Hindawi
Journal of Applied Mathematics
Volume 2020, Article ID 2658213, 9 pages
https://doi.org/10.1155/2020/2658213

Research Article

Optimal Health Insurance and Trade-Off between Health and Wealth

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Received 11 May 2020; Accepted 6 July 2020; Published 1 August 2020

Academic Editor: Yansheng Liu

Health insurance is considered to be a special type of nonlife insurance with two important features. First, compared with property insurance, health insurance provides valuable hedge against unpredictable shocks to health status, instead of loss on property. Therefore, a modified utility function that describes the trade-off between health and wealth should be applied in optimal indemnity design. Second, in the case that the insured is severely or critically ill, with necessary medical treatment, the insured may not fully recover from an illness or an injury. The doctor usually communicates with the patient to set up a personalized treatment plan and explains clearly about the expected outcome beforehand. Hence, there is some probability that health insurance helps to rescue the insured from disastrous financial burden, but it still yields a lower utility of health. By taking these special features into account, we formulate the optimization problem and characterize the optimal solutions via the Lagrange multiplier method and optimal control technique. Finally, we examine our optimal contracts by numerical illustration. Our research work gives new insights into health insurance design.

1. Introduction

Understanding health insurance claim cost at the individual level is critical for optimal health insurance design. Over the past decades, short-term health insurance has experienced tremendous growth around the globe, due to multiple reasons, for example, population aging and lifestyle change. From 2010 to 2019, health insurance premium grew rapidly in China, increasing from 67.7 billion yuan to 706.6 billion yuan. Under the strategy of "Healthy China 2030," it is imperative to effectively measure the health level and illness incidence of residents. The first version of "China Life Insurance Critical Illness Morbidity Table" was released in 2013. Since January 2019, China Banking and Insurance Regulatory Commission and China Association of Actuaries collaborated on the revision of "China Life Insurance Critical Illness Morbidity Table" and released the revised draft, i.e., 2020 version, on 7th May 2020. Therefore, how to take key factors, including health level, health spending, and illness incidence, into consideration and optimize health insurance design is the scientific problem we intend to investigate in this paper.

Numerous researchers have devoted themselves to analyze the optimal insurance design problem. Pioneered by Arrow [1–3], many articles on optimal insurance problem under various constraints, with consideration of application in personal lines or commercial lines, have been contributed into the literature. Some classical papers include these by Smith [4], Spence and Zeckhauser [5], Raviv [6], Doherty and Schlesinger [7], Gollier and Schlesinger [8], Young [9], and Vercammen [10]. More recent papers have discussed more complex problems, for example, Promislow and Young [11] presented a unifying framework for determining optimal insurance; Huang [12] studied the optimal insurance under the VaR constraint; Zhou and Wu [13] investigated an optimal insurance problem under the insurer's risk constraint; Zhou et al. [14] considered an optimal insurance problem in the presence of the insurer's loss limit; Lu et al.

[15] studied the optimal insurance problem when the insurable risk and uninsurable background risk are positively dependent; Huang et al. [16] discussed the optimal insurance problem with stochastic background wealth; Lu et al. [17] revisited the optimal insurance problem with two mutually dependent risks and generalized the corresponding results in [15]; and Xu et al. [18] showed that an optimal insurance contract covers both large and small losses when the insured's preference is of rank-dependent utility type.

The abovementioned papers do not distinguish health insurance from other "property-casualty" insurance. Compared with traditional short-term property insurance, health insurance provides valuable hedge against unpredictable shocks to health status, instead of loss on property. Therefore, a modified utility function that describes the trade-off between health and wealth should be applied in optimal indemnity design. In the case that the insured is critically ill, with necessary medical treatment, the insured may not fully recover from an illness or an injury. The doctor usually communicates with the patient to decide a personalized treatment plan and explains clearly about the expected outcome beforehand. Hence, there is some probability that health insurance helps or rescues the insured from disastrous financial burden, but it still yields a lower utility of health. Last but not least, deterioration in health status may set off a chain effect on health insurance renewal.

