Elimination of the Impact Produced by DG Units on the Voltage Profile of Distribution Networks

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In this paper, an alternative strategy for real-time control of active distribution network voltage is developed, not by controlling the bus voltage as in the various centralized, decentralized, and local approaches presented in literature but rather by only eliminating the impact produced by active and reactive power of distributed generation (DG) units on the voltage of all network nodes and keeping the traditional voltage control systems dealing with the same constraints of passive systems. In literature, voltage deterioration introduced by DGs has been reported as one of the main obstacles for the interconnection of large amounts of DG units to the existing networks. In this paper, the novel control strategy is based on a sensitivity formula developed to calculate the compensation needed for additional distributed flexible AC transmission system (D-FACTS) devices to push and pull the exact reactive power and to eliminate the impact produced by DGs on the network voltage profile. The criteria of the allocation of the var devices and the required network reinforcement are developed in this paper, considering all possible topology structures, and an innovative codification method is introduced to reduce the needed computation time and communication data to actualize the sensitivity coefficients and get the proposed control approach flexible with network topology reconfiguration. The risk of the conflict of the proposed control system with the traditional voltage equipment is reduced due to the fast capability of D-FACTS devices to regulate their reactive power in finer granularity. A case study of two meshed IEEE 15-bus feeders is introduced to compare the voltage behavior with and without the presence of DG units and to evaluate the total system losses. The proposed method could be used for the interconnection of the first generation units in emerging networks, which does not yet have an active voltage control strategy, as it could be used for DG units not able to be connected to existing centralized control systems and it could also be used as the principal voltage control strategy, with the extension for several neighboring units and the preservation of the traditional voltage control systems.

1. Introduction

Without the presence of a distributed generation unit in low and medium voltage distribution systems, the network voltage magnitudes fall along the feeders, depending only on cable characteristics and the demanded load at each node [1]. The biggest voltage drops happen at the end of each feeder branch. Three traditional voltage devices are used: automatic voltage control relays applied to the onload HV/MV transformer tap changer [2], off-load calibration of MV/LV transformer tap position, and capacitor banks [3]. However, those three pieces of equipment are not responsible for keeping the network voltage within the tolerance limit. This has been wrongly reported in several papers in literature [4, 5]. The way traditional distribution networks are designed and developed guarantees that the voltage magnitudes do not deviate outside the permitted range. Before the introduction, the modification, or the elimination of any load, a feasibility study on cables, lines, or electric devices and distribution utilities was conducted, by evaluating the voltage drop levels considering the worst scenarios of power demands and topology structure. Onload and off-load tap changers of
HV/MV and MV/LV transformers are only used to set transformers’ secondary voltages close to the maximum.

However, with the integration of DGs, traditional methods and devices are no longer able to control the network voltage profile [6], and the steady-state voltage rise has been reported in literature as one of the main obstacles for the interconnection of large amount of generation units to the existing distribution systems [7, 8].

In literature, several centralized [9], decentralized [10], local [11], and distributed control [12] approaches are developed to get the violated voltages return inside the permitted range. Both decentralized and local approaches have been advanced for practical application for voltage rise in active distribution networks. Also, various efforts have been developed to avoid the heavy extension of sensing, communication and control infrastructure is required for the implementation of the centralized approaches, as in the method presented in [13], based on the use of solar and demand forecasts, and also in [14], where the authors present a short-time scheduler to define DG setpoints and to minimize the voltage deviations with respect to the rated values.

The efforts presented in literature focus on adapting, improving, or even totally changing the traditional devices and systems. However, those modifications lead not only to wasting conventional voltage control equipment and methods but also to abandoning all the practical experiments and good practices collected over generations by the utility personnel, in terms of network control operation and development.

In this paper, we propose an alternative solution to mitigate the voltage profile in active distribution systems: the suppression of the impact of the active and reactive power produced or consumed by distributed generation units on the network voltage, in a fast way to avoid the risk of any conflict of the proposed tool with the conventional voltage control devices. The purpose of the proposed approach is not to control the bus voltage, but rather to only eliminate the impacts of DG units on the voltage of all network nodes and keep the traditional systems and devices dealing with the same constraints, as in a passive system.

