In order to address the flight delays and risks associated with the forecasted increase in air traffic, there is a need to increase the capacity of air traffic management systems. This should be based on objective measurements of traffic situation complexity. In current air traffic complexity research, no simple means is available to integrate airspace and traffic flow characteristics. In this paper, we propose a new approach for the measurement of air traffic situation complexity. This approach considers the effects of both airspace and traffic flow and objectively quantifies air traffic situation complexity. Considering the aircraft, waypoints, and airways as nodes, and the complexity relationships among these nodes as edges, a dynamic weighted network is constructed. Air traffic situation complexity is defined as the sum of the weights of all edges in the network, and the relationships of complexity with some commonly used indices are statistically analyzed. The results indicate that the new complexity index is more accurate than traffic count and reflects the number of trajectory changes as well as the high-risk situations. Additionally, analysis of potential applications reveals that this new index contributes to achieving complexity-based management, which represents an efficient method for increasing airspace system capacity.

1. Introduction

With the rapid development of civil aviation transportation, continuous increases in air traffic flow are approaching capacity in air traffic management (ATM) systems. In the existing ATM systems, airspace is divided into several sectors, and the flight safety in each sector is managed by a team of controllers. Thus, the capabilities of current ATM systems are constrained by the levels of traffic situation complexity experienced by air traffic controllers. Under a low-complexity situation, the controller has enough time to resolve conflict and can also minimize the disturbance to the pilots under the premise of flight safety [1]. On the contrary, under a high-complexity situation, the controller is often unable to devise the optimal resolution and may even ignore some potential conflicts, which eventually endangers flight safety. Thus, how to objectively measure air traffic situation complexity has become a concern in the field of ATM. In response to this concern, the concept of air traffic complexity was introduced. Based on long-term predictions of air traffic complexity, the controller can identify highly congested regions in advance and thereby replan aircraft trajectories, thus avoiding unnecessary tactical maneuvering. Based on a medium-term prediction of air traffic complexity, the controller can discover the risks of multiaircraft conflict as early as possible, thereby reducing the difficulty of conflict detection and resolution [2]. In fact, measurement of the complexity of a given traffic situation not only improves the efficiency of ground-based ATM systems but also supports the implementation of satellite-based ATM systems in the future [3].

Traffic density (or traffic count), a basic index of air traffic situations, is commonly used in research of air traffic complexity as a result of its convenience in terms of computations, comprehension, and application [4–6]. However, traffic density alone does not fully reflect the complexity of air traffic situations. For instance, when all aircraft are flying in order on regular airways, the controller can guarantee flight safety, even when faced with a large number of aircraft [1].
On the contrary, if aircraft interact in a complicated manner, the safety level of air traffic may still be very low, even with a very small number of aircraft. In order to adequately consider additional complexity factors, some researchers have introduced the concept of dynamic density [4]. Dynamic density is the linear or nonlinear weighted sum of many air traffic complexity factors, including the physical aspects of airspace (airspace complexity), the characteristics of air traffic flow (traffic complexity), and operational constraints (environmental complexity) [6]. Airspace complexity factors are fixed for a sector, and they include the physical structure of airspace, such as terrain elevation, number of sector sides, sector area shape, number of airways, and number of intersections [5, 7]. Traffic complexity varies as a function of time and depends on features such as altitude changes, speed changes, heading changes, and speed mix, as well as proportions of climbing, declining, and converging aircraft [5]. Environment complexity includes weather conditions, equipment status, and flow restrictions [7]. The combination of airspace complexity, traffic complexity, and environment complexity affects air traffic situations [5]. The main shortage of dynamic density is that the weights of all complexity factors should be subjectively adjusted according to the concrete structural characteristics and traffic characteristics of a specific airspace, as well as the controller’s personal experience, which complicates the comparisons of complexity among sectors. Moreover, since dynamic density is represented by a set of complexity factors, when the complexity of a sector is very high, the controller is unable to identify the decisive factors and is thus unable to take targeted measures to reduce complexity.

