Research Article
Multiple Depots Vehicle Routing Problem in the Context of Total Urban Traffic Equilibrium

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A multidepot VRP is solved in the context of total urban traffic equilibrium. Under the total traffic equilibrium, the multidepot VRP is changed to GDAP (the problem of Grouping Customers + Estimating OD Traffic + Assigning traffic) and bilevel programming is used to model the problem, where the upper model determines the customers that each truck visits and adds the trucks’ trips to the initial OD (Origin/Destination) trips, and the lower model assigns the OD trips to road network. Feedback between upper model and lower model is iterated through OD trips; thus total traffic equilibrium can be simulated.

1. Introduction

The VRP is a generic name referring to a class of combinatorial optimization problems in which a number of vehicles serve the customers. The vehicles leave the depot, serve customers in the network, and return to the depot after completion of their routes. Dantzig and Ramser (1959) first proposed this problem in the literature. VRP is generally defined by a graph $G = (V, E, C)$, where $V = \{v_0, \ldots, v_n\}$ is the set of vertices; $E = \{(v_i, v_j) \mid (v_i, v_j) \in V \wedge i \neq j\}$ is the arc set; and $C = (C_{ij})_{(v_i,v_j) \in E}$ is a cost matrix defined over $E$, representing distances, travel times, or travel costs. The VRP consists in finding a set of routes for $K$ identical vehicles based at the depot(s), such that each of the vertices is visited exactly once, while minimizing the overall routing cost.

Travel cost $(C_{ij})_{(v_i,v_j) \in E}$ is a decisive influence factor for VRP. In early researches on vehicle routing problems [1], the vehicle travel time was computed by the Euclidean distance between two nodes (the customers or the depots) and a given speed. As all known, it is not a reasonable way to use the Euclidean distance to compute travel time between two nodes for real vehicle routing problems due to the nonlinear feature of roadways and time-varying traffic situation. Later on, some researchers, to meet practical requirements, used travel time through the shortest path on road network to represent the cost. After realizing that changes of traffic flows on an urban road network may change the shortest path and thus the travel time between two sites, adopting the travel time on the dynamic shortest path as the travel cost was introduced, and the travel cost between two sites in a corresponding time window is calculated dynamically.

Currently, based on partial traffic equilibrium, some researchers calculate the shortest path travel time in delivery time windows dynamically. For example, in recent years, several work [2] considered the actual traffic situation in the urban road network for real vehicle routing problems. They suppose that, in an urban road network, the traffic situation is just determined by the OD traffic of other vehicles but not the delivery vehicles themselves and the delivery vehicles will choose paths based on the traffic equilibrium of the other vehicles. This method is rational when the delivery vehicles are a small number and their influence on roadways can be neglected. However, if the delivery vehicles are in a large number, and their traffic may influence other vehicles to make some of them change their travel paths, the method could not get the real travel time. As a result, the actual travel time of
the optimal loops/paths under partial equilibrium cannot be obtained (in this paper, loop means the access sequence to customers, and path means links sequence passed by a truck in road network).

When location of depots/facilities and demand flows between depots and facilities are stable, VRP should be solved statically (a priori) rather dynamically. In this case, the routing behavior of delivery trucks is similar to other vehicles. Both take traffic condition into account. For example, delivery trucks will choose the roadways with less traffic, and other vehicles may keep away from the roadways where the traffic is heavily affected by the travelling and unloading of delivery trucks. It means that interactions exist among all vehicles and traffic flows on roadways result from all drivers’ choice of their path. Then, travel times on roadways are determined by links’ capacities and the corresponding traffic flows. Moreover, because different VRP schemes will result in different OD traffic of delivery trucks, the delivery loops/paths also interact with urban OD traffic. Therefore, it can be said that delivery loops/paths, which are obtained under the assumption that other vehicles are not affected by delivery trucks, are not optimized for the real situation.

When the influence of delivery trucks on road traffic cannot be ignored, the VRP should be treated as a traffic problem at macro level rather than a logistics issue at micro level. Then, we must design the delivery loops/paths from the view of total traffic equilibrium, namely, considering the interactions among all vehicles.

In the real world, the phenomena that the delivery trucks give no influence on road traffic in a city have been disappearing because (1) road capacity in most large cities (especially Chinese ones) is at the critical point where a few additional vehicles may cause severe congestion in some roadways and further result in the change of the whole urban road traffic pattern and (2) rapid developed e-commerce changes citizens’ shopping behavior. Online shopping reduces personal travel but increases delivery truck traffic. Currently, the delivery trucks have become a significant part of overall urban traffic, which is shown in Figures 1(a) and 1(b).

Due to the above reasons, the VRP is no more just the problem that assigns customers and travel paths for delivery trucks based on a given OD traffic. One should consider delivery and other vehicles as a whole when studying their path choice behaviors. Thus, one should take the interactions between the delivery trucks and other vehicles into account directly because delivery schemes will change OD trips and then the roadway traffic, while the OD trips and roadway traffic inversely determine the VRP schemes. This is a solution method of VRP in terms of total traffic equilibrium. Here, the “total” traffic equilibrium means the interactions between delivery trucks and other vehicles are considered directly. Under the “total” user equilibrium, neither delivery trucks nor other vehicles have willingness to change their travel paths.

