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Research Article

Different Physical Structures of Solutions for a Generalized Resonant Dispersive Nonlinear Schrödinger Equation with Power Law Nonlinearity

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In this work, we investigate various types of solutions for the generalised resonant dispersive nonlinear Schrödinger equation (GRD-NLSE) with power law nonlinearity. Based on simple mathematical techniques, the complicated form of the GRD-NLSE is reduced to an ordinary differential equation (ODE) which has a variety of solutions. The analytic solution of the resulting ODE gives rise to bright soliton, singular soliton, peaked soliton, compacton solutions, solitary pattern solutions, rational solution, Weierstrass elliptic periodic type solutions, and some other types of solutions. Constraint conditions for the existence of solitons and other solutions are given.

1. Introduction

Solitons have become one of the more attractive topics in the physical and natural science. The reason of this remarkable importance is that this type of nonlinear waves has many applications in the study of nonlinear optics, plasma physics, fluid dynamics, and several other disciplines [1–24]. For example, solitons transport information through optical fibers over transcontinental and transoceanic distances in a matter of a few femtoseconds. Furthermore, they also appear in Bose-Einstein condensates, α-helix proteins in clinical sciences, nuclear physics, and several others. The governing equation of such model is the nonlinear Schrödinger equation (NLSE).

The formation of solitons in nonlinear optics, for example, is mainly due to a delicate balance between dispersion and nonlinearity in a model of NLSE. To analyse the dynamics of solitons, it is worthwhile to focus deeply on one model of the NLS family of equations with higher order nonlinear terms. There are many powerful mathematical tools that have been developed to study the behaviour of solitons in a medium dominated by NLSE. For more details, see [25–31]. In the present work, we will shed light on the study of the generalised resonant dispersive nonlinear Schrödinger equation (GRD-NLSE) with power law nonlinearity.

The model of GRD-NLSE which is studied in the current paper has the form

$$i\left(|\psi|^{n-1}\psi\right)_t + \alpha\left(|\psi|^{n-1}\psi\right)_{xx} + \beta|\psi|^m\psi + \gamma\left\{\frac{\left(|\psi|^n\right)_{xx}}{|\psi|}\right\}\psi = 0, \tag{1}$$

where α, β, and γ are real-valued constants. The dependent variable $\psi(x,t)$ is a complex-valued wave profile. Recently, the GRD-NLSE has been studied by many authors to examine the behaviour of solutions. Several integration schemes have been implemented to construct exact solutions such as ansatz method [13, 14], semi-inverse variational principle [15], simplest equation approach [16], first integral method [17], functional variable method, sine–cosine function method [18], (G'/G)-expansion method [19], trial solution approach [20], generalised extended tanh method [21], modified simple equation method [22], and improved extended tanh-equation method [23].

In this paper, we aim to investigate the solitons and other types of solutions to GRD-NLSE. To achieve our goal, simple integration schemes will be applied to reduce the complicated form of GRD-NLSE to an ODE possessing various types of solutions. Solving the resulting ODE yields different physical structures of solutions for GRD-NLSE such as bright soliton, singular soliton, peaked soliton, compacton solutions, solitary pattern solutions, rational solution, Weierstrass elliptic periodic type solutions, and some other types of solutions.

In the following section, (1) will be simplified to an ODE and then different types of exact solutions to this ODE will be extracted.

2. Mathematical Analysis and Solutions

In order to deal with the complicated form of the GRD-NLSE given by (1), we assume the travelling wave solution of the form

$$\psi(x,t) = U(\xi)\, e^{i(-\kappa x+\omega t+\theta)}, \quad \xi = x + 2\kappa\alpha t. \tag{2}$$

Hence, (1) reduces to the following ordinary differential equation:

$$(\alpha+\gamma)\left(U^n\right)'' - \left(\omega+\alpha\kappa^2\right)U^n + \beta U^{m+1} = 0, \tag{3}$$

where prime denotes the derivative with respect to ξ. Setting $V = U^n$, (3) becomes in the form

$$(\alpha+\gamma)V'' - \left(\omega+\alpha\kappa^2\right)V + \beta V^{(m+1)/n} = 0. \tag{4}$$

Now, multiplying by V' and integrating once yield the following first-order ODE:

$$(\alpha+\gamma)\left(\frac{\mathrm{d}V}{\mathrm{d}\xi}\right)^2 = \left(\omega+\alpha\kappa^2\right)V^2 - \frac{2n\beta}{m+n+1}V^{(m+n+1)/n}, \tag{5}$$

where the constant of integration is taken to be zero.