A decentralized voltage control approach, which is flexible with network topology reconfiguration and based on a sensitivity analysis, is developed in this paper, with the main aim of eliminating the impact of a DG unit. The computation of the sensitivity coefficients presented in [15] and used in [16, 17] is adopted in this paper, which is based on the topological structure of the network and independent of the network operation point.

A criterion for the implementation of the participated D-FACTS devices in the voltage control is developed in Section 2, considering network topology reconfiguration. And a powerful codification method is developed in Section 3 to alleviate the communication between SCADA and the proposed decentralized system and to get the decentralized control aware of any topology reconfiguration with a minimum effort.

The proposed voltage control strategy in this paper could be implemented for the first DG units in emerging networks, as in the case of Morocco and several African countries, which are not yet ready to receive a generation unit or do not yet have a voltage control strategy for active systems.

The proposed method could also be applicable for generation units not able to be connected to an existing centralized voltage control system, as it could be intended as the principal voltage control strategy with a possible extension of the approach to eliminate the impact of several neighboring generation units.

The principal advantages of the proposed approach are as follows:

(i) An efficient control strategy able to avoid as much as possible both disconnection and curtailment of distributed generation

(ii) A control strategy that saves the traditional voltage regulation equipment and keeps the utility personnel dealing with the same constraints

(iii) Ease of implementation, with no need for load demand data or powerful electric meter infrastructure

(iv) With the proposed codification method and with no requirement of any load flow analysis, the proposed approach is able to get implemented for real-time application

The main contributions of this paper are as follows:

(i) The introduction of a novel approach to mitigate the voltage control of active distribution systems

(ii) The development of an analytical method for an optimal allocation of voltage control devices with the consideration of the network topology reconfiguration

(iii) The presentation of a codification method able to reduce the needed communication data to actualize the sensitivity coefficients and get the decentralized control flexible with network topology reconfiguration

2. Impacts of Distributed Generation on the Voltage Profile of Distribution Networks

Let us consider a distribution network with $n$ nodes. Without the presence of any distributed generation, the voltage at a node $i$ depends on the powers injected by or absorbed from all network nodes and the network line and cable characteristics:

$$ V_i = V_i(P_1, P_2, \cdots, P_n, Q_1, Q_2, \cdots, Q_n, R_1, R_2, \cdots, R_n, X_1, X_2, \cdots, X_n). $$

(1)

The distribution networks are always developed in meshed topology but operate in a radial structure. For a defined topology fixed structure, $R_1, R_2, \cdots, R_n, X_1, X_2, \cdots, X_n$ will be constant and the voltage will be impacted by the variation of the active and reactive power at each node.

$$ V_i = V_i(P_1, P_2, \cdots, P_n, Q_1, Q_2, \cdots, Q_n). $$

(2)
The linearized form of the voltage with the Taylor series expansion at each node is shown as follows:

\[
\begin{align*}
\frac{dV_1}{dP_1} P_1 + \cdots + \frac{dV_1}{dP_n} P_n + \frac{dV_1}{dQ_1} Q_1 + \cdots + \frac{dV_1}{dQ_n} Q_n = 0, \\
\frac{dV_2}{dP_1} P_1 + \cdots + \frac{dV_2}{dP_n} P_n + \frac{dV_2}{dQ_1} Q_1 + \cdots + \frac{dV_2}{dQ_n} Q_n = 0, \\
\frac{dV_n}{dP_1} P_1 + \cdots + \frac{dV_n}{dP_n} P_n + \frac{dV_n}{dQ_1} Q_1 + \cdots + \frac{dV_n}{dQ_n} Q_n = 0.
\end{align*}
\]

(3)

With the introduction of a DG unit, connected to a node, which will be named DG, the active and reactive power produced or consumed will impact the voltage of all the network nodes:

\[V_i = V_i(P_1, P_2, \ldots, P_n, Q_1, Q_2, \ldots, Q_n, P_{DG}, Q_{DG}).\]  

(4)

