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Research Article

Random Regret-Minimization Model for Emergency Resource Preallocation at Freeway Accident Black Spots

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The preallocation of emergency resources is a mechanism increasing preparedness for uncertain traffic accidents under different weather conditions. This paper introduces the concept of accident probability of black spots and an improved accident frequency method to identify accident black spots and obtain the accident probability. At the same time, we propose a three-stage random regret-minimization (RRM) model to minimize the regret value of the attribute of overall response time, cost, and demand, which allocates limited emergency resources to more likely to happen accident spots. Due to the computational complexity of our model, a genetic algorithm is developed to solve a large-scale instance of the problem. A case study focuses on three-year rainy accidents' data in Weifang, Linyi, and Rizhao of China to test the correctness and validity of the application of the model.

1. Introduction

Freeways play a very important role in the transport system, which accounted for 2.8% of national highways and carried 1/2 of business passengers turnover and 40% of freight and cargo turnover in China in 2016. However, because of the high speed of vehicles, once the incident happens on the freeway, it will lead to relatively heavy casualties. Generally, in response to these unexpected situations, freeway operational companies often configure certain amount of emergency resources around the accident sites to provide assistance to victims conveniently. Hence, the preallocation of emergency resources, as the first step to rescue, directly impacts the efficiency of the rescue service. The reliable decision support models are heavily needed to improve the freeway service before incidents happen.

Increased attention to literature has been focused on stochastic programming (SP) model in order to address the problem of emergency recourse allocation. Zografos et al. constructed an incident-management program to minimize the incident delay and proved that the deployment of traffic-flow restoration units is essential to finish this goal [1]. Later, Zografos et al. developed decision support systems (DSSs) that can be used for incident response logistics and improved the quality of the decision. He also proposed a mathematical model to decrease incident response time considering users' requirements [2]. Baker et al. introduced criteria for budgets and workload and developed an integer, nonlinear mathematical programming model to allocate emergency medical services, especially for ambulances to meet the government's criterion of response time [3]. Ozbay et al. put forward a flexible optimal dispatching model to the operator with the probabilistic constraint for the potential accident locations, that is, the concept of the quality of service [4]. In order to satisfy the demands with a given flood occurrence probability, Garrido et al. addressed an issue of the delivery supplies and formulated a mathematical optimization model considering the level of inventory of emergency resources and the availability of vehicles [5]. Rawls and Turnquist focused on the special disasters like hurricanes and so forth, which had uncertainty in demand for the stocked supplies. He presented a two-stage stochastic mixed

integer program before the disasters [6]. Zhang developed a multiagent-based decentralized resource allocation approach for different emergency events based on the domain transportation theory [7].

The above literatures have studied a variety of approaches with respect to the allocation problem of different types of emergency resources in specific disasters. Overall, there are four obvious features and problems. Firstly, many efforts in emergency response researches focus on natural disasters in the cities, such as hurricane, earthquake, and flood [5, 6, 8], but relatively fewer scholars pay attention to applying approaches on the freeway. In terms of problems of emergency resources allocation in the cities, due to the complexity and high density of the urban road networks, the researchers mostly select the facility locations and then allocate the resources [9]. However, the freeway network has lower road coverage and density than urban road network and supply spots are relatively fixed and less. Obviously, the existing approaches cannot be adapted to the freeway network. In addition, most scholars have studied how to distribute resources to deal with natural disasters, such as floods and earthquakes, but they ignored the fact that accidents happen more easily on rainy, foggy, and snowy days. It is the lack of researches in this field, which needs more adequate study. Secondly, the goal of emergency resource preallocation is to prepare for the future incidents to promise to give assistance in time. There were some works on the potential accidents or secondary locations, such as introducing the quality of service to evaluate the probability for the future incidents [5, 6], and given probabilities of secondary disasters to allocate resources [9, 10]. These methods of representation can, to some extent, indicate the probability of an accident occurring on a freeway, but they are not precise and specific and have a large number of errors. In particular, the locations of accidents are unfixed so that the accident data is mass and disorderly. Under this condition, allocating emergency resources is really difficult. Hence, it is very necessary to investigate the accident-prone location, black spots, prior to the deployment of emergency resources. Thirdly, in the research of this issue, current researches are basically a single-objective or multiobjective SP model to analyze and solve the problem of the allocation of emergency resources. The ultimate goal of optimization is the least cost or the shortest response time. However, these models have a serious problem: a well-performed attribute can compensate for the performance of other poorer attributes. Therefore, the final decision is not globally optimal. In our paper, a random regret-minimization (RRM) function is introduced, which is a convex function that can make up for the previous deficiency. When the attributes are equally important, the better performance of an attribute is only a half-compensating effect on the overall regret value.

In the study of the identification of accident black spots and the calculation of accident probability, the research of these issues is relatively mature. Deacon et al. focused on the intersection and nonintersection's black spots and distinguished between short highway segments and large segments based on accident statistics [11]. Saccomanno et al. established a multivariate Poisson regression and empirical

Bayesian models for the potential accidents and applied them to the highway [12]. Gregoriades and Mouskos illustrated an integration of Bayesian Networks model with the simulator to assess the accident risk index, which was used to identify accident black spots on road networks [13]. Washington et al. proposed a combination of equivalency calculation only considering property damage and quantile regression technique to identify hot spots in a transportation network [14]. Debrabant et al. addressed issues of discrete distribution and overdispersion data from hospital records from Funen and Denmark to present an autoregressive Poisson-Tweedie model [15]. The above studies used different methods to identify accident black spots. Especially for our paper, identifying the black spots objectively and accurately will help decision maker allocate limited resources to accident-prone sites in the event of insufficient funds and improve traffic safety management.

Regret is one of the common and widespread negative emotions which usually expresses dissatisfaction and disappointment in people's feelings about what has been chosen. Initially, the regret theory (RT) was proposed by Bell et al. in 1982 and applied to the field of economics. The RT represents a status that a decision maker, who engages in commodity trading, tend to avoid the risk of loss as much as possible, in other words, to avoid regret [16]. Over the years, Daskin et al. applied P-minimax regret method to solve the facility location problems and presented a new model that optimizes the worst-case performance over a set of scenarios in 1997 [17].