Therefore, in terms of total traffic equilibrium, the VRP becomes GDAP (problem of Grouping Customers + Estimating OD Traffic + Assigning Traffic), among which “Grouping Customers” means the works that customers are assigned to different depots, and then a multidepot VRP is solved by transforming into a group of single-depot VRPs; “Estimating OD Traffic” means the works that the trip chains of the delivery trucks are added to the given OD; “Assigning Traffic” means the works that the updated OD matrices are assigned onto the road network. Its mathematical description is shown as

\[
\begin{align*}
(i) \quad & \text{VRP} = f(\text{Traffic Flow}) ; \\
(ii) \quad & \text{Traffic Flow} = g(\text{OD}) ; \\
(iii) \quad & \text{OD} = \text{OD}^{(0)} + h(\text{VRP}) .
\end{align*}
\]

(1)

We think the delivery schemes (namely, the being served customers and travel paths of the trucks) should be obtained by solving the three equations simultaneously. Here, first equation is VRP model, the second equation is traffic assignment model, and the third one is the method to put delivery truck trip chains into the OD matrix.
We may use the bilevel programming to model above problem, where the interactions between delivery trucks and other vehicles are considered. The upper model determines the customers that each delivery truck visits and the path of each delivery truck, then we add these delivery trucks OD trips into the initial OD matrix, and the lower model is a traffic assignment model that finds paths for all the vehicles. Interactions between the LM and UM are iterated through updating OD. Thus, the total traffic equilibrium on a road network can be simulated and loops/paths of delivery trucks can be found in convergent outputs.

The rest of the paper is organized as follows. In Section 2, we perform a literature review to summarize the existing researches, in particular to discover their shortcomings. In Section 3, we state our ideas and contributions. In Section 4, we present the model structure of GDAP and design the solution algorithm. In Section 5, we do a case study to demonstrate the solution process and results of the model using data for delivery of marine products in Dalian (China). In Section 6, we summarize the study and consider future research.

2. Literature Review

2.1. Researches on VRP. Over the last three decades, the number of academic publications on the numerous variants of the VRP has increased extensively [3] including a brief review of the development of VRP published in 2009 [4].

After Dantzig and Ramser (1959), a considerable number of variants of VRP have been studied, including (1) the VRP with hard, soft, and fuzzy service time windows (VRPTW); (2) the VRP considering backhauls (VRPB); (3) the VRP considering maximum route length, (DVRP); (4) periodic VRP (PVRP); (5) VRP with multiple trips (VPRMT); (6) split delivery vehicle routing problem (SDVRP) and others; (7) the VRP with minimized emissions [5]. Montoya-Torres et al. [6] illustrate the hierarchy of the VRP variants with Figure 2. According to them, the most impressive growing of VRP study was observed between 2006 and 2014 with a total of 103 publications.

Since many variants of VRP are NP-hard problems, lots of researchers have proposed solution algorithms. For example, Clarke and Wright [7] proposed the Clarke-Wright solving algorithm in 1964, which is the first to solve the multidepot VRP (MDVRP) by a heuristic algorithm. Cordeau et al. [8] described a “tabu” search algorithm for the MDVRP. Simulated annealing (SA) has been employed as well to solve VRP by Wu et al. [9]; Yu et al. [10] proposed an improved ant colony optimization with coarse-grain parallel strategy, ant-weight strategy, and mutation operation for the MDVRP; Vidal et al. [11] propose an algorithmic framework that successfully addresses three vehicle routing problems: the multidepots VRP, the periodic VRP, and the multidepots periodic VRP; Martonák et al. [12] proposed a path-integral Monte Carlo quantum annealing scheme for the symmetric travelling-salesman problem; Crispin and Syrichas [13] apply the quantum annealing algorithm for VRP and study the effectiveness of quantum annealing. Polacek et al. [14] used the variable neighborhood search for the MDVRP. Yu et al. [15] solved the MDVRP with time windows by a two-stage heuristic approach. More detailed VRP solution algorithms...
are presented in the literatures of Montoya-Torres et al. [6]; Lin et al. [16]; Wang et al. [17]; and Zhang et al. [18].

At present, solution of VRP is not only vital in the design of distribution systems in supply chain management, but also important in urban solid waste collection, street cleaning, school bus routing, routing of salespeople, and courier services. Researches can be roughly divided into theoretical papers providing mathematical formulations and exact or approximate solution methods for academic problems and case-oriented papers. More recently, attention has been devoted to more complex variants of the VRP (usually called “rich” VRPs) that are closer to the practical distribution problems than classic VRP models.

There is also literature which puts VRP in real-world context to consider the dynamics of travelling times on an urban road network because in urban areas the travel speeds (or times) typically vary during the day due to differing congestion patterns. Malandraki and Daskin [19] presented mixed integer linear programming formulations of the TDTSP and the TDVRP that treat the travel time functions as step functions. Time-dependent travel times are one of the main challenges in the optimization of vehicle routes for urban goods movements. For example, Ichoua et al. [20] presented a model based on time-dependent travel speeds that satisfies the nonpassing property. Polimeni and Vitetta [21] and Cattaruzza et al. [22] studied time-dependent approaches and supposed that the link cost depends also on time. Actually, for the traditional dynamic VRP, the link travel times are thought to be different in different time windows and then assign tasks to trucks. The interaction between the delivery trucks and other vehicles is not considered directly yet.