Motivated by the above observation, our paper further advances the optimal insurance problem and focuses on health insurance. The objective of our paper is threefold: (i) to develop a general optimization model for health insurance design by considering expected outcome of different treatment plans, (ii) to derive the optimal health indemnities under proper utility function, and (iii) to discuss about the effect of key factors on optimal health insurance contract design.

The problem on health insurance or healthcare has been studied in various papers from various perspectives. For example, Phelps [19] provided theoretical analysis and empirical estimates of demand for health insurance; Besley [20] explored the trade-off between risk sharing and incentives to consume increased medical care inherent in reimbursement insurance; Blomqvist [21] investigated the properties of optimal non-linear insurance schedules; Hall and Jones [22] analyzed the rise in health spending; Ellis et al. [23] examined the efficiency-based arguments for second-best optimal health insurance with multiple treatment goods and multiple time periods; Gerfin [24] discussed the effects of cost sharing instruments of modern health insurance on healthcare demand; Ho et al. [25] studied the optimal health insurance design for chronic diseases; and Zheng et al. [26] examined moral hazard and adverse selection effects in healthcare utilization using hospital invoice data. All the aforementioned research works have paid great attention to the examination of "excess health insurance," optimal income taxation, explanation of health spending from the economic point of view, moral hazard and adverse selection, etc. The optimal short-term health insurance problem under a more realistic model setup, including utility of health and wealth and expected treatment outcome, has not been addressed completely.

To the best of our knowledge, our work is among the first efforts to solve individual-level optimal health insurance. Inspired by the existing literature on utility of health and utility of wealth (e.g., [21, 27–32]), we employ the utility function $U(h, w) = h - \log(w)$ to describe the trade-off between health and wealth, where h denotes health status and w denotes wealth. To solve our optimization problem, we extend the method proposed by Arrow [1–3], Raviv [6], Gollier and Schlesinger [8], Zhou et al. [14], and Fan [33].

To summarize, the contribution of our paper is threefold. First, we formulate the utility and expected treatment outcome mathematically and develop the corresponding optimization problem. Second, we derive the optimal solutions by applying the Lagrange multiplier method and optimal control technique. Third, we illustrate the optimal indemnity contract by numerical examples under explicit loss functions and gain new insights into health insurance design.

The remainder of this paper is organized as follows: Section 2 sets up the optimal health insurance model under the utility of health and wealth, with consideration of shocks to the insured's health status. In Section 3, we derive the optimal solution via the Lagrange multiplier method and optimal control technique. The optimal health insurance design is influenced by the deterioration in the insured's health status. Section 4 is devoted to illustrate the optimal insurance design, both analytically and numerically. Section 5 concludes the paper.

2. The Model

In this section, we set up the optimal health insurance model under utility of health and wealth. Then, we establish the corresponding optimization problem for the insured who aims to maximize the expected utility, under the actuarial pricing constraint.

2.1. Utility of Health and Wealth. In general, the insured's well-being is measured by the utility of health and wealth. The trade-off between the two factors is a central issue for social policy and household risk management. A large body of literature has investigated the problem both theoretically and numerically (e.g., [21, 29–32]).

Levy and Nir [30] compared various forms of utility functions, including

(i) logarithmic preference: $U(h, w) = h \bullet \log(aw)$

(ii) power preference: $U(h, w) = h \bullet ((w + A)^{1-a}/(1 - a))$

(iii) negative exponential preference: $U(h, w) = h \bullet (B - e^{-bw})$

where h denotes health status; w denotes wealth; a, b, and B are given scalars; and $-A$ is interpreted as the minimal consumption level required for existence. Both the Standard Gamble (SG) approach and the Time Trade-Off (TTO) approach imply that $U(h, w)$ must be linear in h. Analysis of the health-wealth trade-off choices by survey reveals that the utility function $U(h, w) = h - \log(aw)$ provides the best description of choices not only at the aggregate level but also at the individual level. In our research work, we apply the logarithmic utility function as follows, to describe the trade-off between health and wealth.

$$U(h, w) = h \bullet \log (w), \tag{1}$$

where $0 \leq h \leq 1$, with 0 representing death and 1 representing perfect health; $w > 0$.

