any function Ψ, (12b), of dependent invariants is a dependent invariant and a constant for each η. The physical function's explicit time and space dependence is determined solely by the composite homology.*

$$\Psi = \Psi\left(\rho(\eta)^{\theta_1}, B(\eta)^{\theta_2}, T(\eta)^{\theta_3}, v(\eta)^{\theta_4}\right); \quad \forall\theta_1,\theta_2,\theta_3,\theta_4 \tag{12b}$$

This would apply to (12a), the quantities in brackets in (12a) being the composite homology. Furthermore Γ, (12c), is a three-parameter family of functions of physical variables that is invariant for every consistent solution of (13)-(16). This follows since there are four-theta parameters and one δ constraint.

$$\Gamma = \left[\frac{r^{k\theta_1+w\theta_2+m\theta_3+\theta_4}}{t^{\epsilon\theta_1+l\theta_2+n\theta_3+\theta_4}}\right]\Psi;$$

$$\Psi = \rho(\eta)^{\theta_1}\,B(\eta)^{\theta_2}\,T(\eta)^{\theta_3}\,v(\eta)^{\theta_4}$$

$$\eta^{Q} = \frac{r^{k\theta_1 + w\theta_2 + m\theta_3 + \theta_4}}{t^{\epsilon\theta_1 + l\theta_2 + n\theta_3 + \theta_4}} \quad Q = k\theta_1 + w\theta_2 + m\theta_3 + \theta_4$$
$$\delta = \frac{(\epsilon\theta_1 + l\theta_2 + n\theta_3 + \theta_4)}{(k\theta_1 + w\theta_2 + m\theta_3 + \theta_4)} \tag{12c}$$

There are strong rationals for physical symmetry transformation such as the Lorentz transformation. There is no a priori physical law that requires sets of physical variables to be mapped (evolve) by these automorphism symmetries. Collapsing space and time into a single independent variable η has induced these symmetries. The symmetry is a modeling assumption. If the similarity hypothesis is true for the particular problem under study, it makes possible comparisons between calculated emergent radiation characteristics and small angle measurements about a line of constant η to test the similarity hypothesis. This includes the trajectories of contact surfaces, shocks and plasma sheaths, and density time profiles. No differential equations have to be solved at this point to accomplish these comparisons, if Lemma 1 is found to apply. If large angle measurements need to be made in the vicinity of the shock line, an approximation procedure based on an extension of the AT concept could be used. This is developed further on.

Comment on Lemma 1. It is likely that in real physical problems many cases would involve a weakly broken automorphism symmetry. The measured homology parameters might be close too but not precisely the calculated values from (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f). This might suggest consideration of a calculation that is a perturbation approach to the similarity equations. Depending upon measurements, this could involve a perturbation to the shock path or any of the dependent invariants. The perturbation would likely break the strong internal symmetries of (12b) and (12c).

Using the transformation equations (10a), (10b), (10c), and (10d), a symmetrization of (1)-(4) is being modeled. Furthermore in reducing the PDEs to ODEs we are not modeling the full physical equations subject to boundary values that might lead to a unique solution in a strict mathematical sense. Furthermore, the process predicate (switch on waves) has detached the shock from initial condition of its formation. The notion of constructing a solution to the ODEs that satisfies some arbitrary initial value problem becomes at best moot unless there is a well-defined contact surface or line of constant η on which there has been measured values or observations.

With these caveats in mind, the PDEs (1)-(4) are transformed into the ODEs (13)-(16). We note that for explosion "t"" increases and implosions "t" decreases but it always remains positive. Since the fundamental equations are time translation invariant one could introduce a new time variable τ=T_s-t, t < T_s for implosions where T_s is the implosion time.

$$(v - \delta)\,\eta\partial_\eta\varrho + (k+2)\,\varrho v + \eta\varrho\partial_\eta v = 0 \tag{13}$$

$$\varrho\left[(v-\delta)\,\eta\partial_\eta v + v\,(v-1)\right] + \eta^{q_1}M_0\left[(m+k)\,\varrho T + \eta\partial_\eta\varrho T\right] + \eta^{q_2}M_1\left[2\mathrm{w}B^2 + \eta\partial_\eta B^2\right] = 0 \tag{14}$$

$$\left[E_0\varrho\left((mv-n)\,T + (v-\delta)\,\eta\partial_\eta T\right) + E_1 T\varrho\left(2v + \eta\partial_\eta v\right)\right] + \eta^{q_3}F_0\left((a+1)\,F + \eta\partial_\eta F\right) = 0 \tag{15}$$

$$D_0^{-1}\eta^{(-q_4)}\left[w^2 B + (2\mathrm{w}+1)\,\eta\partial_\eta B + \eta^2\partial_{\eta_\eta}^{\ 2}B\right] = (v-\delta) \cdot \eta\partial_\eta B + \left((w+2)\,v - l + \eta\partial_\eta v\right)B \tag{16}$$

where $E_0 = \rho_0 c_v T_0$; $E_1 = R\rho_0 T_0$; $M_0 = RT_0$; $M_1 = B_0^2/2\mu\rho_0$; $D_0 = \mu\sigma$

The continuity equation does not generate a constraint. The constraints from the momentum, energy, and magnetic diffusion are indicated below, respectively. Equation (17b2) is an alternate and more restrictive form than (17b1). Equation (17f) replaces (17c) in the case one uses (8) as a radiation model.