Taniguchi and Yamada [23] studied the vehicle routing and scheduling procedures using advanced information systems and freight transport systems in urban areas. Ando and Taniguchi [24] studied travel time reliability in vehicle routing based on the data obtained by probe vehicles. Conrad and Figliozzi [25] proposed Traffic Queuing Algorithm and Arrival and Departure Time Algorithms to quantify the impacts of congestion on time-dependent real-world urban freight distribution networks; Delflorio et al. [26] applied the performance indicators to compare different service settings and introduced a simulation approach to build the demand; Muñuzuri et al. [27] believed that modelling urban freight transport is difficult and highly data-demanding and proposed a trip generation model to achieve the estimation of an origin-destination matrix for freight transport in a city. Çetinkaya et al. [28] introduced a new variant of VRP, namely, the Two-Stage Vehicle Routing Problem with Arc Time Windows, which generally emerges from both military and civilian transportation in Turkey. They divided the network into three layers (facility, depots, and customers) and routing operations into two successive layers (i.e., between facility-depots, and depots-customers) and proposed a trip generation model to achieve the estimation of an origin-destination matrix for freight transport in a city.

Çetinkaya et al. [28] introduced a new variant of VRP, namely, the Two-Stage Vehicle Routing Problem with Arc Time Windows, which generally emerges from both military and civilian transportation in Turkey. They divided the network into three layers (facility, depots, and customers) and routing operations into two successive layers (i.e., between facility-depots, and depots-customers). Tang et al. [29] proposed a VRP model subject to travel time reliability constraint. Mancini [30] took into account rush hours traffic congestion and addressed a VRP on a real road network with time-dependent travel speed expressed by a polynomial function. Despite the difficulty to work with these kinds of function, this way more precisely represents congestion evolution behavior. However, this study calculates the actual travel time of road network based on partial traffic equilibrium without considering the interaction between the delivery trucks and other vehicles directly. Chiabaut et al. [31] introduced a general methodology to anticipate and evaluate the impacts of urban logistics on the global performance of a transportation network.

Real-world applications of VRP often include two important dimensions: evolution and quality of information [32]. Evolution of information relates to the fact that in some problems the information available to the planner may change during the execution of the routes. Quality of information reflects possible uncertainty on the available data. In addition, depending on the problem and the available technology, vehicle routes can either be designed statically (a priori) or dynamically.

In real-world applications, static design is more important when VRP is between the layers of depots and facilities. In a small period of time (e.g., a quarter or a month), spatial distribution of depots/facilities, demand flows between depots and facilities of the urban solid waste collection, street cleaning, school bus routing, routing of salespeople, and courier services may hardly change. Thus one delivery scheme may be used for the whole quarter or month. This is similar to bus transit service, where daily personal travel demand is relatively stable and the redesign of bus routes day by day and the dynamic scheduling of buses hour by hour are not necessary.

2.2. Researches on Interactions between Other Vehicles and Delivery Trucks. Taniguchi et al. [33] considered the interactions between the background traffic and the service trucks of the logistics terminals when locating the terminals. They used a bilevel programming model, and the upper-level problem describes the behavior of the planner for minimizing the total cost. The lower level problem describes the behavior of each company and each truck in choosing optimal logistics terminals and transportation routes. The model explicitly takes into account traffic conditions in the network and was successfully applied to an actual road network in the Kyoto-Osaka area in Japan. They only dealt with the location problem not VRP problem because each service truck runs between a terminal and only one depot.

Researchers in Taniguchi team also adopted bilevel programming model to integrate a supply chain network (SCN) with a transportation network (TN) in terms of traffic equilibrium. Among them, Yamada et al. [34] proposed a strategic transportation planning model for designing interregional freight TN and freight terminal development. The lower level is a multimodal multiclass user traffic assignment model, while the best combination of actions, in the upper-level problem, is determined. Yamada et al. [35] proposed a supply chain-transport super-network equilibrium (SC-T-SNE) model, in which the behaviors of six entities—manufacturers, wholesalers, retailers, freight carriers, demand markets, and TN users—are interpreted. With the behavior of TN users including delivery trucks being incorporated, the model allows for endogenously determining transport costs based
on freight carriers’ decision-making, as well as for investigating mutual effects between behavioral changes in SCNs and the TN. Notably, the effects of traffic conditions in the road network on the behavior of the entities on each SCN and vice versa were explored. Yamada and Febri [36] further presented a discrete network design problem, with the assumption of SCN-TN interaction. They developed their discrete optimization model for a TN with equilibrium constraints. Their research studies mainly network planning problems (i.e., facility location, link construction) with equilibrium constraints, rather than delivery management. With the OD of TN users, the same as the OD of goods, the truck path choice behavior differs from that in VRP where delivery loop of a truck is a trip chain. Thus, with the same equilibrium constraints, the key issues and model structure are different.

3. Motivation and Contribution

When location of depots/facilities and demand flows between depots and facilities are stable, VRP should be solved statically (a priori) rather dynamically. In this case, the routing behavior of delivery trucks is similar to other vehicles. Both take traffic condition into account. For example, delivery trucks will choose the roadways with less traffic, and other vehicles may keep away from the roadways where the traffic is heavily affected by the travelling and unloading of delivery trucks. It means that interactions exist among all vehicles and traffic flows on roadways result from all drivers’ choice of their path. Then, travel times on roadways are determined by links’ capacities and the corresponding traffic flows. Moreover, because different VRP schemes will result in different OD traffic of delivery trucks, the delivery loops/paths also interact with urban OD traffic. Therefore, it can be said that delivery loops/paths, which are obtained under the assumption that other vehicles are not affected by delivery trucks, are not optimized for the real situation.