2.2. The Health Insurance Problem. Suppose that an economic agent, called insured or policyholder, with health status h and wealth w, faces exogenous shocks to health status which would incur a positive continuous random financial loss $X \in (0, M]$ defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The individual purchases health insurance against the loss by paying a nonnegative premium π to an insurer in return for a so-called indemnity $I(X)$, with $\rho \geq 0$ denoting the loading factor. We assume that $I(X)$ is increasing with $I(0) = 0$, $0 \leq I(x) \leq x$, $\forall x \geq 0$. Let $\mathscr{I}$ be the set of all indemnity functions which satisfy the condition. Meanwhile, we assume that each individual is restricted to being covered by a single plan and receive "necessary" health service if he/she is sick or injured. According to Arrow [2] and Raviv [6], for a risk-neutral insurer, if the cost of offering the insurance is proportional to the expected value of the indemnity $E[I(X)]$, then the insurer requires

$$(1 + \rho) E[I(X)] \leq \pi. \tag{2}$$

We also assume that the insured's preferences are given by $U(h, w) = h - U(w)$, where $U(w)$ is increasing, strictly concave, and twice differentiable w.r.t. w. With health insurance in force, if shocks to health status occur, the insured will take medical treatment in a hospital and get reimbursement for the cost and related loss from the insurer because the medical treatment does not always cure the health problem for various reasons. In other words, the indemnity of health insurance helps or rescues the insured from disastrous financial burden, but it still yields a lower utility of health. Taking this situation into consideration, we assume three cases as follows, during the term of health insurance:

Case 1. No shock to the insured's health status occurs.

Case 2. Shocks to the insured's health status occur due to mild diseases, and the medical treatment cures the insured, with probability p_1.

Case 3. Shocks to the insured's health status occur due to severe illness or critical illness, and the medical treatment does not fully cure the insured, with probability p_2.

Generally speaking, severe illness and critical illness usually result in heavier medical cost than mild illness. Moreover, critically ill patients may not fully recover by necessary treatment. To achieve better results, the doctor usually communicates with the patient to set up a personalized treatment plan and explains clearly about the expected outcome beforehand. In our paper, we intend to formulate the situation mathematically.

Under the third case, the insured's health status decreases from h to $h - \theta$, where θ is a measure of the difference between the original health status before shocks occur and the health status after medical treatment ends. We assume that Θ is a discrete random variable and takes value in $[\theta_l, \theta_u]$, $0 < \theta_l < \theta_u \leq h$. For convenience, given that shocks to the insured's health status occur, we define the related probabilities as follows:

$$\begin{aligned} p_1 &= \text{Prob}[\Theta = 0], \\ p_{\theta_i} &= \text{Prob}[\Theta = \theta_i], \\ p_2 &= \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i}. \end{aligned} \tag{3}$$

Then, the insured's expected utility can be calculated as follows:

$$\begin{aligned} &E[U(X, \Theta, I(X), \pi)] \\ &\quad \equiv (1 - p_1 - p_2) U(h, w - \pi) \\ &\quad + p_1 \int_0^M U(h, w - x + I(x) - \pi) dF(x \mid \Theta = 0) \\ &\quad + \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i} \int_0^M U(h - \theta_i, w - x + I(x) - \pi) dF(x \mid \Theta = \theta_i), \end{aligned} \tag{4}$$

where $F(x \mid \Theta = \theta_i)$ is the conditional distribution of X given $(\Theta = \theta_i)$, provided that shocks to the insured's health status occur. We also define that $f(x \mid \Theta = \theta_i)$ as the probability density function of X given $(\Theta = \theta_i)$, conditional on occurrence of shocks to the insured's health status.

