

## Research Article

# Modeling the Control of Zika Virus Vector Population Using the Sterile Insect Technology

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This work is aimed at formulating a mathematical model for the control of mosquito population using sterile insect technology (SIT). SIT is an environmental friendly method, which depends on the release of sterile male mosquitoes that compete with wild male mosquitoes and mate with wild female mosquitoes, which leads to the production of no offspring. The basic offspring number of the mosquitoes' population was computed, after which we investigated the existence of two equilibrium points of the model. When the basic offspring number of the model ( $M_0$ ), is less than or equal to 1, a mosquito extinction equilibrium point ( $E_2$ ), which is often biologically unattainable, was shown to exist. On the other hand, if ( $M_0 > 1$ ), we have the nonnegative equilibrium point ( $E_1$ ) which is shown to be both locally and globally asymptotically stable whenever ( $M_0 > 1$ ). Local sensitivity analysis was then performed to know the parameters that should be targeted by control intervention strategies and result shows that female mating probability to be with the sterile male mosquitoes ( $\rho_s$ ), mating rate of the sterile mosquito ( $\beta_2$ ), and natural death rates of both aquatic and female mosquitoes ( $\mu_A + \mu_F$ ) have greater impacts on the reduction and elimination of mosquitoes from a population. Simulation of the model shows that enough release of sterile male mosquitoes into the population of the wild mosquitoes controls the mosquito population and as such can reduce the spread of mosquito borne disease such as Zika.

## 1. Introduction

Zika virus is a disease that is always spread in human population by the bite of an infected mosquito. It was discovered in 1947 from a sentinel monkey in the Zika forest in Uganda [1]; it was later isolated from humans in Nigeria in 1954 [2, 3]. World Health organization declared Zika virus a public health emergency of public concern in February 2016. Research on Zika virus continues as close to 3.6 billion people are living in risk areas for transmission [1]. The most common way to contact Zika virus is from the bites of an infected mosquito. Two species of mosquitoes spread the virus to people, the yellow fever mosquitoes (*Aedes aegypti*) and the Asian tiger mosquitoes (*Aedes albopictus*). Both are native to Texas [4]. During the period of 1960-1980 human infections, typically accompanied by mild illness, were found across Africa and Asia. The first large outbreak of disease causing Zika infection occurred in the island of Yap, feder-

ated states of Micronesia in 2007, indicating that the virus had moved from southeast Asia across the Pacific [5]. Zika belongs to the Flavivirus family, and it is transmitted through daytime-active *Aedes* mosquitoes, such as *A. aegypti* and *A. albopictus* [6]. The recent Zika outbreak in Brazil with over 1.5 million estimated cases from 2015 to 2016 received significant attention globally. The main reasons are its large number of infections, rapid transmission, and the increasing rate of reported microcephaly coincided with the infection. The incidence became public health emergency and followed by the warning announcement from the World Health Organization [7]. Among symptomatic patients, the most common symptoms include popular rash, fever, typically low-grade arthralgia, fatigue, nonpurulent conjunctivitis myalgia, and headache. While other symptoms like retroorbital pain, oedema, vomiting, sore throat, uveitis, and lymphadenopathy are less frequent [4]. A typical feature of Zika virus infection is the popular rash that is often pruriginous and starts on

the face and or trunk and then spreads throughout the body but may be focal and fugacious [8–10]. The main aim of this research work is to formulate a mathematical that controls the population of mosquitoes that causes Zika disease. The objectives are as follows:

- (i) Formulate a mathematical model to control mosquito population
- (ii) To determine the basic offspring number of the mosquito population
- (iii) To determine equilibrium points of the models
- (iv) To investigate the local and global stability analysis of the equilibrium points
- (v) To conduct sensitivity analysis on the model to see which parameters should be targeted by control intervention strategy, which can lead to reduction and elimination of mosquitoes from a population with time
- (vi) To perform numerical simulations of the model to support the analysis

In this study, we divide the vector population into the aquatic class (eggs, larva, and pupae), while the nonaquatic is divided into the male mosquitoes ( $M_M$ ), female mosquitoes not yet laying eggs ( $F_M$ ), female nonsterile mosquitoes ( $F_{NM}$ ), sterile male mosquitoes ( $M_S$ ), and sterile female mosquitoes ( $F_{SM}$ ).

The importance of this study cannot be overemphasized as humanity will welcome contributions for the curative and preventive measures of one of the world most silent killer infections in human history as the Zika virus transmits silently in the absence of severe disease thus allowing the infection to go undetected. The work of [11] presented a mathematical model for Zika virus cross infection between mosquitoes and human. A mathematical model for the transmission dynamics of Zika virus infection with combined vaccination and treatment intervention is formulated by [12], while [13] formulated a mathematical model on the prevention and control of Zika as a mosquitoborne and sexually transmitted disease. A mathematical model, analysis, and simulation of the spread of Zika with influence of sexual transmission and preventive measures are presented by [14]. A deterministic model for the transmission dynamic of Zika that takes into account the aquatic and nonaquatic stages of development is presented by [15, 16]. The work of [17] presented a dynamical model of asymptotical carrier of Zika virus with optimal control strategies; results show that the endemic equilibrium was investigated to be locally and globally stable whenever the basic reproduction number was greater than one  $R_0 > 1$ . A dynamical model of the Zika virus with control strategies is presented by [18]; the Zika model was said to be locally asymptotically stable whenever the basic reproduction number  $R_0 < 1$  for the disease free equilibrium and  $R_0 > 1$  for the endemic equilibrium case. A theoretical model for Zika virus transmission is presented by [19]; the Pontryagin's maximum principle was used to determine the necessary conditions

for effective control of the disease. Since it is wise to control the mosquito population than the human population, we therefore present in this present work a mathematical model that reduces/eliminates mosquito population that transmits Zika infection.

*1.1. Sterile Insect Technology (SIT).* Around the world, farmers fight the same battles against insects and other pests that not only damage their crops, causing high losses, but also transmit diseases to millions of livestock and humans. At the same time, many insects are becoming resistant to insecticides, while consumers are more aware of the negative effects of pesticides on public health, beneficial organisms, and the environment. Chemicals have been and are still extensively used all over the world to control wild mosquitoes' population. However, in the long run, mosquitoes can develop resistance to chemical products. Besides, [7] only allows a limited number of insecticides in view of polluting disasters. As a viable alternative, nonpolluting method is presented in this work. The sterile insect technique is an environmentally friendly insect pest control method involving the mass-rearing and sterilization, using radiation, of a target pest, followed by the systematic area-wide release of the sterile males by air over defined areas, where they mate with wild females resulting in no offspring and a declining pest population. The sterile insect technique is among the most environment friendly insect pest control methods ever developed. Irradiation, such as with gamma rays and X-rays, is used to sterilize mass-reared insects so that, while they remain sexually competitive, they cannot produce offspring [15, 20]. The sterile insect technology (SIT) is a type of control for mosquitoes that does not harm the environment. Millions of male mosquitoes are produced in a special factory, sterilized with radiation, and then released into the field (endemic areas) at regular intervals; there they mate with their wild females and as a result there are no offspring as they lay eggs without hatching [20]. If sufficient sterile males are released, the next generation will have fewer wild mosquitos' population and as such the mosquito's population is being controlled [21].