When the influence of delivery trucks on road traffic cannot be ignored due to the recurring congestion and sophisticated e-commerce business models [37], VRP problem should be treated as a traffic problem at the macro level rather than a logistics issue at the micro level. Then, we must design the delivery loops/paths from the view of total traffic equilibrium, while considering the interactions among all vehicles and traffic generation/attraction in traffic zones.

Our contributions in this study are as follows.

(1) Under the total traffic equilibrium, transforming the multidepot VRP to GDAP (the problem of Grouping Customers + Estimating OD Traffic + Assigning traffic) to take the interactions between delivery trucks and other vehicles into account to obtain delivery schemes under “the total traffic equilibrium.”

(2) Solving GDAP with bilevel model. Based on the feedback loop of “the problem of Grouping Customers - determining the delivery routes - updating OD traffic - assigning OD traffic – re-grouping…” firstly the customers are divided into several groups and secondly the delivery loops/paths for the groups are obtained and the initial OD matrix is updated, and thirdly links’ traffic flows are calculated by user equilibrium traffic assignment model.

(3) Carrying out a case study with actual data in Dalian to examine and verify the method and provide some useful findings.

(4) Evaluating the delivery schemes based on not only the delivery time but also the traffic situation of the entire road network.

4. Model Structure

4.1. Model Assumptions

(A1) Study area consists of continuous but nonoverlapping traffic zones, the OD trips of vehicles other than delivery trucks do not change, but the paths between origins and destinations are not fixed, which will be determined based on UE Theory.

(A2) Demand of each costumer is given.

(A3) Depots’ supply amounts are big enough.

(A4) Length of delivery loop is shorter than the truck’s maximum range.

(A5) All delivery loop is the same type, with the loading capacity given.

(A6) Loading and discharging times during delivery are ignored.

(A7) One truck is equivalent to 3 per car units.

(A8) Drivers know the travel times of all roadways and try to choose the shortest path.

4.2. The Upper Model. The variables are defined as follows:

- $Z^n$: the total delivery time of all delivery trucks in $n$th round of grouping, which is the objective value of the upper model;
- $x^n_{ij}$: 0-1 variable, $x^n_{ij} = 1$ means vehicle $k$ via path $i-j$ in $n$th round of grouping; it is the decision variable to determine the route of a delivery truck, and further the entire delivery network; $x^n_{ij}$ is decision variable, which determines the OD trips of delivery trucks, the path between two customers will be determined through UE in the lower model;
- $\tilde{R}$: the set of origins of other vehicles;
- $\tilde{R}$: the set of origins of delivery trucks (depots or customers);
- $R$: the set of origins of all vehicles, $R = \tilde{R} + \tilde{R}$;
- $\tilde{S}$: the set of destinations $f$ of other vehicles;
- $\tilde{S}$: the set of destinations $f$ of delivery trucks (depots or customers);
- $S$: the set of destinations $f$ of all vehicles, $S = \tilde{S} + \tilde{S}$;
- $K$: the set of delivery trucks;
- $T^n_{ij}$: the travel time of truck $k$ from $i$ to $j$ in $n$th round of grouping;
- $q_k$: the demand of costumer $j$;
- $Q$: load capacity of truck $k$;
\(d_{ij}\): length of delivery path from site \(i\) to site \(j\);

\(D\): maximum travel range of a truck;

\(M_r\): the number of delivery trucks in depot \(r\);

\(\kappa_{aij}^n\): (0-1) variable, \(\kappa_{aij}^n = 1\) means path \(i-j\) via link \(a\) in \(n\)th round of grouping.

The upper model is as follows:

\[
\begin{align*}
\text{min: } & \quad Z_1^n = \sum_{i \in \hat{R}} \sum_{k \in K} \sum_{j \in S} (x_{ijk}^n \cdot t_{ijk}^n + x_{jki}^n \cdot t_{jki}^n) + \sum_{i \in S} \sum_{k \in K} \sum_{j \in S} x_{ijk}^n \cdot t_{ijk}^n \\
\text{s.t.: } & \quad \sum_{e \in \hat{E}} x_{eij}^n q_j + \sum_{i \in S} x_{ijk}^n q_j \leq Q, \quad \forall k \in K \\
& \quad \sum_{e \in \hat{E}} x_{eij}^n d_{ij} + \sum_{j \in S} x_{jrk}^n d_{jr} + \sum_{i \in S} x_{ijk}^n d_{ij} \leq D, \quad \forall k \in K \\
& \quad x_{rjk}^n = x_{jrk}^n \leq 1, \quad \forall r \in \hat{R}, \forall k \in K \\
& \quad \sum_{e \in \hat{E}} \sum_{k \in K} x_{eij}^n + \sum_{j \in S} \sum_{k \in K} x_{jrk}^n = 1, \quad \forall j \in \hat{S} \\
& \quad \sum_{k \in K} \sum_{j \in S} x_{ijk}^n \leq M_r, \quad r \in R \\
& \quad \sum_{i \in \hat{R}} \sum_{j \in R} x_{ijk}^n = 0, \quad k \in K \\
& \quad x_{ijk}^n = \begin{cases} 
1 & \text{Path of truck } k \text{ from } i \text{ to } j \text{ in } n \text{th round of grouping} \\
0 & \text{Otherwise}
\end{cases} \\
& \quad t_{ijk}^n = \sum_{a} \tau_{ia}^n \cdot \kappa_{aij}^n.
\end{align*}
\]  

