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Research Article

Time Coefficient Estimation for Hourly Origin-Destination Demand from Observed Link Flow Based on Semidynamic Traffic Assignment

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Day-long origin-destination (OD) demand estimation for transportation forecasting is advantageous in terms of accuracy and reliability because it is not affected by hourly variations in the OD distribution. In this paper, we propose a method to estimate the time coefficient of day-long OD demand to estimate hourly OD demand and to predict hourly traffic for urban transportation planning of a large-scale road network that lacks discrete-time rich traffic data. The model proposed estimates the time coefficients from observed link flows given a proven day-long OD demand based on a bilevel formulation of the generalized least square and semidynamic traffic assignment (OD-modification approach). The OD-modification approach is formulated as a static user-equilibrium assignment with elastic demand, based on the residual demand at the end of each period. Our model does not require setting many parameters regarding the OD demand matrices and the discrete-time dynamic traffic assignments. Applying the model to large-scale road network demonstrates that it efficiently improves estimation accuracy because the 24-hour time coefficients of survey data are slightly biased and may be modified properly. In addition, the methods that partially relax the assumption of OD-modification approach and transform the estimated demand into demand based on departure time are examined.

1. Introduction

The four-step prediction technique on a one-day unit to predict day-long origin-destination (OD) demand and link flow is generally used worldwide for urban and transportation planning. For one-day activity, a normal pattern exists, in which people commute and shop in the morning and return home in the evening every day on weekdays. Therefore, the estimated day-long OD demand has the advantage of accuracy and reliability, because it is not affected by hourly variations in OD distribution.

However, hourly traffic prediction is important for transportation analysis. Thus, a simple estimate of hourly link flow or OD demand which multiplies the average measured time coefficients by the day-long values of link flow or OD demand is sometimes adopted for practical use in a large-scale road network that lacks real-time data. In this paper, the time coefficients of OD demand are calculated as follows:

$$E_{rs}^{n} = \frac{G_{rs}^{n}}{Q_{rs}}, \qquad \sum_{n=1}^{24} E_{rs}^{n} = 1, \tag{1}$$

where E_{rs}^{n} is the time coefficient of the OD pair rs for the period n ($n = 1, 2, \ldots, 24$), G_{rs}^{n} is the hourly OD demand of OD pair rs for period n, and Q_{rs} is the day-long OD demand of OD pair rs.

Such estimated hourly values cannot ensure user equilibrium on route-choice behavior and prediction accuracy. Accurately estimating hourly link flow and OD demand of a large-scale road network by harnessing the reliable day-long

OD demand is highly desired. Additionally, the input data of road network for our study can now benefit from a nationwide survey that includes an OD survey with questionnaire and link flow observations with manual counting; however, it does not provide real-time and discrete-time rich traffic data.

We therefore propose a time coefficient estimation (TCoE) model to obtain the hourly OD demand and to analyze hourly traffic predictions and measures for urban transportation planning of a large-scale road network that lacks discrete-time rich traffic data. The proposed TCoE model estimates the time coefficients from observed link flows given a proven day-long OD demand, which in turn is based on a bilevel formulation of the generalized least square and semidynamic traffic assignment. In the model, hourly OD demands are deduced from both the time coefficient and day-long demand. The semidynamic traffic assignment is a static user-equilibrium assignment with elastic demand and accounts for residual demand at the end of study period, as discussed later.

Previous work has developed several approaches of frameworks to estimate OD matrices from observed link flows, for example, entropy maximization [1], maximum likelihood [2, 3], and generalized least squares [4–6]. The generalized least square model has the advantages of relative robustness for fitting and applicability because it allows for errors on input data because link flow observations are not treated as constraints in the formulation.

Recent years have seen the development of a bilevel formulation for OD demand estimation from observed link flow [7–9]. In this approach, the upper program uses the generalized least square to estimate the parameters of OD demand and the lower program is a static traffic assignment for calculating link flow proportion for each OD pair.

Other works have developed dynamic OD demand estimates based on the time-dependent proportional and dynamic assignments operated over continuous discrete periods (i.e., several minutes) [10–14]. Tsekeris and Stathopoulos [15] proposed estimating the dynamic OD matrix by an efficient algorithm in an entropy modelling. Etemadnia and Abdelghany [16] proposed a dynamic OD demand estimation through dynamic traffic assignment on the basis of the least square method. This method provides estimates for about 10-minute intervals at peak hours by dividing an urban network into several subareas. However, because of high computational cost due to dynamic traffic assignment, these models did not apply to large-scale road networks that lack rich traffic data. Cascetta et al. [17] proposed a quasi-dynamic assumption for dynamic estimates of the OD matrix. In this approach, OD demand shares are constant across a reference period (1 to 24 h); the efficiency of this model was tested in an experiment on a limited network on motorways in Italy. By using the quasi-dynamic assumption, this method can reduce the number of unknowns given the same set of observed traffic counts. However, this model requires rich data input such as time-dependent OD data (or inflow data from origin) and link flows and did not apply to a generalized large-scale road network, including a nationwide intercity network. The result of the experiment and examination of the hypothesis depends strongly on the accuracy and richness of the input data and the scale of the network.

Zhou et al. [12] proposed dynamic origin-destination demand estimation using multiday link traffic counts by using the bilevel formulation of a generalized least square and a DTA simulation program that is able to improve the accuracy by using a day-to-day count data. In this model, the upper problem uses two objectives to minimize the deviations between observed and estimated link flows and the deviations between the estimated and the target demands, of which the target demand is a static demand or the sum of dynamic demands. Stathopoulos and Tsekeris [18] investigated the problem of updating dynamic OD demands by exploiting a series of days of link traffic counts without the need for surveys. They analyzed different time-recursive mechanisms by transforming trip departures into equivalent OD trip flows which are based on the assumption of a fixed distribution of trip destinations. The technique to use the day-to-day count data for OD estimation can contribute an efficient OD estimation significantly. However, in this paper, we do not consider the day-to-day OD estimation because we focus on the network that cannot prepare such day-to-day counted data.

When we consider uncertain demand information, Tsekeris and Stathopoulos [19] investigated the problem of estimating dynamic matrices using automatic link traffic counts and the uncertain prior demand information. They proposed an optimization algorithm for the fast estimation of trip departure rates with the incorporated lower and upper bound constraints and applied the algorithm to Athena's networks. Bierlaire [20] proposed the total demand scale as a measure of quality for OD trip tables estimated from link counts. The scale means the range of the total level of demand in the network. In our model, the day-long OD demand is used as the constraint where the total of 24-hour OD demands estimated for each OD pair preserves a given day-long OD demand, although there are no reliable hourly OD demand data by survey and automatic counted data.

Note that such OD demand matrix estimation problems generally require many parameters corresponding to the number of centroids of origin and destinations. This number becomes multiples of the number of study periods. The greater this number is, the shorter the study period is. Such a detailed model may be difficult to apply to large-scale road networks including arterial roads and expressways over 24-hour periods. If the study network for estimating time-varying OD demand is large and has many routes for many OD pairs but cannot provide sufficient data, for example, provided by on-line detection equipment, a different approach may be adopted.

The TCoE model proposed herein uses a bilevel formulation, in which the upper problem is based on the generalized least square to estimate 24-hour time coefficients given the day-long OD demand matrix and observed link flows. Thus, the TCoE model does not require many parameters to be set (e.g., for the origins and destinations of the OD demand matrices) so that the hourly OD demand matrices can be calculated by multiplying the given day-long OD demand for the 24-hour time coefficients estimated by the model. The TCoE model thus efficiently improves estimation

accuracy because the 24-hour time coefficients aggregated from OD surveys are somewhat biased, thereby reducing accuracy. Therefore, the results show that the TCoE model reduces the number of parameters and computational cost but retains high accuracy for the 24-hour model when applied to a large-scale network that lacks rich real-time data. The characteristics of the time coefficients of the OD survey data are discussed in Section 4. To apply this model to a large-scale road network with limited input data, we adopt the OD demand-modification approach as a semidynamic traffic assignment for the lower problem of TCoE.