## 2. Model Assumptions

The following assumptions were made in the cause of the model formulation:

- (1) The mating competitiveness of both sterilized and nonsterilized mosquitoes are not equal, that is,  $\beta_1 \neq \beta_2$
- (2) Vector population is divided into the aquatic class (eggs, larva, and pupae), while the nonaquatic is divided into the male mosquitoes (nonsterile male mosquitoes) ( $M_M$ ), female mosquitoes (females not yet laying eggs) ( $F_M$ ), female nonsterile mosquitoes (those who could lay eggs and hatch due to mating with nonsterile male mosquitoes) ( $F_{NM}$ ), sterile male mosquitoes ( $M_S$ ), and female sterile mosquitoes (those mosquitoes who could lay eggs but do not hatch due to mating with sterile male mosquitoes) ( $F_{SM}$ )

- (3) Eggs, larva, and pupae are considered the aquatic stage of mosquitoes with the same death rate assumed for all
- (4) The aquatic mosquitoes have a density dependent death rate, which is a nonlinear decreasing function; nonlinear function is appropriate for population whose growth can be impeded (delayed) by space or resources [15]
- (5) The nonaquatic mosquitoes do not have the same death rate with the Aquatic mosquitoes
- (6) The female mosquitoes do not have the same death rate with the male mosquitoes

2.1. *Model Formulation and Procedures.* The mosquito life cycle is generally divided into two stages, the aquatic and nonaquatic classes.

The population is divided into six compartments consisting of the aquatic mosquitoes ( $A$ ), male mosquitoes ( $M_M$ ), female mosquitoes not yet laying eggs ( $F_M$ ), female nonsterile mosquitoes ( $F_{NM}$ ), female sterile mosquitoes ( $F_{SM}$ ), and sterile male mosquitoes ( $M_S$ ),

The aquatic stage of the mosquitoes which consists of eggs, larva, and pupae population increases from the oviposition by reproductive mosquitoes. It reduces due to natural death of the mosquitoes at the rate of ( $\mu_A$ ) and by density dependence death rate of ( $\mu_\rho$ ).

The female mosquitoes ( $F_M$ ) is recruited at the rate of ( $A\varphi\gamma$ ), where ( $\gamma$ ) is the maturity rate of aquatic mosquitoes to adult mosquitoes and ( $\varphi$ ) is the proportion of emerging females; it is reduced by the mating rate at the level of ( $\beta$ ) for female mosquitoes to be with wild male mosquitoes or sterile male mosquitoes with mating probabilities ( $\rho_\omega$ ) and ( $\rho_s$ ) respectively. The population is reduced finally by natural death at the rate of ( $\mu_F$ ).

The male mosquitoes ( $M_M$ ) is recruited by the proportion of the emerging male mosquitoes ( $1 - \varphi$ ) that mature to adult mosquitoes at the rate ( $\gamma$ ), which also reduces by natural death ( $\mu_M$ ).

The female nonsterile mosquitoes ( $F_{NM}$ ) population is increased by the female mosquitoes probability to mate with the wild male mosquitoes which is given by the rate ( $\rho_\omega$ ), with mating rate of ( $\beta_1$ ). This population is reduced by natural death at the rate of ( $\mu_F$ ).

The female sterile mosquitoes ( $F_{SM}$ ) population is increased by the wild female mosquitoes probability to mate with the sterile mosquitoes which is given by the rate ( $\rho_s$ ), with mating rate of ( $\beta_2$ ). The class reduces by natural death at the rate of ( $\mu_F$ ).

The sterile male mosquitoes ( $M_S$ ) are released into the population at the rate ( $\Lambda_2$ ). However due to some environmental and geographical factors that may affect the mixing of sterile and wild mosquitoes, such as location of mosquitoes breeding site, it is convenient to assume that only a fraction ( $p$ ) of the released mosquitoes will join wild mosquitoes population. Secondly, because of the differences in physiology of wild and sterile mosquitoes, a parameter ( $q$ ) is used to cap-

ture the mean mating competitiveness of sterile mosquitoes, so that the actual number of sterile male mosquitoes competing with wild mosquitoes is ( $pqM_S$ ), and as such, the available injected sterile male mosquitoes ( $M_S$ ) into the wild population of mosquitoes that can competitively mate with wild female mosquitoes is ( $pq\Lambda_2$ ). The population reduces by natural death at the rate of ( $\mu_S$ ).

2.2. *Mathematical Model.* The mathematical equations that incorporate the above assumptions are given by

$$\left. \begin{aligned} \frac{dA}{dt} &= \Lambda_1 F_{NM} - \gamma A - \mu_A A - \mu_\rho A^2 \\ \frac{dF_M}{dt} &= \varphi \gamma A - (\beta_1 \rho_\omega + \beta_2 \rho_s) F_M - \mu_F F_M \\ \frac{dM_M}{dt} &= (1 - \varphi) \gamma A - \mu_M M_M \\ \frac{dF_{NM}}{dt} &= \beta_1 \rho_\omega F_M - \mu_F F_{NM} \\ \frac{dF_{SM}}{dt} &= \beta_2 \rho_s F_M - \mu_F F_{SM} \\ \frac{dM_S}{dt} &= pq \Lambda_2 - \mu_S M_S \end{aligned} \right\} \quad (1)$$

The aquatic mosquitoes population ( $A$ ) can be considered having logistic growth with carrying capacity ( $k$ ) so that  $A(t) \leq K$ . When the population of the aquatic mosquitoes' class is higher than its carrying capacity the growth rate becomes negative, that is, the population goes down. When the population is equal to its carrying capacity ( $k$ ) then the growth rate becomes stagnated. The population grows when it is less than its carrying capacity.

Secondly, the released sterile male mosquitoes' population ( $M_S$ ) can be decoupled from the model (1). Since it is independent of other compartments, the size of its population is controlled by human intervention, and as such, it is independent from the rest of the population. The ( $M_S$ ) population has the solution of the form:

$$M_S(t) = e^{-\mu_S t} \left[ M_S(0) + \int_0^t e^{\mu_S(t)} pq \Lambda_2(t) dt \right] \quad (2)$$

where  $M_S(0)$  is the released sterile male mosquito at time  $t = 0$ .