2.1. Solitary Wave Solution. Here, we aim to obtain the solitary wave solution of (1). Therefore, separating variables and integrating (5) give

$$\int \frac{\mathrm{d}V}{V\sqrt{1-\left(2n\beta/(m+n+1)\left(\omega+\alpha\kappa^2\right)\right)V^{(m-n+1)/n}}} = \sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}\,\xi, \tag{6}$$

which leads to

$$\frac{2n}{n-m-1}\cdot\tanh^{-1}\left(\sqrt{1-\frac{2n\beta}{(m+n+1)\left(\omega+\alpha\kappa^2\right)}V^{(m-n+1)/n}}\right) = \sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}\,\xi. \tag{7}$$

Equation (7) can be manipulated to yield

$$V(\xi) = A\left\{\operatorname{sech}^2\left[B\xi\right]\right\}^{n/(m-n+1)}, \tag{8}$$

which represents a solitary wave with the amplitude

$$A = \left\{\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{2n\beta}\right\}^{n/(m-n+1)}, \tag{9}$$

and the inverse width

$$B = \frac{n-m-1}{2n}\sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}, \tag{10}$$

where

$$\left(\omega+\alpha\kappa^2\right)(\alpha+\gamma) > 0. \tag{11}$$

Eventually, the nontopological 1-soliton solution with power law nonlinearity to (1) is given by

$$\psi(x,t) = A_0\left\{\operatorname{sech}^2\left[B(x+2\kappa\alpha t)\right]\right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{12}$$

where the amplitude, $A_0 = A^{1/n}$, and width of the ψ profile are given by (9) and (10), respectively. The constraint given by (11) must stay valid in order for the soliton solution to exist.

2.2. Peakon Solution. In this subsection, we intend to find the peaked soliton of (1). Hence, we substitute the peakon assumption

$$V(\xi) = pe^{-r|\xi|}, \tag{13}$$

into (5) and solve the resulting equation to find that

$$r = \sqrt{\frac{\omega+\alpha\kappa^2-\beta}{\alpha+\gamma}}, \tag{14}$$

with the constraint $m = n-1$ and p can be any selective real number. Eventually, the peakon solution to (1) is given by

$$\psi(x,t) = \left\{pe^{-\sqrt{(\omega+\alpha\kappa^2-\beta)/(\alpha+\gamma)}|x+2\kappa\alpha t|}\right\}^{1/n} e^{i(-\kappa x+\omega t+\theta)}. \tag{15}$$

In case p is a negative constant so another type of peakon is presented, namely, antipeakon.

Next, replacing the constant p by $p\,\mathrm{sign}(\xi)$ in (13) and substituting into (5) give a new solution to (1) called shock-peakon solution which can be written in the form

$$\psi(x,t) = \left\{p\,\mathrm{sign}(\xi)\, e^{-\sqrt{(\omega+\alpha\kappa^2-\beta)/(\alpha+\gamma)}|\xi|}\right\}^{1/n} e^{i(-\kappa x+\omega t+\theta)}, \tag{16}$$

where $\xi = x+2\kappa\alpha t$ and $\mathrm{sign}(\xi) = \xi/|\xi|$. This type of peakons is a discontinuous wave so it is a shock wave. As we can see from

the computation it has a discontinuous first-order derivative at $\xi = 0$.