Fujita et al. [21] and Matsui and Fujita [22] have proposed a time-of-day user-equilibrium (TUE) traffic assignment of the OD demand-modification approach as a semidynamic traffic assignment. TUE is based on Wardrop's user-equilibrium (UE) principle in which drivers choose the shortest routes for conventional UE assignments [23]. TUE divides a continuous OD demand for one-day unit into demands of each study period (1 or 2 hour units). It semidynamically estimates hourly traffic flow by period by considering the residual traffic volume at the end of each period. TUE is formulated as a static UE assignment with elastic demand that modifies the OD demand to consider the residual traffic volume. That is, in the TUE, the residual traffic in the current period is added semidynamically to the demand in the next period.

The semidynamic UE assignment formulation, which considers the residual traffic at the end of each period, has been proposed in three approaches: the OD demand-modification approach [21, 24], the link flow modification approach [25, 26], and the vertical queue approach [27, 28]. Because the OD-modification approach is more applicable and incurs less computational cost than the other approaches, it was extended to several models that consider a toll road with diversion [29, 30] and a mode-choice function [31].

The TCoE model uses a static UE assignment to obtain link flow proportions of hourly OD demand in the lower problem. The assignment adopted is the same model as the TUE with toll road [30] except that it uses fixed hourly demand without considering residual demand in the OD-modification approach. However, the treatment of residual demand in the TCoE model is almost the same as the OD-modification approach because the OD demand-modification approach is considered in the upper problem of the TCoE model, not treated in the lower problem of UE assignment. Employing the OD-modification approach in TCoE allows us to apply it to a large-scale road network that lacks rich traffic data to reduce a computational cost and to analyze hourly traffic situations (including peak hours), all of which are useful for urban transportation planning.

In this paper, we first review the semidynamic concept for the OD-modification approach by comparing it with the OD modification in the TCoE model. In addition, a partial relaxation of the assumption about the length of the study period in the OD-modification approach is developed for practical use. Next, we develop the TCoE model from observed traffic flow by using the generalized least square under a given day-long OD demand and the OD-modification approach. Third, before application to the study network, we clarify that a slight bias exists in the time coefficients of OD demand aggregated from survey. Finally, we demonstrate that the TCoE model can improve the accuracy of estimates of hourly link flow compared with the traffic assignments adopting the initial hourly OD demand aggregated from the survey. In addition, the method transforming the estimated hourly OD demand into OD demand based on departure time is also examined.

2. Concept of Semidynamic OD Demand-Modification Approach and TCoE Model

2.1. Outline. The TCoE is a model that justifies the 24-hour time coefficients under a given day-long OD demand by minimizing the least square error between hourly observed link flow and estimated link flow. Conventional estimates of the OD demand matrix based on observed link flows require link flow proportions estimated by traffic assignments. The TCoE model also uses the semidynamic OD-modification approach to obtain link flow proportions for each hour of OD demand. The residual demand for the semidynamic approach in the TCoE model is not operated in the lower problem of traffic assignment but rather the upper problem of the TCoE model.

This section reviews the semidynamic concept in the OD-modification approach of Fujita et al. [21] and describes the basic formulation for the OD-modification approach by comparing it with the OD modification in the TCoE model. In addition, a partial relaxation of the assumption about the length of the study period in the OD-modification approach is newly proposed for practical use in order to compare the results of applying the models to a large-scale road network.

2.2. Formulation of OD Demand-Modification Approach and TCoE Model. When an hour is set for a study time period, the OD-modification approach semidynamically assigns hourly OD demand for a day by each hour based on the UE principle that drivers select a route with minimum time. Let T (=60 min) be the length of a period and let G_{rs}^n be the hourly OD demand between OD pair rs ($\in R, S$) in period n ($\in N$). G_{rs}^n is aggregated based on departure time (hourly OD-dep). Furthermore let λ_{rs}^n be the travel time for OD pair rs in period n.

The OD-modification approach assumes that the maximum travel time between OD pair rs is less than T ($\lambda_{rs}^n < T$) and that the hourly OD demand G_{rs}^n departs from the origin uniformly at the rate (G_{rs}^n/T) during period n.

Figure 1 shows the modification of hourly OD demand. This figure shows an OD pair rs with only a path and some links, the hourly OD demand G_{rs}^n, and the travel time λ_{rs}^n for the path during period n.

Even though many paths exist between the OD pair rs, the explanation of this figure is the same except for exchanging G_{rs}^n to a path demand because the OD-modification approach adopts the UE assignment in which all paths used have the same travel time.

Here, part of the demand of G_{rs}^n departing from the time ($T - \lambda_{rs}^n$) at the end of period n does not arrive at its destination. Thus, some traffic does not arrive at its destination at the end of period n after traveling along the path between OD pair rs. This nonarrived traffic is $\lambda_{rs}^n G_{rs}^n/T$.

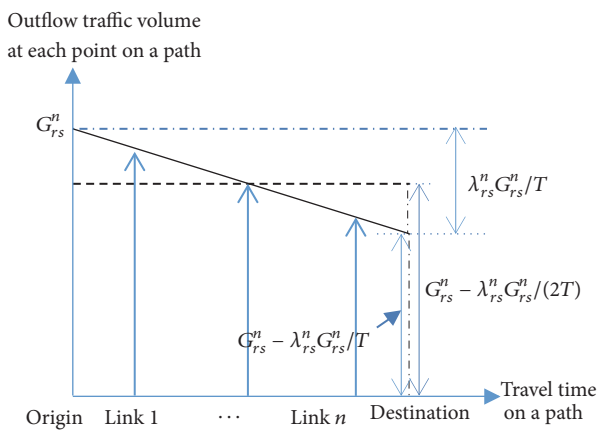


Figure 1: Modification image of residual demand at the end of period in OD modification approach.

Therefore, the traffic that arrives at its destination is

$$G^n_{rs} - \frac{\lambda^n_{rs}G^n_{rs}}{T}. \tag{2}$$

At this time, link flows, which presume observed link flows, at several points along a path between the OD pair are expressed along the downward-slopping solid line in Figure 1. The OD-modification approach cannot describe the variation of link flows midway along a path because of static assignment, although it can uniformly assign the same traffic volume as the demand to all links along the path.

Therefore, to minimize the error between observed and assigned link flows midway along the path, the OD-modification approach modifies G^n_{rs} to the level of demand along the dotted line in Figure 1 at

$$G^n_{rs} - \frac{\lambda^n_{rs}G^n_{rs}}{(2T)}, \tag{3}$$

which subtracts half of the nonarrived traffic from the current OD demand. Furthermore, the subtracted OD demand $q^n_{rs} = \lambda^n_{rs}G^n_{rs}/(2T)$, which is a residual OD demand in the current period n, is assigned to the next period.

We average the burden of a residual demand in both the current and next periods by using the parameter of 1/2 in q^n_{rs} due to the uniform burden of residual demand in a static assignment. That is, when we select the other proportion (for example, 1/3) of nonarrived traffic in a current period n, 2/3 of nonarrived traffic may be loaded as the residual demand in the next period.

