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Research Article

Application of Optimal Control Theory to Newcastle Disease Dynamics in Village Chicken by Considering Wild Birds as Reservoir of Disease Virus

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In this study, an optimal control theory was applied to a nonautonomous model for Newcastle disease transmission in the village chicken population. A notable feature of this model is the inclusion of environment contamination and wild birds, which act as reservoirs of the disease virus. Vaccination, culling, and environmental hygiene and sanitation time dependent control strategies were adopted in the proposed model. This study proved the existence of an optimal control solution, and the necessary conditions for optimality were determined using Pontryagin's Maximum Principle. The numerical simulations of the optimal control problem were performed using the forward–backward sweep method. The results showed that the use of only the environmental hygiene and sanitation control strategy has no significant effect on the transmission dynamics of the Newcastle disease. Additionally, the combination of vaccination and environmental hygiene and sanitation strategies reduces more number of infected chickens and the concentration of the Newcastle disease virus in the environment than any other combination of control strategies. Furthermore, a cost-effective analysis was performed using the incremental cost-effectiveness ratio method, and the results showed that the use of vaccination alone as the control measure is less costly compared to other control strategies. Hence, the most effective way to minimize the transmission rate of the Newcastle disease and the operational costs is concluded to be the timely vaccination of the entire population of the village chicken, improvement in the sanitation of facilities, and the maintenance of a hygienically clean environment.

1. Introduction

Newcastle disease (ND) is a contagious viral disease that affects domestic and other wild avian species [1–4]. It is a seasonal disease of poultry and occurs mostly during the wet seasons [5, 6]. Among other avian species, the ND in chicken is caused by the avian paramyxovirus of serotype 1 (APMV-1) [1, 4, 5, 7–9]. The transmission routes of ND in village chicken include air, direct contact with infected chicken, and contaminated water, food, droppings or discharges of the infected chicken, and other equipment in the flock [4, 7]. Another topic of interest is the interaction of domestic chicken with other wild birds as the second transmission route of the ND among the avian species [7, 10, 11]. The symptoms of the ND in chicken include paralysis in legs and wings, coughing, head twitching, greenish white diarrhea, difficulty in breathing, nasal and eye discharges, decreased egg production, and loss of weight [4, 7, 12]. ND is a very fatal disease, causing approximately 100% death rate in young birds [13]. The main goal of this study is to develop a deterministic model of ND transmission and apply the optimal control theory to study the effect of the control practices of the disease while minimizing the cost of its implementation.

Recently, several scholars in the epidemiology field have used the optimal control theory to study various diseases such as vector-borne diseases [14], Malaria [15–17], Rift Valley fever [18], Tungiasis disease [19], ND [20], and HIV-AIDS [21]. However, to the best of our knowledge, only one study focused on the optimal control of ND in chicken [20] by incorporating three control strategies: vaccination, human education campaign, and treatments of the infected human.

The analysis from this work showed that the combination of these three strategies is the best way to control the spread of ND. Reference [12] modeled the transmission of ND in the presence of wild birds as the reservoir of the disease. The results of this paper suggested that contaminated environment plays a crucial role in the transmission of Newcastle diseases in the village chicken population and hence reducing the rate of spread is better to increase the clearance rate of NDV from the environment.

This study extended the model by [12] by incorporating three time-dependent control measures: vaccination of the susceptible chicken, culling of the infected chicken, and environmental hygiene and sanitation. Vaccination is applied to protect the susceptible chicken against the ND for a sufficient period [4, 22]. The current available vaccines for ND virus are thermostable vaccines such as I-2 strain and the heat-resistant V4 (NDV4-HR) vaccine [7]. Culling involves the identification of the infected chicken and its removal from the population by killing and deposing the remains safely [23, 24]. The environmental hygiene and sanitation involves increasing the awareness of people about biosecurity measures to free the environment of the ND virus. This is achieved by ensuring cleanliness in the chicken's yard and its surroundings so as to avoid indirect transmission of the virus to the chicken from the unhygienic environment [4]. Furthermore, environmental hygiene and sanitation can be applied by safely disposing the manure from the infected chickens, avoiding contaminated feeds, and improving sanitation for the caretakers of chickens. Application of these three control measures helps in the reduction and possibly eradication of the ND in the village chicken but has economic implications. Owing to the scarcity of resources for most village chicken growers, the design of a control strategy that is economically viable but also reduces the adverse effects of the ND in chicken flocks is of paramount importance.

2. Mathematical Model

2.1. Model Formulation. In the current study, the deterministic model for ND in the village chicken by [12] is extended by adding a compartment of the vaccinated chicken $V(t)$ and the time-dependent control variables $u_i(t)$ for $(i = 1; 2; 3)$. In this model, variable $u_1(t)$ measures the control efforts to reduce infections to the susceptible chicken through vaccination. According to [30], the LASOTA vaccine provides 100% protection to chicken against ND. Also according to [31, 32], protection of the birds against ND reaches 90-100% efficacy rate to all kinds of the vaccines used in the study. Therefore, the vaccine efficacy to be 100% depends on the vaccine brand either local produced (live vaccines) or imported brands [26, 31]. In our case, we consider the heat-resistance vaccine which is administered to chicken in drinking water and is applied four times in a year. Variable $u_2(t)$ measures the control efforts aimed to reduce transmission of infections from infected chicken to susceptible chicken by identifying and killing of the infected chicken. The measure of all the efforts of controlling the spread of ND geared toward sanitation and improved hygiene of the environment $H(t)$ is represented by variable $u_3(t)$. The environment $H(t)$ is considered to be the surroundings including all equipment in the chicken yard which can accommodate the NDV in different weather conditions. NDV can survive in the environment but its activeness to cause severity of the disease to the hosts depends on weather conditions. Studies have shown that the NDV become active during the summer seasons [5]. Vaccination removes $\upsilon u_1(t)S_c(t)$ chicken from the susceptible class to the vaccinated compartment where υ is the efficacy rate of vaccines with value ranging from $0 < \upsilon < 1$. Culling removes $\tau u_2(t)I_c(t)$ from the infected chicken population where τ is the culling rate considered between $0 < \tau < 1$. Environmental hygiene and sanitation reduces the avian paramyxovirus-1 (APMV-1) from the environment at the rate σu_3 where σ is the clearance rate of virus through environmental hygiene and sanitation which varies between $0 < \sigma < 1$. Hence we assume that environmental hygiene and sanitation leads to the increased clearance rate of the virus from the environment given by $(\mu_v + \sigma u_3(t))H(t)$ with μ_v as the natural hibernation rate of the NDV. Chicken population $N_c(t)$ is divided into four subpopulations: the susceptible chicken $S_c(t)$, the latently infected $E_c(t)$, the severely infected chicken $I_c(t)$, and the vaccinated village chicken population $V(t)$. It is assumed that the chicken population does not have a mildly infected class which means that chicken are not carrier of the NDV. Thus, the total village chicken population becomes $N_c(t) = S_c(t) + E_c(t) + I_c(t) + V(t)$. The chicken is recruited by birth into a susceptible class at a density dependent rate $\Lambda_1 N_c(t)$. The disease incidence term for the village chicken is given by

$$\lambda_1\left(I_c(t), I_r(t), H(t)\right) = \left(\psi\frac{I_c(t)}{N_c(t)} + b\frac{I_r(t)}{N_b(t)} + \frac{dH(t)}{k+H}\right)S_c(t) \quad (1)$$

where ψ, b, and d are the transmission rates of infection to susceptible chicken when they come into contact with the severely infected chicken, mild infected wild birds, and environment, respectively. After a few days, individuals in the latently infected population of the village chicken progress to the severely infected population at the rate $\gamma E_c(t)$. The severely infected population of village chicken is reduced at the rate $(\mu + \delta_c + \tau u_2)I_c(t)$. The model assumes that the vaccine has a warning rate η and thus a portion $\eta V(t)$ of the vaccinated chicken becomes susceptible again due to the loss of the acquired immunity.