2.3. Compacton and Solitary Pattern Solutions. In order to obtain compacton and solitary patterns solutions of (1) we multiply (5) by $V^{-(m+n+1)/n}$ to arrive at

$$(\alpha+\gamma)\left[V^{-(m+n+1)/2n}\frac{\mathrm{d}V}{\mathrm{d}\xi}\right]^2 = \left(\omega+\alpha\kappa^2\right)V^{(n-m-1)/n} - \frac{2n\beta}{m+n+1}, \tag{17}$$

from which we find

$$(\alpha+\gamma)\left(\frac{2n}{n-m-1}\right)^2\left[\frac{\mathrm{d}V^{(n-m-1)/2n}}{\mathrm{d}\xi}\right]^2 = \left(\omega+\alpha\kappa^2\right)V^{(n-m-1)/n} - \frac{2n\beta}{m+n+1}. \tag{18}$$

Then, (18) can be simplified by assuming $\phi = V^{(n-m-1)/2n}$ to obtain

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 = \frac{1}{(\alpha+\gamma)}\left(\frac{n-m-1}{2n}\right)^2 \cdot\left[\left(\omega+\alpha\kappa^2\right)\phi^2 - \frac{2n\beta}{m+n+1}\right]. \tag{19}$$

Solving (19), we obtain the following periodic type solutions:

$$\begin{aligned}\psi(x,t) &= \left\{A_1\cos^2\left[B_1(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)},\\ \psi(x,t) &= \left\{A_1\sin^2\left[B_1(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)},\end{aligned} \tag{20}$$

which are valid when $(\omega+\alpha\kappa^2)(\alpha+\gamma) < 0$, where

$$A_1 = \frac{2n\beta}{(m+n+1)\left(\omega+\alpha\kappa^2\right)}, \qquad B_1 = \left(\frac{n-m-1}{2n}\right)\sqrt{-\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}. \tag{21}$$

In case of $n > m+1$, we arrive at the compacton solutions in the form

$$\begin{aligned}\psi(x,t) &= \left\{A_1\cos^2\left[B_1(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)}, \quad \left|B_1(x+2\kappa\alpha t)\right| \le \frac{\pi}{2},\\ \psi(x,t) &= 0, \quad \text{otherwise,}\end{aligned} \tag{22}$$

and

$$\begin{aligned}\psi(x,t) &= \left\{A_1\sin^2\left[B_1(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)}, \quad \left|B_1(x+2\kappa\alpha t)\right| \le \pi,\\ \psi(x,t) &= 0, \quad \text{otherwise.}\end{aligned} \tag{23}$$

Now, when the constant $(\omega+\alpha\kappa^2)(\alpha+\gamma) > 0$, (19) admits the solitary pattern solutions

$$\psi(x,t) = \left\{A_1\cosh^2\left[B_2(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{24}$$

$$\psi(x,t) = \left\{-A_1\sinh^2\left[B_2(x+2\kappa\alpha t)\right]\right\}^{1/(n-m-1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{25}$$

where A_1 is give by (21) and

$$B_2 = \left(\frac{n-m-1}{2n}\right)\sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}. \tag{26}$$

2.4. Exponential Solution. In case of $m = n-1$, then (5) will be reduced to

$$\left(\frac{\mathrm{d}V}{\mathrm{d}\xi}\right)^2 = \frac{\omega+\alpha\kappa^2-\beta}{\alpha+\gamma}V^2. \tag{27}$$

Separating variables and integrating (27) give

$$V(\xi) = \left\{e^{c\pm\sqrt{(\omega+\alpha\kappa^2-\beta)/(\alpha+\gamma)}\xi}\right\}, \tag{28}$$

where c is the constant of integration. As a result, (1) possesses an exponential type solution in the form

$$\psi(x,t) = \left\{e^{c\pm\sqrt{(\omega+\alpha\kappa^2-\beta)/(\alpha+\gamma)}(x+2\kappa\alpha t)}\right\}^{1/n} e^{i(-\kappa x+\omega t+\theta)}, \tag{29}$$

when $m = n-1$.