4.3. The Lower Model. The variables used are defined as follows:

\(Z_2\): objective value, which is the sum of travel times on all links;

\(t_{ia}^n\): travel time on link \(a\) in the \(n\)th round of assignment;

\(x_{aij}^n\): traffic flow on link \(a\) in the \(n\)th round assignment;

\(f_{rak}^n\): traffic flow on path \(k\) between OD\((r, s)\) in the \(n\)th round of assignment;

\(q_{rsk}^n\): traffic flow between OD\((r, s)\) in the \(n\)th round of assignment;

\(\delta_{trak}^n\): (0-1) variable, if link \(a\) is on path \(k\) from \(r\) to \(s\) in \(t\) the \(n\)th round of assignment, is 1, otherwise, is 0;

\(t_{a}(0)\): free flow travel time of link \(a\);

\(\alpha, \beta\): parameters (here \(\alpha = 0.15\), \(\beta = 4\)).

The lower model is as follows:

\[
\begin{align*}
\text{min: } & \quad Z_2 = \sum_{a} \int_{0}^{t_{a}^n} f_{a}(w) \, dw \\
\text{s.t.: } & \quad \sum_{k} f_{rak}^n = q_{rsk}^n, \quad \forall r, s
\end{align*}
\]
Equation (11) is the objective function; (12), (13), and (14) are flow constraints, which ensure that the traffic flow should not be negativity and satisfy the flow conservation. Equation (15) is the link performance function; (16) is the (0-1) variable.

5. Model Solution

The model can be solved by iterative calculation of “Grouping Customers → Determining the delivery schemes → Changing OD matrix → Assigning OD traffic → Re-Grouping Customers → ...” Here we use the network in Figure 3, where “1-5” and “6-8”, respectively, represent customers and depots, as an example to explain the solution approach.

5.1. Optimizing the Delivery Scheme. We optimize the delivery scheme in the context of fixed links’ travel speeds for the multidepot VRP by Generic Algorithm (GA). Firstly, the multidepot VRP is transformed into several single-depot VRPs by grouping the customers. Due to the grouping, \( L \)-types of customer clusters will be formed. Secondly, several delivery schemes, respectively, for several sets of single-depot VRPs, which are corresponding to the \( L \)-types of customer clusters, are designed. Finally, the optimal delivery schemes of a group of single-depot VRPs are found by comparing the values of the objective functions corresponding to the \( L \)-types of customer clusters.

The grouping works are as follows.

Step 1. Calculate \( TM_{ij} \) and \( TQ_{ij} \) from 3 Depots to 5 Customers to obtain travel time vector; for customer \( j \), it is \([TM6j, TM7j, TM8j, TQ6j, TQ7j, TQ8j]\).

Step 2. Find the minimum \( TM_{ij} \) of each customer \( j \).

Step 3. Compare the minimum \( TM_{6j} \) of customer \( j \) with \( TQ_{6j} \) and \( TQ_{8j} \), if \( TM_{6j} < TQ_{6j} \) and \( TM_{6j} < TQ_{8j} \), then customer \( j \) is served by Depot 6 (Group 1). Otherwise, assign customer \( j \) to Group 4, the depots for the customers of this group have not been determined yet.

Step 4. Assign the customers of Group 4 to Groups 1–3 through enumeration method, and obtain \( L \)-types of customer clusters, for example, Type 1: Depot 6 (Customers 1, 2), Depot 7 (Customer 5), and Depot 8 (Customers 3, 4); Type 2: Depot 6 (Customer 2), Depot 7 (Customers 1, 5), and Depot 8 (Customers 3, 4).

Step 5. Perform terminating the calculation.

Step 6. Perform crossover and mutation operations, and return to Step 2.

For each step, the detailed calculations are as follows.

(i) Chromosome Design. Set the initial population size of feasible solutions to \( m \) and generate the chromosomes of initial population as follows:

(a) Count customers in Group 1 in \( l \)th type of grouping cluster to get \( NC_1l \), and randomly generate an array (Array 1) of customers for Group 1.

(b) Determine the insertion times of ID of Depot 6 to get \( PN_1^l \) (\( PN_1^l = NC_1^l - 1 \)) and then insert “6” into Array 1 \( PN_1^l \) times randomly to get a new array (Array 2).

(c) Add “6” to the head and end of Array 2 to ensure the truck starting and returning to Depot 6. Get the final array for Group 1 (Array 3).

(d) Repeat the above works for Group 2 and Group 3.

(e) Group the final arrays of Groups 1, 2, and 3 in turn to get an individual set of the initial population. With this method, \( m \) individual sets are produced in the initial population.