By considering continuous periods, the residual OD demand of the previous period also flows in the current network, so it must be added to the current OD demand. Therefore, the following equation expresses the hourly OD demand based on the midway (g^n_{rs}: hourly OD-mid) and that averages the error of link flows along a path (this can be obtained by adding the residual OD demand from the previous period (q^{n-1}_{rs}) to the hourly OD-dep (G^n_{rs}) in the current period and subtracting q^n_{rs} in the current period from G^n_{rs}):

$$g^n_{rs} = q^{n-1}_{rs} + G^n_{rs} - \frac{\lambda^n_{rs}G^n_{rs}}{(2T)}. \tag{4}$$

In the above equation, g^n_{rs} is the demand function for the OD-modification approach, the travel time λ^n_{rs} is a variable, and the previous residual demand q^{n-1}_{rs} is a constant in current period and is based on an estimate made in previous period.

When we set $g^n_{rs} = D^n_{rs}(\lambda^n_{rs})$, in which $D^n_{rs}(\cdot)$ is the demand function for the OD modification in (4), the OD-modification approach can be formulated as a nonlinear minimization problem, that is, a static UE assignment with elastic demand, as follows:

$$\begin{aligned}
\min_{x,g} \quad & Z \\
&= \sum_a \int_0^{x^n_a} t_a(\omega)\, d\omega \\
&\quad + \sum_r \sum_s \int_0^{g^n_{rs}} D^{n\,-1}_{rs}(z)\, dz \\
\text{subject to} \quad & \sum_k f^n_{rsk} - g^n_{rs} = 0 \quad \forall n, r, s, \\
& x^n_a = \sum_{k\in K} \sum_{rs\in\Omega} (\delta^{nrs}_{ak} f^n_{rsk}) \quad \forall n, a, \\
& f^n_{rsk} \geq 0,\ x^n_a \geq 0,\ g^n_{rs} \geq 0 \quad \forall n, r, s, k, a,
\end{aligned} \tag{5}$$

where x^n_a is the flow for link a ($\in A$) during period n, $t_a(\cdot)$ is a link-cost function on link a, f^n_{rsk} is the flow for path k between OD pair rs during period n, δ^{nrs}_{ak} is a link-path incidence variable (=1 if path k for OD pair rs during period n includes link a; =0 otherwise), G^n_{rs} is the hourly OD demand based on departure time, and $D^{n\,-1}_{rs}(\cdot)$ is the inverse of the demand function for OD modification.

Note that, after reformulating the Lagrange function above, we can obtain the optimality conditions. The user-equilibrium conditions can be obtained by applying Karush-Kuhn-Tucker conditions with regard to path flows to the Lagrange function. By applying the Karush-Kuhn-Tucker conditions with regard to g^n_{rs}, we can deduce the demand function for the OD-modification approach in (4).

References [29, 30] explored the basic model above with the extended OD modification (i.e., a semidynamical UE assignment) to an urban road network including expressways with toll load. This model adopted a diversion function based on a binary logit model between both routes with and without the toll expressway. This extended model was applied to a large-scale road network and demonstrated good accuracy and practicability. In this paper, we examine the characteristics of several time-of-day assignments of OD modification approach for a large-scale road network. We use this extended model (hereafter TUE) as the OD-modification approach. For comparison, we also adopt the TUE with fixed hourly demand (TUE-f) to eliminate the residual demand in TUE and assign hourly OD demand separately in each hour,

which is the same model as a day-long UE assignment with toll road [32] when using day-long OD demand.

As mentioned earlier, the OD modification is the semi-dynamic assignment method that modifies the hourly OD-dep (G_{rs}^n) into the hourly OD-mid (g_{rs}^n) to minimize the error of estimated link flow averaged midway over a path along the dotted line in Figure 1 for considering residual demand. Note that the TCoE model proposed herein also estimates the hourly OD-mid (g_{rs}^n) under a given day-long OD demand and operates the residual OD demand in almost the same way as the OD-modification approach because the TCoE model modifies the hourly OD-dep (G_{rs}^n) into the hourly OD-mid (g_{rs}^n) to minimize the error between the observed and estimated link flows midway along a path as a bilevel problem with elastic hourly demand given day-long OD demand. In Section 5, we apply the TCoE model to a large-scale road network that lacks rich real-time data and demonstrate the validity of the OD modification for the TCoE model by analyzing the accuracy and comparing the results to TUE assignments.

2.3. Partial Relaxation of Assumption ($\lambda < T$) in OD-Modification Approach for Practical Use. The OD-modification approach assumes that the length of the period must be set longer than the maximum travel time. However, the model may be hard to treat if we cannot set the period length sufficiently long in practical use, which may force us to give up the application, change the strategy of not satisfying the assumption half way through calculation, or recalculate after resetting the length of the period. Therefore, provided we keep the accuracy for practice use, we consider a partial relaxation of the assumption for trips longer than the length of the period. This is done as follows.

From (4) in Section 2.2, when the travel time λ_{rs}^n becomes longer than the period, the rate of the current OD demand $G_{rs}^n - \lambda_{rs}^n G_{rs}^n/(2T)$ in the terms of twice and third in (4) is modified by more than half of G_{rs}^n and the assignment theoretically loads a demand in the current period which is less than the rate of the next period. However, the links around the origin should have sufficient burden because all the current OD demand G_{rs}^n departs in the current period. Therefore, for uniform treatment with the burden of the current OD demand in a static assignment, in case there are some long trips that exceed the length of the period, we average the burdens of both the OD demands of the current and next periods and at least load half of G_{rs}^n in the current period. However, this special handling must be controlled within the range to avoid influencing the accuracy of assignment by suitably setting the length of the period.

To keep the current theoretical OD demand $G_{rs}^n - \lambda_{rs}^n G_{rs}^n/(2T)$ greater than or equal to half of G_{rs}^n, we replace the nonnegative constraint $g_{rs}^n \geq 0$ defined in the formulation of the OD-modification approach with $g_{rs}^n \geqq q_{rs}^{n-1} + G_{rs}^n/2$. In the iteration algorithm, we calculate g_{rs}^n after exchanging λ_{rs}^n for T when $\lambda_{rs}^n > T$. Note that TUE-f with fixed demand does not need this treatment for $\lambda_{rs}^n > T$. This treatment is verified through an application in Sections 4 and 5.

3. Formulation of TCoE Model and Calculation Method of Hourly OD Based on Departure Time

We now present the formulation and solution algorithm for the TCoE model proposed herein. Additionally, we propose a calculation method of the hourly OD demand based on departure time (hourly OD-dep) from the hourly OD demand based on midway (hourly OD-mid) estimated by the TCoE model.

3.1. Formulation of Time Coefficient Estimation Model from Observed Traffic Flow. The TCoE model justifies the time coefficients given a day-long OD demand by minimizing the least square error between estimated link flow and observed link flow. Generally, estimates of the OD demand matrix based on observed link flow require the link flow proportion for each OD pair, which is estimated by traffic assignment.

In this study, we use a set of 24-hour time coefficients in a day for a pair of departure and arrival areas as a pattern of the time coefficients. The TCoE model can significantly reduce the number of operation variables regarding time coefficients from a pattern to a few dozen patterns of time coefficients with high accuracy, as shown in the result of the application in Section 5. That is, the TCoE model does not need many variables for all OD pairs. The TCoE model employs only a pattern of time coefficients for the whole study area or, when a study area is divided into several subareas, several patterns for the subarea pairs in dual directions.

We set a departure subarea to i in set K and an arrival subarea to j in set L. The upper problem of TCoE minimizes square errors between the observed and estimated hourly link flows under the given link flow proportion as follows:

$$\min_{E} \quad Z = \sum_{n}\sum_{a}\left(\sum_{kl}\sum_{r\in K}\sum_{s\in L} P_{a,rs}^n E_{kl}^n Q_{rs} - \hat{x}_a^n\right)^2 \tag{6}$$

$$\text{s.t.} \quad \sum_{n} E_{kl}^n = 1, \; E_{kl}^n \geq 0 \quad \forall n, k, l, \tag{7}$$

where Q_{rs} is the day-long OD demand for OD pair rs, $\hat{x}_a^n$ is the observed link flow for link a in period n, $P_{a,rs}^n$ is the flow proportion of link a for hourly OD demand for OD pair rs in period n, and E_{kl}^n is the time coefficient for departure area K and arrival area L in period n.