The risk-averse insured aims to maximize expected utility against unpredictable shocks to health status. With the indemnity function $I(X) \in \mathscr{I}$ as the decision variable, the optimal health insurance model can be formalized as follows:

Problem P_1

$$\begin{aligned} &\text{Maximize} \quad E[U(X, \Theta, I(X), \pi)] \\ &\text{subject to} \quad \pi \geq (1 + \rho) E[I(X)] \end{aligned} \tag{5}$$

3. Solution Scheme

In this section, we apply the classical approaches, i.e., the Lagrange multiplier method and optimal control technique, to solve the optimization problem in two steps. In the first subsection, we keep the insurance premium fixed and solve the optimal health insurance problem. In the second subsection, we proceed to determine the global optimal solution.

3.1. The Optimal Health Insurance for a Fixed Premium. We keep the insurance premium fixed and rewrite the constraint condition in Problem P_1 as follows:

$$(1+\rho)E[I(X)] = (1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]} p_{\theta_i}\int_0^M I(x)dF(x\mid\Theta=\theta_i) + (1+\rho)p_1\int_0^M I(x)dF(x\mid\Theta=0) \le \pi. \tag{6}$$

With the help of the Lagrange multiplier method and optimal control technique, we establish the following proposition:

Proposition 1. *Under the assumption that* $w-\pi-((p_1h\bullet f(x\mid\Theta=0)+\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}(h-\theta_i)\bullet f(x\mid\Theta=\theta_i))/(\lambda(1+\rho)p_1f(x\mid\Theta=0)+\lambda(1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}f(x\mid\Theta=\theta_i)))\ge 0$ *and* $U(h,w)=h-\log(w)$, *the solution to optimization Problem* P_1, *for a fixed* π, *is*

$$I^*(x)=(x-d)^+,\quad d=w-\pi-\frac{p_1h\bullet f(x\mid\Theta=0)+\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}(h-\theta_i)\bullet f(x\mid\Theta=\theta_i)}{\lambda(1+\rho)p_1f(x\mid\Theta=0)+\lambda(1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}f(x\mid\Theta=\theta_i)}, \tag{7}$$

where d *is nonnegative deductible and satisfies* $(1+\rho)E[I^*(X)]=\pi$.

Proof. $I(x)$ is written by I for notational simplicity. Let

$$\begin{aligned} L = {} & p_1\int_0^M h\bullet\log(w-x+I-\pi)dF(x\mid\Theta=0) \\ & + \sum_{\theta_i\in[\theta_l,\theta_u]} p_{\theta_i}\int_0^M (h-\theta_i)\bullet\log(w-x+I-\pi)dF(x\mid\Theta=\theta_i) \\ & - \lambda\left((1+\rho)p_1\int_0^M IdF(x\mid\Theta=0) \right. \\ & \left. + (1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]} p_{\theta_i}\int_0^M IdF(x\mid\Theta=\theta_i)-\pi\right), \end{aligned} \tag{8}$$

where λ is the Lagrange multiplier. The Hamiltonian corresponding to Problem P_1 is

$$\begin{aligned} H = {} & p_1h\bullet\log(w-x+I-\pi)f(x\mid\Theta=0) \\ & + \sum_{\theta_i\in[\theta_l,\theta_u]} p_{\theta_i}(h-\theta_i)\bullet\log(w-x+I-\pi)f(x\mid\Theta=\theta_i) \\ & - \lambda(1+\rho)p_1\mathrm{If}(x\mid\Theta=0) - \lambda(1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]} p_{\theta_i}\mathrm{If}(x\mid\Theta=\theta_i). \end{aligned} \tag{9}$$

Since $U(.,.)$ is concave in I, it is easy to show that H is also concave in I. Let the first derivative of H w.r.t. I equal to zero; we then have

$$\begin{aligned} & \frac{p_1h\bullet f(x\mid\Theta=0)+\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}(h-\theta_i).f(x\mid\Theta=\theta_i)}{w-x+I-\pi} \\ & = \lambda(1+\rho)p_1f(x\mid\Theta=0)+\lambda(1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}f(x\mid\Theta=\theta_i). \end{aligned} \tag{10}$$