The population of wild birds is divided into four subpopulations: the susceptible population $S_b(t)$, the latent population $E_b(t)$, the severely infected wild bird population $I_b(t)$, and the mildly infected wild bird population, $I_r(t)$, which gives its total population as $N_b(t) = S_b(t) + E_b(t) + I_b(t) + I_r(t)$. Here it is assumed that wild birds have strong immunity against Newcastle disease; hence, it has a mild class which is considered as the carrier population. The susceptible population of wild bird is recruited at the rate $\Lambda_2 N_b(t)$ through birth. The susceptible wild birds acquire NDV when they interact with the severely infected wild birds $I_b(t)$, the mildly infected wild birds $I_r(t)$, and the unhygienic environment $H(t)$ and move

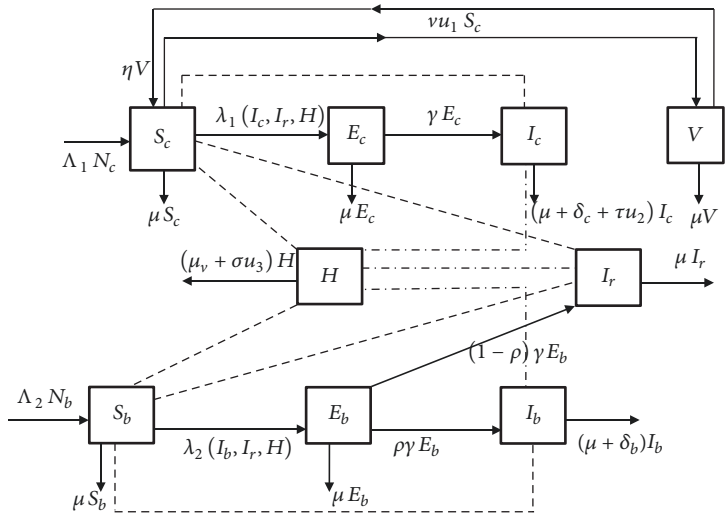

Figure 1: Flow diagram showing the dynamics of ND with vaccination, culling, and environmental hygiene and sanitation control measures.

to the latently infected class at the transmission rate defined by

$$\lambda_2 \left(I_b(t), I_r(t), H(t)\right) = \left(\frac{\varphi I_b(t) + a I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)}\right) S_b(t) \quad (2)$$

where φ, d, and a are the transmission coefficients measuring the transmission of infection upon contact or exposure between the susceptible and severely infected wild birds, mildly infected wild birds, and unhygienic environment, respectively. After one or two weeks of exposure period $(1/\gamma)$, we assume proportion ρ of the latently infected wild bird population progresses toward the severely infected population and the remaining proportion, $1 - \rho$, progresses to the mildly infected population of wild birds. The model assumes that village chicken and wild birds do not recover from the ND but die at disease-induced mortality rates of δ_c and δ_b, respectively. We further assume that the mildly infected wild birds do not die because of infection but at a natural death rate of μ. In our model, parameter κ represents half saturation constant of the ND virus in the environment. All parameters as well as state variables are assumed to be nonnegative. It is also assumed that the hosts (chicken and wild birds) do not recover once infected with the Newcastle disease. Graphically, we represent the interactions between village chicken, wild birds, and ND virus-infested environment by the schematic flow diagram in Figure 1.

Following the aforementioned assumptions and descriptions, the transmission model for the ND in the village chicken with controls is represented by system (3) of nonlinear differential equations.

2.2. Model Equations

$$\begin{aligned}
\frac{dS_c(t)}{dt} &= \Lambda_1 N_c(t) + \eta V - \left(\psi \frac{I_c(t)}{N_c(t)} + b\frac{I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)} + \mu + \nu u_1(t)\right) \cdot S_c(t) \\
\frac{dE_c(t)}{dt} &= \left(\psi \frac{I_c(t)}{N_c(t)} + b\frac{I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)}\right) S_c(t) - (\mu + \gamma) E_c(t) \\
\frac{dI_c(t)}{dt} &= \gamma E_c(t) - \left(\delta_c + \mu + \tau u_2(t)\right) I_c(t) \\
\frac{dV(t)}{dt} &= \nu u_1(t) S_c(t) - (\mu + \eta) V(t) \\
\frac{dS_b(t)}{dt} &= \Lambda_2 N_b(t) - \left(\frac{\varphi I_b(t) + a I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)} + \mu\right) S_b(t) \\
\frac{dE_b(t)}{dt} &= \left(\frac{\varphi I_b(t) + a I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)}\right) S_b(t) - (\mu + \rho\gamma) E_b(t) \\
\frac{dI_b(t)}{dt} &= \rho\gamma E_b(t) - \left(\delta_b + \mu\right) I_b(t) \\
\frac{dI_r(t)}{dt} &= (1 - \rho)\gamma E_b(t) - \mu I_r(t) \\
\frac{dH(t)}{dt} &= \alpha_c I_c(t) + \alpha_b \left(I_b(t) + I_r(t)\right) - \left(\mu_v + \sigma u_3(t)\right) \cdot H(t)
\end{aligned} \quad (3)$$

with initial conditions, $S_c(0) > 0$, $E_c(0) \geq 0$, $I_c(0) \geq 0$, $V(0) \geq 0$, $S_b(0) > 0$, $E_b(0) \geq 0$, $I_b(0) \geq 0$, $I_r(0) \geq 0$, $H(0) \geq 0$, $u_1(0) \geq 0$, $u_2(0) \geq 0$, $u_3(0) \geq 0$; $N_c(t) = S_c(t) + E_c(t) + I_c(t) + V(t)$; $N_b(t) = S_b(t) + E_b(t) + I_b(t) + I_r(t) + V(t)$.

3. The Cost Functional

In this section, we present the cost functional that balances the competing objectives of minimizing the costs associated with the preventions of village chicken from the transmissions of ND. Our purpose is to minimize the number of the severely infected village chicken $I_c(t)$ and concentration of NDV in the environment $H(t)$ while minimizing the cost of the controls. Therefore, to reach this goal we formulate an optimal cost functional of our problem as follows:

$$\mathcal{J} = \min_{u_i(t)\in\mathcal{U}} \int_{t_0}^{t_f} \left(A_1 u_1(t) S_c(t) + A_2 u_2(t) I_c(t) + A_3 u_3(t) + \frac{1}{2}\sum_{i=1}^{3} D_i u_i^2(t) \right) dt \tag{4}$$

subject to the state equation (3), the initial condition $X(0) = X_0$ and the set of controls $u_i(t) \in \mathcal{U}$. In (4), the time-dependent control constraints are Lebesgue measurable *i.e*, $\{u_i(t) \in \mathcal{U} \mid 0 \le u_i(t) \le 1\}$ at the time interval $t \in [t_0, t_f]$. A_1, A_2, and A_3 are per unit costs associated with the control of the susceptible village chicken, severely infected village chicken, and the unhygienic environment, respectively, while D_1, D_2, and D_3 are relative or additional cost weights for each control measure $u_1(t)$, $u_2(t)$, and $u_3(t)$, respectively. This control program is applied within the prescribed time horizon, that is, from initial time t_0 to final time t_f. A_1 includes the costs of buying and administering the vaccines while A_2 includes identification (laboratory tests), veterinary experts, transportation, and the disposing area as well as the official permit for killing the infected chicken. On the other hand A_3 includes the costs of buying the cleaning equipment, paying laborers for disposing the wastes and carcases of the infected chicken. We choose quadratic cost function in the objective functional the same way as in [16, 19, 33–36] so as to indicate the nonlinear control costs arise at higher intervention levels. The optimal control is the set $(u_1^*(t), u_2^*(t), u_3^*(t))$ such that

$$\mathcal{J}\left(u_i^*(t)\right) = \min_{u_i(t)\in\mathcal{U}} \left\{ \mathcal{J}\left(u_i(t)\right) : 0 \le u_i(t) \le 1 \right\}; \quad for\ i = 1,2,3 \tag{5}$$