2.3. *Model Variables.* Descriptions of the model variables used is presented in Table 1 below:

2.4. *Model Parameters.* Descriptions of the model parameters used are presented in Table 2 below.

### 3. Analysis of the Mosquito Model

3.1. *Basic Offspring Number of the Mosquito Population ( $M_0$ ).* This is denoted by ( $M_0$ ). It is defined as the number of offspring produced by a single female mosquito that mates with a nonsterile male mosquito in its entire lifespan [15].

TABLE 1: Model variables and descriptions.

S/N	Variables	Descriptions
1	$A$	Aquatic class of mosquitoes
2	$M_M$	Male mosquitoes
3	$F_M$	Female mosquitoes not yet laying eggs
4	$F_{NM}$	Female nonsterile mosquitoes (can lay and hatch eggs)
5	$F_{SM}$	Female sterile mosquitoes (can lay but do not hatch eggs)
6	$M_S$	Sterile male mosquitoes

TABLE 2: Model parameters and descriptions.

S/N	Parameters	Descriptions
1	$\Lambda_1$	Oviposition rate of fertilized female mosquitoes
2	$\varphi$	Proportion of emerging female mosquitoes
3	$1 - \varphi$	Male mosquitoes emerging population
4	$\beta_i$	Mating rate, where $i = 1, 2$ .
5	$\gamma$	Maturity rate of mosquitoes
6	$\mu_M$	Natural death rate of wild male mosquitoes
7	$\mu_S$	Natural death rate of sterile mosquitoes
8	$\mu_\rho$	Density dependent death rate of the aquatic mosquito class
9	$\mu_A$	Natural death rate for aquatic mosquitoes
10	$\mu_F$	Natural death rate for female mosquitoes
11	$\rho_\omega$	Female mosquitoes probability to mate with wild male mosquitoes
12	$\rho_S$	Female mosquitoes probability to mate with sterile male mosquitoes
13	$p$	Fraction of the released sterile mosquitoes, that can join the wild mosquitoes,
14	$q$	Mean mating competitiveness of the sterile male mosquitoes.

An oviposition occurs after a wild male mosquito mates with a female mosquito, which fertilizes and lays eggs. The mean duration spent in aquatic stage by mosquito is given by  $1/(\mu_A + \gamma)$  where  $\gamma$  is the maturity rate of the aquatic mosquitoes in adult mosquitoes. ( $\varphi$ ) is the fraction of aquatic mosquitoes that become females, then the probability that an egg survives the aquatic stage and becomes an adult female mosquito is given by

$$\frac{\varphi\gamma}{(\mu_A + \gamma)} \tag{3}$$

Also, the mean duration spent by a female mosquito in the female not yet laying eggs mosquitoes class ( $F_M$ ) is given by  $1/(\mu_F + \beta_1\rho_W + \beta_2\rho_S)$ . The rate at which a mosquito in ( $F_M$ ) class moves to the nonsterile female mosquitoes ( $F_{NM}$ ) through mating with the male mosquito ( $F_{NM}$ ) class is then given by

$$\frac{\beta_1\rho_W}{(\mu_F + \beta_1\rho_W + \beta_2\rho_S)}. \tag{4}$$

Similarly, the average lifespan of the nonsterile female mosquitoes ( $F_{NM}$ ) class during its life time is given by

$$\frac{\Lambda_1}{\mu_F}, \tag{5}$$

where  $\Lambda_1$  is the oviposition rate. To get the basic offspring number of the mosquito population; we multiply equations (3), (4), and (5) together.

The basic offspring number of the mosquito population is given by

$$M_0 = \frac{\Lambda_1\varphi\gamma\beta_1\rho_W}{\mu_F(\mu_A + \gamma)(\mu_F + \beta_1\rho_W + \beta_2\rho_S)}. \tag{6}$$

The biological implication of  $M_0 > 1$  is that, the mosquito population persist, otherwise if  $M_0 \leq 1$  then the mosquito population goes to extinction and the human to vector or vector to human transmission can be eliminated.

3.2. Existence of Equilibria in Mosquito Population. Solving equation (1) by setting the right hand side of the equation to zero, we have the following equilibria represented by

$$E_0 = \begin{bmatrix} A^0 & F_M^0 & M_M^0 & F_{NM}^0 & F_{SM}^0 \\ A^0, & \frac{\varphi\gamma A^0}{[\beta_1\rho_w + \beta_2\rho_s + \mu_F]}, & \frac{(1-\varphi)\gamma A^0}{\mu_M}, & \frac{\beta_1\rho_w\varphi\gamma A^0}{\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F]}, & \frac{\beta_2\rho_s\varphi\gamma A^0}{\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F]} \end{bmatrix}. \tag{7}$$

From the first equation of (1), which represents the aquatic class, we have that

$$\mu_\rho(A^0)^2 + (\gamma + \mu_A)A^0 - \Lambda_1 F_{NM}^0 = 0, \tag{8}$$

substituting the expression of  $F_{NM}^0$  in equation (8) we have:

$$\mu_A(A^0)^2 + (\gamma + \mu_A) \left[ 1 - \frac{\Lambda_1\beta_1\rho_w\varphi\gamma}{\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F][\gamma + \mu_A]} \right] A^0 = 0, \tag{9}$$

where

$$\frac{\Lambda_1\beta_1\rho_w\varphi\gamma}{\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F][\gamma + \mu_A]} = M_0, \tag{10}$$

equation (10) represents the basic offspring number of the mosquito population, hence:

$$\begin{aligned} \mu_\rho(A^0)^2 + (\gamma + \mu_A)[1 - M_0]A^0 &= 0, \\ \therefore (A^0)^2 + \frac{(\gamma + \mu_A)}{\mu_\rho}[1 - M_0]A^0 &= 0, \end{aligned} \tag{11}$$

similarly, we have

$$A^0 \left[ A^0 + \frac{(\gamma + \mu_A)}{\mu_\rho}[1 - M_0] \right] = 0, \tag{12}$$

where the roots of  $(A^0)$  are controlled by the magnitude and value of  $M_0$ .