$$\eta^{(q_1)} = \frac{r^{m-2}}{t^{n-2}}; \quad (q_1) = (m-2); \qquad \delta = \frac{(n-2)}{(m-2)} \tag{17a}$$

$$\eta^{(q_2)} = \frac{r^{2\mathrm{w}-2-k}}{t^{2\mathrm{l}-2}}; \quad (q_2) = 2\mathrm{w} - 2 - k; \qquad \delta = \frac{(l-1)}{(w-1-k/2)} \tag{17b1}$$

$$\eta^{q_1} = \eta^{q_2}; \quad q_1 \equiv q_2; \qquad k + m = 2\mathrm{w}, \qquad n = 2\mathrm{l} \tag{17b2}$$

$$\eta^{(q_3)} = \frac{r^{a-(1+k+m)}}{t^{b-n-1}}; \quad q_3 = a - (1+k+m); \qquad \delta = \frac{(b-n-1)}{(a-(1+k+m))} \tag{17c}$$

$$\eta^{(q_4)} = \frac{r^{(\zeta m)+2}}{t^{(\zeta n)+1}}; \quad q_4 = (\zeta m) + 2; \qquad \delta = \frac{((\zeta n)+1)}{((\zeta m)+2)} \tag{17d}$$

$$\eta^{(q_5)} = \frac{r^{w-k}}{t^{l}}; \quad q_5 = w - k; \qquad \delta = \frac{l}{(w-k)} \tag{17e}$$

$$\eta^{(q_6)} = \frac{r^{3\mathrm{m}-2\mathrm{k}-2}}{t^{3\mathrm{n}-1}}; \quad q_6 = 3\mathrm{m} - 2\mathrm{k} - 2; \qquad \delta = \frac{(3\mathrm{n}-1)}{(3\mathrm{m}-2\mathrm{k}-2)} \tag{17f}$$

Note that ζ=3/2 and ζ=0 correspond, respectively, to SH and a constant conductivity. Equations (13)-(16) are the transformed continuity, momentum, energy, and field diffusion,

Table 1

	k	δ	v_0	w	l	m	n	ς	k	δ	v_0	w	l	m	n	ς
NO RADIAL FLOW	0	0.8	0	1.156	1.125	2.313	2.250	1.5	0	0.5	0	2.250	1.625	4.5	3.25	0
RADIAL FLOW	-1	0.8	0.4	0.656	1.125	2.313	2.250	1.5	-1	0.5	0.25	1.750	1.625	4.5	3.25	0

respectively. Equations (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f) are the constraints. Since we must match similarity structures across the shock front along a line of constant η further constraints may be imposed on the homologies.

Corollary 2. *For the particular class of transformations characterized by equations (10a), (10b), (10c), and (10d), the origin is a singular point. By Lemma 1, it will be the case that dependent invariant functions are multivalued at an intersection point of η lines. Hence they are not uniquely defined at this point. For this reason, the solution space should exclude a small region centered about the origin (r=0,t=0). Depending on the circumstances, it may be possible to generate similarity transformations that do not have this problem.*

6. Homology Analysis

It is necessary to find a consistent set of parameters (n, m, δ, ζ, l,w,k) to satisfy the conditions of equations (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f). There is no unique way of doing this. The goal is to obtain (13)-(16) solely a function of η. Powers of η to the qth power will necessarily appear in these equations. In obtaining an homology solution to transform the system, it tacitly assumes the governing constraint properties are valid throughout the entire domain. Plasmas are unlikely to be that consistent. This situation is not reflected in global similarity solution applications. This study is focused on locally approximate solutions in the vicinity of luminous contact surfaces. We consider several cases.

In both Cases 1(a) and 1(b), the conductivity is assumed to be very large so explicit solution of the entire diffusion equation can be ignored but still characterized by the conductivity homology. The parameters ζ,k and n are free.

Case 1(a). For ζ=3/2, σ corresponds to the SH law. The parameter "m" is determined from (17a) and (17d). Using (17a) determines δ. From (17b2), use k+m=2w and n=2l which makes the kinetic and magnetic pressure terms completely homogeneous in η, i.e., $\eta^{q_1} = \eta^{q_2}$; q_1=q_2. Flow chart (18) follows:

$$m = \frac{(5n-2)}{4} \longrightarrow \delta = \frac{(n-2)}{(m-2)} \longrightarrow l = \frac{n}{2} \longrightarrow w = \frac{(k+m)}{2} \tag{18}$$

Case 1(b). For ζ =0, σ is a constant. Then from (17d), necessarily δ=1/2. Equation (17a) defines "m" and as before use (17b2) for l and w. Flow chart (19) follows

$$\delta = \frac{1}{2} \longrightarrow m = 2(n-1) \longrightarrow l = \frac{n}{2} \longrightarrow w = \frac{(k+m)}{2} \tag{19}$$

Since the radiation is not a priori specified, for any values of k, m, n, and δ, a relationship exists between a and b from (17c). The flowcharts are not unique and the parameter space has many solutions. The solutions generated by the transformation of equation (10a), (10b), (10c), and (10d) are a symmetry approximation and are used to build an approximate model. Some solutions using (18) and (19) are indicated in Table 1.

Table 1 contains values for k=0 and -1. Because of (28) this corresponds to a zero and nonzero preshock velocity. In some modeling cases a nonzero preshock velocity might be useful. In explosion geometries, near instantaneous preshock radiative heating will begin an expansion before the arrival of the shock front. In implosion geometries as radiation streams into the preshock region there may be a preshock radially inward or outward flow or both in different parts of the region, depending on the nature of the energy transfer process. Photon and neutron mean free paths (MFP) and resultant energy deposition and possibly energy release will influence this.

Case 2. Homology solutions that include an expression of the radiation flux according to (8) and (9) in the energy equation can be constructed. Additionally, the magnetic diffusion can be used for both constant and SH conductivity approximations. Other sequences built from (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f) are possible. The calculation sequence, in flowchart (20), produces consistent homology solutions for this combination. Table 2 summarizes some of these.