For example, the initial population is coded by natural coding method, and each chromosome is encoded by three Gene Segments (Gene Segment 1, Gene Segment 2, and Gene Segment 3), which represent the code of Groups 1, 2, and 3.
respectively. In the case of Chromosome \([621674783858],[6216]\) is Gene Segment 1 representing that a truck from Depot 6 delivers goods to Customer 2 and Customer 1 in turn; \([747]\) is Gene Segment 2 representing that a truck from Depot 7 delivers goods to Customer 4 and returns to Depot 7; \([83858]\) is Gene Segment 3 representing that the first truck from Depot 8 delivers goods to Customer 3 and returns to Depot 8 and the second truck from Depot 8 delivers goods to Customer 5 and returns to Depot 8; that is why in this chromosome, number 8 appears twice.

(2) Fitness Calculation. Calculate the fitness values by (2).

(3) Selection Operator. The roulette wheel method is used for the selection operation based on the fitness values \(f(i)\).

Firstly, calculate the probability \((p(i) = f(i)/\sum f(i))\) of each chromosome; (2) calculate the cumulative probability \((q(i) = \sum_{1}^{i} p(i))\) of each chromosome; (3) generate a random number \(r \in [0, 1]\); if \(r < q(1)\), select chromosome 1; if \(q(i - 1) < r \leq q(i)\), \(i \geq 2\), select chromosome \(i\).

(4) Crossover Operation. Perform a crossover on the same Gene Segment. Take Chromosome A- \([621674783858]\) and Chromosome B- \([62167378458]\) as example. Firstly, remove the number for each distribution center and obtain Chromosome A1- \([21435]\) and Chromosome B2- \([21345]\). Secondly, determine two crossing points (the underlined gene locus). Next, the gene between these two points are exchanged to obtain Chromosome A2- \([21335]\) and Chromosome B2- \([21445]\). Then change the repeating number into the missing number to obtain Chromosome A3- \([21345]\) and Chromosome B3- \([21435]\). Finally, add the numbers, which represent the distribution center, into the original position to obtain Chromosome A4- \([621673784588]\) and Chromosome B4- \([621674783588]\).

(5) Mutation Operator. Perform a mutation within the same Gene Segment, for example, Chromosome A- \([621674783858]\). Firstly, determine a mutating point (the underlined gene locus). Next, the mutating point will be changed into another customer number of this group, \([621674783838]\). Then, change the repeating number into the missing number and then obtain \([621674785838]\).

5.2. Estimate Demand (Renew the OD Matrix). Determine the OD trips of the delivery trucks based on the optimal grouping pattern, and then add the OD trips of the delivery trucks to the former OD matrix.

5.3. Assign Traffic. Frank-Wolfe (FW) approach is used to solve the lower model, which is a normal user equilibrium traffic assignment model, as follows.

Step 1 (initialization). Set \(C_{ij}^{0} = C_{ij}(0), \forall \text{link}(i, j)\), do all-or-nothing assignment to obtain a set of feasible flows \(\{x_{ij}^{0}\}\), and set \(n = 1\).

Step 2 (link-impedance update). Set \(C_{ij}^{n} = C_{ij}(x_{ij}^{n}), \forall \text{link}(i, j)\).

Step 3 (direction finding). Repeat all-or-nothing assignment with \(C_{ij}^{n} = C_{ij}(x_{ij}^{n}), \forall \text{link}(i, j)\) to additional link flows \(\{y_{ij}^{n}\}\).

Step 4 (move-size determination). Solve \(\sum_{i}(y_{ij}^{n} - x_{ij}^{n}) \times C_{ij}[x_{ij}^{n} + \lambda(y_{ij}^{n} - x_{ij}^{n})] = 0\) to obtain \(\lambda\).

Step 5 (flow update). One has \(x_{ij}^{n+1} = x_{ij}^{n} + \lambda(y_{ij}^{n} - x_{ij}^{n})\).

Step 6 (convergence judgment). If \(\sum_{i}(x_{ij}^{n+1} - x_{ij}^{n})^{2}/\sum_{i}x_{ij}^{n} < \varepsilon\) (\(\varepsilon\): a given threshold), stop calculation and output \(x_{ij}^{n+1}\). Otherwise, set \(n = n + 1\), and go to Step 2.

In this paper, the main model (the upper one) is proposed to describe VRP. The decision variable for the upper model is \(x_{i,j}^{n}\), which is finite and discrete. In this case, the optimal solution must exist. Actually, only the uniqueness of the lower model can be proved [38], while uniqueness of the upper model cannot be guaranteed.

6. Case Study

6.1. Needed Data. The retailing delivery of aquatic products in the Xigang district in Dalian (China) is used to do the case study. As shown in Figure 4(a), there are four depots, 27 retailers (customers). The daily demands of the 27 customers are listed in Table I. For the delivery, both the aquatic products and cold keeping materials (such as refrigerators, ice, and sea-water) must be loaded on the delivery trucks; thus the actual capacity (rated one) of the trucks is 0.35 t (1 t).

The study area is divided into 31 traffic zones (Figure 4(a)), and the road network is shown in Figure 4(b). Other vehicle OD trip matrices during time 7:30–8:30 come from the personal trip survey of Dalian in 2011. Assigning the initial OD matrix on the road network, we can get the initial link flow \(X_{ij}^{0}\) and travel time \(t_{ij}^{0}\).

6.2. Solution Analysis. The solution of the optimization model, which is the delivery schemes under total traffic equilibrium, is shown in Figure 5(a), where the customers served by different trucks/depots are illustrated by different shaped and colored points, respectively. The distance travelled by all delivery trucks is 109.7 km and the corresponding travelling time is 329.1 minutes. Figure 5(b) shows the delivery schemes when the interactions between the delivery trucks and other...
Figure 4: Traffic zones, depots, costumers, and road network.