Now, by Substituting  $(A^0)$  into (7) we have a positive equilibrium point of the model in terms of the basic offspring number of the mosquito ( $M_0$ )

$$E_1 = \left. \begin{aligned} [A^0 \ F_M^0 \ M_M^0 \ F_{NM}^0 \ F_{SM}^0] &= \left. \begin{aligned} \frac{(\gamma + \mu_A)(M_0 - 1)}{\mu_\rho}, & \frac{\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_\rho[\beta_1\rho_w + \beta_2\rho_s + \mu_F]}, \\ \frac{(1 - \varphi)\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_\rho\mu_M}, & \frac{\beta_1\rho_w\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_\rho\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F]}, & \frac{\beta_2\rho_s\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_\rho\mu_F[\beta_1\rho_w + \beta_2\rho_s + \mu_F]} \end{aligned} \right\}. \end{aligned} \tag{13}$$

If  $M_0 \leq 1$ , the only biologically meaningful root of equation (13) is  $A^0 = 0$ , which corresponds to the trivial equilibrium given by

$$E_2 = (A^0, F_M^0, M_M^0, F_{NM}^0, F_{SM}^0) = (0, 0, 0, 0, 0). \tag{14}$$

This is also called the mosquito extinction equilibrium, which is of no use due to the absence of mosquitoes in the population. On the other hand, if  $M_0 > 1$ , the system has equation (13) as the nonzero (positive) equilibrium point, where

$$A^0 = \frac{(\gamma + \mu_A)}{\mu_\rho}(M_0 - 1) > 0. \tag{15}$$

### 3.3. Local Stability Analysis of the Nonnegative Equilibrium Point of the Model

**Theorem 1.** *The nonnegative equilibrium point of the mosquito population model is locally asymptotically stable if  $M_0 > 1$ .*

*Proof.* To show this, we obtain the Jacobian matrix ( $J$ ) of the model (1) and then we use the properties of eigenvalues or the principle of  $M$ -matrix to check for its stability.

The Jacobian of model (1) is given as

$$J = \begin{pmatrix} -(2A^0\mu_\rho + \mu_A + \gamma) & 0 & 0 & \Lambda_1 & 0 \\ \varphi\gamma & -(\mu_F + \beta_1\rho_w + \beta_2\rho_s) & 0 & 0 & 0 \\ (1 - \varphi)\gamma & 0 & -\mu_M & 0 & 0 \\ 0 & \beta_1\rho_w & 0 & -\mu_F & 0 \\ 0 & \beta_2\rho_s & 0 & 0 & -\mu_F \end{pmatrix}. \tag{16}$$

By substituting the expression for the aquatic mosquito population ( $A^0$ ) which is given by  $A^0 = ((\gamma + \mu_A)/\mu_\rho)(M_0 - 1)$  into the Jacobian matrix ( $J$ ) in (16), we have that ( $J$ ) can be expressed as:

$$\begin{pmatrix} -[2(\gamma + \mu_A)(M_0 - 1) + \mu_A + \gamma] - \lambda_1 & 0 & 0 & \Lambda_1 & 0 \\ \varphi\gamma & -(\mu_F + \beta_1\rho_\omega + \beta_2\rho_S) - \lambda_2 & 0 & 0 & 0 \\ (1 - \varphi)\gamma & 0 & -\mu_M - \lambda_3 & 0 & 0 \\ 0 & \beta_1\rho_\omega & 0 & -\mu_F - \lambda_4 & 0 \\ 0 & \beta_2\rho_S & 0 & 0 & -\mu_F - \lambda_5 \end{pmatrix} = 0. \tag{17}$$

The characteristics equation of the system (17) is expressed as  $|J - \lambda I| = 0$ , where  $(\lambda) = \lambda_1, \lambda_2, \lambda_3, \lambda_4$  and  $\lambda_5$  are the eigenvalues and  $(I)$  is the identity matrix for system (17).

The eigenvalues were computed using maple 2015, result shows that  $\lambda_1 = -[2(\gamma + \mu_A)(M_0 - 1) + \mu_A + \gamma] < 0$  or negative if  $M_0 > 1, \lambda_2 = -(\mu_F + \beta_1\rho_\omega + \beta_2\rho_S), \lambda_3 = -\mu_M, \lambda_4 = -\mu_F$ , and  $\lambda_5 = -\mu_F$ .

Since all the eigenvalues have negative real parts, we therefore conclude using the principle of  $M$ -matrix that the nonnegative equilibrium point  $(E_1)$  of the model is locally asymptotically stable provided  $M_0 > 1$ . This result epidemiologically implies that the mosquito population will grow or persist with respect to the initial condition (population) of the mosquito.

### 3.4. Global Stability Analysis of the Nonnegative Equilibrium Point of the Model

**Theorem 2.** *if  $M_0 > 1$ , the nonnegative equilibrium point  $(E_1)$  of the model (1) is globally asymptotically stable.*

*Proof.* To establish the global stability of this equilibrium  $(E_1)$ , we construct the following Lyapunov function following the method used in [22].

$$\begin{aligned} M(A^* F_M^* M_M^* F_{NM}^* F_{SM}^*) &= \left( A - A^* - A^* \log \frac{A}{A^*} \right) \\ &+ \left( F_M - F_M^* - F_M^* \log \frac{F_M}{F_M^*} \right) \\ &+ \left( M_M - M_M^* - M_M^* \log \frac{M_M}{M_M^*} \right) \\ &+ \left( F_{NM} - F_{NM}^* - F_{NM}^* \log \frac{F_{NM}}{F_{NM}^*} \right) \\ &+ \left( F_{SM} - F_{SM}^* - F_{SM}^* \log \frac{F_{SM}}{F_{SM}^*} \right). \end{aligned} \tag{18}$$

The derivative of  $(M)$  along the solution path of (1) by direct calculation gives

$$\begin{aligned} \frac{dM}{dt} &= \left( \frac{A - A^*}{A} \right) \frac{dA}{dt} + \left( \frac{F_M - F_M^*}{F_M} \right) \frac{dF_M}{dt} \\ &+ \left( \frac{M_M - M_M^*}{M_M} \right) \frac{dM_M}{dt} + \left( \frac{F_{NM} - F_{NM}^*}{F_{NM}} \right) \frac{dF_{NM}}{dt} \\ &+ \left( \frac{F_{SM} - F_{SM}^*}{F_{SM}} \right) \frac{dF_{SM}}{dt}, \end{aligned} \tag{19}$$

where

$$\left. \begin{aligned} \frac{dA}{dt} &= \Lambda_1 F_{NM} - \gamma A - \mu_A A - \mu_P A^2 \\ \frac{dF_M}{dt} &= \varphi\gamma A - (\beta_1\rho_\omega + \beta_2\rho_S) F_M - \mu_F F_M \\ \frac{dM_M}{dt} &= (1 - \varphi)\gamma A - \mu_M M_M \\ \frac{dF_{NM}}{dt} &= \beta_1\rho_\omega F_M - \mu_F F_{NM} \\ \frac{dF_{SM}}{dt} &= \beta_2\rho_S F_M - \mu_F F_{SM} \end{aligned} \right\}. \tag{20}$$