In 2008, Chorus et al. presented random regret-minimization (RRM) model rooted in RT at the first time and provided several useful features for travel demand analysis [18]. Later, the RRM model was gradually improved through in-depth studies of many scholars and applied to more and more fields. We introduce the latest publications and give an overview of them. In 2010, Chorus developed a new discrete choice model to improve the RRM 2008 model with respect to foregone alternatives and revealed a promising performance of the new RRM-model through the application of travel mode choices, travel information acquisition choices, parking lot choices, and shopping location choices [19]. Dekker et al. introduced the concept of need-satisfaction into the hybrid choice model and better understood the behavioral processes underlying leisure activity participation based on the RRM [20]. In the same year, Hess et al. stated the contrast between RRM model and the Random Utility Maximization (RUM) model and found out that the differences between two models, apart from the factor of alternatives, were driven by the datasets [21]. Van Cranenburgh et al. proposed new methodological insights on RRM models, which are the μRRM model and the Pure-RRM model illustrated by reanalyzing ten datasets [22]. Guevara et al. developed an approach to achieve consistency, asymptotic normality, and relative efficiency of the estimators while sampling of alternatives and testing their approaches were more practical than a truncated model through real data experiments [23]. Jang et al. defined the regret as a function of physical attributes of choice alternatives and proposed a nonlinear psychophysical representations of the perception

of attributes levels, which greatly enriched the RRM [24]. Rasouli and Timmermans compared original specification of RRM with the logarithmic specification of RRM and found that although logarithmic specification became theoretically inferior to original specification, the results of case study illustrated that the original specification outperformed the new specification for the collected data [25]. In 2018, Rasouli and Timmermans summed up the issues of recent researches in this field and developed a new sight of Chorus and Van Cranenburgh models [26]. Dekker and Chorus interpreted the choice probability as a well-behaved approximation and developed a measure of consumer surplus for RRM [27].

We give a brief overview of these studies on the RRM models and summarize three characteristics of these studies. First, incremental changes have been suggested and new insights of the RRM model have been added. An original RRM model just considered the choice alternatives. Later, a logarithmic specification was expressed. The scholars developed different specifications from various angles to provide a good deal of insight into the fundamental theory involved. Second, the empirical comparisons with RUM constitute above all publications, because some peers have questioned the performance of the RRM model. Therefore, their compromise effects and the differences have been argued and observed. Third, the applications of the RRM model have involved various fields including marketing decisions, travel route choices, travel information acquisition choices, parking lot choices, and shopping location choices. With its application enlarged, its utility gets more and more important. Thus, in this paper, we aim to broaden the range of the application of the RRM model continually and use it in the emergency management.

In this paper, we put forward a three-stage RRM model to solve the emergency resource preallocation problem on the freeway from the perspective of regret based on the above research basis. Before the decision is made, we first identify the black spots on the freeway using the improved accident frequency method and then calculate the probability of an accident. In particular, because of the different external conditions such as the geographic location of the freeway and the condition of the road surface, the black spots in the discussed area are not completely consistent under different weather conditions. Thus, only the accident data under similar weather conditions can be collected.

We attempt to address the emergency resource allocation problem with different approaches from other researches, which is a three-stage RRM model. Since the response time is the most vital factor in the rescue, a minimum response time model is established in the first stage with the specified coverage constraint about time. We use a genetic algorithm to generate the initial plan set with regard to the number of resources dispatching from supply locations to black spots. In the second stage, a new solution set is generated based on the original set of plans, combining the probability of accidents occurring in black spots under specific circumstances. The third stage is comparison of plans to select the final decision. RRM model is formulated comparing three attributes of every plan, i.e., response time, storage and procurement costs, and the demand. The plan with the minimal regret value is regarded as our final decision.

The rest of this paper is organized as follows. In Section 2, we describe the mathematical formulation including two parts, which is the identification of black spots and a three-stage RRM model. In Section 3, we illustrate the use of the model through a case study for a three-stage RRM model for emergency resources preallocation in anticipation of rainy day in three cities in China. Section 4 provides conclusions and directions for further work.

2. Methodology

This section introduces the identification of freeway black spots using the approach of improved accident frequency at first and then describes the model development, that is, a three-stage RRM model, which promises the minimization of regret value, considering the probability of black spots. Our goal is to find the most optimized plan to allocate resources from supply locations to black spots on the freeway.

2.1. The Identification of Black Spots. The diagnosis of accident black spots does not only solve the existing traffic safety problems, but also provide first-hand information for the prediction and prevention of traffic accidents. The black spots seriously reduce the service quality of the road network, since the cumulative number of incidents at each black spot accounts for a high proportion of the total number of accidents, which has a great impact on the overall safety of the transportation system. For emergency management, the identification of black spots on freeways also plays a vital role as it provides guidance about preallocation emergency resources to nearby areas prone to accidents.

To begin, we divide the road section into units so that it is easier to process accident data statistically. And then the improved accident frequency method is used to identify black spots. We select a critical number of accidents as the criteria for identification. If the number of accidents on a certain road section is greater than the critical value, it will be marked as an accident-prone point. The advantage of this method is easy to calculate and select and clear at a glance. It is especially suitable for the processing of massive accident data [28].

The improved accident frequency method is described as follows. Firstly, calculate the average number of accidents λ on a unit.

$$\lambda = \frac{\sum m_i}{n} \tag{1}$$

where m_i is the number of accidents on the road section i and n is the total number of units.

If the confidence level is α, the critical number of accidents R is

$$R = \lambda + u_{(1-\alpha)/2} \cdot \sqrt{\lambda}, \quad i = 1, 2, \ldots, n \tag{2}$$

We compare the number of actual accidents with the critical value R. If it is greater than the critical value R, we can determine that the road section is a black spot.

The "spot" in the black spots can be a point, a road section, an entire road, or an area, so the spot in this paper refers to a road segment. However, the above method will miss some real black spots resulting from concentrated accident locations which were divided into two by a fixed segment; hence, after the completion of the above identification, it is necessary to use the section to cut surface technology in order to correct the results. The principle of cut surface technology is to move one or more adjacent unit sections without exceeding the critical value R to a suitable location in light of their distribution. This supplementary calculation, the nonfixed description of black spots, has greatly improved the accuracy of the accident probability calculation.

2.2. A Three-Stage RRM Model for the Emergency Resources Allocation. The problem of freeway emergency resource preallocation can be considered as a risky decision issue. Existing studies have been focusing on a variety of factors in the SP model. If certain attributes perform well, good-performing attributes will fully compensate for other worse performing attributes. Obviously, this is not acceptable for an emergency rescue decision, because the result of emergency decision does not depend on the longest board, but the shortest board. However, the RRM model can exactly make up for the shortages caused by the ordinary SP model.