Solving (10), we can obtain the optimal solution to Problem P_1, I^*.

$$I^*=(x-d)^+,\quad d=w-\pi-\frac{p_1h\bullet f(x\mid\Theta=0)+\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}(h-\theta_i)\bullet f(x\mid\Theta=\theta_i)}{\lambda(1+\rho)p_1f(x\mid\Theta=0)+\lambda(1+\rho)\sum_{\theta_i\in[\theta_l,\theta_u]}p_{\theta_i}f(x\mid\Theta=\theta_i)}, \tag{11}$$

d is nonnegative deductible and satisfies $(1+\rho)E[(x-d)^+] = \pi$. Thus, we have proved the proposition.

3.2. Determination of the Optimal Health Insurance. In the previous subsection, we derive the optimal indemnity contract $I^*(x)$, for fixed premium π. In this subsection, we proceed to determine the global optimal health insurance.

Proposition 2. *If $I^*(X)$ as (11) is the optimal solution to Problem P_1 under $U(h,w) = h \bullet U(w) = h \bullet (\log(w))$, then $I^*(X)$ should satisfy the following equation:*

$$\begin{aligned} 0 = {} & \mathscr{K} \frac{(1-p_1-p_2)h}{w-(1+\rho)E[I^*]} \\ & + p_1 \int_0^M \frac{h}{w-x+I^*-(1+\rho)E[I^*]} \\ & \times \left(-1_{\{x>d\}} + \mathscr{K}\right) dF(x \mid \Theta = 0) \\ & + \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i} \int_0^M \frac{h-\theta_i}{w-x+I^*-(1+\rho)E[I^*]} \\ & \times \left(-1_{\{x>d\}} + \mathscr{K}\right) dF(x \mid \Theta = \theta_i), \end{aligned} \tag{12}$$

where $\mathscr{K} = (1+\rho)[p_1(1-F(d \mid \Theta = 0)) + \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i}(1-F(d \mid \Theta = \theta_i))]$.

Proof. Let

$$\begin{aligned} N = {} & E[U(X, \Theta, I^*, (1+\rho)E[I^*])] \\ = {} & (1-p_1-p_2)h \bullet \log(w-(1+\rho)E[I^*]) \\ & + p_1 \int_0^M h \bullet \log(w-x+I^* \\ & -(1+\rho)E[I^*]) dF(x \mid \Theta = 0) \\ & + \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i} \int_0^M (h-\theta_i) \bullet \log(w-x+I^* \\ & -(1+\rho)E[I^*]) dF(x \mid \Theta = \theta_i). \end{aligned} \tag{13}$$

From the first-order condition that $\partial N / \partial d = 0$, we have Equation (12). Thus, we prove the proposition.

Thus, we follow Arrow [1–3], Raviv [6], Gollier and Schlesinger [8], and Zhou et al. [14], to apply optimal control theories to solve the optimal health insurance problem via a two-step procedure. Thereafter, we proceed to discuss the results.

Remark 3 (discussion about Proposition 2). Equation (12) can be rewritten as follows:

$$\begin{aligned} & \frac{p_1 h(1-F(d \mid \Theta = 0)) + \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i}(h-\theta_i)(1-F(d \mid \Theta = \theta_i))}{w-d-(1+\rho)E[I^*]} \\ & = \underbrace{\frac{(1-p_1-p_2)h\mathscr{K}}{w-(1+\rho)E[I^*]}}_{\text{marginal utility under Case 1}} \\ & \quad \underbrace{+ p_1 \int_0^M \frac{h\mathscr{K}}{w-x+I^*-(1+\rho)E[I^*]} dF(x \mid \Theta = 0)}_{\text{marginal utility under Case 2}} \\ & \quad \underbrace{+ \sum_{\theta_i \in [\theta_l, \theta_u]} p_{\theta i} \int_0^M \frac{(h-\theta_i)\mathscr{K}}{w-x+I^*-(1+\rho)E[I^*]} dF(x \mid \Theta = \theta_i)}_{\text{marginal utility under Case 3}}, \end{aligned} \tag{14}$$

which could be interpreted economically. Equation (14) shows that the marginal utility cost of paying additional premium is equal to the marginal utility benefit of receiving additional indemnity. Note that Young [9] and Zhou and Wu [13] provided similar analysis. The significant difference of our results lies in the composition of the RHS, which can be decomposed into three parts, i.e., (i) marginal utility under Case 1, (ii) marginal utility under Case 2, and (iii) marginal utility under Case 3.