Thus,

$$\begin{aligned} \frac{dM}{dt} &= \frac{A - A^*}{A} [\Lambda_1 F_{NM} - \gamma A - \mu_A A - \mu_P A^2] \\ &+ \left( \frac{F_M - F_M^*}{F_M} \right) [\varphi\gamma A - [\beta_1\rho_\omega + \beta_2\rho_S] F_M - \mu_F F_M] \\ &+ \left( \frac{M_M - M_M^*}{M_M} \right) [(1 - \varphi)\gamma A - \mu_M M_M] \\ &+ \left( \frac{F_{NM} - F_{NM}^*}{F_{NM}} \right) [\beta_1\rho_\omega F_M - \mu_F F_{NM}] \\ &+ \left( \frac{F_{SM} - F_{SM}^*}{F_{SM}} \right) [\beta_2\rho_S F_M - \mu_F F_{SM}]. \end{aligned} \tag{21}$$

Then,

$$\begin{aligned} \frac{dM}{dt} &= \frac{(A - A^*) [\Lambda_1 F_{NM} - (\gamma + \mu_A + \mu_P A) A]}{A} \\ &+ \frac{(F_M - F_M^*) [\varphi\gamma A - (\beta_1\rho_\omega + \beta_2\rho_S) F_M + \mu_F F_M]}{F_M} \\ &+ \frac{(M_M - M_M^*) [(1 - \varphi)\gamma A - \mu_M M_M]}{M_M} \\ &+ \frac{(F_{NM} - F_{NM}^*) [\beta_1\rho_\omega F_M - \mu_F F_{NM}]}{F_{NM}} \\ &+ \frac{(F_{SM} - F_{SM}^*) [\beta_2\rho_S F_M - \mu_F F_{SM}]}{F_{SM}}. \end{aligned} \tag{22}$$

Therefore,

$$\begin{aligned} \frac{dM}{dt} = & \frac{(A - A^*)[\Lambda_1(F_{NM} - F_{NM}^*) - (\gamma + \mu_A + \mu_P)(A - A^*)]}{A} \\ & + \frac{(F_M - F_M^*)[\varnothing\gamma(A - A^*) - (\beta_1 P_\omega + \beta_2 P_S)(F_M - F_M^*)]}{F_M} \\ & + \frac{(M_M - M_M^*)[(I - \varnothing)\gamma(A - A^*) - \mu_M(M_M - M_M^*)]}{M_M} \\ & + \frac{(F_{NM} - F_{NM}^*)[\beta_1 P_\omega(F_M - F_M^*) - \mu_F(F_{NM} - F_{NM}^*)]}{F_{NM}} \\ & + \frac{(F_{SM} - F_{SM}^*)[\beta_2 P_S(F_M - F_M^*) - \mu_F(F_{SM} - F_{SM}^*)]}{F_{SM}}. \end{aligned} \tag{23}$$

Now, we have that

$$\begin{aligned} \frac{dM}{dt} = & \frac{(A - A^*)\Lambda_1(F_{NM} - F_{NM}^*)}{A} - \frac{(A - A^*)^2\Lambda_1[\gamma - \mu_A]}{A} - \frac{(A - A^*)^3\Lambda_1\mu_P}{A} \\ & + \frac{(F_M - F_M^*)\varnothing\gamma(A - A^*)}{F_M} - \frac{(F_M - F_M^*)^2[\beta_1 P_\omega + \beta_2 P_S + \mu_F]}{F_M} \\ & - \frac{\mu_M(M_M - M_M^*)^2}{M_M} + \frac{(M_M - M_M^*)(I - \varnothing)\gamma(A - A^*)}{M_M} \\ & + \frac{(F_{NM} - F_{NM}^*)\beta_1 P_\omega(F_M - F_M^*)}{F_{NM}} - \frac{(F_{NM} - F_{NM}^*)^2\mu_F}{F_{NM}} \\ & + \frac{(F_{SM} - F_{SM}^*)\beta_2 P_S(F_M - F_M^*)}{F_{SM}} - \frac{(F_{SM} - F_{SM}^*)^2\mu_F}{F_{SM}}. \end{aligned} \tag{24}$$

Collecting the positive and negative terms, we obtained  $dM/dt = M - N$ , where  $(M)$  represent the positive terms and  $(N)$  represent the negative terms

$$\begin{aligned} M = & \frac{(A - A^*)\Lambda_1(F_{INM} - F_{INM}^*)}{A} + \frac{(F_M - F_M^*)}{F_M}\varnothing\gamma(A - A^*) \\ & + \frac{(M_M - M_M^*)}{M_M}(I - \varnothing)\gamma(A - A^*) + \frac{(F_{NM} - F_{NM}^*)}{F_{NM}}\beta_1 P_\omega(F_M - F_M^*) \\ & + \frac{(F_{SM} - F_{SM}^*)}{F_{SM}}\beta_2 P_S(F_M - F_M^*). \end{aligned} \tag{25}$$

Similarly,

$$\begin{aligned} N = & \frac{(A - A^*)^2}{A} \left[ \Lambda_1(\gamma - \mu_A) + (A - A^*)\Lambda_1\mu_P \right] \\ & + \frac{(F_M - F_M^*)^2}{F_M} [\beta_1 P_\omega + \beta_2 P_S + \mu_M + \mu_F] \\ & + \frac{(M_M - M_M^*)^2}{M_M} \mu_M + \frac{(F_{NM} - F_{NM}^*)^2}{F_{NM}} \mu_F \\ & + \frac{(F_{SM} - F_{SM}^*)^2}{F_{SM}} \mu_F \end{aligned} \tag{26}$$

Therefore, if  $M < N$ , then  $dM/dt$  will be negative definite along the solution path of the system. Thus, this implies that  $dM/dt < 0$  and  $dM/dt = 0$  only at a point where

$$\left. \begin{aligned} A = \frac{(\gamma + \mu_A)(M_0 - 1)}{\mu_P}, F_M = \frac{\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_P[\beta_1\rho_W + \beta_2\rho_S + \mu_F]}, M_M = \frac{(1 - \varphi)\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_P\mu_M}, \\ F_{NM} = \frac{\beta_1\rho_W\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_P\mu_F[\beta_1\rho_W + \beta_2\rho_S + \mu_F]}, F_{SM} = \frac{\beta_2\rho_S\varphi\gamma(\gamma + \mu_A)(M_0 - 1)}{\mu_P\mu_F[\beta_1\rho_W + \beta_2\rho_S + \mu_F]} \end{aligned} \right\}. \tag{27}$$

Therefore, the largest compact set  $\{(A, F_M, M_M, M_{NM}, F_{SM}) \in \Omega, dM/dt = 0\}$  is just the singleton set  $(E_1)$  where  $(E_1)$  is the nonnegative equilibrium point of the model. Now according to LaSalle's invariant principle, it means that the  $(E_1)$  is globally asymptotically stable in  $\Omega$  if  $M < N$ , hence  $(E_1)$  will be stable if  $M_0 > 1$ . This result epidemiologically implies that the mosquito population will grow or persist in respective of the initial population.