The complexity of air traffic systems has been intensively studied from the perspective of complex systems. The basic intrinsic characteristics of air traffic situations (e.g., relative distance and relative speed) can be computed based on aircraft track information (e.g., position and speed). The between-aircraft interaction relationships are then mathematically described, and the complexity of a single aircraft pair can be computed. Indicators such as fractal dimension and Lyapunov exponent are then used to describe intrinsic traffic disorder, which serve as a measure of air traffic complexity [8, 9]. Traffic disorder is an objective measure of traffic situation complexity. This index is independent of a specific sector and can therefore be used to compare degrees of complexity among different sectors. However, this index is feasible for the computation of long-term tracking and is not suited to predict traffic situations in real-time operational settings. Regarding the impacts of abrupt disturbances in traffic flow caused by the entrance of another aircraft into the airspace, Lee et al. proposed a complexity model based on traffic flow disturbance, defined the complexity degree as a measure of control activities faced by a controller in response to emergency, and validated this method via simulation [10]. As an important traffic structure, aircraft clusters (groups of proximate aircraft) were originally studied to improve the performance of conflict resolution and were later used to measure sector-independent airspace congestion in future air traffic operations [11–13]. A methodology to generate complexity maps, taking into account the probability of conflict and a pair-wise intersection between aircraft flows flying along two straight lines, was proposed [14].

Numerous systems in the nature can be described as networks. Complex networks are a method for abstraction and description of complex systems, and they highlight the topological characteristics of system structures. Generally, a system can be characterized through a complex network if the component units (maybe of different properties) of a system can be abstracted into interrelationships between units and nodes [15, 16]. There are many studies that apply complex networks to air traffic and can be divided into macroscopic and microscopic types. From the macroscopic perspective, airspace units, including airports and airway points, are considered as nodes, while the air traffic flow between nodes is regarded as edges. The evaluation of network robustness, connectivity, and other indices can provide guidance for airspace design and planning, and it is focused on airspace structural characteristics [17–20]. From the microscopic perspective, aircraft are considered as nodes, while between-aircraft conflict or proximity relationships are regarded as edges. This helps to explore traffic behavioral characteristics, including flight conflict pattern and local congestion, providing assistance for air traffic control services, and it is focused on traffic flow characteristics [21–23]. Based on the above analyses, use of the complex network theory offers some meaningful conclusions on airspace structure and traffic flow characteristics. However, there is little research combining both of them to uniformly characterize air traffic situations.

The above studies focused on the complexity of air traffic situations from different perspectives. They agreed that airspace structural and traffic characteristics are the basic characteristics of traffic situation complexity. However, existing methods are unable to simply and efficiently integrate airspace structural characteristics and traffic characteristics and are thus unable to uniformly describe the complexity of different sectors, which is unfavorable for the further application of complexity (e.g., implementation of complexity-based management). The objective of this paper is to propose a new method for the air traffic complexity metric to overcome the limitations of existing methods.

In this study, we propose to build a unified air traffic situation complexity model, which considers the effects of not only airspace structure, but also traffic characteristics. In Section 2, we introduce the algorithm for definitions and computation of between-unit (e.g., aircraft and aircraft, aircraft and waypoint, aircraft and route segment) proximity and the complexity of air traffic situations. Then, with different units as nodes, if between-unit complexity occurs, there is an edge between the nodes, and complexity is considered as the weight of this edge. To clarify the modeling of air traffic weighted networks, we provide a simple example. Finally, based on the total number of relationships in air traffic situations, we propose a new method for the air traffic complexity metric. In Section 3, we selected same-day historical radar data at different sectors and statistically analyze and discuss the relationships between complexity and some key air traffic indices, including traffic count, trajectory change, potential flight conflict, and aircraft acceptance rate.
We also validated that the new method can be used to describe the main characteristics of air traffic situations.