For the radiation flux to conform to the T^4 law of (8), set a=4m-k-1 and b=4n in (17c) to obtain (17f). It is useful to note that models express the mean free path as a product of different powers of the density and temperature may be constructed differently in the pre- and postregions as long as the homologies cancel across the shock. Choose "n" and k as independent parameters. Express "m" as a function of n and k using (17a) and (17f). Use (17a) from the momentum equation to express δ as a function of n and m. The diffusion equation, which is part of the solution set, places no constraint on B but the momentum equation does. Thus use (17b2) to establish w and l. Then use relation (17d) from the diffusion equation to express ζ as a function of n, δ, and m. Find solution sets for ζ =0 and 1.5 by searching k, n parameter space. Some parameters satisfying (20) are indicated in Table 2.

$$m = \frac{[n(4-2k)+4k+2]}{5} \longrightarrow \delta = \frac{(n-2)}{(m-2)} \longrightarrow l = \frac{n}{2} \longrightarrow w = \frac{(k+m)}{2} \longrightarrow \zeta = \frac{(2\delta-1)}{(n-m\delta)} \tag{20}$$

These calculation sequences (18), (19), and (20) are obviously not unique. Solutions were obtained by putting these relations

TABLE 2

K	N	M	δ	L	W	ς
0.75	2.5	2.25	2	1.25	1.5	1.5
0.75	3	2.5	2	1.5	1.63	1.5
-3	2.5	3	0.5	1.25	0	0
-3	3	4	0.5	1.5	0.5	0

in a spreadsheet. Many solutions that are available in a spreadsheet analysis make sense and many more do not. Part of the modeling process is to select appropriate homologies to model the problem. The fact that homology solutions to transform the system of equations can be found does not imply that they usefully represent a physically meaningful problem. Observational validation has been the known historical requirement of scaling models.

7. The Preshock Model

In subsequent sections the consequences of extrapolating the basic observation of AT are examined. The spectral character of the radiation flux is left unspecified. The expectation is that measurements of the emergent radiation, (12a), compared with their pre- and postshock similarity functional forms and homologies, (44a)-(46), would provide input on the characteristics of the plasma.

In modeling the core, if there is a strong developing thermal gradient, there would be some density variation and fluid movement to compensate for the pressure imbalance, the result of the energy streaming into the core. Furthermore experimental systems are open-ended and the cylinder being finite would incur end losses. These are difficult nuances to accommodate in a similarity representation. The process used to model the core, (21)-(25), is indicated below. In building core models, the implications of Corollary 2 are applicable. Since the problem involves a standalone RSO system, initial conditions are still an issue. If we measure dependent variable values at (r_o, t_o) then the values at this point are the initial condition at $\eta_o = r_o/t_o^{\delta}$ for developing the solution to the shock front $\eta_s = r_s/t_s^{\delta}$.

(1) Density Profile. When the preshock state is strongly influenced by near instantaneous long range field effects, the density can be parametrically postulated, obtained by observation of a naturally occurring event or structured as a prepared experimental state. When building a density model structure, for any the above reasons, it may be required to cancel homologies involving powers of the space variable. This poses no problem. A function of η can be chosen to always produce bounded values of the real physical variable as indicated.

$$\rho = r^k \rho_0 \eta^{-k} g(\eta);$$
$$\varrho(\eta) = \eta^{-k} g(\eta)$$
$$\eta\partial_\eta \ln \varrho(\eta) = -k + \eta\partial_\eta \ln g(\eta) = H(\eta)$$
$$\text{if } g(\eta) = \exp A\eta;$$
$$H(\eta) = [A\eta - k] \tag{21}$$

This example will address the case of A=0 and H=-k. Consistent with this, the physical density can be a constant (k=0) or spatially uniform and increasing in time (k≠0). Exponential density profiles have received some recent study and, for A≠0, such exponential profiles can be addressed.

(2) Obtain a Velocity Profile from the Continuity Equation Using the Modeled Density Profile in (21). Since A=0, the velocity is determined by the density homology and the dimensionless shock speed. Equation (22) is a linear ODE that can be put in standard form and solved for arbitrary functions H(η) and in particular for constant H.

$$\upsilon(H(\eta) + k + 2) + \eta\partial_\eta \upsilon = \delta H(\eta)$$
$$\upsilon(\eta) = \left(\frac{\eta}{\eta_r}\right)^{-\Phi}\left[\upsilon_0 + \int\left(\frac{\eta}{\eta_r}\right)^{\Phi}\left(\frac{\delta H}{\eta}\right)d\eta\right]; \tag{22}$$
$$\Phi = (k + 2 + H)$$

H=-k in this model and the physical velocity should trend to zero as you approach the origin.

$$\upsilon = \upsilon_0\left(\frac{\eta}{\eta_r}\right)^{-2} - \frac{k\delta}{2}; \quad \upsilon_0 = 0;$$
$$v(r,t) = -\frac{kr\delta}{2\mathrm{t}} \tag{23}$$

(3) The Magnetic Field Is Obtained from an Infinite or Limiting Large Conductivity Model. In the limit, the inverse diffusion parameter D_0 goes to infinity. A second-order equation would become a first-order equation. If it is of interest to model the evolution of the field capture in the core with initially small finite conductivity, the complete diffusion equation (16) may be used in this region. First substitute (23) into the limiting large conductivity approximate form of the similarity diffusion equation (16). Then in the case of a constant radial velocity field, (24) results in (25). The solution is a monomial.

$$\partial_\eta \ln B(\eta) = -\left[\frac{((w+2)k\delta - 2l)}{\delta(k+2)}\right]\eta^{-1} \tag{24}$$

$$B(\eta) = B(\eta_o)\left(\frac{\eta}{\eta_o}\right)^{-\beta}; \quad \beta = \frac{[(w+2)k\delta + 2l]}{\delta(k+2)} \tag{25}$$

(4) With a Modeling Structure for the Density, Velocity, and Magnetic Field, the Momentum Equation (26) Is in Standard Format and May Be Integrated to Determine the Temperature Field

$$T(\eta) = \exp^{-\int_{\eta_o}^{\eta}((m+k+H)/\eta)d\eta}\left[T(\eta_0) + \int_{\eta_o}^{\eta}\exp^{\int_{\eta_o}^{\eta}((m+k+H)/\eta)d\eta}Q(\eta)\,d\eta\right]$$
$$Q(\eta) = \left[\left(-\frac{M_1\varrho^{-1}}{M_0}\right)\left(2\mathrm{wB}^2 + \eta\partial_\eta B^2\right)\eta^{q_2-q_1-1} - M_0^{-1}\left((v-\delta)\eta\partial_\eta v - v + v^2\right)\eta^{-(q_1+1)}\right] \tag{26}$$

Using the model values for the velocity and density equations (21)-(23) in (26) results in (31) and can be expressed as a two-term polynomial.