Figure 5: Delivery loops and travel paths.
vehicles are not considered, namely, the delivery schemes under partial traffic equilibrium. In this case, the total travel distance of the delivery trucks is 94.8 km and the total travel time is 425.2 minutes. Although the distance of the schemes under total traffic equilibrium is 15.7% longer, its travel time is 29.2% shorter. Therefore, we cannot say that the schemes obtained under partial traffic equilibrium are the real optimal ones and should not be adopted for the real work.

To demonstrate the validity of the proposed model, we set some scenarios by changing OD matrices and the numbers of delivery trucks, respectively and then do sensitivity analysis. For building the scenarios, we adjust vehicle OD trip matrix of Dalian in 2011 with \( \eta_1 \in [0.2, 2] \) by step = 0.2) and multiply the number of delivery vehicles by \( \eta_2 \) by step 0.2. Then, we use the 0.2, 0.4, ..., 2.0 times of the initial OD traffic and the 0.2, 0.4, ..., 2.0 times of the delivery trucks to test the model and give some findings, respectively. The comparing indices are the overlapping ratio \( \rho_1 \) (17) of the delivery loops/paths, which means the sensitivities of traffic volumes to the delivery loops, and the ratio of customers not changing service depot \( \rho_2 \) (18), which means the sensitivities of traffic volumes to the groupings, and the total travel time \( T_d \) of the delivery trucks.

\[
\rho_1 = \frac{\text{length of overlapping routes in different scenarios}}{\text{total length of routes under the situation of free flow traffic}} \tag{17}
\]

\[
\rho_2 = \frac{\text{number of the customers not changing service depot}}{\text{number of customers}} \tag{18}
\]

The indices are shown in Figure 6. When \( \eta_1 = 0.2 \) or \( \eta_1 = 0.4, \rho_1 = 1 \) and \( \rho_2 = 1 \). It can be seen that the delivery schemes under the total traffic equilibrium and the situation of free flow traffic are the same. In this case (namely, \( \eta_1 = 0.0, 0.2, 0.4 \)), the total travel times \( T_d \) of the delivery trucks hardly change, and the loops/paths of the delivery trucks are totally the same.

When \( \eta_1 = 0.6, \rho_1 < 1, \rho_2 = 1 \). It means that the delivery loops/paths under the total traffic equilibrium and the situation of free flow traffic are not the same any more. For example, we can see the difference in the delivery loops/paths from Depot 1 to Customer 21. The length of the path of

![Figure 6: \( \rho_1, \rho_2 \), and \( T_d \) in different scenarios.](image-url)

<table>
<thead>
<tr>
<th>Costumer</th>
<th>1 Wal-Mart (Zhongshan Rd)</th>
<th>2 METRO (Shugang Rd)</th>
<th>3 Tesco (Changchun Rd)</th>
<th>4 Shunmin Supermarket (Changchun Rd)</th>
<th>5 Daqing Supermarket (Changchun Rd)</th>
<th>6 Zhongbei Supermarket (XiaN Rd)</th>
<th>7 New Mart (Yuouyi Street)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>0.19</td>
<td>0.17</td>
<td>0.18</td>
<td>0.21</td>
<td>0.18</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>Costumer</td>
<td>8 Dalian seafood Supermarket</td>
<td>9 Champs Supermarket</td>
<td>10 Future Supermarket (Changchun Rd)</td>
<td>11 Tesco Supermarket (Changchun Rd)</td>
<td>12 Dashang Supermarket (XiaN Rd)</td>
<td>13 Wal-Mart Supermarket (Victory Rd)</td>
<td>14 Tesco Supermarket (Victory Rd)</td>
</tr>
<tr>
<td>Amount</td>
<td>0.2</td>
<td>0.15</td>
<td>0.21</td>
<td>0.18</td>
<td>0.16</td>
<td>0.18</td>
<td>0.17</td>
</tr>
<tr>
<td>Costumer</td>
<td>15 Tesco (Jiefang Road)</td>
<td>16 Carrefour Supermarket</td>
<td>17 Carrefour Supermarket (Changchun Rd)</td>
<td>18 Hualian Supermarket (XiaN Rd)</td>
<td>19 Wilson Supermarket (Victory Rd)</td>
<td>20 Sanbao Supermarket (Victory Rd)</td>
<td>21 Lihua Supermarket (Victory Rd)</td>
</tr>
<tr>
<td>Amount</td>
<td>0.15</td>
<td>0.15</td>
<td>0.18</td>
<td>0.2</td>
<td>0.19</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>Costumer</td>
<td>22 Yongfu fresh Supermarkets (Baishan Rd)</td>
<td>23 Tesco Supermarkets</td>
<td>24 All Poly Supermarkets (Changchun Rd)</td>
<td>25 YiFeng Supermarket (XiaN Rd)</td>
<td>26 Food Supermarket (XiaN Rd)</td>
<td>27 Triumph Supermarket (XiaN Rd)</td>
<td>85.0 Seafood Mall (XiaN Rd)</td>
</tr>
<tr>
<td>Amount</td>
<td>0.2</td>
<td>0.15</td>
<td>0.09</td>
<td>0.21</td>
<td>0.1</td>
<td>0.1</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 1: The demands of the customers (Ton/Day).
changes of the traffic volume. In case of free flow, the trucks travel along the loops/paths with the shortest travel distance (or time). As the traffic volume increment the path with the shortest road distance may not be that with the shortest travel time. According to the congestion degrees, some other paths may become the shortest travel time ones. Therefore, the delivery schemes under total traffic equilibrium are practical.