2. Method

2.1. Local Complexity Metrics

2.1.1. General Description. Since ATM system can be considered to be a collection of interrelated components, ATM system can be characterized as a time-evolving network. In current ATM systems, there are mainly three types of relationships: aircraft-aircraft, aircraft-route, and aircraft-waypoint, and aircraft-route segment. And the complexity of ATM system is mainly embodied in spatial and temporal complexities. The most common metric for spatial complexity is separation. It is embodied in spatial and temporal complexities. The most important concept of keeping an aircraft outside a minimum distance from another aircraft to reduce the risk of those aircraft colliding, which can also apply to terrain, waypoint, and airway. The spatial approaching rate can be used to measure the temporal complexity, which reflects the effect of time pressure on air traffic control. Different local complexity metrics that describe both spatial and temporal complexities for the aforementioned three types of relationships, that is, aircraft-aircraft, aircraft-waypoint, and aircraft-route, will be discussed in detail in the following sections.

2.1.2. Complexity Measurement for Aircraft and Aircraft Relationships. The core task of ATM is to ensure that between-aircraft separation does not violate the separation standard. Currently, the most commonly used horizontal and vertical separation standards are 5 nautical miles and 1,000 feet, respectively (10 km and 300 m, resp., in China). According to the specified separation standard, a protection zone for aircraft can be set, such as a cylinder, ellipsoid, or spherical protection zone [20]. A conflict situation occurs when an aircraft protection zone is invaded by other aircraft (loss of separation). Based on the ellipsoid protection zone model, in this study, we introduced the concept of ellipsoid distance to reflect the between-aircraft proximity relationship (Figure 1) [3, 24].

\[ E_{i,j}(t) = \sqrt{\frac{\Delta x_{i,j}(t)^2}{a^2} + \frac{\Delta y_{i,j}(t)^2}{b^2} + \frac{\Delta z_{i,j}(t)^2}{c^2}}, \]  

where \( \Delta x_{i,j}(t), \Delta y_{i,j}(t), \) and \( \Delta z_{i,j}(t) \) are the longitudinal, lateral, and vertical separation between aircraft \( i \) and \( j \) at time \( t \); and \( a \) and \( b \) are the semimajor axis and semiminor axis of the ellipsoid model, respectively. In this model, \( a \) and \( b \) can be set according to the sensitive degree of conflict risks, and, in this study, they were set to be 10 km and 300 m, respectively, according to the separation standards. Clearly, at \( E_{i,j}(t) \leq 1 \), the protection zone of aircraft \( i \) will be invaded by aircraft \( j \), and the two aircraft will conflict. At \( E_{i,j}(t) > \sqrt{3} \), aircraft \( j \) is outside of the protection zone of aircraft \( i \), and no conflict will occur. If \( E_{i,j}(t) \) is within \( (1, \sqrt{3}) \), aircraft \( j \) may have invaded the protection zone of aircraft \( i \), or it may be outside of the protection zone but very close to the boundary. In general, ellipsoid distance reflects the spatial proximity relationship between aircraft. A smaller ellipsoid distance means that the two aircraft are spatially closer and the corresponding traffic situation is more complex.

In real operations, time factors significantly affect the controller’s work load and the choice and effectiveness of control strategies. For instance, during detection and resolution of potential conflicts, the time duration available for the controller determines the difficulty level of resolution. The between-aircraft ellipsoid distance changing rate \( V_{i,j}^A(t) \) (the spatial approaching rate) is then computed to reflect the time factor during the approaching process. Admittedly, at \( V_{i,j}^A(t) > 0 \), or, namely, at time \( t \), the spatial distance between aircraft \( i \) and \( j \) is enlarged, the traffic situation is diverging, and, otherwise, it is converging. If the absolute value of \( V_{i,j}^A(t) \) is larger, the corresponding diverging or converging situation is more evident:

\[ V_{i,j}^A(t) = \frac{E_{i,j}(t) - E_{i,j}(t-1)}{E_{i,j}(t-1)}. \]  

The complexity of air traffic situations is essentially the between-aircraft proximity spatially and temporally. Based on the temporal and spatial between-aircraft proximity, between-aircraft complexity, \( C_{A,i,j}(t) \), is defined as follows:

\[ C_{A,i,j}(t) = \begin{cases} 1 + \beta A V_{i,j}^A(t), & E_{i,j}(t) \geq 1 \\ \frac{1}{E_{i,j}(t)}, & E_{i,j}(t) < 1 \end{cases} \]  