This model is a bilevel problem in which the upper problem is the above minimization and the lower problem is the TUE-f assignment. Therefore, the upper problem estimates the time coefficients under the given link flow proportions and the lower problem estimates link flow proportions by using the TUE-f assignment with the hourly OD demand, which is calculated by multiplying the given day-long OD demand by the time coefficients of the upper problem. As mentioned in Section 2.2, the TCoE model gains link flow proportion for each OD pair and each period estimated by the TUE-f assignment, which is basically the same model as the day-long UE assignment except that it uses hourly OD demands and parameters related to the hourly link-cost function.

3.2. Solution Algorithm for TCoE Model. The time coefficients of the solution that minimizes the optimum function Z can be obtained as a convergence value after alternately calculating the upper and lower problems, and the hourly OD demand and link flows are also estimated simultaneously. The first-order condition for the above problem can be read as the Lagrangian function integrated with constraint conditions as follows:

$$\begin{aligned}\psi &= \sum_n \sum_a \left(\sum_{kl} E_{kl}^n \sum_{r\in K} \sum_{s\in L} P_{a,rs}^n Q_{rs} - \hat{x}_a^n \right)^2 \\ &\quad - \sum_{kl} \nu_{kl} \left(1 - \sum_n E_{kl}^n \right), \\ E_{kl}^n &\geq 0 \quad \forall n, k, l,\end{aligned} \tag{8}$$

where ν_{kl} is a Lagrange multiplier for origin area k and destination area l. When we set a departure area to i in set K and an arrival area to j in set L and deviate ψ by E_{ij}^n and ν_{ij}, we obtain

$$\begin{aligned}\frac{\partial \psi}{\partial E_{ij}^n} &= 2 \sum_a \left\{ \left(\sum_{kl} E_{kl}^n \sum_{r\in K} \sum_{s\in L} P_{a,rs}^n Q_{rs} - \hat{x}_a^n \right) \right. \\ &\quad \left. \cdot \left(\sum_{r\in I} \sum_{s\in J} P_{a,rs}^n Q_{rs} \right) \right\} - \nu_{ij} \\ &= 2 \sum_a \left\{ \left(\sum_{kl} E_{kl}^n \sum_{r\in K} \sum_{s\in L} P_{a,rs}^n Q_{rs} \right) \right. \\ &\quad \left. \cdot \left(\sum_{r\in I} \sum_{s\in J} P_{a,rs}^n Q_{rs} \right) \right\} - 2 \sum_a \left(\hat{x}_a^n \sum_{r\in I} \sum_{s\in J} P_{a,rs}^n Q_{rs} \right) \\ &\quad - \nu_{ij} \quad \forall n, i, j.\end{aligned} \tag{9}$$

From the Karush-Kuhn-Tucker condition, the optimum solution satisfies

$$\begin{aligned}\frac{\partial \psi}{\partial E_{ij}^n} &= 0 \quad \text{if } E_{ij}^n > 0 \\ \text{or } \frac{\partial \psi}{\partial E_{ij}^n} &\geq 0 \quad \text{if } E_{ij}^n = 0, \\ \frac{\partial \psi}{\partial \nu_{ij}} &= \sum_n E_{ij}^n - 1 = 0 \\ &\qquad \forall n, i, j.\end{aligned} \tag{10}$$

The numerical solution for this problem is to solve the simultaneous equations (9).

Additionally, we calculate the following iterative steps under the nonnegative constraint condition $E_{kl}^n \geq 0$ in (7).

Step 1. Obtain the optimum if all E_{kl}^n satisfy the following. Otherwise, set h to 0 and all E_{kl}^n to $E_{kl}^n(h)$ and then go to Step 2.

$$\begin{aligned}&E_{kl}^n > 0, \\ &\text{or } \frac{\partial \psi}{\partial E_{ij}^n} \geq 0 \quad \text{when } E_{kl}^n = 0.\end{aligned} \tag{11}$$

Step 2. Exclude $E_{k'l'}^n$ from the set of $E_{kl}^n(h)$; that is, $E_{k'l'}^n(h) < 0$ and set $E_{k'l'}^n(h+1) = 0$.

Keep excluding $E_{k'l'}^n$ if $(\partial\psi/\partial E_{k'l'}^n)|_{E_{k'l'}^n(h-1)} \geq 0$ for $E_{k'l'}^n$; that is, $E_{k'l'}^n(h-1) = 0$.

Otherwise, add $E_{k'l'}^n$ to the set of $E_{kl}^n(h)$.

Solve the simultaneous equation (9) under new set of $E_{kl}^n(h)$ and gain $E_{kl}^n(h+1)$.

Step 3. Set $E_{kl}^n(h+1)$ as the optimum if all $E_{kl}^n(h+1)$ satisfy (9). Otherwise, set $h = h + 1$ and go to Step 2.

The optimum time coefficients obtained are multiplied by day-long OD demand to estimate the hourly OD demand which is used for TUE-f assignment in the next step. The link flow proportions given from the result of TUE-f assignment are applied to obtain the new time coefficients for the upper problem of the TCoE model. The solution of time coefficients and hourly OD demand to minimize the square errors of link flows can be obtained by converging the values of time coefficients through these calculations.

Conversely, when setting only a pattern of time coefficients in the study area, the optimum time coefficients can be calculated by using only link flows from the TUE-f assignment without link flow proportion for each OD pair as follows.

When $x_a'^n$ is set to an estimated link flow for link a in period n given by the TUE-f assignment and E'^n is set as a pattern of time coefficients for the study area, we get

$$\sum_{r\in I} \sum_{s\in J} P_{a,rs}^n E'^n Q_{rs} = x_a'^n \quad \forall n, a. \tag{12}$$

By substituting above E'^n, (9) is transformed into

$$\frac{\partial \psi}{\partial E'^n} = 2E^n \sum_a \left\{ \left(\frac{x_a'^n}{E'^n} \right)^2 \right\} - 2 \sum_a \left(\hat{x}_a^n \frac{x_a'^n}{E'^n} \right) - \nu = 0 \quad \forall n. \tag{13}$$

We get a pattern of time coefficients when (13) is solved with $x_a'^n$ estimated by the TUE-f assignment.

The optimum solution for the time coefficient can be obtained by the convergence of E'^n in upper and lower iterative calculations. The conventional OD demand estimation models have to use the link flow proportion for each OD pair. However, this method can do the calculation only by using estimated link flows, even if link flow proportions are not estimated due to limits of computer capacity for large-scale road networks.

3.3. Calculation Method of Hourly OD-dep from the Result of TCoE Model. Hourly OD demand obtained by TCoE is the hourly OD-mid that minimizes the error of estimated and observed link flows midway along paths between OD pairs.