The changes in voltage at each node will depend on the variation of the two new variables of introduced DG: \(P_{DG}\) and \(Q_{DG}\), as demonstrated in equation (5):

\[
\begin{align*}
\frac{dV_1}{dP_{DG}} P_{DG} + \frac{dV_1}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_1}{dQ_{Ai}} Q_{Ai} &= 0, \\
\frac{dV_2}{dP_{DG}} P_{DG} + \frac{dV_2}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_2}{dQ_{Ai}} Q_{Ai} &= 0, \\
\frac{dV_n}{dP_{DG}} P_{DG} + \frac{dV_n}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_n}{dQ_{Ai}} Q_{Ai} &= 0.
\end{align*}
\]

(5)

3. The Proposed Approach

The purpose of the proposed approach in this paper is to add “m” devices “A1, A2, Am”, able to regulate their reactive power, in such a way that the impact of the DG at the nodes “i1, i2, im” is equal to zero. Several advanced devices with the ability to act on their reactive power have been presented in literature as D-FACTS and supercapacitors [18].

\[
\begin{align*}
\frac{dV_{i1}}{dP_{DG}} P_{DG} + \frac{dV_{i1}}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_{i1}}{dQ_{Ai}} Q_{Ai} &= 0, \\
\frac{dV_{i2}}{dP_{DG}} P_{DG} + \frac{dV_{i2}}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_{i2}}{dQ_{Ai}} Q_{Ai} &= 0, \\
\frac{dV_{im}}{dP_{DG}} P_{DG} + \frac{dV_{im}}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_{im}}{dQ_{Ai}} Q_{Ai} &= 0.
\end{align*}
\]

(6)

The size and site and also the number of those pieces of equipment should be optimized.

Now, we will prove that the impact of the DG unit has been eliminated for the voltage of all network points. To do so, we consider that the additional D-FACTS devices have eliminated the impacts of the DG unit on the voltage of the nodes \(i1, \ldots, im\). So for node \(j\) from the nodes \(i1, \ldots, im\), \(A\) is

\[
\frac{dV_j}{dP_{DG}} P_{DG} + \frac{dV_j}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_j}{dQ_{Ai}} Q_{Ai} = 0. \tag{7}
\]

For node “x” different from the nodes \(i1, \ldots, im\), the impact of the DG unit and the additional devices \(A1, \ldots, A\) is

\[
\frac{dV_x}{dP_{DG}} P_{DG} + \frac{dV_x}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dV_x}{dQ_{Ai}} Q_{Ai} = 0. \tag{8}
\]

By the application of the Chasles relation,

\[
\frac{dV_x}{dQ_{DG}} Q_{DG} = \frac{dV_x}{dP_{DG}} P_{DG} + m \sum_{i=1}^{m} \frac{dV_x}{dQ_{Ai}} Q_{Ai}, \tag{9}
\]

And \(\emptyset_x\) becomes

\[
\emptyset_x = \frac{dV_x}{dP_{DG}} P_{DG} + m \sum_{i=1}^{m} \frac{dV_x}{dQ_{Ai}} Q_{Ai}. \tag{10}
\]

So

\[
\emptyset_x = \frac{dV_x}{dP_{DG}} P_{DG} + m \sum_{i=1}^{m} \frac{dP_{DG}}{dQ_{Ai}} Q_{Ai}. \tag{11}
\]

So equation (7) becomes

\[
\frac{dV_j}{dP_{DG}} P_{DG} + \frac{dV_j}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dP_{DG}}{dQ_{Ai}} Q_{Ai} = 0. \tag{12}
\]

So

\[
\frac{dP_{DG}}{dQ_{DG}} Q_{DG} + m \sum_{i=1}^{m} \frac{dP_{DG}}{dQ_{Ai}} Q_{Ai} = 0. \tag{13}
\]

And equation (10) becomes

\[
\emptyset_x = \frac{dV_x}{dP_{DG}} P_{DG} + \frac{dV_x}{dQ_{DG}} Q_{DG} = 0. \tag{14}
\]

So by the elimination of the impact of a DG unit on the voltage of some certain nodes, the voltage of all network points will become independent of the small variation of
the active and reactive power introduced by the DG unit; in
the next section, an analytical approach based on the work
presented in [17] is discussed, with an improvement by
considering all the possible network topology structures.