2.5. Other Solutions. Now, we aim to extract more types of solutions to (1) using straightforward mathematical approach. Thus, we reduce (5) to a simple ODE by means of the following transformation. Letting

$$V^{m-n+1/n} = W^2, \tag{30}$$

we find

$$V = W^{2n/(m-n+1)}, \tag{31}$$

from which we reach

$$\mathrm{d}V = \frac{2n}{m-n+1}\mathrm{d}W^{2n/(m-n+1)-1}. \tag{32}$$

Substituting (31) and (32) into (5) results in the following equation:

$$\left(\frac{dW}{d\xi}\right)^2 = \frac{1}{(\alpha+\gamma)}\left(\frac{m-n+1}{2n}\right)^2 \cdot \left[\left(\omega+\alpha\kappa^2\right)W^2 - \frac{2n\beta}{m+n+1}W^4\right]. \tag{33}$$

Solving this equation, one can obtain the following types of solutions.

2.5.1. Rational Type Solution. In case of $\omega = -\alpha\kappa^2$, (1) admits the rational solution of the form

$$\psi(x,t) = \left\{\frac{-2n(m+n+1)(\alpha+\gamma)}{\beta(m-n+1)^2(x+2\kappa\alpha t+c)^2}\right\}^{1/(m-n+1)} \cdot e^{i(-\kappa x+\omega t+\theta)}, \tag{34}$$

where c is the constant of integration.

2.5.2. Complex Type Solution. The solution to (33) brings about the following complex solution for (1):

$$\psi(x,t) = \left\{-\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{n\beta}\frac{1}{\epsilon i\sinh\left[((m-n+1)/n)\sqrt{\left(\omega+\alpha\kappa^2\right)/(\alpha+\gamma)}(x+2\kappa\alpha t)\right]-1}\right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{35}$$

where $\epsilon = \pm 1$ and $i = \sqrt{-1}$. This solution demands

$$\left(\omega+\alpha\kappa^2\right)(\alpha+\gamma) > 0. \tag{36}$$

2.5.3. Bright Soliton Solutions. The solution to (33) leads to the following forms of bright solitons for (1):

$$\psi(x,t) = \left\{\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{2n\beta}\operatorname{sech}^2\left[\frac{m-n+1}{2n} \cdot \sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}(x+2\kappa\alpha t)\right]\right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}. \tag{37}$$

This soliton is valid for

$$\left(\omega+\alpha\kappa^2\right)(\alpha+\gamma) > 0.$$

$$\psi(x,t) = \left\{\frac{a\operatorname{sech}^2(x+2\kappa\alpha t)}{b\operatorname{sech}^2(x+2\kappa\alpha t)+c\tanh(x+2\kappa\alpha t)-2b}\right\}^{2/(m-n+1)} \cdot e^{i(-\kappa x+\omega t+\theta)}, \tag{38}$$

which is valid when

$$\left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right) = 4, \qquad \left(\frac{m-n+1}{2n}\right)^2\frac{2n\beta}{(m+n+1)(\alpha+\gamma)} = \frac{4b^2-c^2}{a^2}, \tag{39}$$

where a, b, c are arbitrary constants.

$$\psi(x,t) = \left\{\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{n\beta}\frac{1}{\cosh\left[((m-n+1)/n)\sqrt{\left(\omega+\alpha\kappa^2\right)/(\alpha+\gamma)}(x+2\kappa\alpha t)\right]+1}\right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}. \tag{40}$$

Solution (40) is valid when

$$\left(\omega+\alpha\kappa^2\right)(\alpha+\gamma) > 0. \tag{41}$$

2.5.4. Singular Soliton Solutions. The solution to (33) gives rise to the following forms of singular solitons for (1):

$$\psi(x,t) = \left\{\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{2n\beta}\operatorname{csch}^2\left[\frac{m-n+1}{2n} \cdot \sqrt{\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}(x+2\kappa\alpha t)\right]\right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}. \tag{42}$$