We then apply the Pontryagin's Maximum Principle as described in [37–39], to find the optimal solution of system (3). Firstly, the Hamiltonian function $\mathcal{H}(t, X, u, \lambda)$ is formulated by introducing the adjoint variable, $\lambda(t)$, which saves as the Lagrangian multiplier of our optimal control system (3) constrained with the control variables $\{u_i(t) \in \mathcal{U} \mid 0 \le u_i(t) \le 1\}$, $t \in [t_0, t_f]$ and the state variables $X(t) \in \mathbb{R}^9$. We then apply the Pontryagin's Maximum Principle necessary conditions to find the optimal solution $\mathcal{J}(u_i^*(t))$ of system (3).

3.1. Existence of the Control Problem. An optimal control problem exists if the five necessary conditions that define the optimal solutions $\mathcal{J}(u_1^*(t), u_2^*(t), u_3^*(t)) = \min_{u_i(t)\in\mathcal{U}}\{(\mathcal{J}(u_1(t), u_2(t), u_3(t))\}$ of problem (3) derived using the Pontryagin's Maximum Principle are satisfied.

Theorem 1. *Given an optimal problem $\mathcal{N}(t, X(t), u_i(t))$ of system (3), subject to its initial boundary condition for state variable $X(t) \in \mathbb{R}^9$ and a control variable $u_i(t) \in \mathcal{U}$, then there exists an optimal solution $\mathcal{J}(u_i^*(t))$ such that $\mathcal{J}(u_i^*(t)) = \min_{u_i(t)\in\mathcal{U}}\{(\mathcal{J}(u_i(t))\}$ for $i = 1, 2, 3$ if the following necessary conditions are satisfied.*

(1) *The set of controls and the corresponding state variables are nonempty.*

(2) *The control set $\mathcal{U}$ is convex and closed.*

(3) *The right hand side of the state system is bounded by the linear function in the state and control variables.*

(4) *The integrand of the objective function is convex.*

(5) *There exist constant numbers $a_1, a_2 > 0$ and $\omega > 1$ such that the integrand of the objective function is bounded below by $a_1(|u_1| + |u_2| + |u_3|)^{\omega/2} - a_2$.*

Proof. The existence of an optimal control is verified by conditions stated in [40]. From our optimal problem $\mathcal{N}(t, X(t), u_i(t))$ of system (3), the set of all state variables $X(t) \in \mathbb{R}^9$ and the control variables $\{u_i(t) \in \mathcal{U} \mid 0 \le u_i(t) \le 1\}$, $t \in [t_0, t_f]$ are nonnegative; hence, the first condition is satisfied. By the definition, the optimal solution $u_i^*(t)$ is convex and bounded in $\mathcal{U}$ and thus the second condition is also satisfied [18, 41, 42].

The optimal system (3) is bounded which determines the compactness needed for the existence of the optimal control [15] and hence the third condition holds. In addition, the integrand in the functional (4) is clearly convex on the control set $\mathcal{U}$ which proves the fourth condition. According to [42], since the state variables are bounded, therefore the integrand is also bounded below by

$$A_1 u_1(t) S_c(t) + A_2 u_2(t) I_c(t) + A_3 u_3(t) + \frac{1}{2}\sum_{i=1}^{3} D_i u_i^2(t) \ge a_1 \left(\sum_{i=1}^{3} |u_i(t)| \right)^{\omega/2} - a_2 \tag{6}$$

for $i = 1, 2, 3$ which satisfies the last condition. With those five conditions satisfied we therefore conclude that there exist control variables $u_1^*(t)$, $u_2^*(t)$, and $u_3^*(t)$ such that

$$\mathcal{J}\left(u_1^*(t), u_2^*(t), u_3^*(t)\right) = \min_{u_i(t)\in\mathcal{U}} \left\{ \left(\mathcal{J}\left(u_1(t), u_2(t), u_3(t)\right)\right) \right\} \tag{7}$$

□

3.2. Characterization of the Optimal Control. Here we apply the Pontryagin's Maximum Principle to derive the necessary conditions that optimal control solutions must satisfy [39, 43, 44]. To obtain the minimum Lagrangian of the optimal problem we establish the Hamiltonian $\mathcal{H}(X, u, \lambda)$ as follows:

$$\mathcal{H}(X, u, \lambda) = A_1 u_1(t) S_c(t) + A_2 u_2(t) I_c(t) + A_3 u_3(t) + \frac{1}{2}\sum_{i=1}^{3} D_i u_i^2(t) + \sum_{j=1}^{9} \lambda_j F_j \tag{8}$$

where F_j stands for the right hand side of the j^{th} equation of the optimal control problem (3). Now, in the expanded form (8) become

$$\begin{aligned}\mathcal{H}(X,u,\lambda) &= A_1u_1(t)(t)S_c(t) + A_2u_2(t)(t)I_c(t)\\ &+ A_3u_3 + \frac{1}{2}D_1u_1^2(t) + \frac{1}{2}D_2u_2^2(t) + \frac{1}{2}D_3u_3^2(t)\\ &+ \lambda_{S_c(t)}\left(\Lambda_1N_c(t) + \eta V(t)\right.\\ &- \left(\psi\frac{I_c(t)}{N_c(t)} + b\frac{I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)} + \mu + \upsilon u_1(t)\right)\\ &\cdot S_c(t)\Big) + \lambda_{E_c}\left(\left(\psi\frac{I_c(t)}{N_c(t)} + b\frac{I_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)}\right)\right.\\ &\cdot S_c(t) - (\mu+\gamma)E_c(t)\Big) + \lambda_{I_c}(\gamma E_c(t)\\ &- (\delta_c + \mu + \tau u_2(t))I_c(t)) + \lambda_V(\upsilon u_1(t)S_c(t)\\ &- (\mu+\eta)V(t)) + \lambda_{S_b}\left(\Lambda_2N_b(t)\right.\\ &- \left(\frac{\varphi I_b(t) + aI_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)} + \mu\right)S_b(t)\Big)\\ &+ \lambda_{E_b(t)}\left(\left(\frac{\varphi I_b(t) + aI_r(t)}{N_b(t)} + \frac{dH(t)}{\kappa + H(t)}\right)S_b(t)\right.\\ &- (\gamma+\mu)E_b(t)\Big) + \lambda_{I_b}(\rho\gamma E_b(t) - (\delta_b + \mu)I_b(t))\\ &+ \lambda_{I_r}((1-\rho)\gamma E_b(t) - \mu I_r(t)) + \lambda_H(\alpha_cI_c(t)\\ &+ \alpha_b(I_b(t) + I_r(t)) - (\mu_v + \sigma u_3(t))H(t))\end{aligned} \tag{9}$$