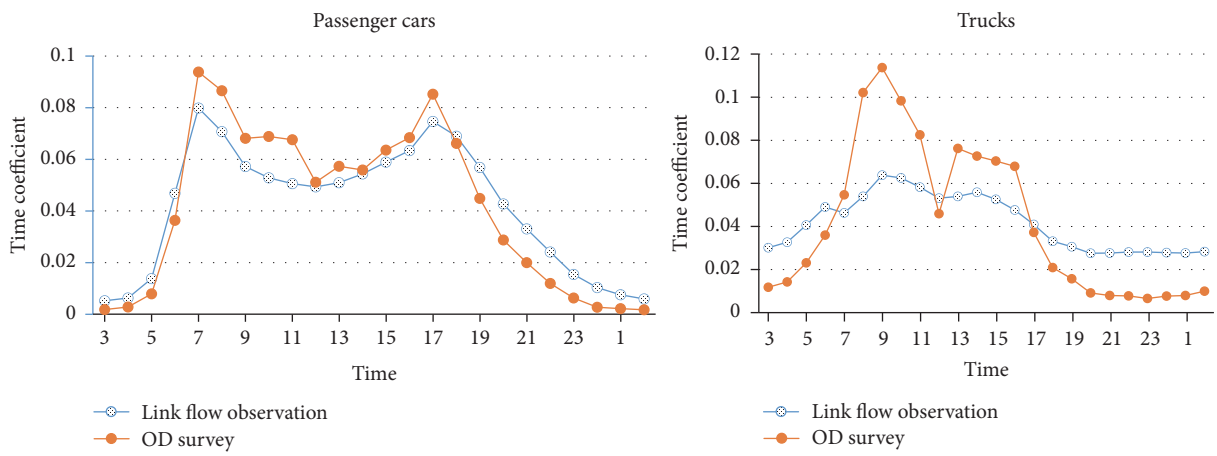


Figure 2: Hourly variation patterns by survey type and vehicle type.

Therefore, the hourly OD demand by the TCoE model differs a little from OD demand aggregated based on departure time (hourly OD-dep) from survey data. When the hourly OD-dep is required for practical use, we explain the method to calculate the hourly OD-dep from the TCoE result.

When the hourly OD demand by TCoE is assumed to be the hourly OD-mid g_{rs}^{n}, as mentioned in Section 2, the hourly OD-dep G_{rs}^{n} in period n is obtained by transforming (4) as follows:

$$G_{rs}^{n} = \frac{\left[g_{rs}^{n} - \lambda_{rs}^{n-1'} G_{rs}^{n-1} / (2T)\right]}{\left[1 - \lambda_{rs}^{n'} / (2T)\right]} \quad \forall n, r, s, \tag{14}$$

where G_{rs}^{n} is the hourly OD demand based on departure time (hourly OD-dep) for OD pair rs in period n, g_{rs}^{n} is the hourly OD demand based on the midway value (hourly OD-mid), taking into account the residual demand for OD pair rs in period n, and $\lambda_{rs}^{n'}$ is the minimum travel time for OD pair rs in period n but is set to $\lambda_{rs}^{n'} = T$ when $\lambda_{rs}^{n} > T$ (T is the period). Here the residual demand in period n is expressed by $\lambda_{rs}^{n'} G_{rs}^{n} / (2T)$.

By using the above equation, hourly OD-dep can be calculated from hourly OD-mid of the TCoE model. In Section 5, we apply the hourly OD-dep to the OD-modification approach (TUE assignment). We also analyze the hourly variation pattern and examine the validity of this method.

4. Basic Analysis of Time Coefficients for Hourly Origin-Destination Demands

The road traffic census is one of the most important national traffic surveys in Japan. This survey includes observations of hourly link flow on arterial road and expressway and questionnaire surveys of origin destination for each automobile trip nationwide. In this section, we examine the hourly variation pattern of OD demand and compare it with the hourly variation pattern of link flows observed and assigned by TUE.

4.1. Comparison between Link Flow and Hourly OD-dep from Survey Data. Figure 2 shows hourly variation patterns of OD survey and link flow observation for passenger cars and trucks. The OD survey shows an hourly OD variation pattern that is a fluctuation of the 24-hour time coefficients for OD demand, as mentioned in Section 1. Similarly, the link flow observation shows the hourly link flow variation pattern of a fluctuation of 24-hour time coefficients.

Figure 2 compares hourly variation patterns for the study area (i.e., the Chukyo metropolitan area including the region of the Aichi prefecture, with 7.2 million people, a part of Mie, also with 1.1 million people, and the Gifu prefecture, also with 1.2 million people). The OD survey in Figure 2 is the average variation pattern from the hourly total of all OD survey data departing from and arriving to the study area aggregated based on their departure time. This was divided by the total of day-long OD demand within study area. The link flow observation is the average hourly total of link flow of all survey data within the study area, divided by the total of day-long link flows within the study area.

Note that the OD survey gives larger values in the daytime, but the link flow survey gives larger values in the nighttime. The reason for these differences is that the OD variation pattern tends to be underestimated at nighttime and overestimated at peak hours because data is missing from questionnaires especially at nighttime, although OD variation pattern is naturally a little different from link flow variation due to the different survey type. This bias can be seen notably in trucks because trucks usually make more trips at nighttime than other vehicles. We examine this bias of OD variation pattern in view of the results of traffic assignment for a real network in the next section.

4.2. Application of Initial OD Demand into TUE and Considerations

4.2.1. Outline. The hourly OD demand based on departure time aggregated by the road traffic census 2010 is hereafter called "initial OD." We examine the characteristics of the

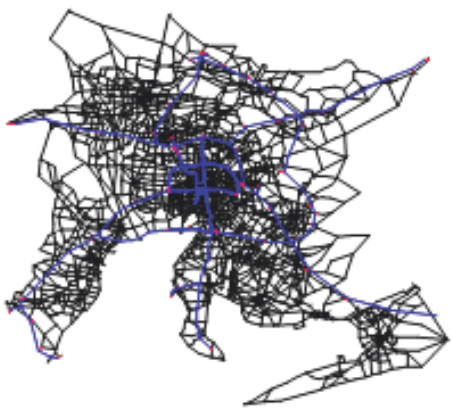


Figure 3: Study network.

hourly variation pattern of the initial OD by applying it to the TUE, TUE-f, and TCoE mentioned in Sections 2 and 3. Additionally, to examine the bias of initial OD in a day-long unit, we also execute the day-long UE assignment over the same study network as for TUE.

The study network is composed of 484 zones, 6683 links, and 4468 nodes, which is the Chukyo metropolitan network based on the road traffic census 2010, as shown in Figure 3. The observed link flow for examining assignment accuracy uses 292 links with 24 hours of data from the road traffic census 2010. The expressway diversion function and the link-cost function in all types of UE assignment in this paper are the same type of function that is used and examined in the previous application [33–35].

The study network (and study area) to which TUE is applied is a large road network within the Chukyo metropolitan area that is also generally used for day-long traffic assignments for practical use. This network connects several cities inside the study area and also simply connects with the outside network including the main cities throughout Japan outside the study area. A cordon line defines the boundary of the study area. When the TUE assignment is applied to such a large network connected with the outside network, new centroids must be set along the cordon line as origins of inflow traffic into the study network. In addition, the periods during which outside traffic departs from its initial origins should be adjusted to the periods that outside traffic enters the study network from new centroids along the cordon line. However, because adjusting the origin and periods for outside trips requires significant manpower, we adjust the initial departure time of outside trips to the periods departing from the cordon line according to the travel time between the initial origin and the cordon line. By using the adjusted departure time for outside trips, hourly OD demand is aggregated from survey data. In the application of TUE, TUE-f, and TCoE, the hourly OD demand from the outside network is assigned as a fixed OD demand separately without residual demand in each hour. In the next chapter, computational time and the PC used for all calculations are described.