This soliton is valid for

$$\left(\omega+\alpha\kappa^2\right)(\alpha+\gamma) > 0.$$

$$\psi(x,t)$$

$$= \left\{ \frac{a \operatorname{csch}^2 (x + 2\kappa\alpha t)}{b \operatorname{csch}^2 (x + 2\kappa\alpha t) + c \coth (x + 2\kappa\alpha t) + 2b} \right\}^{2/(m-n+1)} \cdot e^{i(-\kappa x + \omega t + \theta)}, \tag{43}$$

which is valid when

$$\left(\frac{m-n+1}{2n}\right)^2 \left(\frac{\omega + \alpha\kappa^2}{\alpha + \gamma}\right) = 4,$$
$$\left(\frac{m-n+1}{2n}\right)^2 \frac{2n\beta}{(m+n+1)(\alpha+\gamma)} = \frac{4b^2 - c^2}{a^2}, \tag{44}$$

where a, b, c are arbitrary constants.

$$\psi(x,t) = \left\{ -\frac{(m+n+1)(\omega+\alpha\kappa^2)}{n\beta} \frac{1}{\cosh\left[((m-n+1)/n)\sqrt{(\omega+\alpha\kappa^2)/(\alpha+\gamma)}\,(x+2\kappa\alpha t)\right] - 1} \right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{45}$$

Solution (45) is valid when

$$(\omega + \alpha\kappa^2)(\alpha + \gamma) > 0. \tag{46}$$

2.5.5. Singular Periodic Solutions. The solution to (33) provides the following variety of singular periodic solutions for (1).

$$\psi(x,t) = \left\{ \frac{(m+n+1)(\omega+\alpha\kappa^2)}{2n\beta} \sec^2 \left[\frac{m-n+1}{2n} \cdot \sqrt{-\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}\,(x+2\kappa\alpha t) \right] \right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{47}$$

$$\psi(x,t) = \left\{ \frac{(m+n+1)(\omega+\alpha\kappa^2)}{2n\beta} \csc^2 \left[\frac{m-n+1}{2n} \cdot \sqrt{-\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}}\,(x+2\kappa\alpha t) \right] \right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}. \tag{48}$$

Equations (47) and (48) demand

$$(\omega + \alpha\kappa^2)(\alpha + \gamma) < 0. \tag{49}$$

$$\psi(x,t) = \left\{ \frac{a \sec^2 (x + 2\kappa\alpha t)}{b \sec^2 (x + 2\kappa\alpha t) + c \tan (x + 2\kappa\alpha t) - 2b} \right\}^{2/(m-n+1)} \cdot e^{i(-\kappa x + \omega t + \theta)}, \tag{50}$$

$$\psi(x,t) = \left\{ \frac{a \csc^2 (x + 2\kappa\alpha t)}{b \csc^2 (x + 2\kappa\alpha t) + c \cot (x + 2\kappa\alpha t) - 2b} \right\}^{2/(m-n+1)} \cdot e^{i(-\kappa x + \omega t + \theta)}. \tag{51}$$

Both solutions (50) and (51) are valid for

$$\left(\frac{m-n+1}{2n}\right)^2 \left(\frac{\omega + \alpha\kappa^2}{\alpha + \gamma}\right) = -4,$$
$$\left(\frac{m-n+1}{2n}\right)^2 \frac{2n\beta}{(m+n+1)(\alpha+\gamma)} = -\frac{4b^2 + c^2}{a^2}, \tag{52}$$

where a, b, c are arbitrary constants.

$$\psi(x,t) = \left\{ -\frac{(m+n+1)(\omega+\alpha\kappa^2)}{n\beta} \frac{1}{\epsilon\cos\left[((m-n+1)/n)\sqrt{-(\omega+\alpha\kappa^2)/(\alpha+\gamma)}\,(x+2\kappa\alpha t)\right] - 1} \right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{53}$$

$$\psi(x,t) = \left\{ -\frac{(m+n+1)(\omega+\alpha\kappa^2)}{n\beta} \frac{1}{\epsilon\sin\left[((m-n+1)/n)\sqrt{-(\omega+\alpha\kappa^2)/(\alpha+\gamma)}\,(x+2\kappa\alpha t)\right] - 1} \right\}^{1/(m-n+1)} e^{i(-\kappa x+\omega t+\theta)}, \tag{54}$$