#### 4. Sensitivity Analysis of the Model

Sensitivity analysis is performed on the model to know which parameters should be targeted towards control intervention strategies, that is to say, which parameter value should be increased or decreased in order to reduce or control the population of mosquito. It is a tool applied to study the change of an output of a model due to change in the input parameters [15].

Mathematically, the normalized sensitivity index of the mosquito basic offspring number with respect to  $\Lambda_1$  is given by  $\chi_{\Lambda_1}^{M_0} = (\partial M_0 / \partial \Lambda_1) \times (\Lambda_1 / M_0)$ . The sensitivity index of the remaining parameters can be computed in the same way as that of  $\Lambda_1$ . Using the data in Table 3, we provide in Table 4 the sensitivity index of the parameters.

$$\chi_{\Lambda_1}^{M_0} = \frac{\varphi\gamma\beta_1\rho_\omega\Lambda_1\mu_F(\mu_A + \gamma)(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)}{\mu_F(\mu_A + \gamma)(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)\varphi\gamma\beta_1\rho_\omega\Lambda_1} = 1,$$

$$\chi_{\varphi}^{M_0} = \frac{\gamma\beta_1\rho_\omega\varphi\Lambda_1\mu_F(\mu_A + \gamma)(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)}{\mu_F(\mu_A + \gamma)(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)(\varphi\gamma\beta_1\rho_\omega\Lambda_1)} = 1,$$

$$\chi_{\gamma}^{M_0} = \frac{\beta_1\rho_S\rho_\omega\varphi\Lambda_1[(\mu_A + \gamma)]\gamma\mu_F(\mu_A + \gamma)(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)}{\mu_F(\mu_A + \gamma)^2(\beta_1\rho_\omega + \beta_2\rho_S + \mu_F)(\varphi\gamma\beta_1\rho_\omega\Lambda_1)} = \rho_S,$$

$$\begin{aligned} \chi_{\beta_1}^{M_0} &= \frac{\beta_1 \rho_\omega \varphi \Lambda_1 (\mu_F + \beta_2 \rho_S) \beta_1 \mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}{\mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)^2 (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \\ &= \frac{(\mu_F + \beta_2 \rho_S)}{(\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}, \\ \chi_{\rho_\omega}^{M_0} &= \frac{\beta_1 \rho_\omega \varphi \Lambda_1 (\mu_F + \beta_2 \rho_S) \beta_1 \mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}{\mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)^2 (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \\ &= \frac{(\mu_F + \beta_2 \rho_S)}{(\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}, \\ \chi_{\rho_S}^{M_0} &= \frac{-\beta_1 \beta_2 \rho_\omega \varphi \Lambda_1 \gamma (\mu_F \rho_S) (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}{\mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)^2 (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \\ &= \frac{-\beta_2 \rho_S}{(\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}, \\ \chi_{\beta_2}^{M_0} &= \frac{-\beta_1 \rho_\omega \rho_S \varphi \Lambda_1 \gamma (\mu_F \beta_2) (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}{\mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)^2 (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \\ &= \frac{-\beta_2 \rho_S}{(\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}, \\ \chi_{\mu_F}^{M_0} &= \frac{-\beta_1 \rho_\omega \varphi \Lambda_1 \mu_F^2 (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}{\mu_F^2 (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)^2 (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \\ &= \frac{-[2\mu_F + \beta_2 \rho_S + \beta_1 \rho_\omega]}{(\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)}, \\ \chi_{\mu_A}^{M_0} &= \frac{-\beta_1 \rho_\omega \varphi \Lambda_1 \gamma [\mu_A \mu_F (\mu_A + \gamma) (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F)]}{\mu_F (\mu_A + \gamma)^2 (\beta_1 \rho_\omega + \beta_2 \rho_S + \mu_F) (\varphi \gamma \beta_1 \rho_\omega \Lambda_1)} \quad (28) \\ &= \frac{-\mu_A}{(\mu_A + \gamma)}. \end{aligned}$$

**5. Simulations and Discussion of Results**

The values tabulated in Table 3 above are used for the model simulation using MATLAB 7.0.

5.1. *Discussions of Results.* Using parameter values in Table 3, some numerical simulations for the model given by (1) are presented. Figure 1 shows the impact of mating probability of female mosquitoes ( $\rho_s$ ) with sterile male mosquitoes on the total population of female mosquitoes, where an increase in such probability tremendously reduces the population of female mosquitoes. Similar dynamics is observed on the total population of female nonsterile mosquito population as depicted in Figure 2. On the other hand, increase in the mating probability of female mosquitoes with wild male mosquitoes increases the population of female nonsterile mosquitoes as presented by Figure 3, while it reduces the population of female sterile mosquitoes as shown in Figure 4. Total mosquito population (both males and females) is the function of female mosquito mating probability ( $\rho_s$ ) and mating rate of sterile male mosquitoes ( $\beta_2$ ) which are presented by Figure 5 (where the population reduces with increase in the female mosquito mating probability) and Figure 6 (with total

TABLE 3: Numerical values of variables/parameters used.

S/N	Variables/parameters	Numerical values	Source
1	$A$	3000	Assumed
2	$M_M$	200	Assumed
3	$F_M$	700	Assumed
4	$F_{NM}$	300	Assumed
5	$F_{SM}$	150	Assumed
6	$\Lambda_1$	70	[15]
7	$\varphi$	0.6	[15]
8	$\beta_1$	0.6	Assumed
9	$\beta_2$	0.4	Assumed
10	$\gamma$	0.00006	[23]
11	$\mu_M$	0.000025	[24]
12	$\mu_\rho$	0.00002	[12]
13	$\mu_A$	0.00003	[25]
14	$\mu_F$	0.000057	Assumed
15	$\rho_W$	0.8	Assumed
16	$\rho_S$	0.2	Assumed

TABLE 4: Sensitivity index of the parameters.