Theorem 2. *Given $\mathcal{U}^* = \{u_1^*(t), u_2^*(t), u_3^*(t)\}$ is the set of the optimal control and S_c^*, E_c^*, I_c^*, V^*, S_b^*, E_b^*, I_b^*, I_r^* and H^* are the corresponding solutions minimizing $J(u_i)$ over $\mathcal{U}$, then there exists a costate variable, $\lambda(t)$, such that*

$$\begin{aligned}&\frac{d\lambda_{S_c}}{dt} = -\frac{\partial\mathcal{H}}{\partial S_c},\ \frac{d\lambda_{E_c}}{dt} = -\frac{\partial\mathcal{H}}{\partial E_c} =, \ldots, = \frac{d\lambda_H}{dt}\\ &\quad = -\frac{\partial\mathcal{H}}{\partial H}\quad (\textit{adjoint condition})\end{aligned} \tag{10}$$

$$\lambda_{S_c}(t_f) = \lambda_{E_c}(t_f) =, \ldots, \lambda_H(t_f) = 0 \quad (\textit{transvesality condition}) \tag{11}$$

$$\frac{\partial\mathcal{H}}{\partial u_i} = 0 \quad \textit{at } u_i^* = 0,\ i = 1,2,3,\ (\textit{optimality condition}) \tag{12}$$

From (9) and (10) we get the following adjoint equations:

$$\begin{aligned}\frac{d\lambda_{S_c}}{dt} &= -\frac{\partial\mathcal{H}}{\partial S_c} = -A_1u_1^*(t) + (\lambda_{S_c} - \lambda_{E_c})\Delta_1\\ &+ \lambda_{S_c}(\mu + \upsilon u_1^*(t) - \Lambda_1) - \lambda_V u_1^*(t)\\ \frac{d\lambda_{E_c}}{dt} &= -\frac{\partial\mathcal{H}}{\partial E_c} = -\lambda_{S_c}\Lambda_1 - \lambda_{I_c}\gamma + (\mu+\gamma)\lambda_{E_c}\\ &+ (\lambda_{S_c} - \lambda_{E_c})\frac{\psi S_c^*(t)(N_c^*(t) - I_c^*(t))}{N_c^2(t)}\\ \frac{d\lambda_{I_c}}{dt} &= -\frac{\partial\mathcal{H}}{\partial I_c} = -A_2u_2^*(t) - \lambda_{S_c}\Lambda_1 + (\lambda_{S_c} - \lambda_{E_c})\\ &\cdot\frac{\psi S_c^*(t)(N_c^*(t) - I_c^*(t))}{N_c^2(t)} + \lambda_{I_c}(\delta_c + \mu + \tau u_2^*)\\ &- \lambda_H\alpha_c\\ \frac{d\lambda_V}{dt} &= -\frac{\partial\mathcal{H}}{\partial V} = -\lambda_{S_c}\Lambda_1 + \mu\lambda_V - (\lambda_{S_c} - \lambda_V)\eta\\ &- (\lambda_{S_c} - \lambda_{E_c})\frac{\psi I_c^*(t)S_c^*(t)}{N_c^2(t)}\\ \frac{d\lambda_{S_b}}{dt} &= -\frac{\partial\mathcal{H}}{\partial S_b} = -\lambda_{S_b}\Lambda_2 + \mu\lambda_{S_b}\\ &+ \frac{(\lambda_{S_c} - \lambda_{E_c})bI_r^*(t)S_c^*(t)}{N_b^2(t)} + \Delta_2\\ \frac{d\lambda_{E_b}}{dt} &= -\frac{\partial\mathcal{H}}{\partial E_b} = -\lambda_{S_c}\Lambda_2 + \lambda_{E_b}(\gamma+\mu) - \lambda_{I_r}\gamma\\ &- (\lambda_{I_b} - \lambda_{I_r})\gamma\rho - \frac{(\lambda_{S_c^*} - \lambda_{E_c})bI_r^*(t)S_c^*(t)}{N_b^2(t)} - \Delta_3\\ \frac{d\lambda_{I_b}}{dt} &= -\frac{\partial\mathcal{H}}{\partial I_b} = -\lambda_{S_b}\Lambda_2 - \lambda_H\alpha_b + \lambda_{I_b}(\delta_b + \mu)\\ &- \frac{(\lambda_{S_c} - \lambda_{E_c})bI_r^*(t)S_c^*(t)}{N_b^2(t)} + \Delta_4\\ \frac{d\lambda_{I_r}}{dt} &= -\frac{\partial\mathcal{H}}{\partial I_r} = -\lambda_{S_b}\Lambda_2 - \alpha_b\lambda_H + \lambda_{I_r}\mu\\ &+ \left(\frac{bS_c^*(\lambda_{S_c} - \lambda_{E_c})}{N_b^2}\right) + (\lambda_{S_b} - \lambda_{E_b})S_b(t)\\ &\cdot\left(\frac{a(N_b^*(t) - I_r^*(t)) - \varphi I_b^*(t)}{N_b^2(t)}\right)\\ \frac{d\lambda_H}{dt} &= -\frac{\partial\mathcal{H}}{\partial H} = (\lambda_{S_c} - \lambda_{E_c})\frac{d\kappa S_c^*(t)}{(\kappa + H^*(t))^2}\\ &+ (\lambda_{S_b} - \lambda_{E_b})\frac{d\kappa S_b^*(t)}{(\kappa + H^*(t))^2} + (\mu_v + \sigma u_3^*)\lambda_H\end{aligned} \tag{13}$$

whereas

$$\Delta_1 = \left(\psi\frac{I_c^*(t)(N_c^*(t) - S_c^*(t))}{N_c^2(t)} + \frac{bI_r^*(t)}{N_b(t)} + \frac{dH^*(t)}{\kappa + H^*(t)}\right),$$

$$\Delta_2 = \left(\lambda_{S_b} - \lambda_{E_b}\right) \cdot \left(\frac{\left(N_b^*(t) - S_b(t)\right)\left(\varphi I_b(t) + aI_r(t)\right)}{N_b^2(t)}\right)$$

$$\Delta_3 = \frac{\left(\lambda_{S_b} - \lambda_{E_b}\right)\left(\varphi I_b^*(t) + aI_r^*(t)\right)S_b^*(t)}{N_b^2(t)},$$

$$\Delta_4 = \left(\lambda_{S_b} - \lambda_{E_b}\right)S_b(t) \cdot \left(\frac{N_b(t)\varphi - \left(\varphi I_b(t) + aI_r(t)\right)}{N_b^2(t)}\right) \tag{14}$$

From (9) and (12) we obtain the optimal solution given by

$$u_1^*(t) = \frac{1}{D_1}\left(v\left(\lambda_{S_c} - \lambda_V\right) - A_1\right)S_c^*(t) \tag{15}$$

$$u_2^*(t) = \frac{1}{D_2}\left(\tau\lambda_{I_c} - A_2\right)I_c^*(t) \tag{16}$$

$$u_3^*(t) = \frac{1}{D_3}\left(\lambda_H \sigma H^*(t) - A_3\right) \tag{17}$$

With the transversality condition $\lambda_r(t_f) = 0$ ($for\ r = X(t) \in \mathbb{R}^9$) and the boundedness condition of our control variables, $\mathcal{U} = \{u_i^*(t) \mid 0 \le u_i^*(t) \le u_{max}^*(t)\}$ (with $i = 1, 2, 3$). Now, let us consider the control bound, $0 \le u_i^*(t) \le 1$. By using the bounds for the control $u_1^*(t)$ we get the following solution:

$$u_1^*(t) = \begin{cases} 0 & \text{if } \frac{1}{D_1}\left(v\left(\lambda_{S_c} - \lambda_V\right) - A_1\right)S_c^* \le 0 \\ \frac{1}{D_1}\left(v\left(\lambda_{S_c} - \lambda_V\right) - A_1\right)S_c^* & \text{if } 0 \le u_1(t) \le 1 \\ 1 & \text{if } \frac{1}{D_1}\left(v\left(\lambda_{S_c} - \lambda_V\right) - A_1\right)S_c^* \ge 1 \end{cases} \tag{18}$$

and therefore $u_1^*(t)$ is expressed as

$$u_1^*(t) = \min\left\{1,\ \max\left(0,\ \frac{1}{D_1}\left(v\left(\lambda_{S_c(t)} - \lambda_V\right) - A_1\right)S_c^*(t)\right)\right\} \tag{19}$$