4. Allocation of Voltage Regulation Devices

The proposed method in this section for the identification
of the site and size of voltage regulation devices that needed
to eliminate the impact of a DG unit on the network voltage
profile is built on the progress of the optimal approach developed
in [19], where such a method is used to identify the optimal
allocation of capacitor banks to reduce voltage deviation with-
out the consideration of network topology reconfiguration.

To calculate the sensitivity coefficients, we consider the
formula presented in [15] and used in [16, 17]:

\[
\begin{align*}
\frac{\partial V_i}{\partial P_j} &= -\frac{1}{V_{nom}} L_{ij}^r x_{ij}, \\
\frac{\partial V_i}{\partial Q_j} &= -\frac{1}{V_{nom}} L_{ij} x_{ij}, \\
\end{align*}
\]

(15)

where \( L_{ij} \) are

(i) for \( i = j \) the sum of the branch lengths forming the
path from the origin (node 0) to node \( i \)

(ii) for \( i \neq j \) the sum of the branch lengths forming the
path from the origin to the common node of the
paths formed by the origin and nodes \( i \) and \( j \)

From this formula, we can observe that with network
topology reconfiguration, the highest values of \( \frac{\partial V}{\partial P} \) and \( \frac{\partial V}{\partial Q} \) will correspond to the nodes presenting the
longest common path from the substation to the DG node,
which means that the voltage magnitudes of the nearest
nodes to the DG are the most impacted parts due to the active
and reactive power variations, which will be confirmed
through following the numerical application of equation
(14), for two meshed IEEE 15-bus feeders, driven from two
separate HV/MV substations A and B, as shown in Figure 1,
and three topology structures obtained by changing
the open point: structure no. 1 (open point) at the node
"15A-5B," structure no. 2 at the node "3B," and topology
no. 3 at the node "2A."

The results of the sensitivity coefficients obtained are
arranged in a descending order, and the most 5 impacted
nodes are reported in Table 1.

Table 1 confirms that the order of the most impacted
nodes by the DG unit is still the same with network topology
structure reconfiguration and the most impacted parts are
the nearest nodes to the DG connection point. We combine
this powerful observation with the analytical approach presented
in [13], wherein the authors developed three steps to
identify the optimal size and site of capacitor banks in radial
distribution networks.

We consider a number of D-FACTS devices to install,
considering the ability of the available communication
support for a real-time application and also the length and
the costs of developing coaxial cable or fiber-optic links.
Developing an own communication infrastructure makes
the proposed approach able to reduce the risks of network
failures and cyberattacks; however, when the number of
voltage regulation devices is high, more communication
infrastructure is required to be developed.

A powerful optimization method presented in [20], based
on a numerical method to identify the optimal number of
distributed generation units to install, could be adopted to
calculate the number of voltage regulation devices to install.

The optimal optimization objective in this paper is to
identify the minimal capacity of capacitor banks to eliminate
the impact of a DG unit on the voltage of all network points.
To do so, we consider the following steps:

Step 1. Define the number of voltage regulation devices.

Step 2. Place the D-FACTS devices at the nearest nodes to
the DG unit.

Step 3. D-FACTS sizes are varied from a zero value in
constant steps until the impacts of the DG on the voltage of
those nodes are equal to zero, considering all possible
network topology structures and scenarios of load demand
and active power production.

Step 4. The minimum size able to eliminate the impact of
the DG unit is taken as the optimal size.

The estimation of the voltage magnitudes at each node
can be computed by the direct application of the backward/-
forward sweep approach, presented in [21].

After the determination of the optimal number, site, and
size of the reactive power sources to eliminate the impact of a
DG unit on the network voltage, in the next section, the ther-
mal limits of lines and transformers are evaluated to identify
the required network reinforcement action to take in order to
guarantee a proper functioning of the proposed control
strategy.

5. Network Reinforcement

Let us consider the three nodes "\( n \)," "\( n - 1 \)," and "\( n + 1 \)," as
shown in Figure 2.