RRM was proposed by Chorus based on classical regret theory in 2008. RRM is a general choice model that deals with multiple alternatives with multiattributes and it can weigh the performance of multiple attributes to avoid the regret. The regret results from the unselected schemes that perform better than the selected scheme [18]. Consider the traveler who faces a choice between alternatives i, j, and k. The alternatives i, j, and k have the attributes of x, y, and z; that is, $i = \{x_i, y_i\}$, $j = \{x_j, y_j\}$. The regret is obtained by the comparison of that alternative with the best of other two alternatives:

$$R_i = \max\left\{R_{ij}, R_{ik}\right\} \tag{3}$$

$$R_j = \max\left\{R_{ji}, R_{jk}\right\} \tag{4}$$

$$R_k = \max\left\{R_{ki}, R_{kj}\right\} \tag{5}$$

Take an example of the calculation of R_{ij}:

$$R_{ij} = \varphi_x\left(x_i, x_j\right) + \varphi_y\left(y_i, y_j\right) + \varphi_z\left(z_i, z_j\right) \tag{6}$$

where $\varphi_x, \varphi_y, \varphi_z$ is an attribute-regret function using the following formulas to obtain. Let β be the estimated parameter of the attribute.

$$\varphi_x\left(x_i, x_j\right) = \max\left\{0, \beta_x \cdot \left(x_j - x_i\right)\right\} \tag{7}$$

$$\varphi_y\left(y_i, y_j\right) = \max\left\{0, \beta_y \cdot \left(y_j - y_i\right)\right\} \tag{8}$$

$$\varphi_z\left(z_i, z_j\right) = \max\left\{0, \beta_z \cdot \left(z_j - z_i\right)\right\} \tag{9}$$

We finally select the minimum regret value from R_i, R_j, R_k: $\min\{R_i, R_j, R_k\}$.

Based on the above theory, we propose a three-stage RRM emergency resource preallocation at freeway black spots. The first stage is that the SP model generates a preliminary plan based on the response time. The second stage is to form a new set of solutions by combining the probability of accidents occurring at black spots. The third stage is to compare the schemes obtained in the second stage by the calculation of the RRM model and finally choose the scheme with the smallest regret value.

Stage 1: Generate a Preliminary Plan Based on Response Time. In emergency rescue, the response time is the most important decisive factor. Certain supply locations do not cover many black spots, so that rescue personnel cannot reach the black spots within the response time range specified by the system. Therefore, we need to make a preliminary match between supply locations and black spots. Due to the vast area, i.e., the large number of supply locations and black spots, preliminary screening can simplify calculation and increase the speed of operations.

In the first step, we propose a stochastic programming model with the smallest response time. We assume that the road from supply locations to the black spots is travelable and the shortest distance. Supply can be prepositioned at the location i, and black spots can be defined as j. We use t_{ij} to denote emergency resource transportation time from supply location i to black spot j, t_0 to denote the rescue system specified response time, and $\overline{t_l}$ to denote the average processing time after receiving the alarm at the depot i. Let l_{ij} be the shortest distance from a supply location i to the black spot j, and let v^k be the average transportation speed under special scenario k ($k \in K$): $t_{ij} = l_{ij}/v^k$. In addition, the parameter θ_{ij} represents whether accident black spots j are within the coverage of the depot i and a_i represents the maximum stock capacity of the supply location i. We use x_{ij} ($x_{ij} \in X_n$) to denote the number of emergency resources dispatched from depot i to black spot j. Finally, an initial scheme set S_1 is calculated by a genetic algorithm and X_n is the element of the initial scheme set S_1.

The first stage of the stochastic programming model is given as

$$\min \quad \sum_{i=1}^{n_i}\sum_{j=1}^{n_j}\theta_{ij}t_{ij}x_{ij} \tag{10}$$

subject to

$$\sum_{j=1}^{n_j}x_{ij} \le a_i \tag{11}$$

$$\theta = \begin{cases} 0 & t_{ij} > t_0 - \overline{t_i} \\ 1 & t_{ij} \le t_0 - \overline{t_i} \end{cases} \tag{12}$$

$$x_{ij} \geq 0,$$

$$x_{ij} \in X_n, \qquad (13)$$

$$x_{ij} \text{ is integer}$$

The objective function of the first stage (4) incorporates the total response time in order to provide an incentive to execute the resources preparedness at the lowest time within the constraint of response time coverage. Constraint (5) states that the number of emergency resources in depot i must be within the maximum inventory capacity a_i at that point. Constraint (6) states that if the black spot j is within the coverage of supply location i, $\theta_{ij} = 1$; otherwise, $\theta_{ij} = 0$. Constraint (7) requires x_{ij} must be a positive integer. After the calculation of the genetic algorithm, several different solution sets will be generated. The solution set will be stored in the form of a matrix X_n, which is a set of solutions, and finally, form the initial solution set S_1 ($X_n \in S_1$).

Stage 2: A New Solution Set S_2 Based on the Accident Probability of Black Spots. Through formulas (1) and (2), we can obtain the results of identifying black spots, so that we can calculate the relative accident probability p_j^k of black spots under the circumstance k and form a diagonal matrix P. Each element X_n in the set S_1 is weighted by this diagonal matrix P to form a new element X_n'. Formula (8) is as follows:

$$X_n' = X_n \cdot P \qquad (14)$$

Through the above formula, we can obtain a new set of solutions S_2 ($X_n' \in S_2$).

Stage 3: The Best Solution Based on RRM Model. After the calculation of the above two steps, the initial set S_1 is generated by statistic programming model on the basis of geographical location and response time. In addition, a new solution set S_2 is generated based on the accident probability of black spots. However, the two stages only consider the response time and accident probability but do not comprehensively consider the costs of storage and procurement of emergency resources, and the demand for black spots. Some emergency resources cannot be stored permanently and need to be replaced regularly; therefore, decision makers hope to use the lowest cost to maximize benefits. We define that there are m plans obtained by stage 2. Let C_s, C_b, respectively, be the inventory costs and the procurement costs of emergency resources, and let C be the attribute of the total cost. The supply locations in the rescue network have different resource allocation standards and construction costs resulting in different rescue service [29]. In this paper, we divide the supply locations into three grades. Grade 1 is the top of them, which means that it has the greatest rescue ability. Otherwise, Grade 3 is the bottom of them, which has the worst rescue service. We denote $G(i)$ as the service level of the depot i,

$$G(i) = \begin{cases} 3 & \text{Grade 1} \\ 2 & \text{Grade 2} \\ 1 & \text{Grade 3.} \end{cases} \qquad (15)$$

The purpose of the emergency resources preallocation is that, in the event of a traffic accident, the supply locations can distribute enough resources to meet the needs of the accident site in the shortest possible time. Hence, in addition to considering the response time and cost, we also consider whether the scheme can meet the requirements of black spots in the RRM model. We denote W as the shortage of commodity at black spots and denote d_j as the demand of black spot j.