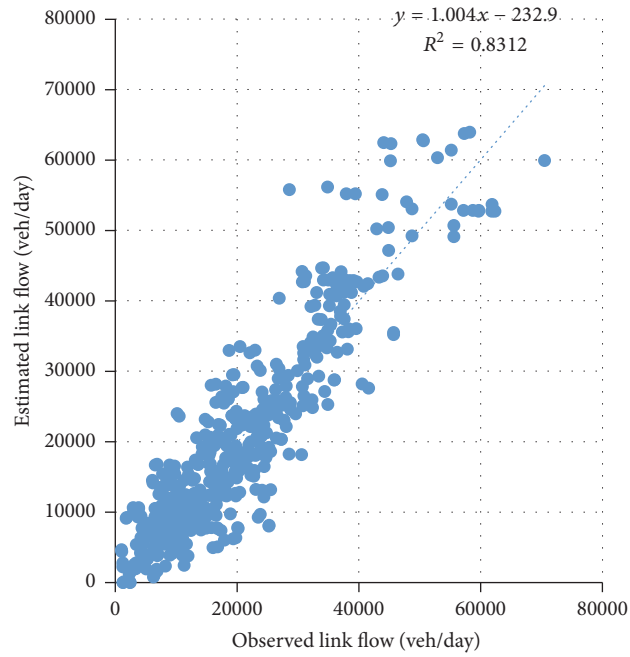


Figure 4: Result of day-long UE assignment.

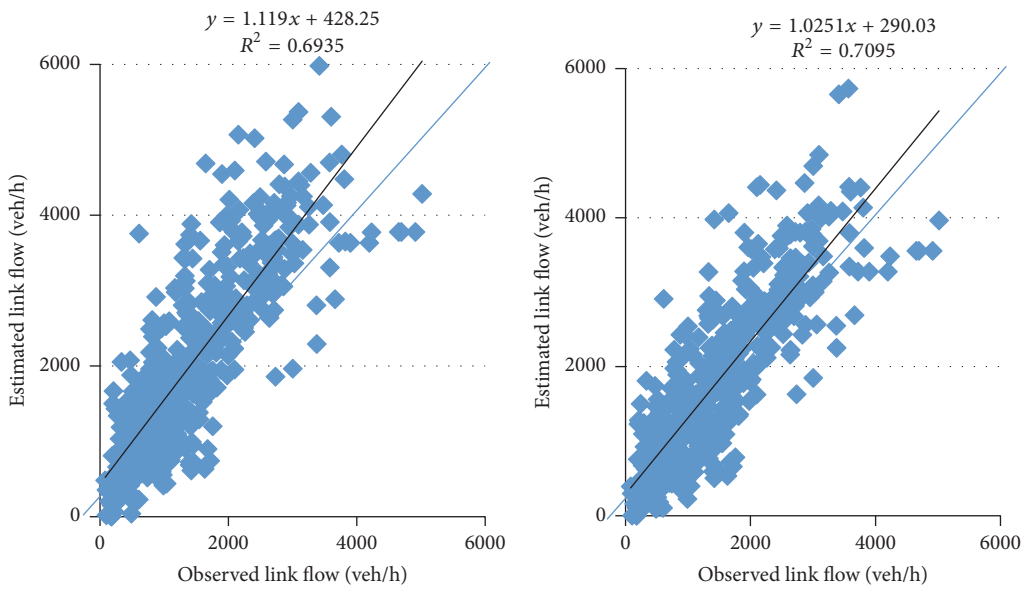
4.2.2. Result of Day-Long UE Assignment. Figure 4 shows the result of the day-long UE assignment. The day-long UE predicts link flows with good accuracy and little bias in data variation.

4.2.3. Result of Hourly UE Assignment by Initial OD. Figure 5 shows the result of the link flow estimated at 7:00 by TUE and TUE-f assignments with the initial OD. The assignment of TUE-f largely overestimates link flows compared with observed link flows. This is attributed to the initial OD at peak hours being naturally overestimated from the questionnaire survey because of the bias mentioned in Section 4.1. Although the assignment result of TUE also has some bias of overestimation at peak hours, the trend overestimated by TUE is less than that of TUE-f because TUE can realistically modify the initial OD with the residual demand between the current and next periods. Figure 6 shows the result of TUE and TUE-f assignments at 22:00, which underestimate link flows compared with observed link flows. The initial OD also seems to be underestimated at nighttime.

Therefore, from the comparison of hourly variation patterns in Section 4.1 and the result that little bias exists in the day-long UE assignment (as shown in Figure 4) and the result of TUE and TUE-F assignments, the bias in assignment almost corresponds to the characteristics of overestimating or underestimating the hourly variation patterns of the survey data. The TCoE model should decrease the bias of survey data by modifying the time coefficients that minimize the error of hourly link flows.

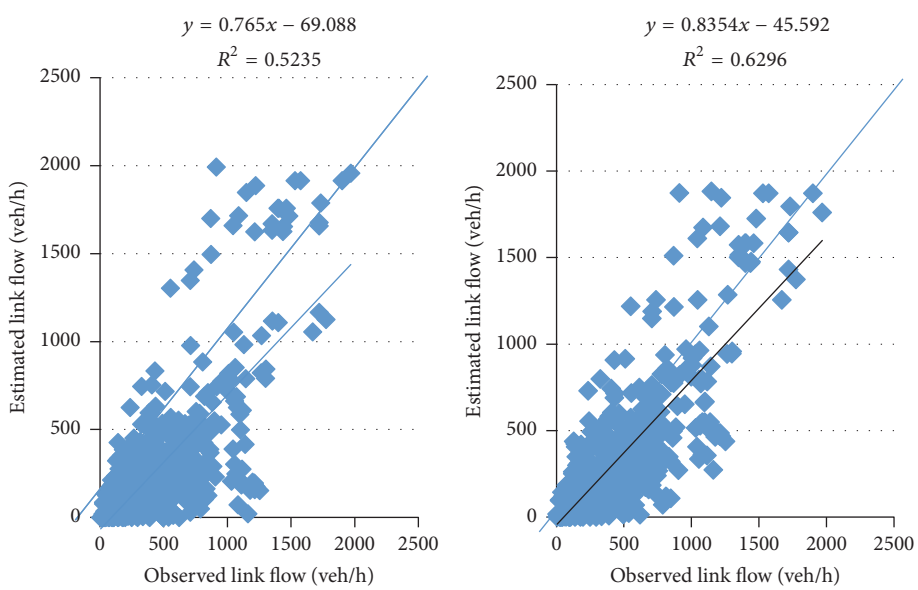
5. Assignment Result and Consideration for TCoE

5.1. Application of TCoE to Large-Scale Road Network. In this section, we examine the validity of the TCoE model by



(a) TUE-f (without residual demand) (b) TUE (with residual demand)

Figure 5: Result of hourly UE assignment with initial OD at 7:00.



(a) TUE-f (without residual demand) (b) TUE (with residual demand)

Figure 6: Result of hourly UE assignment with initial OD at 22:00.

applying it to the same road network as in Section 4. First, the hourly OD demand and link flow estimated by TCoE with two patterns of 24-hour time coefficients for a passenger car and a truck are examined over the entire network. Several conditions for the study network are the same as those used for the TUE assignment in Section 4. The results of the TCoE model and the TUE assignments that use the initial OD are compared and examined by analyzing the RMS errors between observed link flow and estimated link flow.

The convergence criterion for bilevel problems of TCoE is set as follows: the difference of the totals of current and previous steps of RMSEs is less than or equal to 0.002. This criterion is applied separately to two vehicle types. The convergence of the TCoE model is judged when the criteria for the two vehicle types are satisfied. The total computational time for above convergence is about 240 minutes with a personal computer (Intel(R) Core(TM) 4.00 GHz processor with 64 GB RAM), at which the iteration number for bilevel

Table 1

(a) Comparison of RMS error for link flows for all vehicles

	7:00	8:00	9:00	22:00	Total in a day
TUE-f (with initial OD)	936	646	470	340	10177
TUE (with initial OD)	706	766	534	276	9450
TCoE	571	532	434	264	8695

(b) Comparison of RMS error for link flows for the vehicles on expressway

	7:00	8:00	9:00	22:00	Total in a day
TUE-f (with initial OD)	1297	839	640	340	12145
TUE (with initial OD)	881	1020	698	346	10177
TCoE	732	749	597	302	11423

problem is four times. The iterative algorithms of the upper problem and the lower problem (TUE) are implemented in FORTRUN and MATLAB, respectively. The average computational time for TUE in a peak hour is approximately 10 minutes in the condition that the iteration number for UE traffic assignment is limited up to 20 times. The average travel time for all OD pairs within study region is about 35 minutes in peak hours. The proportion of traffic which cannot reach its destination within a period to all OD demand in the study region is about 3% and the proportion of residual demand to all OD demand in each hour is about 25–35%.