S/N	Parameters	Parameters value	Sensitivity index
1	$\Lambda_1$	70	1.00000
2	$\varphi$	0.6	1.00000
3	$\gamma$	0.00006	0.20000
4	$\beta_1$	0.6	0.142944
5	$\rho_\omega$	0.8	0.142944
6	$\rho_S$	0.2	-0.14284
7	$\beta_2$	0.4	-0.14284
8	$\mu_F$	0.000057	-1.00001
9	$\mu_A$	0.00003	-0.33333

population increasing as mating rate of sterile male mosquitoes reduces), respectively. Thus, the numerical simulations conform with the expected biological behavior of both male and female mosquito populations in the presence of sterilization.

The basic results are therefore enumerated below:

- (i) The nonnegative equilibrium point of the model ( $E_1$ ) was confirmed to be locally and globally asymptotically stable if  $M_0 > 1$ . This result epidemiologically implies that the mosquito population will grow with respect of and in respect to the initial population of the mosquitoes
- (ii) An increase in the female mosquito mating probability ( $\rho_\omega$ ) to be with the wild male increases the

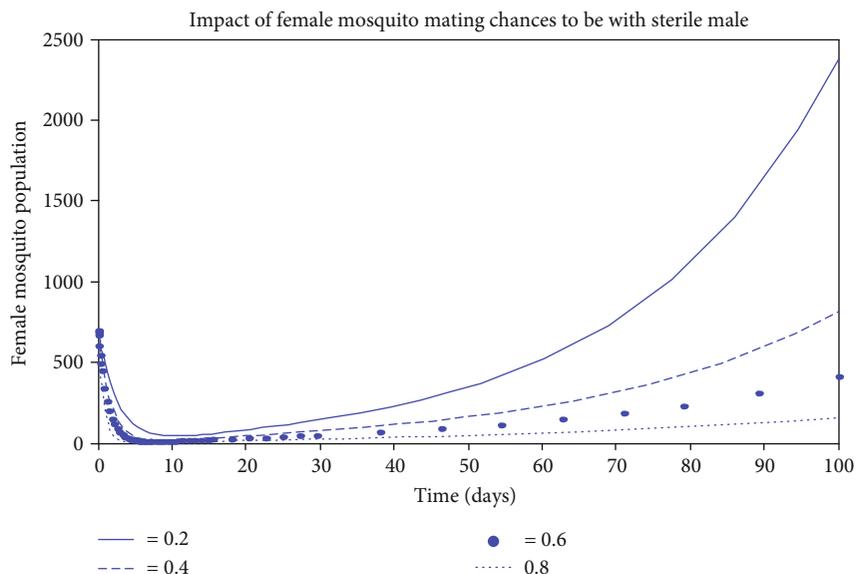


FIGURE 1: Impact of female mosquito mating probability ( $\rho_s$ ) to be with the sterile male on the female mosquitoes; as this rate increases, the population of the female mosquitoes is reduced.

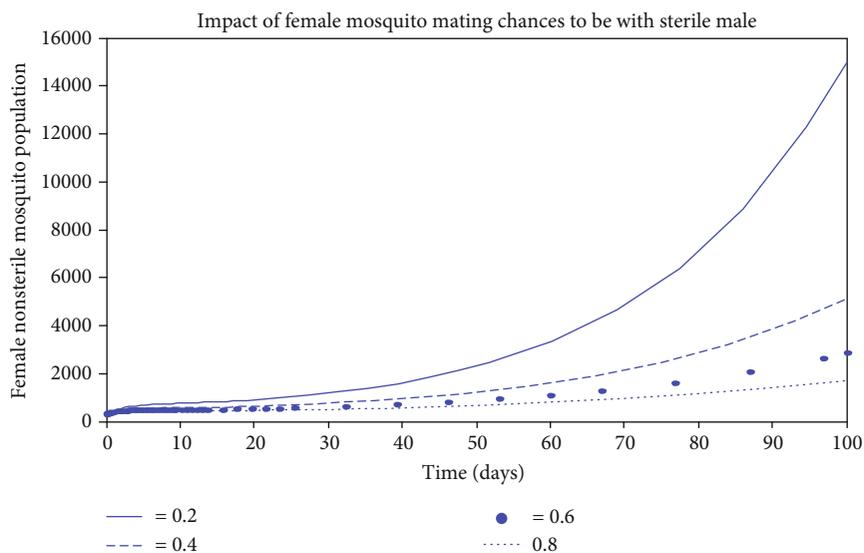


FIGURE 2: Impact of female mosquito mating probability ( $\rho_s$ ) to be with the sterile male on the female nonsterile mosquitoes; as this rate increases, the population of the female nonsterile mosquitoes is reduced.

population of the female nonsterile mosquitoes as seen in Figure 3

- (iii) An increase in the female mosquito mating probability ( $\rho_s$ ) to be with the sterile male reduces the population of the female mosquitoes, female nonsterile mosquitoes, and sterile female mosquito as seen in Figures 1, 2, and 4, respectively
- (iv) An increase in the female mosquito mating probability ( $\rho_s$ ) to be with the sterile male reduces the population of the total mosquito as seen in Figure 5

- (v) An increase in the mating rate ( $\beta_2$ ) of the sterile mosquitoes also reduces the population of the total mosquito population as seen in Figure 6
- (vi) Increasing the female mating probability to be with the sterile male mosquitoes ( $\rho_s$ ), the mating rate of the sterile mosquito ( $\beta_2$ ) and the natural death rates of both the aquatic and female mosquitoes ( $\mu_A, \mu_F$ ) reduce the value of the basic mosquito offspring number ( $M_0$ ), thereby, leading to the reduction and elimination of mosquitoes from a population with time

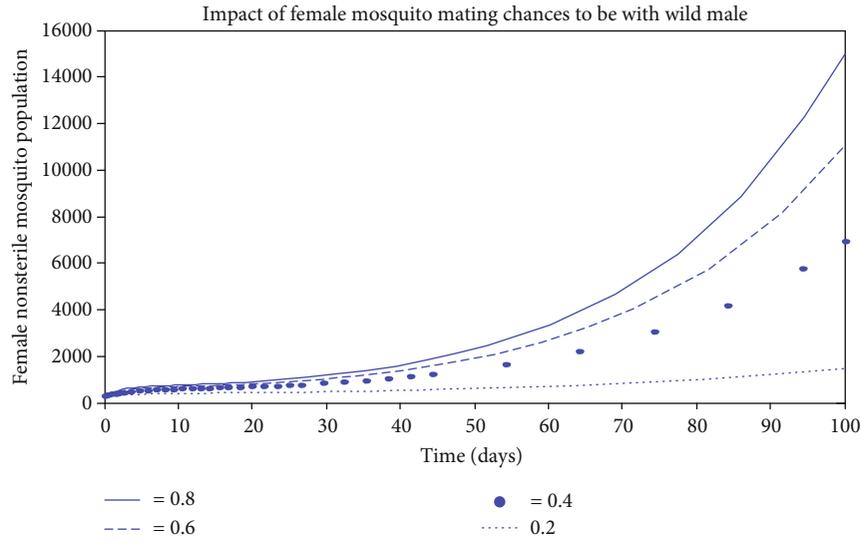


FIGURE 3: Impact of female mosquito mating probability ( $\rho_w$ ) to be with the wild male on the female nonsterile mosquitoes; an increment in this rate increases the population of the female nonsterile mosquitoes.