(5) The Energy Equation Can Be Used to Establish a Consistent Heuristic Functional Form for the Radiation Flux in the Core. There is no specification of the spectral form and homology. The radiation is treated as a moderate to weak loss mechanism. The radiation flux equation can be expressed from the energy equation in standard form.

$$F(\eta) = \left(\frac{\eta}{\eta_0}\right)^{-(a+1)}\left[F(\eta_0) + \int\left(\frac{\eta}{\eta_0}\right)^{(a+1)}Q_E(\eta)\,d\eta\right]$$
$$Q_E = -\frac{\eta^{-q_3-1}\left[E_0\varrho\left((mv-n)T + (v-\delta)\eta\partial_\eta T(\eta)\right) + E_1 T\varrho\left(\eta\partial_\eta v + 2v\right)\right]}{F_0} \tag{27}$$

The calculation, driven by the density profile, is consistent. The homology of a weak radiation loss mechanism does not drive the dynamics. The values of the variables in the preshock model at $\eta = \eta_o$ are the values at observation or measurement points and the preshock values to use in the jump equations are the functions evaluated at $\eta = \eta_s$. Solution of the jump equation at the shock front, along a line of constant η, provides the initial value to perform the postshock integration. Without observation there are free parameters in the calculation. The solutions (constructed model) are summarized in (28)-(31). The equations, particular to the assumption of the preshock model, can use any of the parameter sets from Table 1.

$$v = -\frac{k\delta}{2} \tag{28}$$

$$\varrho = \varrho_o\left(\frac{\eta}{\eta_o}\right)^{-k}; \tag{29}$$

$$B(\eta) = B(\eta_o)\left(\frac{\eta}{\eta_o}\right)^{-\beta}, \quad \beta = \frac{((w+2)v - l)}{(v-\delta)} \tag{30}$$

$$T = \left(\frac{\eta}{\eta_o}\right)^{-m}\left[T_0 + \int_{\eta_o}^{\eta}\left(\frac{\eta}{\eta_o}\right)^{m}Q(\eta)\,d\eta\right]$$
$$Q(\eta) = \varrho_0^{-1}B(\eta_0)^2\left(\frac{\eta}{\eta_0}\right)^{-(k+2\beta)}\left[2(\beta-w)\frac{M_1}{M_0}\eta^{q_2-q_1-1}\right] - \frac{\eta^{-q_1-1}}{M_0}\left(k\frac{\delta}{2}\left(1+k\frac{\delta}{2}\right)\right) \tag{31}$$

Although the process seems somewhat unorthodox, it is in reality not conceptually different from a strong blast model which assumes limiting values. The long range near instantaneous fields will condition the preshock fluid before the arrival of the mechanical shock. Building an appropriate preshock model, by any of the processes indicated above, acknowledges that reality. The solution set is complete and displays the dependence explicitly on the homology numbers. The expense has been to leave unspecified the spectral structure of the internal core radiation flux although the radiation flux can be calculated as a loss. More to the point, the dynamics are not being determined by the radiation homology which in this approximation is not considered a dominant mechanism. The process is sufficiently simple that many computer experiments can be conducted in a spreadsheet varying the structure of the density profile and admissible homologies resulting in corresponding structures for the dependent invariants. Using preshock model equations (28)-(31) and postshock model equations (41a), (41b), and (41c)-(42), the homology parameters can be compared with experimental data using (44a)-(48).

In the infinite conductivity case, D_0→ ∞, the fluid in the core is entrained by the field lines. The fluid is necessarily compressed and both the kinetic and magnetic pressures increase. The increase in postshock density will also occur, among other reasons, due to geometric factors. If the conductivity in the core remains finite, D_0 ≈ O(1), the gas can slip away from the fields and the shock can continue to trap some flux. The field will slip through as well and will build behind the shock front where the conductivity is highest. The full magnetic diffusion equation needs to be considered. With a finite conductivity, in particular a tensor conductivity, the analysis quickly becomes more complicated. Tensor conductivity may be an important issue in studying emergent radiation characteristics such as cyclotron and synchrotron signatures. The models considered are spatially uniform both with and without a temporally increasing density. It is apparent from (21) that other structures, such as a density gradient, simulating a snowplough model profile can be built. This process differs substantially from a foil implosion which is simply compressing field where the density ρ_0 interior to the foil is negligible. The strong shock approximation might be relevant in that type of problem. At some point the shock might eventually dissipate or bounce but the similarity analysis is limited because of Corollary 2.

8. Shock Jump Conditions

The shock jump conditions arise from the conservation equations across the sharp transition and are given in (32b)-(32d). U is the shock velocity. To find a solution across the jump equations it is presumed that the preshock values and shock speed are known, supplemented by a caloric and state equations; the postshock values are then determined from the jump equations. When limiting strong shock waves are assumed, the necessity of structuring a more detailed preshock model is preempted. Equations (32a) summarize this for a polytropic gas. The compression is specified in this limit by the specific heat ratio.

$$\begin{aligned} \mu^2 &= \frac{(\gamma-1)}{(\gamma+1)} \\ \frac{\rho_b}{\rho_f} &= \frac{1}{\mu^2} = \varphi \\ v_b &= U\left(1-\frac{1}{\varphi}\right) = U\left(1-\mu^2\right) \end{aligned} \tag{32a}$$

With two-sided similarity this generally is not necessarily the case. In an RSO standalone event, the data classically discussed that determines the transition is not immediately available. In a very detailed computer calculation the value of the long range fields in this region and their effects could be calculated as the shock structure forms. This does not preempt the need to make measurements. In lieu of this, observations are required in the core to execute the similarity calculation.