According to formula (3), we put forward to RRM model for emergency resource preallocation problem.

$$\min \quad R_n(k) \qquad (16)$$

$$R_1 = \max \{R_{12}, R_{13}, \cdots, R_{1m}\} \qquad (17)$$

$$R_2 = \max \{R_{21}, R_{23}, \cdots, R_{2m}\} \qquad (18)$$

$$\vdots \qquad (19)$$

$$R_m = \max \{R_{m1}, R_{m2}, \cdots, R_{m(m-1)}\} \qquad (20)$$

$$R_{n_1 n_2} = \varphi_T\left(T_{n_1}, T_{n_2}\right) + \varphi_C\left(C_{n_1}, C_{n_2}\right) \\ + \varphi_W\left(W_{n_1}, W_{n_2}\right) \qquad (21)$$

$$\varphi_T\left(T_{n_1}, T_{n_2}\right) = \max\left\{0, \beta_T \cdot \left(T_{n_2} - T_{n_1}\right)\right\} \qquad (22)$$

$$\varphi_C\left(C_{n_1}, C_{n_2}\right) = \max\left\{0, \beta_C \cdot \left(C_{n_2} - C_{n_1}\right)\right\} \qquad (23)$$

$$\varphi_W\left(W_{n_1}, W_{n_2}\right) = \max\left\{0, \beta_W \cdot \left(W_{n_2} - W_{n_1}\right)\right\} \qquad (24)$$

$$T_n = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} t_{ij} x_{ij} \qquad (25)$$

$$C_n = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} (C_s + C_b) G_i x_{ij} \qquad (26)$$

$$W_n = \sum_{j=1}^{n_j} (d_j - x_j) \qquad (27)$$

subject to

$$\sum_{j=1}^{n_j} x_{ij} \leq a_i \qquad (28)$$
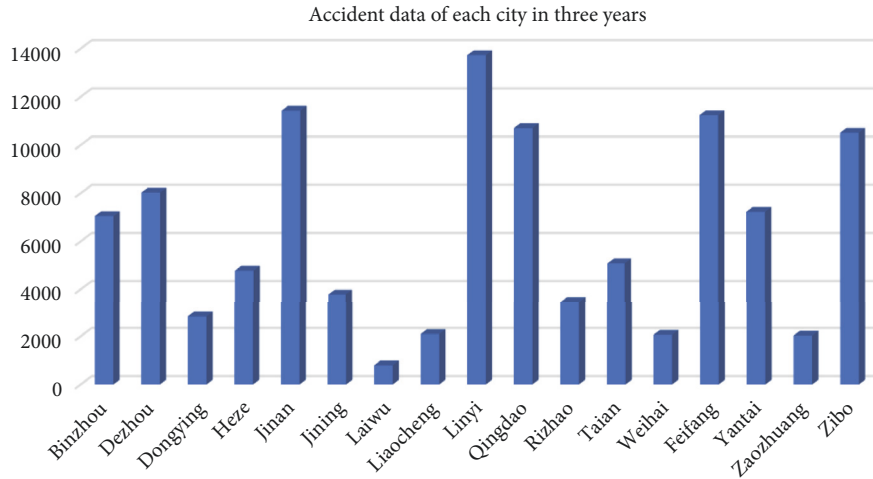
$$\sum_{i=1}^{n_i} \sum_{j=1}^{n_j} x_{ij} \leq A \qquad (29)$$

Accident data of each city in three years

FIGURE 1: The accident data for each city in three years.

$$x_{ij} \geq 0,$$

$$x_{ij} \in X'_n, \tag{30}$$

$$x_{ij} \text{ is integer}$$

where $R_n(k)$ is the regret value of plan n under the scenario k, and $R_1, R_2, \cdots, R_m$, respectively, is the regret value of the plan $1, 2, \cdots, m$. Let $\varphi_T, \varphi_C, \varphi_W$ be the attribute-regret function. We use β_T to denote the estimated parameters of response time, β_C to denote the estimated parameters of the total cost, and β_W to denote the estimated parameters of unfilled demand.

The goal of the objective function (16) is to select the minimal regret value from all schemes. Equations (17), (18), and (20), respectively, indicate that the regret value of scheme $1, 2, \cdots, m$ equals the regret associated with the comparison of that alternative with the best of the other alternatives. Equation (21) represents the binary regret associated with alternative n_1 when compared to alternative n_2. Equations (22), (23), and (24) state that either alternative n_1 performs better than n_2 in terms of the response time attribute, the total cost attribute, and the demand attribute, in case there is no attribute-regret, or alternative n_1 performs worse than n_2, in case the regret value associated with these attributes is a linear function of the difference in attribute values. Equation (25) indicates the meaning of the attribute of the response time. The total cost attributes including storage cost and acquisition cost, as well as considering the rescue service level of the depot i, are represented by formula (26). The difference between the resources dispatched to the black spots and its demand is stated by formula (27). Constraint (28) requires that the number of emergency resources in depot i must be within the maximum inventory capacity at that point. Constraint (29) states that the total number of resources in the study area cannot exceed the total number of emergency resources provided by the government. Constraint (30) requires that x_{ij} must be a positive integer and belong to X'_n.

Because the parameters belong to different dimensions in $\varphi_T, \varphi_C, \varphi_W$ and the numerical differences are relatively large, it is not appropriate to use them directly in the RRM model. Therefore, it is necessary to carry out the nondimensionalization of $(T_{n_2} - T_{n_1})$, $(C_{n_2} - C_{n_1})$, and $(W_{n_2} - W_{n_1})$, called the normalization of parameter data, that is, convert the parameters into relative dimensionless numbers. In this paper, a standardized method is used to perform a dimensionless treatment of $(T_{n_2} - T_{n_1})$, $(C_{n_2} - C_{n_1})$, and $(W_{n_2} - W_{n_1})$, and they are separately converted into dimensionless standardized data to eliminate the impact of the data dimension.

Standardized formula is as follows:

$$y_i = \frac{x_i - \overline{x}}{s} \tag{31}$$

where $\overline{x} = (1/n) \sum_{i=1}^{n} x_i$ and $s = \sqrt{(1/(n-1)) \sum_{i=1}^{n} (x_i - \overline{x})^2}$.