Remark 4 (extensions of application scenario). In our model setup, we classify three possible situations that could occur to the insured, such as (i) healthy state, (ii) mild disease, and (iii) severe or critical illness; and then, we solve the optimal health insurance under the framework of utility of health and wealth. Our model could be extended in different application scenarios. Herein, we give two examples. First, suppose there is a health insurance package to cover a family over a given period. Due to different demographical characteristics, each family member faces different health risks. Our model could be modified to analyze the heterogeneity of the health status shock that may occur during the insurance term. Second, life insurance can be regarded as an extreme situation in which case we should set $\theta = h$. As a usual plan design in life insurance industry, the health insurance could be considered an extended coverage rider on normal life insurance. Therefore, the optimization problem on a combination of a primary life insurance coverage with a health insurance rider should be carefully considered.

4. Numerical Illustrations

In this section, we use a simple numerical example to illustrate the optimal health insurance contract in the first and second subsections. Then, in the third subsection, we examine the impact of shocks to the insured's health status and the safety loading factor on the optimal indemnity design, i.e., the optimal deductible level.

4.1. Illustration Model Setting. To investigate the optimal indemnity contract design for an individual insured, our paper builds an optimal health insurance framework under individual utility of health and wealth. For illustration purpose, we assume a simple model setting as follows:

Case 1. The insured keeps health during the insurance term with probability $1 - p_1 - p_2$.

Case 2. The insured gets mild illness and recovers after necessary treatment, i.e., $\Theta = 0$, with probability p_1.

Case 3. The insured gets severe illness or critical illness and his/her health status degenerates by $\Theta = \theta$ after necessary treatment, with probability p_2.

Therefore, under utility $U(h, w) = h \bullet \log(w)$, the insured's expected utility is

$$\begin{aligned} &E[U(X, \Theta, I(X), \pi)] \\ &\quad \equiv (1 - p_1 - p_2) h \bullet \log(w - \pi) \\ &\qquad + p_1 \int_0^M h \bullet \log(w - x + I(x) - \pi) dF(x \mid \Theta = 0) \\ &\qquad + p_2 \int_0^M (h - \theta) \bullet \log(w - x + I(x) - \pi) dF(x \mid \Theta = \theta). \end{aligned} \tag{15}$$

We apply the optimal control theory to solve the optimization problem

$$\begin{aligned} &\text{Maximize} \quad E[U(X, \Theta, I(X), \pi)] \\ &\text{subject to} \quad \pi \geq (1 + \rho) E[I(X)] \end{aligned} \tag{16}$$

and obtain the optimal health insurance, i.e., optimal deductible insurance

$$I^* = (x - d)^+, \tag{17}$$

where the optimal deductible d^* is determined by

$$\begin{aligned} 0 = {} & \mathcal{J} \frac{(1 - p_1 - p_2) h}{w - (1 + \rho) E[I^*]} + p_1 \int_0^M \frac{h}{w - x + I^* - (1 + \rho) E[I^*]} \\ & \times \left(-1_{\{x > d\}} + \mathcal{J} \right) dF(x \mid \Theta = 0) \\ & + p_2 \int_0^M \frac{h - \theta}{w - x + I^* - (1 + \rho) E[I^*]} \\ & \times \left(-1_{\{x > d\}} + \mathcal{J} \right) dF(x \mid \Theta = \theta), \end{aligned} \tag{18}$$

where $\mathcal{J} = (1 + \rho)[p_1(1 - F(d \mid \Theta = 0)) + p_2(1 - F(d \mid \Theta = \theta))]$.