Figure 1: Illustration of the ellipsoid-shaped protection zone model.
where $\beta_A$ is the adjustment coefficient for between-aircraft spatial proximity. Here, with the reduction of $E_{i,j}(t)$, $C_A^{i,j}(t)$ gradually increases and will be greater than or equal to one when two aircraft are in conflict. On the other hand, piecewise functions are used to guarantee that, for traffic situations with the same spatial proximity, a larger spatial approaching rate (larger divergence) corresponds to smaller situation complexity, and vice versa. In summary, this formula can effectively represent the complexity relationship between aircraft.

A scenario is then used to validate this equation. We selected a separation violation event from an air traffic control sector. This event involved two aircraft and their traffic situations are shown in Figure 2(a). At first, aircraft XXX1191 was declining and aircraft XXX5223 was leveling off, and their vertical separation was gradually reduced. Before $t_0$, there was potential conflict, but the controller did not detect it. At time $t_0$, the horizontal and vertical separation were both smaller than the minimum separation, thus inducing a flight conflict. At time $t_1$, the controller realized the conflict and chose a resolution strategy based on heading change. At time $t_2$, the between-aircraft horizontal separation was beyond the minimum separation, and the conflict was resolved. Between-aircraft horizontal and vertical separation during the occurrence of this event are shown in Figure 2(b). The time evolution of between-aircraft Euclidean distance and complexity are shown in Figure 2(c). Clearly, with the reduction of between-aircraft space distance, complexity gradually increases, but, in most cases, between-aircraft complexity is smaller than 1. Upon the occurrence of conflict, the between-aircraft proximity relationship is very strong, and corresponding complexity begins to increase beyond 1. During the resolution of conflict, with the gradual increase of horizontal or vertical separation, between-aircraft complexity
gradually decreases. Existing methods of calculating the relationship based on the Euclidean distance cannot provide us with satisfactory results, which overlook the effect of vertical separation and approaching rate. Thus, between-aircraft complexity relationship can be used to precisely describe between-aircraft interaction strength. Additionally, since the computation of complexity involves the spatial approaching rate, the complexity metric also reflects some abnormality of space proximity. For instance, at time $t_3$ in Figure 2(c), complexity suddenly increases to 0.9. The reason is that, at this time, the two aircraft are very close spatially, with a very severe approaching trend (the between-aircraft vertical separation suddenly decreases), and accordingly at very high risk. If at this time the controller realizes the high complexity of this situation, he or she could adopt a resolution by changing velocity or altitude to avoid conflict in time. Thus, the complexity-based method is able to describe between-aircraft space proximity and reflect the essence of traffic situation complexity.

2.1.3. Complexity Measurement for Aircraft and Waypoint Relationship. In current ATM systems, airspace is divided into several neighboring sectors. When an aircraft flies from one sector to another, the controllers must coordinate with other downstream sectors. Generally, for aircraft farther away from sector boundaries, coordination is less urgent, and the impact on the controller is less severe. When the aircraft are close to the entry points on the sector boundary, the controller usually has to issue higher amplitude instruction in order to satisfy the transfer agreement and in-trail spacing restriction. For example, sharper turns and more aggressive speed reductions may be necessary. Meanwhile, the controller may have insufficient time to resolve the potential conflict near sector boundaries, thus increasing the complexity of air traffic situations. It is believed that the difficulty is two times greater to resolve conflicts within 10 nm versus 20 nm from sector boundaries [9]. Moreover, flights away from 10 min of the transfer point would aggravate the controller’s workload load [25]. In current ground-based ATM systems, airplanes should follow planned airways. The airways in airspace usually have multiple intersection points, near which the flight situations are generally very complex. The controller must specifically monitor the convergence of traffic flows near the intersection points as well as the heading adjustment of individual aircraft. Thus, the proximity relationships of aircraft with some key waypoints (e.g., intersection points, metering fix) in the sector severely affect the complexity of air traffic situations. Let the key waypoint corresponding to aircraft $i$ at time $t$ be point $i^p$, and let the space proximity relationship between them be $D_{i,j}^p(t)$, which is computed as follows:

$$
D_{i,j}^p(t) = \begin{cases} 
  e^{-|d_{i,j}^p(t) - D_{i,j}^p|/D_{i,j}^p}, & d_{i,j}^p(t) > D_{i,j}^p \\
  1, & d_{i,j}^p(t) \leq D_{i,j}^p,
\end{cases}
$$