3. Case Study

We present a case study to demonstrate our approach. Three-year accidents' data of three big cities in Shandong province is analyzed and processed to verify the effectiveness of the three-stage RRM model. Although our model can cover several types of emergency resources, in this case study, we only consider single type resources (wreckers) for the sake of clarity in the representation.

3.1. Identify Black Spots and Calculate the Accident Probability of Black Spots. In this paper, we analyze the freeway accident data on rainy days in Shandong province in years from 2014 to 2016 and count the accident distributions of each city. The results are as in Figure 1.

As shown in Figure 1, we choose Linyi with the largest number of accidents as our study area. Meanwhile, we select its adjacent cities Weifang and Rizhao to identify three cities' accident black spots on the freeway.

TABLE 1: The number of accidents on the unit road section.

No.	Original Pile No.	Final Pile No.	m_i
1	K69+909	K71+000	3
2	K71+000	K72+000	0
3	K72+000	K73+000	3
⋮	⋮	⋮	⋮
52	K120+000	K121+000	2
53	K121+000	K122+000	4
54	K122+000	K123+000	5
⋮	⋮	⋮	⋮
101	K188+000	K189+000	21
102	K189+000	K191+000	13
103	K191+000	K192+000	27
⋮	⋮	⋮	⋮
121	K232+000	K233+000	30
122	K233+000	K234+000	27
123	K234+000	K236+414	39
	Total		1109

FIGURE 2: Accident locations marked on the map of ArcGIS.

Weifang contains four freeways, which are G18, G25, G20, and S16. Rizhao includes four freeways, which are G15, G1511, G25, and S7801. Linyi has four freeways, which are G2, G20, G22, and G25.

We calibrate the latitude and longitude positions of accidents in three years on the map of ArcGIS. As shown in Figure 2, we can see that the actual accident points are marked on the map. However, due to the large volume of accident data and its inconsistent locations, their distributions are very fragmented. If we do not classify them, it is very difficult to allocate emergency resources in such a disorganized situation. Therefore, we need to process the three-year accidents' data on the freeway and identify the black spots from them.

Take the G20 as an example to describe the identification process. G20 has a total length of 419 kilometers, which starts from east of Qingdao and passes through Weifang, Zibo, and Jinan. In this paper, we only identify the black spots of G20 in the Weifang section.

(1) Determination of the Critical Number of Accidents. We decide 1 km as a unit length and divide G20 into several unit lengths, 123 unit road sections in total. The number of accidents m_i occurring on each unit i is shown in Table 1.

The average number of accidents λ on a unit is

$$\lambda = \frac{\sum m_i}{n} = \frac{1109}{123} \approx 9 \qquad (32)$$

If the confidence level is 95%, the critical number of accidents R is

$$R = \lambda + u_{(1-\alpha)/2} \cdot \sqrt{\lambda} = \lambda + 1.96\sqrt{\lambda} \approx 15 \qquad (33)$$

(2) Obtain Accident Black Spots through the Primary Election. We compare the actual number of accidents on the G20 with the critical value R and then obtain the results of black spots through primary selection. There are 31 accident black spots in total as shown in Table 2.

(3) The Modification of Black Spots. According to the accidents occurring on the adjacent road sections, we use "section interception technology" to modify the original units and merge adjacent black spots, and then we obtain the results that include 14 black spots in Table 3.

We also calculate the critical value R and identify black spots for other freeways in the same method. The results are in Table 4.

We totally identified 88 accident black spots on the freeway in the three cities using the above method.

(4) The Calculation of Accident Probability of Black Spots. After screening out the black spots, we need to calculate their accident probabilities. In the study area, there are seven freeways with different traffic volumes and number of accidents. We have mentioned before that the accident

TABLE 2: Preliminarily select black spots.

Original Pile No.	Final Pile No.	The amount of accidents	Original Pile No.	Final Pile No.	The amount of accidents
K99	K100	16	K206	K207	20
K135	K136	20	K207	K208	18
K136	K137	16	K208	K209	16
K144	K145	18	K209	K210	16
K157	K158	21	K210	K211	17
K158	K159	23	K211	K212	42
K159	K160	16	K212	K213	22
K165	K166	17	K213	K214	34
K168	K169	26	K216	K217	47
K177	K178	24	K220	K221	40
K188	K189	21	K225	K226	38
K191	K192	27	K230	K231	37
K194	K195	18	K232	K233	30
K197	K198	16	K233	K234	27
K200	K201	18	K234	K235	39

TABLE 3: The results of the modification.

Black spots j		The number of accidents
Original Pile No.	Final Pile No.	
K101	K102	36
K107	K108	36
K131	K133	36
K136	K141	96
K142	K144	36
K147	K151	60
K157	K158	36
K169	K172	60
K177	K186	156
K189	K191	84
K194	K196	36
K197	K199	36
K200	K206	144
K213	K216	132

FIGURE 3: The percentage of accidents on every freeway.

probability of black spots is just relative value based on the proportion of three-year accidents in this region. As Figure 3 shows, the percentage of accidents on every freeway is counted as in Figure 3.

We can get the results of the accident probability on the basis of the above percentage in Table 5.

According to the accident probability, the summary graph is shown in Figure 4.

We mark the black spots with different proportions that depend on the accident probability on the map of ArcGIS in Figure 5. The results are more obvious, clear, and orderly than before.

3.2. The Three-Stage RRM Model of Allocating Emergency Resource. We apply the three-stage RRM model to the freeway in Linyi, Weifang, and Rizhao based on the accident data in 2013-2016. We take the wrecker as an example under the natural environment where the rainfall intensity is less than $2.5\ mm \cdot h^{-1}$, and the average speed v is 82.4 km/h to verify the RRM emergency resource preallocation model.

(1) The First Stage: Generate a Preliminary Plan. The goal of this stage is to generate a preliminary plan based on the minimum of response time through the genetic algorithm. We change the initial scale of the population, the number of iterations, and so on, in order to obtain several schemes.

Freeway operation company requires that the maximum response time in the system t_0 is 30 minutes and the average process time $\overline{t_l}$ is 5 minutes. According to the judging conditions $t_0 - \overline{t_l} = 25min$, we can get the value of θ_{ij} whether supply location i can cover the black spot j. Finally, we get 15 schemes, which compose the plan set S_1. It is stipulated that government provides 75 wreckers for three cities that are stored in road administration brigades and road administration squadrons and their rescue grade, respectively, are 1 and 2. The road administration brigades

TABLE 4: The result of the identification of black spots in the study area.