Furthermore, the traffic of delivery trucks may also change the travel behaviors of other vehicles. To demonstrate this, we set some scenarios by setting the OD matrix of other vehicles as $OD \times \eta_1$ and multiplying the number of delivery vehicles by $\eta_2$ ($\eta_2 \in [0.2, 2]$ by step 0.2). Then we solved the model for each scenario.

The obtained results are shown in Figure 9. It can be seen that the travel times of other vehicles positively relate to the number of trips of themselves and the number of delivery trucks.

In many Chinese large cities, in order to manage traffic, trucks are banned from travelling in the morning and evening rush hours. With the information in Figure 9, a city can manage the traffic more efficiently in terms of trucks banning. For example, if setting 25 minutes as the critical point of the average travel time of all trips, then we get the following three analyses.

**Case 1.** It is the green point in Figure 9, where $\eta_1 = 0.6$ and $\eta_2 = 1.0$. The corresponding average travel time is 23.9 minutes (shorter than 25 minutes); therefore, there is no need for banning the delivery trucks in the rush hours.

**Case 2.** It is the blue triangle in Figure 9, where $\eta_1 = 0.6$ and $\eta_2 = 1.4$. The corresponding average travel time is 26.4 minutes (larger than 25 minutes). In this case, if the delivery trucks are banned from travelling in the rush hours, the average travel time of all other vehicle trips will decrease to 22.3 minutes (the right black point shown in Figure 9), which shows the effectiveness and necessity of the banning.

**Case 3.** It is the red square in Figure 9, where $\eta_1 = 1.2$ and $\eta_2 = 1.0$. The corresponding average travel time is 45.1 minutes (much larger than 25 minutes). In this case, if the delivery trucks are banned from travelling in the rush hours, the average travel time of all other vehicle trips will decrease to 40.6 minutes (the left black point shown in Figure 9), which is also much bigger than 25 minutes. Thus, we can say that it is not very necessary to ban the delivery trucks because of the tiny effects.

In addition, VRP schemes based on partial traffic equilibrium may worsen the service level of the whole road network and cause severe congestion, because some roadways on the delivery paths may already have large amount of traffic to be at saturation, and a few delivery trucks may lead to severe congestion. Figure 10 demonstrates the influence of the delivery trucks on road traffic. When the delivery is done based on VRP scheme under partial traffic equilibrium, the congestions on Link 1, Link 2, and Link 3 get worse.

![Figure 7: Delivery routes from Depot 1 to Customer 21 by two methods.](image7)

![Figure 8: Delivery paths and service depots of Customer 8 in the two schemes.](image8)
(Figure 10(a)). If delivery is done based on delivery scheme under total traffic equilibrium, congestions will not happen on Link 1, Link 2, and Link 3 and the traffic flow on the entire network is more balanced. It indicates that our method is more realistic and helpful as it fully uses the road network.

Moreover, the results also show that in the situation of the total equilibrium the travel flow and travel speed on Link 2 are 1699 pcu/h and 13.4 km/h. However, in the situation of partial equilibrium, the corresponding figures are 1746 pcu/h and 12.5 km/h, respectively. It proved that without taking them into account, more delivery trucks will choose Link 2, which will be more congested after the delivery trucks join in. Thus, the shortest path under partial equilibrium on it is a fake one.

In terms of calculation time for the solution, for the partial traffic equilibrium method it is 13 seconds and for the total traffic equilibrium method it is 256 seconds (CUP: Core i5, 1.5 G). It is obvious that to simulate the interactions between all vehicles and between delivery schemes and OD trips, iterative computations should be done; thus our method costs more time for solution calculation. However, since GDAP is not a dynamic problem but prior simulation of total traffic equilibrium with bilevel model and real-time calculation is not needed, the computing speed and efficiency are not important.

7. Conclusion

In this paper, we change multidepot VRP into GDAP, which is an alternative way of designing routes for a multidepot VRP and then modelling urban goods transport. A bilevel
programming is proposed to model the GDAP, and the interaction among the path choice behaviors of all vehicles and the interaction between delivery schemes and OD trip matrix can be simulated. Then the solution for multidepot VRP is obtained by hybrid grouping method, genetic algorithm, and Frank-Wolf algorithm. The delivery schemes outputs from the model are those under total traffic equilibrium and thus are realistic, which can make full use of road capacity and balance the traffic flow in the entire network.

Compared with the solution under partial traffic equilibrium, the solution under the total traffic equilibrium can not only shorten the delivery time, but also help to avoid traffic congestions on the roads which are near saturation. The bilevel model has good adaptability for optimizing MDVRP.

It can be used as a reference for further study on combination optimization problems.

We recommend that further research be done comparing our new method with other methods for real life situations. More research could also be done on algorithms for cases where loads on trucks are not constant.

The limits of the method are that (1) it is difficult to get the OD of other vehicles; (2) the GA cannot assure the global optimal solution of the delivery schemes.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