The electric current between the terminal node "\( n \)" and
the next node "\( n - 1 \)" is

\[
j^k_n = \text{conj} \left( \frac{P_n + j \ast Q_n}{V_n^k} \right).
\]

(16)

The electric current between a sending node \( (n - i) \) and a
receiving node \( (n - i - 1) \) is

\[
j^k_{n-i} = -\text{conj} \left( \frac{P_{n-i} + j \ast Q_{n-i}}{V_{n-i}^k} \right) + \sum_r j^k_{n-i,r} \quad r = 1 \ldots n - i - 1,
\]

(17)

where \( \sum_r j^k_{n-i,r} \) is the current in branches emanating from the
node "\( n - i \)"
For each branch,

\[ J_{n+1}^k \leq J_{\text{Limit} - \text{max}} \]  \hspace{1cm} (18)

If the constraint (17) is not satisfied, network reinforcement action should be applied.

For the thermal line limit, the cable section should be increased to the next standard section and the evaluation should be repeated for the new section.

### 6. The Codification Method

The value of the sensitivity coefficients depends on the topology network, and for each network reconfiguration, the proposed tool should be able to actualize those coefficients. In order to reduce the needed computation time and the needed data to be transmitted and get the proposed approach able for a real-time application, in the next section, we propose a codification approach able to reduce the efforts needed to actualize the sensitivity coefficients with network topology reconfiguration.

The idea is to identify all possible network topology structures, obtained by changing the state of all network sections, to give a code for each topology, to compute the values of the sensitivity coefficients for each possible topology structure, and to save all these data at the proposed tool calculator. Each time a topology change happens, only the code of the new structure will be sent from the MV SCADA.

This codification will save not only the size of the data to send but also the required computation time to actualize the sensitivity coefficient values.

### 7. The Algorithm

The purpose of the proposed approach is not to control the bus voltage, but rather to only eliminate the impacts of DG units on the voltage of all network nodes and keep the traditional systems and devices dealing with the same constraints.

As proven in Section 2, the elimination of the impact of the DG unit on the voltage of certain network nodes involves
the elimination of the network voltage. The control law of the proposed approach is the solution of equation (6).

The matrix format of equation (6) is

\[
\begin{bmatrix}
\frac{\partial V_{i1}}{\partial P_{DG}} \\
\vdots \\
\frac{\partial V_{in}}{\partial P_{DG}} \\
\frac{\partial V_{i1}}{\partial Q_{DG}} \\
\vdots \\
\frac{\partial V_{in}}{\partial Q_{DG}}
\end{bmatrix}
\begin{bmatrix}
dP_{DG} \\
\vdots \\
dQ_{DG}
\end{bmatrix}
+ \begin{bmatrix}
\frac{\partial V_{i1}}{\partial Q_{DG}} \\
\vdots \\
\frac{\partial V_{in}}{\partial Q_{DG}} \\
\frac{\partial V_{i1}}{\partial QA_{1}} \\
\vdots \\
\frac{\partial V_{in}}{\partial QA_{m}}
\end{bmatrix}
\begin{bmatrix}
dQ_{A1} \\
\vdots \\
dQ_{Am}
\end{bmatrix}
= 0.
\]

(19)

The required small changes in the reactive power to be pushed or pulled by D-FACTS devices to eliminate the expected voltage changes for small changes on the active and reactive power of the DG unit is given by

\[
\begin{bmatrix}
dQ_{A1} \\
\vdots \\
dQ_{Am}
\end{bmatrix}
= -\begin{bmatrix}
\frac{\partial V_{i1}}{\partial QA_{1}} & \cdots & \frac{\partial V_{i1}}{\partial QA_{m}} \\
\vdots & \ddots & \vdots \\
\frac{\partial V_{in}}{\partial QA_{1}} & \cdots & \frac{\partial V_{in}}{\partial QA_{m}}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{\partial V_{i1}}{\partial P_{DG}} \\
\vdots \\
\frac{\partial V_{in}}{\partial P_{DG}} \\
\frac{\partial V_{i1}}{\partial Q_{DG}} \\
\vdots \\
\frac{\partial V_{in}}{\partial Q_{DG}}
\end{bmatrix}
\begin{bmatrix}
dP_{DG} \\
\vdots \\
dQ_{DG}
\end{bmatrix}.
\]

(20)

The algorithm is shown as follows:

Step 1: Identify all network possible topologies.
Step 2: Calculate the sensitivity coefficients for each topology structure using equation (15), and save all those values at the proposed control relay.
Step 3: Compute the needed reactive power to be pushed or pulled by the reactive power sources using equation (20).
Step 4: If a change occurs in network infrastructure, for example, the addition/elimination of a network node, a branch, and so on, go to step 5; otherwise, go to step 6.
Step 5: Actualize the network data.
Step 6: If a network topology occurs, go to step 7; otherwise, go to step 3.
Step 7: The MV SCADA identifies the code of the new topology structure and sends it to the control relay. The control relay actualizes the sensitivity coefficients. Go to step 3.