Table 1: Parameter values for the illustration model.

Parameter	Symbol	Value
The insured gets mild illness and $\Theta = 0$	p_1	0.5
The insured gets severe illness or critical illness and $\Theta = \theta$	p_2	0.1
Safety loading	ρ	0.2
Initial wealth	w	15
Initial health status	h	0.9
Severity of health degeneration after treatment	θ	0.3
Intensity parameter under Case 3	m_0	0.2
Intensity parameter under Case 2	m_1	0.1
The maximum health expenditure	M	10

Now, we assume the loss X follows truncated exponential distribution conditional on occurrence of shocks to his/her health status, and the density function is

$$f(x \mid \Theta) = \frac{m e^{-mx}}{1 - e^{-mM}} 1_{x \in (0, M]}, \tag{19}$$

where the intensity parameter m varies w.r.t. Θ. As such, we have

$$\begin{aligned} f(x \mid \Theta = 0) &= \frac{m_0 e^{-m_0 x}}{1 - e^{-m_0 M}} 1_{x \in (0, M]}, \\ f(x \mid \Theta = \theta) &= \frac{m_1 e^{-m_1 x}}{1 - e^{-m_1 M}} 1_{x \in (0, M]}. \end{aligned} \tag{20}$$

4.2. Simulation Results. Throughout the numerical analysis, unless otherwise stated, the basic parameters are given in Table 1, including probability that each case occurs, health status, indemnity design, and density parameter for loss distribution.

We compute the optimal indemnity for the insured under the illustration model setting and obtain the results as follows:

$$I^*(x) = \begin{cases} 0, & 0 < x < 4.370, \\ x - 4.370, & 4.370 \leq x \leq M, \end{cases} \tag{21}$$

as shown in Figure 1.

4.3. Sensitivity Analysis. According to our model setting, the probability of getting mild illness or severe/critical illness, the insured's current health status, the insured's current net wealth, and the current health systems and medical resources in the community are all predetermined factors, which would not experience great change during the short term of health insurance. Therefore, we focus on the impact of the two key factors, i.e., the degree of health degeneration after necessary treatment θ and the safety loading for health insurance ρ.

Figure 2 depicts the effect of θ on d^*. We observe that d^* increases with respect to θ in a linear shape when θ takes value from 0.1 to 0.5. The sensitivity result confirms the

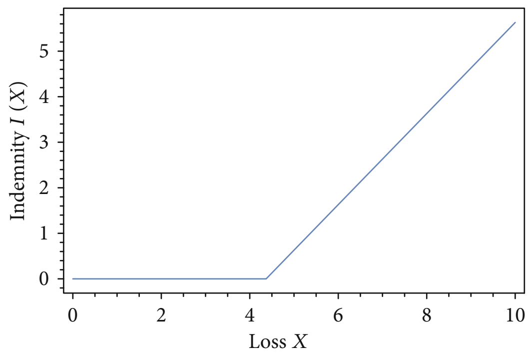

Figure 1: Optimal indemnity for the insured under the illustration model setting.

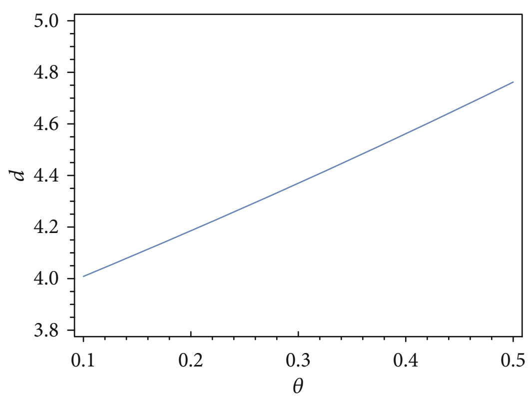

Figure 2: Sensitivity of d^* with respect to θ.

theoretical analysis and qualitative influence of θ on the optimal indemnity through utility of health and wealth, i.e., the trade-off between health and wealth. In addition, θ also affects the deductible level via heavy healthcare expenditures for treatment of severe/critical illness. Therefore, the insurer should set up various information channels and predict the medical cost. Figure 3 depicts the effect of ρ on d^*. We observe that a greater ρ leads to a higher d^* along a curve. d^* varies dramatically from 0.737 to 7.030 when ρ takes value from 0.0 to 0.7. In reality, the proportional safety loading depends highly on the type of insurance. As such, the insurer must be prudent in product positioning and pricing.