Table 1(a) shows the RMS errors of link flows by TUE-f, TUE, and TCoE. Comparing them for 24 hours and all vehicle types shows that the TUE-f assignment is much less accurate than the TCoE model. Conversely, the TCoE model can significantly improve its estimation accuracy by modifying the hourly OD demand. From the RMS error for all vehicles for the TCoE model, the accuracy was improved especially for peak hours. The accuracy at 7:00 for the TCoE model decreases by about 40% of RMS errors when compared with that of TUE-f with initial OD demand. The TCoE model can increase the accuracy for all periods uniformly because it modifies time coefficients for all periods simultaneously. Table 1(b) also shows the RMSE for link flows for the vehicles on Nagoya expressways. It indicates that TCoE model can also estimate link flows on expressways with the highest accuracy among models. Figures 7 and 8 show the scatter diagram of link flows at 7:00 and 22:00 estimated by TCoE. Comparing them with Figures 5 and 6, these results indicate that the TCoE model can cancel the bias for overestimating at peak hour and underestimating at nighttime relative to the initial OD, as noticed in Section 4.1, and thereby greatly improve the estimation accuracy.

5.2. Comparison of Assignment Results in TCoE and TUE with Hourly OD-dep from TCoE. In Section 3.3 we discuss the method to calculate hourly OD demand based on departure time (hourly OD-dep) from the hourly OD demand based on the midway (hourly OD-mid) of the TCoE estimation. We now apply the hourly OD-dep from the TCoE model to the TUE and analyze the hourly variation pattern to test the validity of the method.

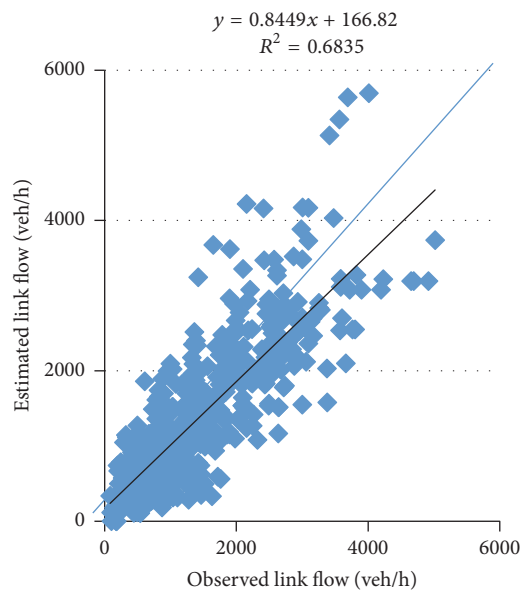


Figure 7: Result of TCoE at 7:00.

Figure 9 compares hourly variation patterns of the hourly OD-dep obtained by the calculation method in (14), the hourly OD-mid estimated by the TCoE model, and the initial OD by survey. These hourly variation patterns are sums of the trips within the study area. As seen in this figure, the variation pattern for hourly OD-dep has a higher value during the peak hour than the hourly OD-mid because the TCoE model estimates the hourly OD-mid that is justified to fit the link flow midway along each path and not fit the link flow near each origin. In comparison with initial OD by survey, the calculated hourly OD-dep reduced the bias more by retraining the time coefficients during the peak hour and increasing them at 6:00 and at nighttime.

Figure 10 compares the results of link flows assigned by the TUE with hourly OD-dep from TCoE model and the TCoE model directly, for which the link flows are summed up for all vehicles at 7:00 and 22:00. Based on this comparison, because the coefficient of determination for each period

Table 2: RMS errors for TCoE with basic model and 3-subarea model.

Period	7:00	8:00	9:00	22:00	Total in a day
TCoE (3 subareas)	567	526	431	252	8605
Difference (TCoE with 3 subareas-basic model)	−4	−6	−3	−12	−90

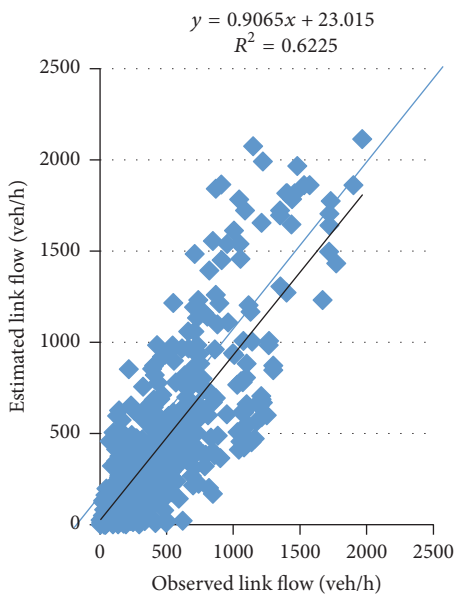


Figure 8: Result of TCoE at 22:00.

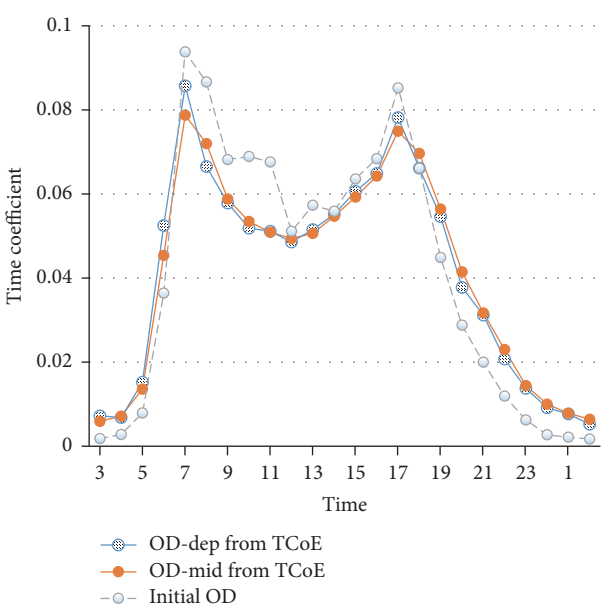


Figure 9: Comparison of hourly variation patterns.

exceeds 0.98 and the slope is almost 1.0, these results are almost the same. From Table 1 and Figure 10, when the TUE assigns the hourly OD-dep, the TUE can estimate traffic volumes with the same accuracy as the estimate of TCoE model in which the TCoE model has the best accuracy among assignments.

Additionally, from Table 1, the TUE with initial OD also has better accuracy than TUE-f. Therefore, the calculated hourly OD-dep from the TCoE model is more accurate than the initial OD for hourly OD demand based on departure time.

5.3. Application in Each Direction and Several Subareas for TCoE. Now the TCoE model is applied to several subareas and dual directions into which the study area is divided. In the study area, Chukyo metropolitan area contains most of Aichi prefecture (including Nagoya city) and part of Mie and Gifu prefectures, which are the commuting areas for Nagoya city. For an analysis that increases the number of time coefficients in the TCoE, we set three subareas and directions (Aichi area to Aichi area, Aichi area to outside of Aichi area, outside of Aichi area to Aichi area, and outside of Aichi area to outside of Aichi area). Therefore, we apply the TCoE under the condition that the time coefficients are set to the above three subareas (four patterns) for two vehicle types, with the other conditions being the same as the application in Section 5.1.