The flow chart of the proposed method is presented in Figure 3.

---

**Figure 3: Flowchart diagram.**

**Table 2: Site and size of voltage regulation devices.**

<table>
<thead>
<tr>
<th>Site</th>
<th>Size (kVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>464</td>
</tr>
<tr>
<td>A2</td>
<td>1,112</td>
</tr>
<tr>
<td>A3</td>
<td>1,112</td>
</tr>
</tbody>
</table>

---
8. Simulation Test

To simulate the proposed approach, we consider the network presented in Figure 1, composed of two IEEE 15-bus feeders, driven from two separate HV/MV substations A and B, and a DG unit of 1500 kVA, connected between the nodes “12A” and “13A.”

Three voltage regulation devices are proposed to be added, and the results of the optimal site and size of the additional D-FACTS are shown in Table 2. After the evaluation of the thermal limits of network lines, no reinforcement actions are required.

For topology structure no. 1, the obtained simulation results are regrouped as shown in Table 3.

The first column of Table 3 represents the voltage magnitudes before the connection of the DG unit. And for each active power variation, two columns are given: the voltage magnitudes without the application of the proposed approach and the results obtained after the application of the approach.

As we can observe, for small active power variation, the proposed approach is able to eliminate the impacts of the DG and get the voltage magnitudes equal to the state of the system before the connection of the generation unit.

For bigger variation values, the impact is only reduced, and in all cases, the total active and reactive power losses tend to increase. Another shortcoming of the proposed method is the case of network topology change happening at the section of a node where a voltage regulation device is installed; for example, in the case study, this change can happen if a fault is detected at the branch "DG-13A." Two solutions can be used in this case: the disconnection of the DG unit or the overdimensioning of the D-FACTS capacity, in such a way that the production of the reactive power of the isolated equipment is insured.

9. Conclusion

In this paper, an alternative voltage control for public distribution networks with the presence of distributed generation is proposed, based on the elimination of the impact produced by a DG unit and inability of the traditional voltage control devices and methods to deal with the same constraints.

The simulation results had proven the ability of the proposed method to eliminate the impacts of the small variation of active power produced by a DG unit on the voltage magnitudes of all network nodes; however, the total network losses of the networks tend to increase.

For future scope, further technical, economic, and environmental evaluation efforts will be conducted to evaluate the efficiency of the proposed approach:

(i) Impacts on fault currents, protection devices, and voltage stability

(ii) Evaluation of the additional voltage regulation devices, network reinforcement actions, and saving training fee costs, in comparison to the saving costs of saving the current existing voltage control equipment.

Nomenclature

DG: Distributed generation
MV: Medium voltage
LV: Low voltage
D-FACTS: Distributed flexible AC transmission system
$P_{DG}$: Active power produced by the DG
$Q_{DG}$: Reactive power produced or consumed by the DG

<table>
<thead>
<tr>
<th>Node</th>
<th>Without</th>
<th>$\Delta_{DG} = 1$ kVA</th>
<th>$\Delta_{DG} = 3$ kVA</th>
<th>$\Delta_{DG} = 5$ kVA</th>
<th>$\Delta_{DG} = 10$ kVA</th>
<th>$\Delta_{DG} = 50$ kVA</th>
<th>$\Delta_{DG} = 500$ kVA</th>
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</tr>
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$P_{\text{loss}}$: Active power loss
$Q_{\text{loss}}$: Reactive power loss

Table 3: Simulation results.
\[ V_i: \text{ Voltage magnitude at node } i \]
\[ J_{ij}: \text{ Electric current between the terminal node } i \text{ and node } j. \]

**Data Availability**

The code Matlab developed and used to perform the proposed method is available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**References**