In this two-sided similarity analysis we have built a preshock model to use with (32b) through (32d). The preshock variables, (29)-(31), evaluated at the shock line are the values that are the necessary conditions. The shock moves on a line of constant η, r=.$\eta_s t^{\delta_i}$; $\partial_t r = \delta \eta_s t^{\delta-1} = \delta r/t$. Now U=r$\delta$/t. In a standalone event, these values are based on observations at $\eta = \eta_o$. Observations or measurements have to be made at an interior point. If observations are not available, there are free modeling parameters. The quantity "m" is the mass flow through the shock.

$$\begin{aligned} \left[\rho(v-U)\right]_f^b &= 0; \\ m &= \left|\rho(v-U)\right| \end{aligned} \tag{32b}$$

$$\left[\rho(v-U)^2 + P + \frac{B^2}{2\mu}\right]_f^b = 0 \tag{32c}$$

$$\left[\frac{v^2}{2} + E + \frac{P}{\rho} + \frac{B^2}{\mu\rho} + \frac{F}{m}\right]_f^b = 0 \tag{32d}$$

In formulating (32d), the transition is expressed in terms of a change in total enthalpy. The rate of work done by Lorentz force on the bulk fluid, (jXB·v) has been included. The fluid is assumed infinitely conducting at the jump interface. These approximations have been used by many authors and relaxing these conditions to include finite conductivity, ion slippage, and nontangential fields leads to considerably more complicated equations. The post- and preshock model variables are matched across the shock front on the shock line of constant $\eta = \eta_s$. To form the similarity jump equations (33a)-(33c), substitute the homology functions of equations (10a), (10b), (10c), and (10d) and use the constraints (17a), (17b2), and (17c) in the jump equations (32b)-(32d).

$$\begin{aligned} &\left[\varrho(\upsilon-\delta)\right]_f^b = 0; \\ &\{m\} = \left|\varrho(\upsilon-\delta)\right|; \quad m = \frac{\rho_0 r^{k+1}\{m\}}{t} \end{aligned} \tag{33a}$$

$$\left[\varrho(\upsilon-\delta)^2 + M_0\eta_s^{q_1}\varrho T + \eta_s^{q_2}M_1B^2\right]_f^b = 0 \tag{33b}$$

$$\left[\frac{\eta_s^{-q_1}\upsilon^2}{2} + \left(\frac{E_0}{\rho_0} + M_0\right)T + \frac{2M_1B^2}{\varrho} + \frac{FF_0\eta_s^{q_3}}{(\{m\}\rho_0)}\right]_f^b = 0 \tag{33c}$$

There are terms, $\eta_s{}^q$, that appear in two-sided similarity. These terms have to be included and are the result of (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f) and the two-sided similarity process. Since the evaluation of the jump is along a line of constant η=η_s these terms become multiplicative constants. The preshock and postshock radiation models can be studied as quite different structures. In particular this would apply to the mean free path dependence on the temperature and density. This is possible because we matched the homology in both regions. The value of η_s is a property of the shock trajectory and must be determined either from a measurement, (48), or considered a free modeling parameter. It is a key parameter in building models.

The velocity is expressed as a function of the compression from the continuity equation. The quotient of the magnetic field divided by the density is constant. Substituting the expressions into the momentum equation allows a solution for the compression ratio. The continuity jump, the isothermal limit, frozen in field modeling assumptions, and the momentum are indicated in (34)-(37).

$$\text{Continuity;}\quad v_b = \left(\frac{1}{\varphi}\right)\left(v_f - \delta\right) + \delta \quad \varphi = \frac{\varrho_b}{\varrho_f} \tag{34}$$

$$\text{Isothermal assumption;}\quad T_b = T_f \tag{35}$$

$$\text{Frozen field assumption;}\quad B_b = B_f\varphi \tag{36}$$

$$\text{Momentum Jump}\quad \varphi = \frac{\left[-\left(P_f^k + P_f^m\right) + \sqrt{\left(P_f^k + P_f^m\right)^2 + 4\mathrm{KP}_f^m}\right]}{2P_f^m};\quad P_f^m > 0 \tag{37}$$

where $K(\eta) = \varrho_f(v_f - \delta)^2 P_f^k = M_0\eta_s^{(q_1)}\varrho_f T_f(\eta)$; $P_f^m = M_1\eta_s^{(q_2)}B_f^2(\eta)$

The compression φ is determined from the cubic polynomial derived from the momentum equation. So φ =1 is a root and a degenerate case corresponding to a smooth density transition or no shock transition. The remainder is a quadratic and only the positive branch is acceptable. For an isothermal approximation the temperature is assumed continuous. While the radiation flux is usually considered continuous, it need not be unless it is assumed proportional to the gradient in the temperature. The postshock radiation flux amplitude follows from (33c). Because of the assumptions of (35) and (36), the jump in flux is only a function of the change in kinetic energy and magnetic pressure. It is easy to see from (38) the conditions under which the flux would be continuous.

$$F_b = F_f - \left[\frac{F_0\eta_s^{(q_3)}}{\rho_0\varrho_f\left(v_f - \delta\right)}\right]^{-1} \cdot \left[\eta_s^{-q_1}\left(\frac{1}{2v_b^2} - \frac{1}{2v_f^2}\right) + 2M_1\left(\frac{B_f^2}{\varrho_f}\right)(\varphi - 1)\right] \tag{38}$$

The jump equations provide v_b, ρ_b, B_b, T_b, and F_b. The values so obtained can be used to continue the model into the postshock region. If the preshock gas is available for measurement, then there would be an experimental imperative to fashion a preshock model to reflect those measurements. Thinking in terms of preshock and postshock fission or fusion events triggered by the RSO shock, it might be important to build a pre- and postshock model that likely would not be the same.