Name	G15	G18	G2	G20	G22	G25	G1511
The average number of accidents λ	3	4	8	9	3	5	5
The critical value R	7	8	14	15	7	10	10
The number of black spots	4	7	21	14	7	22	13

TABLE 5: The results of accident probability.

No.	Name	Original Pile No.	Final Pile No.	Accident Probability
1		K704	K707	0.012234
2	G15	K712	K713	0.009176
3		K714	K715	0.006117
4		K742	K744	0.006117
5		K363	K364	0.00534
⋮	G18	⋮	⋮	⋮
11		K443	K445	0.00534
12		K542	K543	0.006662
⋮	G2	⋮	⋮	⋮
32		K707	K709	0.006662
33		K101	K102	0.010531
⋮	G20	⋮	⋮	⋮
46		K213	K216	0.038614
47		K101	K103	0.005706
⋮	G22	⋮	⋮	⋮
53		K159	K160	0.005706
54		K1406	K1408	0.005464
⋮	G25	⋮	⋮	⋮
75		K1604	K1606	0.010927
76		K16	K18	0.009786
⋮	G1511	⋮	⋮	⋮
88		K156	K158	0.009786
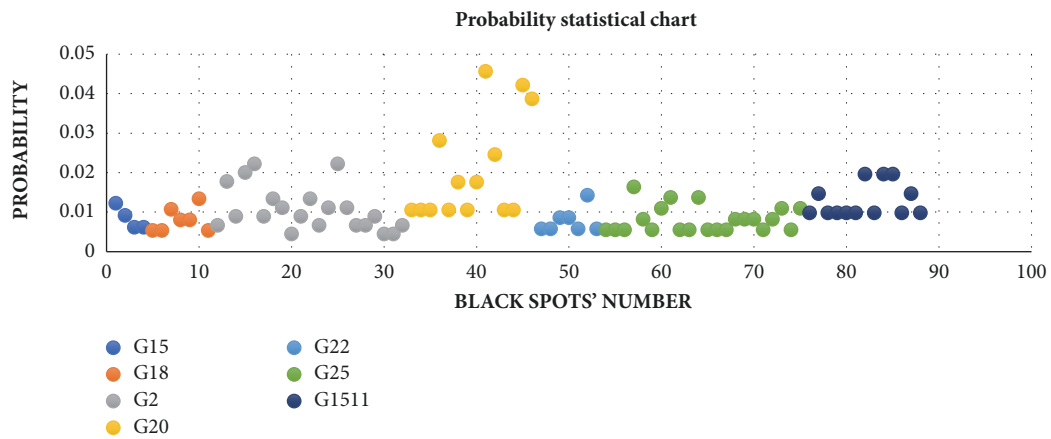
FIGURE 4: Probability statistical chart.

TABLE 6: The preliminary schemes.

Black spot	Scheme No.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	2	2	2	2	2	2	2	2	2	2	0	2	2	2
2	0	0	2	0	0	0	0	0	0	2	0	0	2	0	1
3	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80	2	2	2	0	0	0	1	0	1	0	1	0	0	0	1
81	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
82	1	2	0	1	0	1	1	0	0	0	2	0	2	2	1
83	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
84	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
85	0	0	2	0	1	2	1	2	0	0	0	0	0	0	0
86	2	0	0	2	2	2	2	2	2	2	2	2	2	2	2
87	2	2	2	0	0	0	1	1	0	0	0	2	2	2	2
88	2	2	2	0	0	0	1	0	1	0	1	0	0	0	1
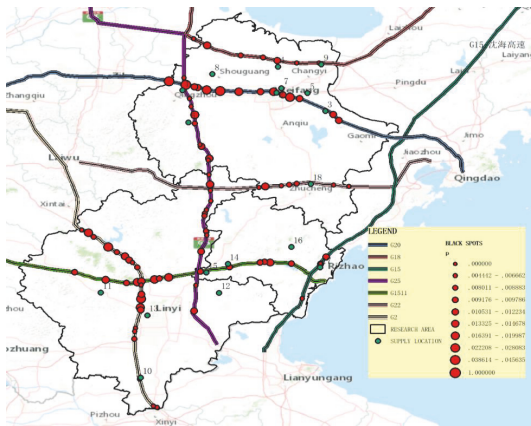
FIGURE 5: The map with final black spots.
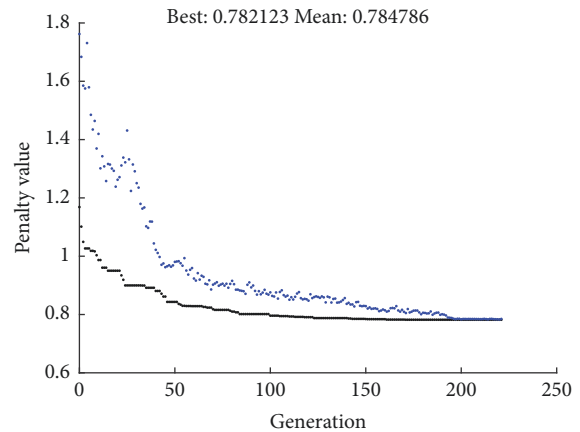
FIGURE 6: The result of genetic algorithm operation.

and road administration squadrons make up the supply location i, and they are 18 in all. We get 15 schemes that can represent how many wreckers allocate in every supply location, which is shown in Table 6.

Figure 6 shows the result of Scheme No. 1 through genetic algorithm operation. The optimal local value is 0.782123 after 221 iterations. Other schemes are obtained by changing the size of the initial population and the methods of hybridization or selection, and so forth.

(2) Generate Scheme Set S_2 on the Basis of Accident Probability. According to the accident probability of black spots in Table 4, we generate a diagonal matrix: $P = \begin{bmatrix} 0.012234 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0.009786 \end{bmatrix}$. Then, we apply formula (8) in Section 2.2 and finally obtain the new scheme set S_2.

(3) Generate the Most Optimal Plan through the Calculation of the RRM Model. In this part, we put every element in the scheme set S_2 into the RRM model to calculate. At first, we use a standard method to perform a nondimensional treatment of $(T_{n_2} - T_{n_1})$, $(C_{n_2} - C_{n_1})$, and $(W_{n_2} - W_{n_1})$, then compare 15 plans, respectively, 210 times to obtain $\varphi_T, \varphi_C, \varphi_W$, and select the lowest regret value $R_n(k)$.