Similarly the controls $u_2^*(t)$ and $u_3^*(t)$ are therefore written as

$$u_2^*(t) = \min\left\{1,\ \max\left(0,\ \frac{1}{D_2}\left(\tau\lambda_{I_c} - A_2\right)I_c^*(t)\right)\right\}$$
$$u_3^*(t) = \min\left\{1,\ \max\left(0,\ \frac{1}{D_3}\left(\lambda_H \sigma H^*(t) - A_3\right)\right)\right\} \tag{20}$$

4. Numerical Solution and Discussion

In this section, we investigate and compare numerical results of the following control strategies when applied for reducing the spread of the ND among the village chicken population. The strategies are

(i) Strategy S: vaccination of the susceptible chicken,

(ii) Strategy U: culling of the infected chicken from their flocks,

(iii) Strategy V: environmental hygiene and sanitation,

(iv) Strategy W: the combination of vaccination and culling,

(v) Strategy X: the combination of vaccination and environmental hygiene and sanitation,

(vi) Strategy Y: the combination of the culling and the environmental hygiene and sanitation,

(vii) Strategy Z: the combination of vaccination, culling, and the environmental hygiene and sanitation.

As explained in [16, 19, 39, 45], the adjoint system is solved by using the backward in time Runge-Kutta scheme with terminal conditions $\lambda_i(t_f) = 0$, where $t_f = 365$ days and initial conditions $S_c(0) = 20000$; $V(0) = 0$; $S_b(0) = 300000$; $E_c(0) = 120$; $I_c(0) = 500$, $E_b(0) = 1000$; $I_b(0) = 400$; $I_r(0) = 500$; $H(0) = 5000$. The controls are considered to be bounded in the interval $u_i(t) \in [0, 1]$ and the weights in the objective functional are estimated to be $A_1 = 0.01$ USD per vaccinated chicken, $A_2 = 20$ USD per culled individual chicken, and $A_3 = 2$ USD per square sanitation level. With initial value of controls $u_i(0)$ and the initial condition $X(0) = X_0$, the state solutions system is solved forward in time using the Runge-Kutta method of order four. The update of the control is done using a convex combination of the current and previous controls to obtain the new solution for X and λ. The method continues by using these new updates aiming at finding a fixed point (X, λ, u). This iterative process terminates when the last and preceding iterations are negligible close and the last iteration is the solution of the optimal problem. The parameter values in the state system and the objective function are obtained from different literatures and others are estimated depending on the epidemiology of the ND as shown in Table 1. In this paper all plots for state variables are in the logarithmic form.

4.1. Strategy S: Control ND with Vaccination (u_1). In this strategy, only vaccination ($u_1(t)$) is used to optimize the objective functional $\mathcal{J}$ in (4) while $u_2(t)$ and $u_3(t)$ are set to zero. The control strategy shows a significant difference in the dynamics of the susceptible chicken, infected chicken,

Table 1: The Parameter values used for running system (3).

Parameter	Value	Source
a	$0.01day^{-1}$	[12]
α_c	$1.667 \times 10^{-3} virus^{-1} chiken^{-1} day^{-1}$	[12]
α_b	$2.0 \times 10^{-5} virus^{-1} wildbird^{-1} day^{-1}$	Estimated
ψ	$0.08.3 \times 10^{-2} - 3day^{-1}$	[12]
μ	$5.80 \times 10^{-4} - 0.00136day^{-1}$	[8, 25]
b	$0.21day^{-1}$	[7, 10]
ϕ	$0.02day^{-1}$	[12]
φ	$0.0001day^{-1}$	Estimated
ρ	$0.8 - 1$	[12]
d	$0.001day^{-1}$	[12]
υ	$0.9 - 1$	[26]
τ	$0 - 1$	Estimated
σ	$0 - 1$	Estimated
μ_v	$0.00219day^{-1}$	[12]
γ	$0.067 - 0.625day^{-1}$	[4, 27]
δ_b	$0.025day^{-1}$	[28]
δ_c	$0.01989day^{-1}$	[20]
Λ_1	0.0001	Estimated
Λ_2	0.0001	Estimated
κ	100000 virus cells/m^3	Estimated
D_1	20 USD /vaccinated individual chicken	Estimated
D_2	10 USD/proportional of chicken culled	Estimated
D_3	10 USD /square of sanitation level	[20, 29]

and the concentration of the NDV in the environment when compared with the case without any control measure (see Figures 2(a)–2(d)). Using this control strategy, the number of severely infected chicken (Figure 2(b)) and the concentration of NDV in the environment (Figure 2(c)) have been significantly reduced. In Figure 2(d) the control $u_1(t)$ is maintained at its maximum value for about 396 days and thereafter decreases to zero. This means that by using this control strategy the vaccination needs to be applied at 100% effort almost throughout the control period.

4.2. Strategy U: Control ND with Culling (u_2) of the Infected Chicken. Here, the culling control ($u_2(t)$) is used to optimize the objective functional $\mathcal{J}$ in (4) while $u_1(t)$ and $u_3(t)$ are set to zero. This strategy shows a significant increase in the number of susceptible chicken (see Figure 3(a)) and a significant reduction in the number of infected chicken and the concentration of the NDV in the environment (Figures 3(b) and 3(c)) when compared with the case with no control measure. According to Figure 3(b), incubation periods of the NDV differ among host populations. When implementing the culling strategy, the removed chicken are those which show the clinical symptoms of the disease. This implies that some infected chicken which do not yet show symptoms will not be removed. Also due to the presence of carrier wild birds and contaminated environment which are other infectious agents, immediately after lowering the culling effort to zero, the disease rebounds. Thus the bending of the curves in Figures 3(b) and 6(b) toward the end of the control means that the culling strategy cannot be maintained at the maximum effort (100%) until the end of the control program. In the Figure 3(d) the control $u_2(t)$ is maintained at its upper bound for about 375 days and thereafter decreases to zero. This shows that the control strategy can be used to reduce the rate of NDV in the population but also needs to be applied at 100% effort almost throughout the control period which may be unachievable goal.

4.3. Strategy W: Control with Vaccination and Culling ($u_1(t)$ and $u_2(t)$). In this strategy, the vaccination ($u_1(t)$) and culling ($u_2(t)$) controls are applied together to optimize the objective functional $\mathcal{J}$ in (4) while $u_3(t)$ is set to zero. We observed in Figures 4(a)–4(c) that due to the combination of these control measures, the number of the susceptible chicken increases while the number of the infected chicken and the concentration of NDV in the environment decreases. The control profile in Figure 4(d) shows that the controls $u_1(t)$ and $u_2(t)$ are maintained at 100% effort for first 396 and 49 days respectively before declining to their lower bound. The results of this strategy favors vaccination over the culling due to the low cost and effectiveness of the vaccines against Newcastle disease. Thus more effort must be invested in the vaccines after killing the infected chicken in the first few days of the control program.