where $\epsilon = \pm 1$. Solutions (53) and (54) imply that

$$\left(\omega + \alpha\kappa^2\right)(\alpha + \gamma) < 0. \tag{55}$$

2.5.6. Weierstrass Elliptic Periodic Type Solutions. The solution to (33) generates Weierstrass elliptic type solutions for (1) in the following forms:

$$\psi(x,t) = \left\{-\frac{2n(m+n+1)(\alpha+\gamma)}{3(m-n+1)^2\beta}\left[3\wp\left((x+2\kappa\alpha t), g_2, g_3\right) - \left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right)\right]\right\}^{1/(m-n+1)} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{56}$$

where the invariants of the Weierstrass elliptic function are given by

$$g_2 = \frac{4}{3}\left[\left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right)\right]^2, \quad g_3 = -\frac{8}{27}\left[\left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right)\right]^3. \tag{57}$$

$$\psi(x,t) = \left\{-\frac{2n(m+n+1)(\alpha+\gamma)}{(m-n+1)^2\beta}\left[\frac{3\wp'\left((x+2\kappa\alpha t), g_2, g_3\right)}{6\wp\left((x+2\kappa\alpha t), g_2, g_3\right) + ((m-n+1)/2n)^2\left((\omega+\alpha\kappa^2)/(\alpha+\gamma)\right)}\right]^2\right\}^{1/(m-n+1)} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{58}$$

$$\psi(x,t) = \left\{\frac{(m+n+1)\left(\omega+\alpha\kappa^2\right)}{2n\beta}\left[\frac{12\wp\left((x+2\kappa\alpha t), g_2, g_3\right) - ((m-n+1)/2n)^2\left(\left(\omega+\alpha\kappa^2\right)/(\alpha+\gamma)\right)}{12\wp\left((x+2\kappa\alpha t), g_2, g_3\right) + 5((m-n+1)/2n)^2\left((\omega+\alpha\kappa^2)/(\alpha+\gamma)\right)}\right]^2\right\}^{1/(m-n+1)} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{59}$$

where both solutions (58) and (59) are valid for the invariants of the Weierstrass elliptic function given by

$$g_2 = \frac{1}{12}\left[\left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right)\right]^2, \quad g_3 = -\frac{1}{216}\left[\left(\frac{m-n+1}{2n}\right)^2\left(\frac{\omega+\alpha\kappa^2}{\alpha+\gamma}\right)\right]^3. \tag{60}$$

Overall, the majority of results obtained here for (1) are very new. In comparison with some previous studies, the solutions given by (34), (35), (37), (42), (47), (48), and (54) are already derived in [18, 23] while the rest of the solutions extracted here are new exact solutions.

It should be noted that the proposed transformation in (31) has led to the ODE (33) which is simpler than that derived in [23]. Further to this, (33) can be converted into the form

$$\frac{d^2W}{d\xi^2} = \frac{1}{(\alpha+\gamma)}\left(\frac{m-n+1}{2n}\right)^2 \cdot \left[\left(\omega+\alpha\kappa^2\right)W - \frac{4n\beta}{m+n+1}W^3\right]. \tag{61}$$

This second-order equation is known to admit the application of many solution methods like the Jacobi elliptic function method [32], the exp-function method [33], the G'/G-expansion method [34], the generalised Kudryashov method [35], etc. As a result, a lot of exact analytic solutions can be constructed to the GRD-NLSE (1).

3. Discussion and Conclusion

This study scoped different physical structures of solutions for GRD-NLSE with power law nonlinearity. Applying a simple mathematical scheme allowed us to simplify the complex form of GRD-NLSE to an ODE. It is found that the constructed ODE is rich in various types of solitons and other solutions for GRD-NLSE. The derived solutions include bright soliton, singular soliton, peaked soliton, compacton solutions, solitary pattern solutions, rational solution, trigonometric function solutions, and Weierstrass elliptic periodic type solutions. All generated solutions are verified by utilising symbolic computation. The results obtained here can be useful to understand the physics of nonlinear optical fibers.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

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