From the RMS errors of estimated link flows in Table 2, the three-subarea model of the TCoE can reduce the errors compared with the basic model of TCoE analyzed in Section 5.1. These results indicate that setting the time coefficient by subarea and direction can increase the estimation accuracy. Figure 11 shows the hourly variation patterns for passenger cars estimated by the three-subarea model. We see the general tendency that the hourly variation pattern of Aichi to Aichi with higher volume of total OD demand results in the lower value at peak hour.

6. Conclusion

The day-long OD demand for transportation forecasting has advantages of accuracy and reliability because it is not affected by hourly variation of OD distribution. In this paper, we proposed the time coefficient estimation (TCoE) model to obtain the hourly OD demand from observed link flows given a proven day-long OD demand. It was constructed based on a bilevel formulation of the generalized least square and the semidynamic traffic assignment (OD-modification approach). Since the hourly OD demand matrices can be calculated by multiplying the given day-long OD demand for the 24-hour time coefficients estimated by the TCoE, TCoE is not needed to set many parameters regarding origins and destinations of the OD demand matrices. TCoE could significantly improve estimation accuracy because the initial OD demand by survey had some bias due to many data

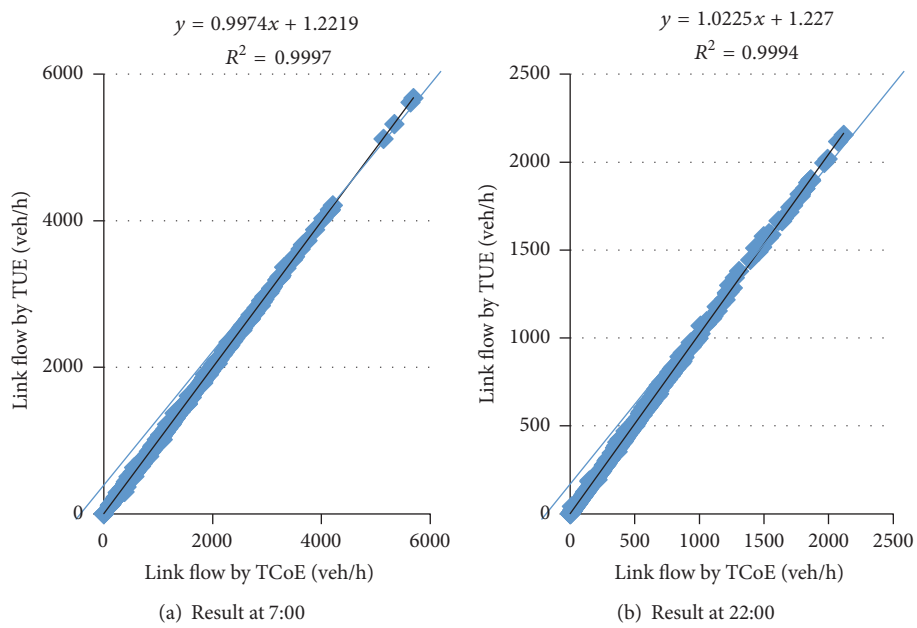


Figure 10: Comparison between results by the TCoE and the TUE with OD-dep from TCoE.

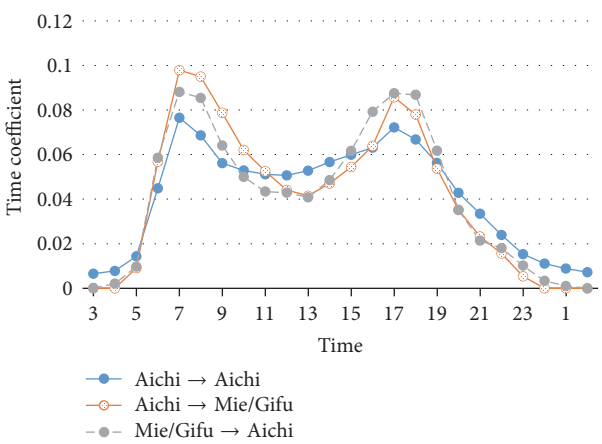


Figure 11: Hourly variation patterns for TCoE result of three subareas.

missing at nighttime, whereas the TCoE could cancel the bias with a few parameters. From the result of assignments, the accuracy at 7:00 for the TCoE reduced by about 40% the RMS errors in comparison with the TUE-f with initial OD demand. Additionally, we adopted the generalized least square formulation for TCoE to improve the accuracy of hourly OD demand because the maximum-entropy formulation requires a prior hourly OD demand and the prior hourly OD demand in our study network has some bias and is not a reliable demand.

We have reviewed the semidynamic concept for the OD-modification approach (TUE) and compared it with the OD modification in the TCoE model and newly proposed a partial relaxation method of the assumption about the study period length in the OD-modification approach. The TUE is formulated as a static user-equilibrium traffic assignment with elastic demand which modifies the OD demand in the current period to consider the residual traffic volume at the end of each period in a congested network. That is, the residual traffic of TUE is semidynamically subtracted from the demand in the current period and added to the demand in the next period corresponding to the degree of congestion in the study network in which the original hourly OD demand is preserved in both the current and next periods. The treatment of residual demand in the TCoE is almost the same as in the TUE because the OD modification is considered in the upper problem in the TCoE but in the lower problem of traffic assignment.

Hourly OD demand obtained by the TCoE is the hourly OD-mid that minimizes the error of estimated and observed link flows midway along paths between OD pairs. Therefore, the hourly OD demand by the TCoE is a little different from OD demand aggregated based on departure time (hourly OD-dep) from survey data. In case the hourly OD-dep is required for practice use, we also explored and examined the method to calculate the hourly OD-dep from the TCoE result.

The OD-modification approach (TUE) assumes that the period length must be set longer than the maximum travel time. Although a partial relaxation of the assumption was proposed, it is difficult to apply TUE into the network that many OD pairs have more travel times than the period length, such as the network with 15 minutes of period length and 30 minutes of average travel time, because in this case almost the traffic cannot reach its destination within the current period and the treatment of residual flows in Figure 1 cannot be applied adequately. Therefore, TUE can analyze the travel time and degree of congestion as an average value for each period by using the link-cost function with traffic capacity but cannot treat a congestion queue. When the queue analysis

in congested network is needed, a combined application that adopts the dynamic traffic assignment in a limited study network and uses the OD demand estimated by TCoE may be considered as an efficient method reducing construction cost of large-scale road network.

However, since TUE is a user-equilibrium traffic assignment with the elastic demand, it can integrate the logit model that expresses travel behaviors such as a route choice between normal roads and toll roads of expressway. Thus, TUE can be properly applied to the route-choice analysis to predict the change of hourly traffic volume after the construction of new bypass road and expressways or after the congestion charging in peak hours. If there are a day-long OD demand for future transportation planning and a traffic assignment system in large-scale road network with the observed link flows (that may be not autocounted data) which have already been prepared, TCoE is able to be applied to the same network only by changing several parameters such as the hourly traffic capacity and simultaneously estimate the hourly OD demands and link flows during 24-hour periods. Since TUE can use a simple expression of intersections in network as a static traffic assignment, TUE has a characteristic to reduce the maintenance cost of the network with high accuracy.

Future research should examine how to set subareas and durations in a study area for good accuracy and efficiency. This paper executed the TCoE model by using as many observed link flows as possible. We could not clarify the relationship between the estimation accuracy and the number of observed links and sizes in study network. The relationship between estimation accuracy and location of observed links should also be analyzed in the future.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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