4.4. Strategy X: Control with Vaccination and Environmental Hygiene and Sanitation ($u_1(t)$ and $u_3(t)$). In this strategy, the vaccination ($u_1(t)$) and environmental hygiene and sanitation

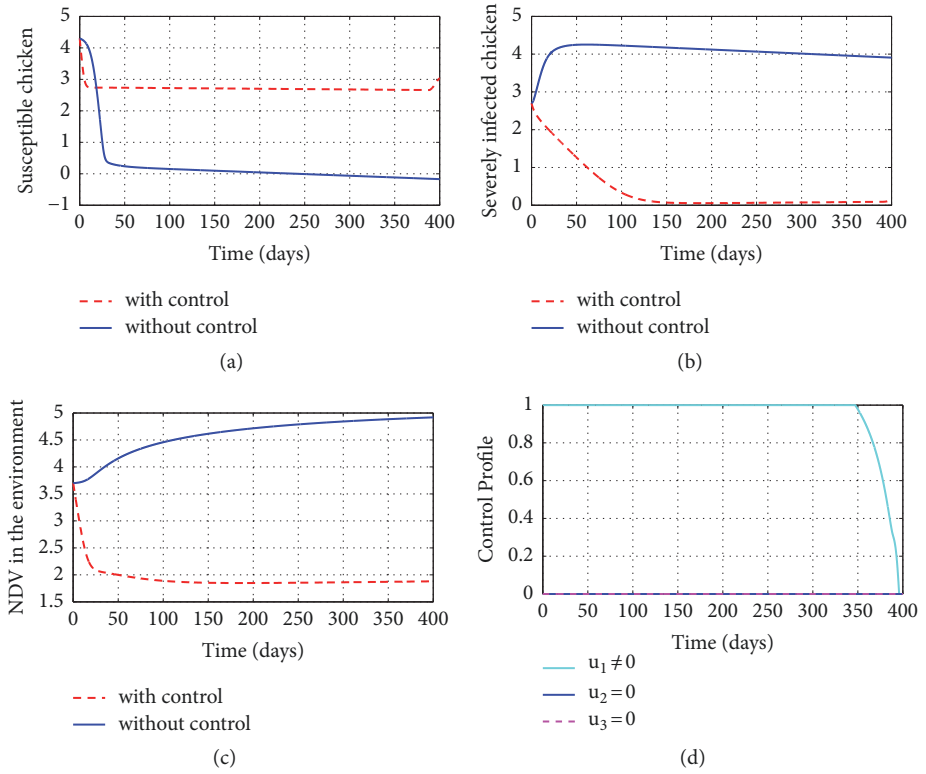

FIGURE 2: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 \neq 0$, $u_2 = 0$, $u_3 = 0$).

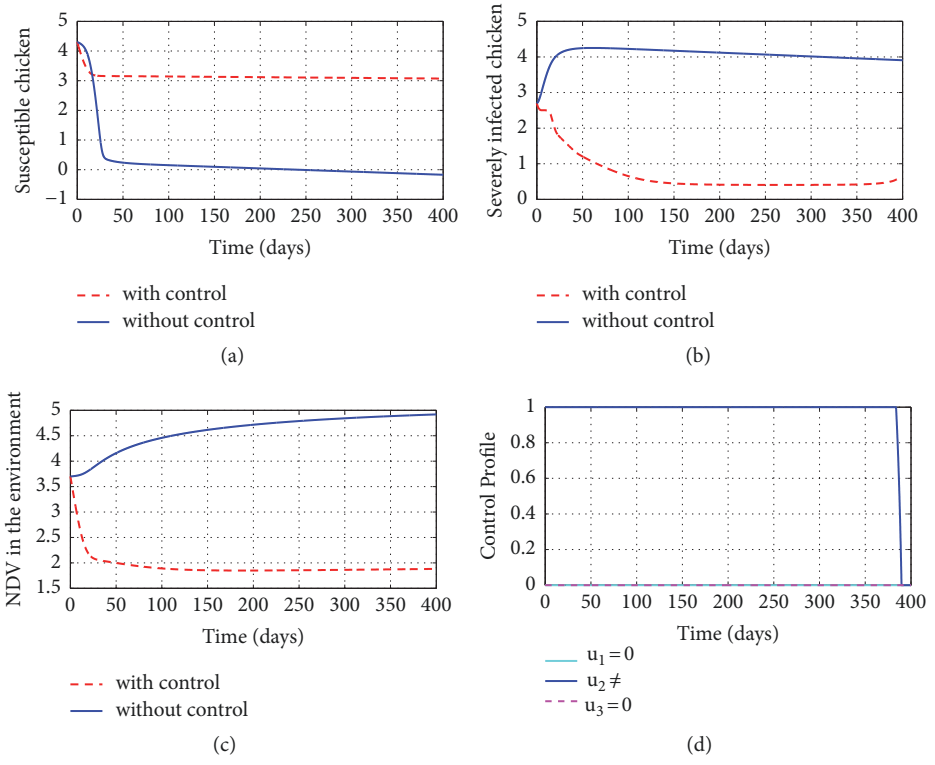

FIGURE 3: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 = 0$, $u_2 \neq 0$, $u_3 = 0$).

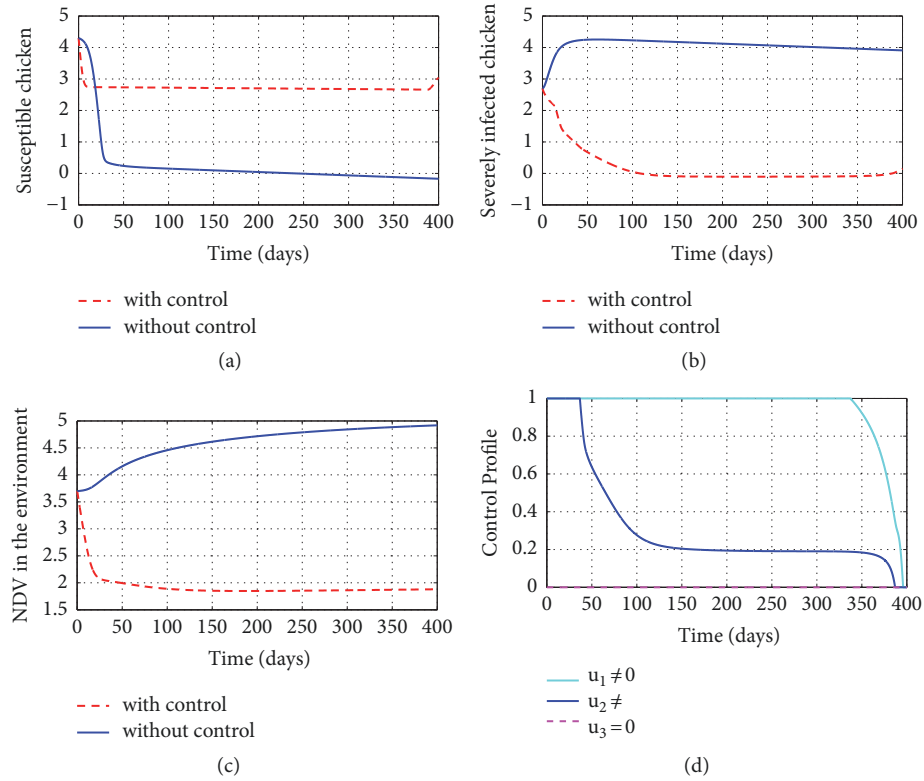

FIGURE 4: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 \neq 0, u_2 \neq 0, u_3 = 0$).

(u_3) controls are applied together to optimize the objective functional $\mathcal{J}$ in (4) while $u_2(t)$ is set to zero. There is a significant difference in the dynamics of susceptible chicken, the infected chicken and the concentration of NDV in the environment when this control strategy is used (See Figures 5(a)–5(c)). The population of susceptible chicken have increased (Figure 5(a)) while we observed a decline in the population of infected chicken and the concentration of NDV from the environment (Figures 5(b) and 5(c)) respectively. The control profile in Figure 5(d) shows that, the vaccination ($u_1(t)$) remains at its upper bound throughout a control period while the environmental hygiene and sanitation control ($u_3(t)$) is maintained at its upper bound for the first 49 days and thereafter steadily decline to its lower bound. From these results we can conclude that to control Newcastle disease it is important to invest more on the provision of the vaccines than the sanitation and hygiene of the environment.