9. Postshock Model

The postshock calculations continue to use the augmented AT process in dealing with the energy equation. The solution proceeds from the shock front using the initial values derived from the jump equations (34)-(37). The solutions are approximations in the vicinity of the shock front, are not intended to be global in nature and to be used for wide angle observations of the luminous fronts. Without calibrating observation of the similarity model we cannot have rest assurance of the validity of the similarity hypothesis.

The continuity equation (22) was written in standard form. The coefficients for the velocity and density are established from the continuity equation and jump values. For this example, the calculation proceeds by truncating an expansion for the density to one term. Close to the shock line, the density model, either postulated or obtained by measurements, will drive the calculation. So ψ and ρ_b=$\varphi\rho_f$ and δ, the dimensionless shock speed, are central parameters. The compression φ is obtained from (37) and involves the preshock state. The velocity is a solution to (22) with A=0 and H=-ψ

$$\begin{aligned} &\varrho \approx a_1\eta^{-\psi};\quad \psi = (k+1);\\ &\Phi = k + 2 + H;\quad H = -\psi;\\ &v = b_0 + b_1\eta^{-1} \end{aligned} \tag{39}$$

The postshock value initial condition is $v = v_b$. With negative powers of η, it is expected that dependent variables would fall off as "r" increases. This need not be the case if the plasma is being driven by a mechanical piston, explosives, or JXB current sheath. The values for the magnetic field follow by using the solution for the velocity in the high conductivity diffusion equation (40) and using the jump condition.

$$\ln B = -\int\left[\frac{\left[-l + (w+2)v + \eta\partial_\eta v\right]}{\eta\left[v - \delta\right]}\right]d\eta \tag{40}$$

If you depart from the shock line into a region where D_0 cannot be ignored then the diffusion equation must be solved in its entirety. The momentum equation and the energy equation are entirely the same as equations (14) and (15). Note that if ψ=k and b_{-1} = 0 we recover the preshock solution for the density and velocity. Substituting the forms (39) into (13) through (16) and remembering D_0 is large yields (41c) and (42). Both the temperature and flux are ordinary differential equations and can be put in standard form. The temperature is obtained from the momentum equation (26). The flux follows from the energy equation (27). Equations (41c) and (42) are integrable for this model choice of homology parameters.

$$\varrho(\eta) = \frac{a_1}{\eta^{(k+1)}}; \quad a_1 = \varrho_b \eta_s^{(k+1)} \tag{41a}$$

$$\upsilon(\eta) = -\delta(k+1) + \left(\frac{\eta_s}{\eta}\right)(\delta(k+1) + \upsilon_b) \tag{41b}$$

$$\begin{aligned} B(\eta) &= B_b \exp - \Omega(\eta, \eta_s); \\ \Omega(\eta, \eta_s) &= \int_{\eta_s}^{\eta} \frac{[(w+2) b_0 - l]\eta + (w+1) b_1}{\eta[(b_0 - \delta)\eta + b_1]} d\eta \end{aligned} \tag{41c}$$

$$\begin{aligned} T(\eta) &= \left(\frac{\eta}{\eta_s}\right)^{-(m-1)} \left(T_s + \int_{\eta_s}^{\eta} \left(\frac{\eta}{\eta_s}\right)^{(m-1)} Q(\eta)\, d\eta\right) \\ Q(\eta) &= -\left(\frac{M_1}{M_0}\right)\left(\frac{\eta}{\eta_s}\right)^{(k+1)}\left(\frac{B_b^2}{\rho_b}\right) \\ &\quad \cdot \eta^{q_2 - q_1 - 1}\left[2\mathrm{w} - 2\eta\partial_\eta(\Omega)\right]\exp - 2\Omega \\ &\quad - \frac{\eta^{-q_1 - 1}\left[b_0^2 - b_0 + (b_0 b_1 + b_1(\delta - 1))\eta^{-1}\right]}{M_0} \end{aligned} \tag{42}$$

The predicate for this entire pre- to postshock approximation modeling process is that a presumed spectral structure for the radiation flux does not drive the process and radiation is a moderate loss mechanism. The changes in the energy are primarily due to PdV work but the process is not isentropic; 0<dF/PdV<<1. Furthermore the density structure function obtained through trial approximation or observation drives the calculation. If the conductivity at the shock front is not limiting large, then Joule heating should be added to the energy equation. Note that since homology solutions exist for the full system, as indicated in Tables 1 and 2, numerical calculations can be accomplished with both radiation and magnetic diffusion. Computer solution would compromise some of the transparency of the homology dependence. However (45)-(48) can still be used to evaluate the homology hypothesis.

If radiation is a dominant process, it would require including radiation pressure in the momentum equation and radiation energy in the energy equation. The approximation process could be reversed. The temperature would be determined by the energy equation and would drive the density and flow velocity through the momentum and continuity equations. This is not likely to be the case in laboratory plasmas.

Had the polytropic and isentropic model been strictly assumed, from the equation of state, the temperature can be expressed explicitly as a function of the density. The continuity equation is a linear ODE and can be integrated, so the velocity can be expressed explicitly as a function of the density. By substitution, the magnetic field is then similarly a function of the density. Although cumbersome, the momentum equation becomes a single equation for the density. Solving that equation determines all the dependent variables consistently. This approximation approach was not taken.