According to the degree of importance of each attribute, we use the expert scoring method to determine the estimated parameter values, $\beta_T = -0.45$, $\beta_C = -0.3$, $\beta_W = -0.25$. The results are shown in Table 7 and Figure 7. In addition to the analysis of the results, except for scheme 15, the maximum regret value of other alternatives is generated from the compassion with $R_{n_1,15}$. Comparing all regret values of each plan, scheme 15 has the lowest total regret value and

TABLE 7: The regret value of each plan.

R_1	$R_{1,2}$	$R_{1,3}$	$R_{1,4}$	$R_{1,5}$	$R_{1,6}$	$R_{1,7}$	$R_{1,8}$	$R_{1,9}$	$R_{1,10}$	$R_{1,11}$	$R_{1,12}$	$R_{1,13}$	$R_{1,14}$	$R_{1,15}$
1.065	0.297	0.143	0.143	0.306	0.143	0.462	0.445	0.429	0.569	0.419	0.286	0.000	0.000	1.065

R_2	$R_{2,1}$	$R_{2,3}$	$R_{2,4}$	$R_{2,5}$	$R_{2,6}$	$R_{2,7}$	$R_{2,8}$	$R_{2,9}$	$R_{2,10}$	$R_{2,11}$	$R_{2,12}$	$R_{2,13}$	$R_{2,14}$	$R_{2,15}$
0.908	0.000	0.071	0.071	0.080	0.071	0.462	0.374	0.357	0.303	0.214	0.214	0.000	0.000	0.908

R_3	$R_{3,1}$	$R_{3,2}$	$R_{3,4}$	$R_{3,5}$	$R_{3,6}$	$R_{3,7}$	$R_{3,8}$	$R_{3,9}$	$R_{3,10}$	$R_{3,11}$	$R_{3,12}$	$R_{3,13}$	$R_{3,14}$	$R_{3,15}$
1.734	0.811	1.036	0.594	1.117	0.704	1.109	0.870	1.074	1.237	1.087	0.242	0.611	0.551	1.734

R_4	$R_{4,1}$	$R_{4,2}$	$R_{4,3}$	$R_{4,5}$	$R_{4,6}$	$R_{4,7}$	$R_{4,8}$	$R_{4,9}$	$R_{4,10}$	$R_{4,11}$	$R_{4,12}$	$R_{4,13}$	$R_{4,14}$	$R_{4,15}$
1.139	0.217	0.442	0.000	0.523	0.110	0.515	0.303	0.480	0.643	0.493	0.143	0.017	0.000	1.139

R_5	$R_{5,1}$	$R_{5,2}$	$R_{5,3}$	$R_{5,4}$	$R_{5,6}$	$R_{5,7}$	$R_{5,8}$	$R_{5,9}$	$R_{5,10}$	$R_{5,11}$	$R_{5,12}$	$R_{5,13}$	$R_{5,14}$	$R_{5,15}$
1.051	0.071	0.143	0.214	0.214	0.214	0.534	0.517	0.500	0.445	0.357	0.357	0.000	0.071	1.051

R_6	$R_{6,1}$	$R_{6,2}$	$R_{6,3}$	$R_{6,4}$	$R_{6,5}$	$R_{6,7}$	$R_{6,8}$	$R_{6,9}$	$R_{6,10}$	$R_{6,11}$	$R_{6,12}$	$R_{6,13}$	$R_{6,14}$	$R_{6,15}$
1.029	0.107	0.332	0.000	0.000	0.413	0.462	0.303	0.370	0.533	0.383	0.143	0.000	0.000	1.029

R_7	$R_{7,1}$	$R_{7,2}$	$R_{7,3}$	$R_{7,4}$	$R_{7,5}$	$R_{7,6}$	$R_{7,8}$	$R_{7,9}$	$R_{7,10}$	$R_{7,11}$	$R_{7,12}$	$R_{7,13}$	$R_{7,14}$	$R_{7,15}$
0.767	0.164	0.461	0.143	0.143	0.470	0.200	0.214	0.570	0.502	0.583	0.286	0.000	0.000	0.767

R_8	$R_{8,1}$	$R_{8,2}$	$R_{8,3}$	$R_{8,4}$	$R_{8,5}$	$R_{8,6}$	$R_{8,7}$	$R_{8,9}$	$R_{8,10}$	$R_{8,11}$	$R_{8,12}$	$R_{8,13}$	$R_{8,14}$	$R_{8,15}$
0.863	0.244	0.469	0.000	0.026	0.549	0.136	0.310	0.434	0.438	0.448	0.071	0.043	0.000	0.863

R_9	$R_{9,1}$	$R_{9,2}$	$R_{9,3}$	$R_{9,4}$	$R_{9,5}$	$R_{9,6}$	$R_{9,7}$	$R_{9,8}$	$R_{9,10}$	$R_{9,11}$	$R_{9,12}$	$R_{9,13}$	$R_{9,14}$	$R_{9,15}$
0.803	0.023	0.248	0.000	0.000	0.329	0.000	0.462	0.231	0.449	0.156	0.000	0.000	0.000	0.803

R_{10}	$R_{10,1}$	$R_{10,2}$	$R_{10,3}$	$R_{10,4}$	$R_{10,5}$	$R_{10,6}$	$R_{10,7}$	$R_{10,8}$	$R_{10,9}$	$R_{10,11}$	$R_{10,12}$	$R_{10,13}$	$R_{10,14}$	$R_{10,15}$
0.605	0.000	0.030	0.000	0.000	0.111	0.000	0.231	0.071	0.286	0.143	0.143	0.000	0.000	0.605

R_{11}	$R_{11,1}$	$R_{11,2}$	$R_{11,3}$	$R_{11,4}$	$R_{11,5}$	$R_{11,6}$	$R_{11,7}$	$R_{11,8}$	$R_{11,9}$	$R_{11,10}$	$R_{11,12}$	$R_{11,13}$	$R_{11,14}$	$R_{11,15}$
0.693	0.000	0.092	0.000	0.000	0.172	0.000	0.462	0.231	0.143	0.293	0.000	0.000	0.000	0.693

R_{12}	$R_{12,1}$	$R_{12,2}$	$R_{12,3}$	$R_{12,4}$	$R_{12,5}$	$R_{12,6}$	$R_{12,7}$	$R_{12,8}$	$R_{12,9}$	$R_{12,10}$	$R_{12,11}$	$R_{12,13}$	$R_{12,14}$	$R_{12,15}$
1.419	0.712	0.937	0.000	0.495	1.018	0.605	1.010	0.700	0.832	1.138	0.845	0.512	0.451	1.491