4.5. Strategy Y: Control with Culling and Environmental Hygiene and Sanitation ($u_2(t)$ and $u_3(t)$). In this strategy, the culling ($u_2(t)$) and environmental hygiene and sanitation (u_3) controls are used to optimize the objective functional $\mathcal{J}$ in (4) while $u_1(t)$ is set to zero. In Figure 6(a) we observe that the number of susceptible chicken increases while in the Figures 6(b) and 6(c) there is significant decrease of both infected chicken and the NDV in the surroundings respectively. The increase of the susceptible chicken is due to the fact that, when the culling and environmental hygiene and sanitation are done effectively the population become free from the disease and thus, recruitment of chicken become higher and death is only through natural death. However, the increase of the clearance rate of NDV in the environment reduces the rate of spread of the disease in chicken population hence lowers the number of infected chickens. Figure 6(d) shows the control profile for u_2 and $u_3(t)$. The control profile for u_3 drops to its lower bound and maintains it until the end of the control period. This result reveals that environmental hygiene and sanitation has little contribution in the control of the Newcastle disease when applied together with the culling strategy.

4.6. Strategy Z: Control with Vaccination, Culling, and Environmental Hygiene and Sanitation ($u_1(t)$, $u_2(t)$, and $u_3(t)$). In this strategy, all controls u_1, u_2, and u_3 are used together to optimize the objective functional $\mathcal{J}$ in (4). In Figure 7(a) it can be seen that susceptible chicken population increases while the infected chicken population and the concentration of NDV in the environment were greatly reduced (Figures 7(b) and 7(c)) respectively. In Figure 7(d), the vaccination control (u_1) is maintained at its maximum value for the whole year while the culling control (u_2) is maintained at its maximum value for 47 days and then decline slowly for 373 days before it further decline to zero. The environmental and sanitation strategy ($u_3(t)$) has very low contributions in this strategy as it maintained at zero throughout the control program. This strategy managed to reduce the rate of transmission of the disease to a very low level and maintain it at this same level for the entire period of the control program.

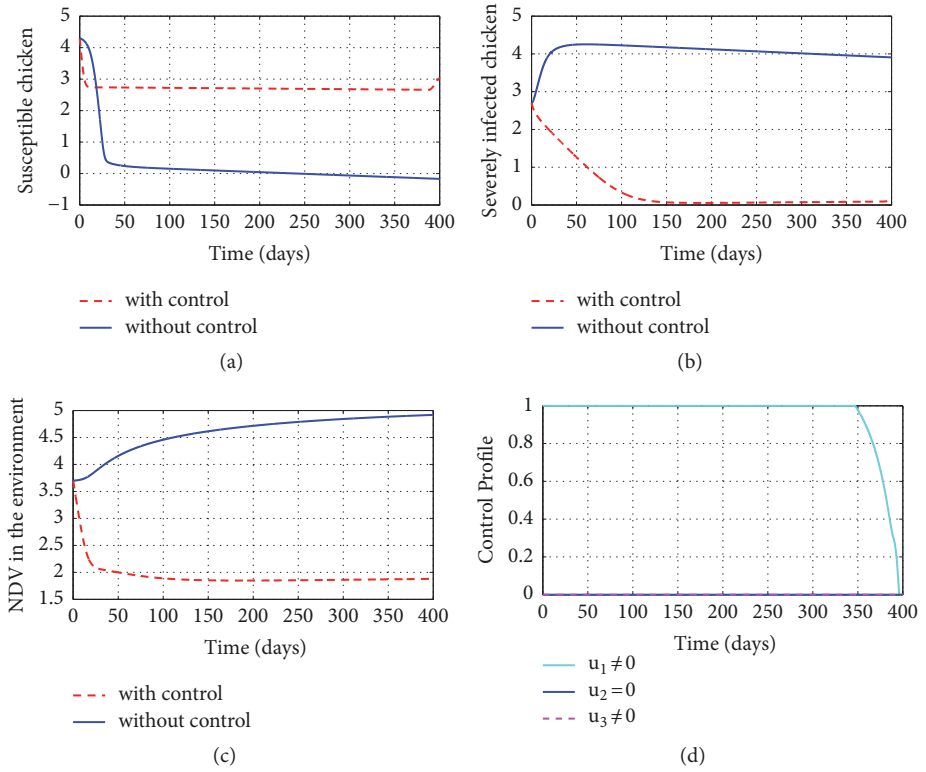

Figure 5: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 \neq 0, u_2 = 0, u_3 \neq 0$).

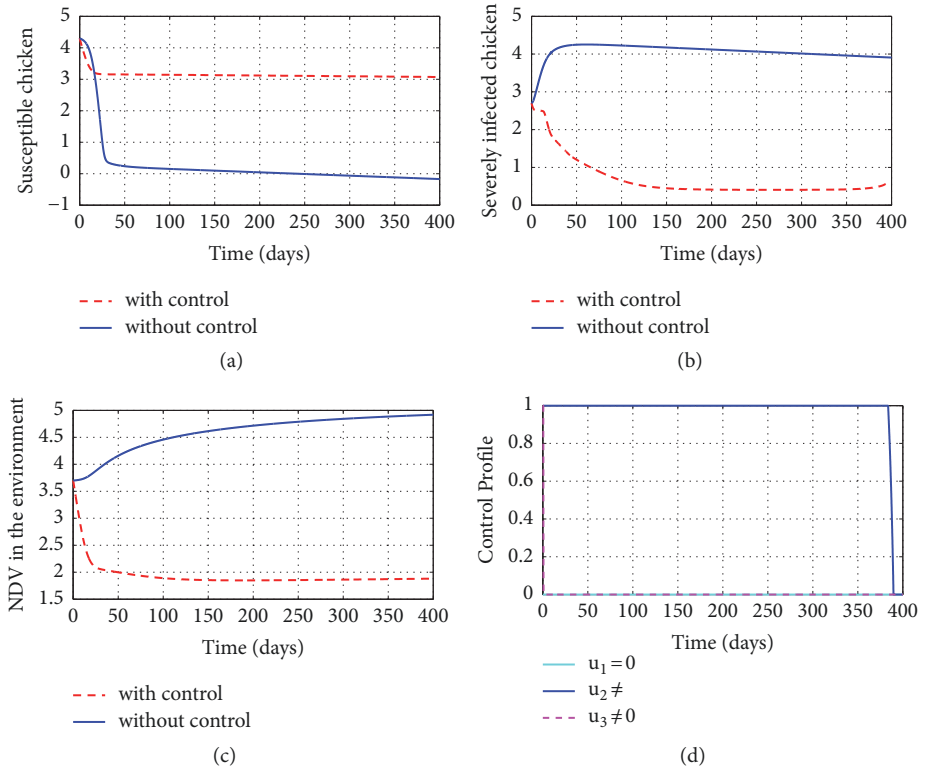

Figure 6: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 = 0, u_2 \neq 0, u_3 \neq 0$).