10. Power Radiated

Corollary 3. *By Lemma 1, any integral between two lines of constant η is a constant. Hence narrow angle and wide angle observations have the same homology if taken between lines of constant η. Let the luminous front under observation (shock) and the power function be described generically by*

$$\begin{aligned} r_s &= \eta_s t_s^{\delta}; \\ P &= \overline{P} r^{\alpha} t^{-\beta} P(\eta) \end{aligned} \tag{43}$$

As a result of Lemma 1, Corollary 3 applies to observational integral equation (44a). The narrow angle and wide angle observations have the same time evolution. Also the use of a computer calculation, instead of the above analytic approximation for the dependent invariants, does not change the time evolution in this case. The amplitude changes. For this reason it might be sufficient to compare the normalized outputs of the time evolution of each of the emergent radiation function. The power radiated from a small interval $d\eta$ about a line of constant η at any time t and from a finite interval are given respectively by (44a). In this formulation, the shock thickness or spatial observation interval is collapsing.

$$P = \overline{P}\Delta\theta t^{(\delta(\alpha+2)-\beta)}\left[P(\eta)\eta^{(\alpha+1)}\right] d\eta; \tag{44a}$$

$$P = \overline{P}\Delta\theta t^{(\delta(\alpha+2)-\beta)}\left[\int_{\eta_1}^{\eta_2} P(\eta)\eta^{(\alpha+1)} d\eta\right] \tag{44b}$$

Corollary 4. *From Corollary 3, the time evolution of the emergent power during convergence depends only on the homology and the geometry since the quantities in brackets, (44a), are a constant in time. Hence the quotient of the logarithm of the measured emergent radiation power at observation times t_1 and t_2 divided by the logarithm of the quotient of the observation times is equal to the homology. The number is unique for each type of radiation in (12a), (12b), and (12c). Using the generic formula (43) and (44a), this can be expressed in*

$$\delta(\alpha+2) - \beta = \frac{\ln(P(t_1)/P(t_2))_{Observation}}{\ln(t_1/(t_2))} \tag{45}$$

This holds whether the process consists of observations or is an analytic or computer model. An estimate of the error in the analytic model may be obtained by comparing the difference of the ratio of the observed and analytic or computed power ratios as per

$$\epsilon_r = \left|1 - \frac{\ln(P(t_1)/P(t_2))_{calculated}}{\ln(P(t_1)/P(t_2))_{Observation}}\right| \tag{46}$$

There is another perspective, depending on the nature of the observing instrument, in which a fixed spatial interval Δ_{r} about the converging front needs to be kept constant. Corollary 3 does not apply to (47). In this case, as convergence occurs, more lines of constant η are added to the field of view and the integral *continues to change. This means the model functions for the dependent variables have to be taken explicitly into account.*

$$P\left(r_j, t\right) = \Delta\theta\overline{P}\int_{r_j-\Delta_r/2}^{(r_j+\Delta_r/2)} r^{\alpha}t^{-\beta}P\left(\eta\right)rdr;$$
$$P\left(r_j, t_j\right) = \overline{P}\Delta\theta t_j^{\delta(\alpha+2)-\beta}\int_{((r_j-\Delta_r/2)/t_j^{\delta})}^{((r_j+\Delta_r/2)/t_j^{\delta})} P\left(\eta\right)\eta^{\alpha+1}d\eta \tag{47}$$

The pre- and postshock functions to be used for the emergent radiation function in (12a), (12b), and (12c) to evaluate (44a)-(47) are, respectively, available in (28)-(31) and (41a), (41b), and (41c)-(42). For narrow angle observation the integral on each side of the front can be replaced by $d\eta=|\eta^{+}-\eta^{-}|$*. This interval should be just sufficient to capture the luminous front, since the approximate calculation of the dependent invariants is only intended in the vicinity of the shock front.*

$$S = \left\{r_j, t_j\right\};$$
$$\langle\delta\rangle = \left\langle\frac{\ln\left(r_j/r_k\right)}{\ln\left(t_j/t_k\right)}\right\rangle; \tag{48}$$
$$\langle\eta_s\rangle = \left\langle\frac{r_j}{t_j^{\langle\delta\rangle}}\right\rangle$$

From a streak photograph, the shock line parameters and deviations can be established from a measurement set "S" as indicated in (48). The brackets indicate averages over "S." The transformation, (10d), depends on the structure of the shock line. There will be a statistical deviation in δ *and* η_s *and if they are too large might negate the similarity hypothesis or suggest consideration of a perturbation correction as indicated in the comment to Lemma 1.*

11. Concluding Remarks

(1) Two-sided similarity models are built to bridge the gap between conservation equations and physical observables in regions coupled by long range and near instantaneous fields. The bulk emergent radiation functions, (12a), (12b), and (12c), are the observables measured closely around a line of constant η (narrow angle) or a small bundle of lines of constant η (wide angle). The homology of the emergent radiation functions alone determines their time evolution in a narrow angle measurement To accommodate wide angle measurement, the AT concept was extended and was used to obtain approximate characterizations of the thermodynamic parameters of the plasma around shock fronts, contact surfaces, or plasma sheaths. Detailed calculation change the absolute amplitude; they do not change the scaling relations. The analytic approximation explicitly displays the dependence on the homology parameters. Complex dependence of the solutions on the similarity transformation parameters can be obscured with individual discreet numerical calculation.

Measurements must play an important role in building and testing scaling models.

(2) The transformation introduced by Boltzmann in the classic treatment of the heat problem involved one dependent variable that satisfied transformable boundary conditions. The automorphism (10a)-(12a), (12b), and (12c) deals with four variables and results in a strong symmetry relationship between them as per Lemma 1. Symmetries such as time translation and space translation invariance are well studied but similarity automorphisms are in a distinctly different class of their own. The techniques impose a strong symmetry on the physics and hence are part of the solution hypothesis. The implication is that arbitrary composite functions of dependent invariants, (12b), are also invariant along composite space time lines. In addition, (12c) indicate that for every solution to (13)-(16) there exists an internal symmetry of a three-parameter family of physical invariant function sets. This powerful symmetry may or may not capture the nature of the physics. If the phenomenon that drives the homology is a dominant physical process then the transformation would likely capture the physics. As per the comment on Lemma 1, it is suggested that when measurement does not precisely conform to homology parameters, as is likely to be the case, the similarity equations might be the subject of a perturbation analysis and the perturbation would break the strong internal symmetries.