5. Conclusions and Further Research

Most research work on optimal nonlife insurance focuses on optimal indemnity contract design for a risk-averse insured who seeks to maximize expected utility against a possible loss on property. Our paper is distinguished from other research output. We studied the optimal health insurance problem under modified utility functions. The solution to the corresponding optimization problem would help the insurer to understand the insured's preferences and inspire improvement in health insurance design. The most important contributions of our work are threefold. First, two key factors, i.e., health and wealth, are taken into account to produce a more realistic utility function. Second, optimal health insurance is obtained against health status shocks. Third, in the presence of exogenous shocks to health, we provide new insights into health insurance design.

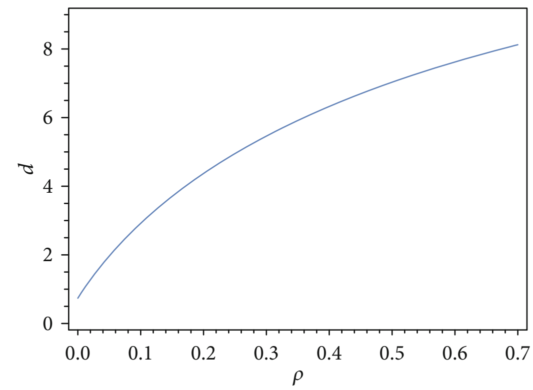

Figure 3: Sensitivity of d^* with respect to ρ.

From the insurer's perspective, it is crucial to predict the medical cost for the insured person, both in the individual level and in the aggregate level, so as to optimize the insurance contract design. Actuarial techniques, such as the individual risk model and aggregate risk model, are applied to understand the health insurance liability that exhibits strong skewness and a significant fraction of zeros. Recently, there is a vast literature investigating the claim modeling strategy for health insurance. Important research work includes but is not limited to Liu et al. [34], Frees et al. [35], Erhardt and Czado [36], Amin and Salem [37], Shi and Zhao [38].

We also observe the emerging trends in medical care industry and health insurance industry as follows: (i) due to technology transformation in insurance industry and evolution in people's lifestyle, it is essential to improve the existing actuarial techniques [39]; (ii) health insurance claim big data could be used to understand the trends in medicine prescriptions for diseases [40]; (iii) with the support of big data of people's disease occurrence rate, actual treatment costs, and demographic characteristics, the insurer is able to formulate personalized solutions for each and every policyholder [39, 41]. However, there are still barriers to conquer for data access [42].

Under the strategy of "Healthy China 2030," it is imperative to effectively measure the health level and illness incidence of residents. Since January 2019, China Banking and Insurance Regulatory Commission and China Association of Actuaries collaborated on the revision of "China Life Insurance Critical Illness Morbidity Table" and released the revised draft on 7th May 2020. The formal 2020 version, including the nationwide morbidity table and Guangdong-Hong Kong-Macao Greater Bay Area morbidity table, is scheduled to be in force soon. Our paper constructs an optimal health insurance framework under individual utility of health and wealth. To further investigate the optimal indemnity contract design for an individual insured, it is of great significance to combine health insurance claim data with people's medical record data, nationwide and regional illness incidence, and then apply a sophisticated data analysis methodology for analysis under our framework. We leave the discussion of this research topic to future study.

Data Availability

We took an arbitrary choice of parameter values in numerical analysis with reference to the existing literature. We did not use raw data in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research work is supported by Featured Innovation Project (Youth Foundation) of Guangdong University of Foreign Studies (19QN21).

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