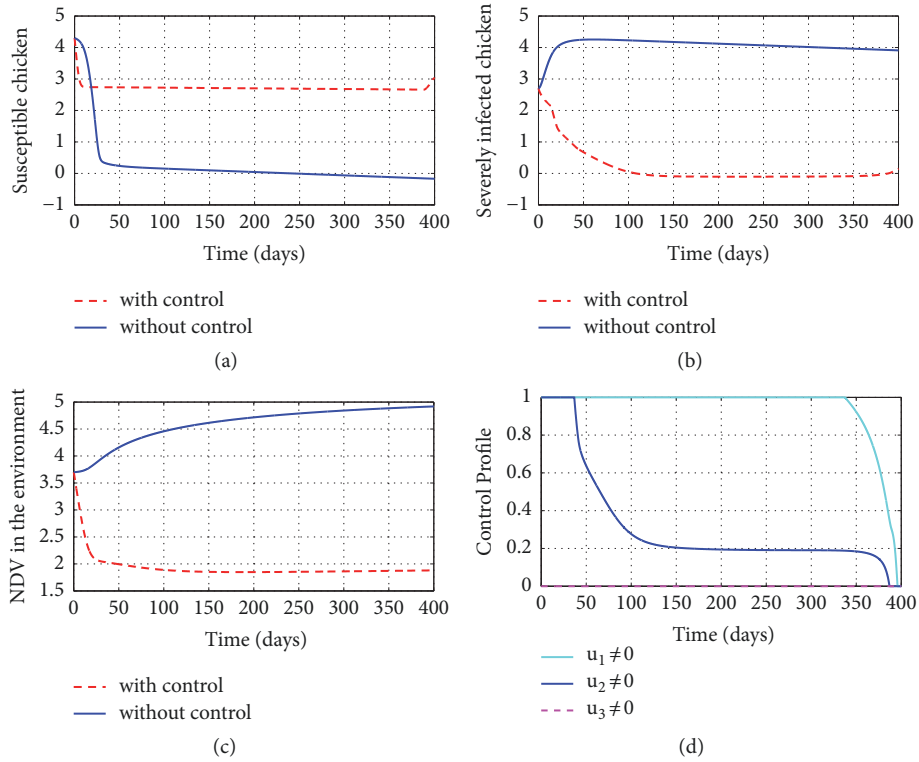

FIGURE 7: Optimal solutions for model variables S_c, I_c, H and the control profiles for u_1, u_2, u_3 with ($u_1 \neq 0, u_2 \neq 0, u_3 \neq 0$).

TABLE 2: The arrangement of the strategies in ascending order of the total infection averted.

Strategy	Infection averted	Total Cost in US Dollar	ICER
No control	0	0	-
Strategy U	3.6719×10^8	4.057×10^5	+0.0011
Strategy Y	3.8262×10^8	4.038×10^5	−0.000123
Strategy S	4.8263×10^8	7.7633×10^3	−0.00396
Strategy W	4.8405×10^8	2.2098×10^5	+0.1502
Strategy Z	4.8435×10^8	2.9658×10^5	+0.0252
Strategy X	4.8449×10^8	3.3525×10^5	+0.27621

4.7. Cost-Effectiveness Analysis. The cost-effective analysis helps identify the control strategy that is less costly toward the control of ND. In this section, the cost-effectiveness analysis is conducted using the incremental cost-effectiveness ratio (ICER) method. In this method, the costs and effects of two interventions competing under the scarcity of resources were compared [34, 36, 46]. To quantify the cost effectiveness of the control strategies, we obtained the ratios for the difference of cost between the two control strategies and the difference of the total number of infections averted by these strategies [34, 47]. By using the parameter values given in Table 1, the ICER numerical values are obtained in Table 2.

Referring to Table 2, the comparison of strategies U and Y shows that the ICER of strategy Y is less than that of strategy U, indicating that strategy U is more costly and less effective. Therefore, we omitted strategy U and recalculated ICER for the remaining strategies.

TABLE 3: The cost and infection averted for controls Y, S, W, Z, and X.

Strategy	Infection averted	Total Cost in US Dollar	ICER
Strategy Y	3.8262×10^8	4.038×10^5	+0.00106
Strategy S	4.8263×10^8	7.7633×10^3	−0.00396
Strategy W	4.8405×10^8	2.2098×10^5	+0.1502
Strategy Z	4.8435×10^8	2.9658×10^5	+0.0252
Strategy X	4.8449×10^8	3.3525×10^5	+0.27621

TABLE 4: The cost and infection averted for controls S, W, Z, and X.

Strategy	Infection averted	Total Cost in US Dollar	ICER
Strategy S	4.8263×10^8	7.7633×10^3	+0.0000161
Strategy W	4.8405×10^8	2.2098×10^5	+0.1502
Strategy Z	4.8435×10^8	2.9658×10^5	+0.0252
Strategy X	4.8449×10^8	3.3525×10^5	+0.27621

From Table 3, the comparison of strategies S and Y shows that the ICER of strategy S is less than that of strategy Y, implying that strategy Y is more costly and less effective. Therefore, we omitted strategy Y and recalculated ICER for the remaining strategies.

The comparison of strategies S and W in Table 4 shows that the ICER of strategy S is less than that of strategy W, indicating that strategy W is more costly and less effective. Therefore, we omitted strategy W and recalculated ICER for the remaining strategies.

Table 5: The cost and infection averted for controls S, Z, and X.

Strategy	Infection averted	Total Cost in US Dollar	ICER
Strategy S	4.8263×10^8	7.7633×10^3	+0.0000161
Strategy Z	4.8435×10^8	2.9658×10^5	+0.167917
Strategy X	4.8449×10^8	3.3525×10^5	+0.276214

Table 6: The cost and infection averted for controls S and X.

Strategy	Infection averted	Total Cost in US Dollar	ICER
Strategy S	4.8263×10^8	7.7633×10^3	+0.0000161
Strategy X	4.8449×10^8	3.3525×10^5	+0.276214

The comparison of strategies S and Z in Table 5 shows that the ICER of strategy S is less than that of strategy W, indicating strategy W to be more costly and less effective. Therefore, we omitted strategy W and recalculated ICER for the remaining strategies.

Using the infection averted and the total costs as in Table 6, the ICER values are computed as follows:

$$\begin{aligned} \text{ICER (Strategy S)} &= \frac{7.7633 \times 10^3}{4.8263 \times 10^8} = +0.0000161 \\ \text{ICER (Strategy X)} &= \frac{3.3525 \times 10^5 - 7.7633 \times 10^3}{4.8449 \times 10^8 - 4.8263 \times 10^8} \\ &= +0.276214 \end{aligned} \tag{21}$$

The comparison of strategies S and X in Table 6 shows that the ICER of strategy S is less than that of strategy X, indicating that the application of vaccination alone as a control measure of ND in village chicken (strategy S) is less costly and more effective than the strategy that combines vaccination and environmental hygiene and sanitation (strategy X).

5. Conclusion

This work presented a mathematical model describing the dynamics of ND in the village chicken population. The purpose of this study was to analyze the effects of vaccination, culling, and environmental hygiene and sanitation control strategies on the village chicken population. The efficiency of each strategy: susceptible, culling, environmental hygiene and sanitation, combination of the vaccination and culling strategies, combination of culling and environmental hygiene and sanitation and the combination of vaccination, culling and hygiene and sanitation control strategies was evaluated using Pontryagin's Maximum Principle. Our analysis revealed that strategy X, a combination of vaccination and environmental hygiene and sanitation, reduces the population of infected chicken and ND virus concentration more than the other control strategies. Therefore, we recommend the adoption of vaccination and environmental hygiene and sanitation strategies for the village chicken growers at 100% effort as the most effective way to combat the devastating effects of ND. Moreover, the cost-effective analysis suggests that strategy S (vaccination) is the most cost effective of all the combinations of strategies under the scarcity of resources. Although strategy S seems to be less costly than other control strategies, we suggest the adoption of strategy X as the best control practice for reducing the risk of the disease to the village chicken population. In the future, the study will be extended by considering the age structure of the chicken and will include the investigation of the role of temperature and humidity on the transmission of ND in the village chicken population.

Data Availability

The data used to support the findings of this study are included within the article. We use a set of parameter values whose sources are from literature and others are estimated depending on the epidemiology of the ND as shown in Table 1. The weights in the cost functional are the average operational costs of vaccines and clearing equipment as in the local markets in Kibaha and Bagamoyo, the districts of a Coastal Region, Tanzania.

Conflicts of Interest

There are no conflicts of interest among the authors.

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