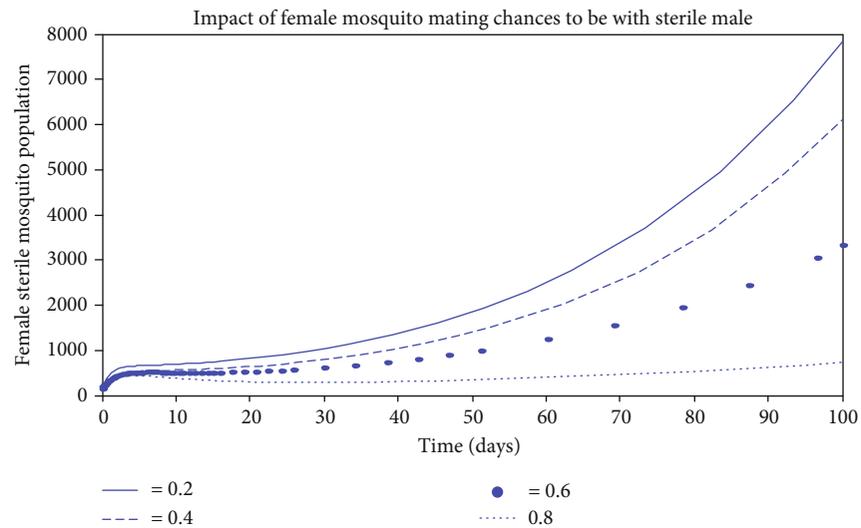


FIGURE 4: Impact of female mosquito mating probability ( $\rho_s$ ) to be with the sterile male on the female sterile mosquitoes; as this rate increases, the population of the female sterile mosquitoes is reduced.

### 6. Conclusion

In this article, we formulated a mathematical model that controls the mosquito population using the sterile insect technology. The research major findings are as follows:

- (i) The nonnegative equilibrium point of the model ( $E_1$ ) was confirmed to be locally and globally asymptotically stable if  $M_0 > 1$
- (ii) An increase in the female mosquito mating probability ( $\rho_s$ ) to be with the sterile male reduces the population of the female mosquitoes, nonsterile female mosquitoes, and sterile female mosquito

- (iii) An increase in the female mosquito mating probability ( $\rho_s$ ) to be with the sterile male reduces the population of the total mosquito
- (iv) An increase in the mating rate ( $\beta_2$ ) of the sterile mosquitoes also reduces the population of the total mosquito population
- (v) Results from sensitivity analysis shows that the female mating probability to be with the sterile male mosquitoes ( $\rho_s$ ), the mating rate of the sterile mosquito ( $\beta_2$ ), and the natural death rates of both the aquatic and female mosquitoes are the most effective parameters for the control of mosquito population

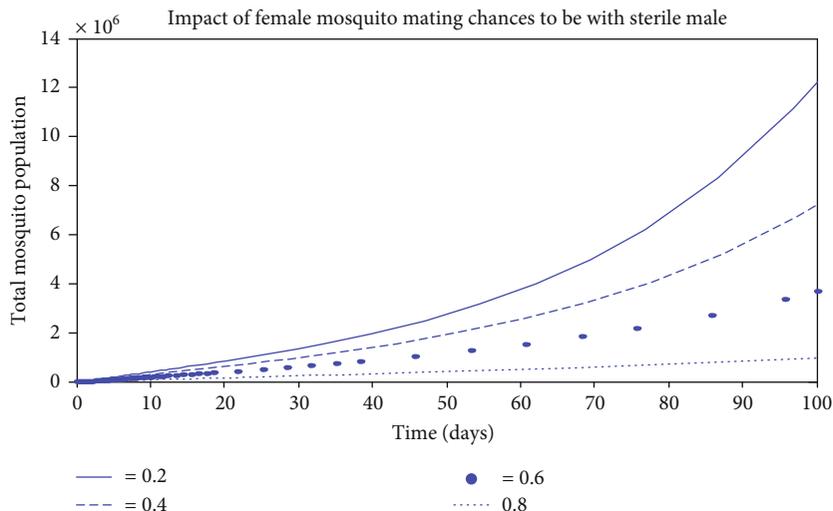


FIGURE 5: Impact of female mosquito mating probability ( $\rho_s$ ) to be with the sterile male on the total mosquito population; an increase in this rate leads to a reduction in the total mosquito population.

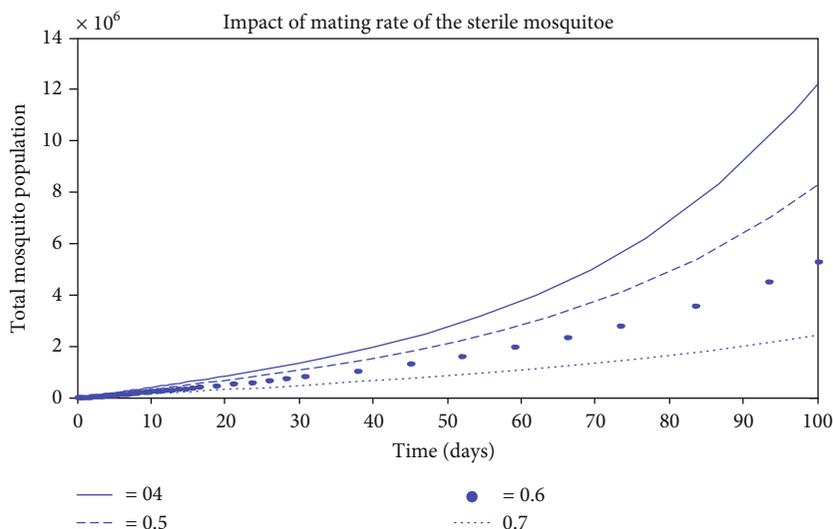


FIGURE 6: Impact of mating rate ( $\beta_2$ ) of the sterile mosquitoes on the total mosquito population; an increase in this rate also leads to a reduction in the total mosquito population.

(vi) Conclusively, releasing sufficient number of the sterile mosquitoes into the wild mosquito population reduces the population of the wild mosquitoes

**Data Availability**

I used hypothetical data in which the sources are dully acknowledged.

**Conflicts of Interest**

The authors declare no conflict of interest in this research.

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