R_{13}	$R_{13,1}$	$R_{13,2}$	$R_{13,3}$	$R_{13,4}$	$R_{13,5}$	$R_{13,6}$	$R_{13,7}$	$R_{13,8}$	$R_{13,9}$	$R_{13,10}$	$R_{13,11}$	$R_{13,12}$	$R_{13,14}$	$R_{13,15}$
1.551	0.486	0.783	0.429	0.429	0.720	0.522	0.784	0.731	0.891	1.055	0.905	0.571	0.286	1.551

R_{14}	$R_{14,1}$	$R_{14,2}$	$R_{14,3}$	$R_{14,4}$	$R_{14,5}$	$R_{14,6}$	$R_{14,7}$	$R_{14,8}$	$R_{14,9}$	$R_{14,10}$	$R_{14,11}$	$R_{14,12}$	$R_{14,13}$	$R_{14,15}$
1.326	0.261	0.557	0.143	0.186	0.566	0.296	0.559	0.462	0.666	0.829	0.679	0.286	0.060	1.326

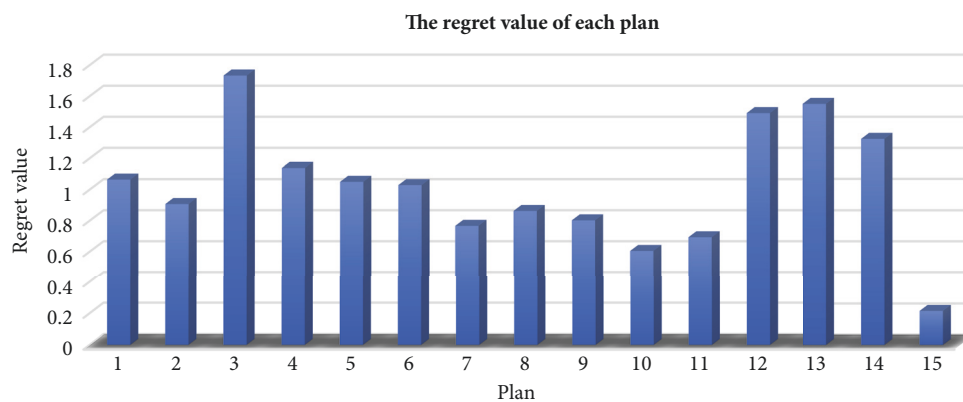| R_{15}	$R_{15,1}$	$R_{15,2}$	$R_{15,3}$	$R_{15,4}$	$R_{15,5}$	$R_{15,6}$	$R_{15,7}$	$R_{15,8}$	$R_{15,9}$	$R_{15,10}$	$R_{15,11}$	$R_{15,12}$	$R_{15,13}$	$R_{15,14}$
0.220	0.000	0.139	0.000	0.000	0.220	0.000	0.000	0.000	0.143	0.109	0.047	0.000	0.000	0.000

FIGURE 7: The regret value of each plan.

is the best option for the decision maker. Since scheme 15 is the best plan in all solutions, it can generate the lowest regret value among them. Therefore, if the decision makers choose other schemes rather than scheme 15, the regret will be larger than the regret of scheme 15.

In order to prove the superiority of our proposed method, we compare it with the SP model considering response time, total cost, and the demand as well and solve it by a genetic algorithm. The final SP solution and the final RRM solution are considered as two alternatives. Then we compare them

TABLE 8: The comparing result.

Selected plan	SP model				RRM model			
	T	C	W	Final Result	φ_T	φ_C	φ_W	R
SP plan	36.98	55.18	45.22	137.38	2.21	2.64	0.69	5.54
RRM plan	55.93	48.79	49.19	153.91	0.00	0.00	0.00	0.00

separately by the SP model and RRM model. The results are shown in Table 8.

As shown in Table 8, although the result of SP plan is smaller than the RRM plan through the calculation of the SP model, the attribute of response time extremely compromises the final result in SP plan. In addition, we can see that the regret of the SP plan is larger than the regret of the RRM model when we choose the SP plan. The regret value of every attribute of the SP plan performs worse than the RRM plan, but we cannot find this result from the calculation of SP model. Therefore, the results of the comparison can confirm that the RRM approach we proposed is better than SP model in emergency resource preallocation problem.

4. Conclusions

A three-stage RRM model considering the accident probability of black spots is established to study the emergency resource preallocation. Firstly, the improved accident frequency method is used to identify the black spots, in order to calculate the accident probability. Although some scholars considered the probabilistic constraints in the model for the resource allocation problem, they assumed a given probability rather than identifying the accident black spots. With no doubt, the identification of the black spot can be beneficial to summarize the mass accident data, which makes the occurring accident probability more authentic and believable.

Secondly, we come up with a three-stage RRM model for the emergency resources preallocation in this paper. Because response time plays the most important role in the rescue, we select the initial plan set through minimization stochastic programming model with response time constraint in the first stage. Then, we combine the accident probability with the initial plan set to obtain a new scheme set. To put it in another way, accident probability is attached as the weight to the initial plan set. In the third stage, we consider the factors of response time, total cost, and demand of black spots comprehensively and regard them as the attribute of RRM model to calculate every scheme's regret value and select the scheme with the smallest regret value.

In the case study, in order to verify the correctness and validity of our approach, we analyze three-year rainy day accidents' data of Linyi, Weifang, and Rizhao, identify 88 black spots, and calculate their relative accident probability. In addition, we calculate the regret value of 15 plans through the RRM model and compare the final selection with the result of the SP model. The case study shows the following results:

(1) The improved accident frequency method is effective for limited accident data. The method extremely simplifies the complexity of data and highly summarizes where accidents occur more. The identification of black spots lays the foundation for the following emergency resource preallocation issue.

(2) The three-stage RRM model for emergency resource preallocation is more advantageous for the emergency decision than common SP model, since the characteristics of semi-compensation in RRM model can consider every attribute comprehensively, while the better performance of attribute can completely compromise the total value in SP model. Therefore, our approach has good applicability to the preallocation of emergency resources on the freeway.

Several avenues present themselves as direction for further work. At first, due to the limitation of data acquisition and the confidentiality of certain data by related departments, there is no specific number of casualties. Hence, we cannot put this factor into the identification of black spots. Actually, the number of casualties cannot be ignored in the issue of identifying black spots. Although the improved accident frequency has relatively errors, this approach has already made the most of available data. In the future, the number of casualties per accident can be fully taken into consideration as one of the identification factors for black spots to improve the accuracy. On the other hand, the RRM model needs to compare every plan with others, which obviously increases the computation time. Hence, how to decrease the computation time using the heuristic algorithm is also the next step of our study.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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