(3) "Switch on waves" are disconnected from initial values by virtue of the long range fields developed after the switch on process occurs. This is especially the case if the radiation field is a "switch on" neutron radiation field. We have discussed Radiation "switch on" in the context of isolated radiation loss mechanisms. Neutron switch on waves are inevitably accompanied by strong thermal radiation environments and when the "switch on" is triggered, the shock is essentially a standalone event. It might seem that the process resembles a classical detonation wave. This would be misleading since there do not exist nearly spontaneous long range influences in classical detonation wave theory.

The preshock and postshock solution can be matched across a shock front (or contact discontinuity) as long as the homologies in both regions cancel or can be absorbed. This provides a form of solution "continuation" of the analytic model. In a standalone event, the shock jump conditions along with the determination of the dimensionless shock speed are insufficient to determine the problem. More data is needed to build models of standalone events. If there are no experiments or measurements then there are free parameters and the solutions must be tested by some other means. In the example calculation, while δ is determined as part of the homology calculation, the shock line $\eta=\eta_s$ would have to be determined by an observation, (48). Also in the preshock model there is an observation point and hence a line of constant $\eta_o = r_o/t_o^{\delta}$ where observations of the dependent variables are taken. The functions calculated from η_o to η_s determine the preshock values. Along with δ, this determines the postshock values by continuation across the jump equations. These solutions (models) can be used in (44a)-(48) for comparisons with measurements of the emergent radiation functions and measurement of the shock trajectory. In a comparison with the similarity homology structure, (17a), (17b1), (17b2), (17c), (17d), (17e), and (17f) may provide corroborative homology design data.

Disclosure

Dr. J. NiCastro* received his Ph.D. degree from the Case Institute of Technology and was formerly associated with Lawrence Radiation laboratory, Science Applications Incorporated, Boeing Corporation, and Sandia Laboratory.

Conflicts of Interest

*The author declares that there are no conflicts of interest regarding the publication of this paper.

References

[1] J. P. Vishwakarma and V. Kumar Pandey, "Self-Similar Flow under the Action of Monochromatic Radiation Behind a Cylindrical MHD Shock in a Non-Ideal Gas," *Journal of Applied Mathematics*, vol. 2, no. 2, pp. 28–33, 2012.

[2] G. Nath and P. K. Sahu, "Similarity solution for the flow behind a cylindrical shock wave in a rotational axisymmetric gas with magnetic field and monochromatic radiation," *Ain Shams Engineering Journal*, 2016.

[3] J. R. NiCastro, "Similarity Analysis of the Radiative Gas Dynamic Equations with Spherical Symmetry," *Physics of Fluids*, vol. 13, no. 8, pp. 2000–2006, 1970.

[4] S. Singh, "Self Similar flow behind a radiative spherical shock wave," *Advances in Applied Science Research*, vol. 5, no. 5, pp. 94–98, 2014.

[5] K. Singh and J. Vishwakarma, "Propagation of Spherical Shock Waves in a Dusty Gas with Radiation Heat Flux," *Journal of Theoretical and Applied Mechanics*, vol. 45, no. 4, pp. 801–817, 2007.

[6] D. Mihalas and B. W. Mihalas, *Foundations of Radiation Hydrodynamics*, Dover Publications, New York, 2nd edition, 1999.

[7] G. Nath, M. Dutta, and R. P. Pathak, "An exact Solution for the propagation of Shock Waves in Self Gravitating Perfect Gas in the Presence of Magnetic Field and Radiative Heat Flux," *AMSE JOURNALS_AMSE IIETA publication-2017 Series; Modeling B*, vol. 86, no. 4, pp. 907–927, 2017.

[8] F. Auluck and J. Tandon, "Radiation effects on the propagation of plane perpendicular Magnetogasdynamic shock in a plasma," *Proceedings of the National Academy of Sciences India*, no. A, 1966.

[9] P. Germain, "Shock waves and shock-wave structure in magneto-fluid dynamics," *Reviews of Modern Physics*, vol. 32, pp. 951–958, 1960.

[10] K. O. Friedrichs and R. Courant, *Supersonic Flow and Shock Waves*, Springer-Verlag, New York, NY, USA, 1976.

[11] G. Guderley, "*Starke kugelige und zylindrische Verdichtungsstosse in der Nahe des Kugelmittelpunktes bzw. Der Zylinderachse., Luftfahrtforschung", band 19, Lfg.9*, 1942.

[12] G. Taylor, "The formation of a blast wave by a very intense explosion," *Proceedings of the Royal Society*, vol. 201, 1950.

[13] J. D. Cole and C. Greifinger, "Similarity solution for cylindrical magnetohydrodynamic blast waves," *Physics of Fluids*, vol. 5, pp. 1597–1607, 1962.

[14] H. P. Greenspan, "Similarity solution for a cylindrical shock-magnetic field interaction," *Physics of Fluids*, vol. 5, pp. 255–259, 1962.

[15] M. A. Heaslet and B. S. Baldwin, "Predictions of the structure of radiation-resisted shock waves," *Physics of Fluids*, vol. 6, pp. 781–791, 1963.

[16] J. F. Clarke, "Radiation-resisted shock waves," *Physics of Fluids*, vol. 5, no. 11, pp. 1347–1361, 1962.

[17] M. Mitchner and M. Vinokur, "Radiation smoothing of shocks with and without a magnetic field," *Physics of Fluids*, vol. 6, no. 12, pp. 1682–1692, 1963.

[18] V. S. Imshennik, "Shock Wave Structure in a Dense High Temperature Plasma," *Soviet Physics—JETP*, vol. 15, pp. 167–174, 1962.

[19] Sutton and Sherman, *Engineering Magnetohydrodynamics*, Mcgraw Hill, 1965.

[20] H. Greim, *Plasma Spectroscopy*, McGraw Hill, 1964.

[21] L. Boltzmann, "Zur integration des diffusiongleichung bei variabeln diffusions coefficienten," *Annals of Physics*, vol. 53, p. 959, 1894.

[22] C. F. Weizsäcker, "Genäherte Darstellung starker instationärer Stoßwellen durch Homologie-Lösungen," *Zeitschrift für Naturforschung A*, vol. 9, no. 4, 1954.