(4)

where $d_{i,j}^p(t)$ is the distance between them at time $t$ (aircraft altitude not considered) and $D_{i,j}^p$ is the reference distance of key waypoint $i^p$, which may differ for different key waypoints. A closer distance between an aircraft and a key waypoint means the traffic situation is more complex. For the convenience of computation, the maximum $D_{i,j}^p(t)$ is set to 1 when $d_{i,j}^p(t) \leq D_{i,j}^p$; that is also why exponential functions are adopted. We further consider the changing rate of the space proximity relationship $V_{i,j}^p(t)$:

$$
V_{i,j}^p(t) = \frac{d_{i,j}^p(t) - d_{i,j}^p(t-1)}{d_{i,j}^p(t-1)}. 
$$

(5)

According to the space proximity and approaching rate between an aircraft and a key waypoint, we can compute the complexity $C_{i,j}^p(t)$ between them as follows:

$$
C_{i,j}^p(t) = (D_{i,j}^p(t))^{1+\beta_p V_{i,j}^p(t)},
$$

(6)

where $\beta_p$ is the adjustment coefficient of the approaching rate between them. It is clear that the range of $C_{i,j}^p(t)$ is $[0, 1]$. When the distance between an aircraft and the waypoint decreases, the corresponding complexity gradually aggravates. Meanwhile, the approaching rate affects complexity modestly; when it is larger than 0, the proximity between aircraft and waypoint is reduced, thus weakening complexity. When it is smaller than 0, the complexity is increased, thus aggravating complexity. When the absolute value of the approaching rate is larger, the influence on degree of complexity is enhanced. We then use an entry point on the sector boundary as an example for illustration, and the situation is shown in Figure 3(a). Specifically, Flight 1 is approaching sector $S_3$ from sector $S_1$; Flight 2 flies from sector $S_3$ to sector $S_1$ and is flying away from the metering fix, whose reference distance $D^p$ is set as 15 km. The Euclidean distance and complexity between aircraft and the metering fix, computed by (4), are shown in Figures 3(b) and 3(c), respectively. Clearly, when an aircraft is approaching a metering fix, the complexity relationship is gradually aggravated and is maximized to 1 when it reaches the preset reference distance of the metering fix. When the aircraft is flying away from a metering fix, the complexity between them declines from 1. Additionally, since the changing rate of the proximity relationship is considered, the complexity degrees of two aircraft at the same Euclidean distance from the metering fix are different. This is because Flight 1 is gradually closer to the metering fix, which aggravates complexity, while Flight 2 is gradually farther from the metering fix, which reduces complexity. This is consistent with reality.

2.1.4. Complexity Measurement for Aircraft and Route Segment Relationship. In current ATM systems, pilots usually follow designated airways with a specific width; therefore, an excessively large distance deviation of aircraft from the route centerline would influence flight safety. When the trajectory is as intended by the controller without deviation from its authorized flight path, the controller can manage more flights [26]. When traffic disorder is used to reflect air traffic complexity, if a larger number of aircraft do not follow the designated airways, the corresponding traffic disorder and the resulting situation complexity are higher [9]. Air traffic can be divided into major traffic flows and outliers (outliers
are trajectories that are deemed “too different” from trajectories assigned flow) [20]. Outlier aircraft are particularly important in assessing air traffic control complexity because they act as perturbations to an otherwise static system [20]. Moreover, an abnormal flight behavior identification method based on track clustering was proposed and applied in the flight phases of takeoff and final approach [27]. In actual control, in order to avoid disturbance onto traffic flow, the controller usually will reduce abnormal behaviors, such as deviations from the expected flight path [28]. However, in case of bad weather, restricted airspace, or highly congested regions, the controller may provide radar navigational guidance and approve deviations. In this case, the controller must spend more energy and time, which aggravates the workload and thus the traffic situation complexity. Thus, the complexity relationship between aircraft and the airway can significantly affect the situation complexity. If aircraft \( i \) is following route segment \( i' \) at time \( t \), then the space proximity between them is \( D_{i,i'}(t) \), which is computed as follows:

\[
D_{i,i'}(t) = \begin{cases} 
  e^{-|d_{i,i'}(t)-D_{i,i'}'|/D_{i,i'}}, & d_{i,i'}(t) > D_{i,i'} \ 
  1, & d_{i,i'}(t) \leq D_{i,i'} \end{cases}, \tag{7}
\]

where \( d_{i,i'}(t) \) is the distance between aircraft \( i \) and route segment \( i' \) at time \( t \) (aircraft altitude not considered); \( D_{i,i'} \) is the reference distance of deviation from route segment \( i' \), which may differ for different segments. In the current airspace in China, the maximum route deviation of most

![Diagram](image)

**Figure 3**
airsps is 10 km. We then further consider the changing rate when aircraft approach a segment, $V_{i,j}(t)$:

$$V_{i,j}(t) = \frac{d_{i,j}(t) - d_{i,j}(t-1)}{d_{i,j}(t-1)}.$$  \hspace{1cm} (8)

According to the space proximity and approaching rate between an aircraft and a route segment, we compute the complexity between them, $C_{i,j}(t)$, as follows:

$$C_{i,j}(t) = (1 - D_{i,j}(t))^{-\beta}V_{i,j}(t),$$  \hspace{1cm} (9)

where $\beta_i$ is the adjustment coefficient of the approaching rate between them. It is clear that the range of $C_{i,j}(t)$ is 0 to 1. As the distance between an aircraft and the segment increases, the corresponding complexity gradually improves.

Meanwhile, the approaching rate also affects complexity. When it is $>0$, the aircraft is deviating from this segment. An aircraft farther away from a segment is more likely to affect the traffic on other airways, and the corresponding traffic situation may be more complex. When it is $<0$, the aircraft is approaching this segment, and the corresponding situation complexity is very likely to be reduced. When the absolute value of the approaching rate is greater, its influence on complexity is enhanced. Here, the actual trajectories of two aircraft were used as examples, as shown in Figure 4(a). When Flight 3 was flying along the airway, one spiral waiting was required; as a result, its trajectory deviated from the airway. When Flight 4 was flying, it had to divert due to bad weather or other reasons; hence, one part of the track deviated from the airway. Based on the current situation...
Table 1: Between-aircraft ellipsoid distances at the current and previous time points.

<table>
<thead>
<tr>
<th></th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
</tr>
</thead>
<tbody>
<tr>
<td>t−1</td>
<td>-</td>
<td>-</td>
<td>6.16</td>
<td>7.41</td>
<td>9.42</td>
</tr>
<tr>
<td>t</td>
<td>6.16</td>
<td>7.41</td>
<td>-</td>
<td>-</td>
<td>9.10</td>
</tr>
<tr>
<td>t−1</td>
<td>9.42</td>
<td>7.52</td>
<td>9.25</td>
<td>8.94</td>
<td>8.69</td>
</tr>
<tr>
<td>t</td>
<td>12.17</td>
<td>10.37</td>
<td>5.14</td>
<td>4.58</td>
<td>-</td>
</tr>
<tr>
<td>t−1</td>
<td>15.11</td>
<td>15.76</td>
<td>6.25</td>
<td>8.36</td>
<td>7.88</td>
</tr>
<tr>
<td>t</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Distances between aircraft and waypoints and between aircraft and segments at the current and previous time points.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
</tr>
</thead>
<tbody>
<tr>
<td>t−1</td>
<td>-</td>
<td>-</td>
<td>49.5</td>
<td>42.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>t</td>
<td>-</td>
<td>-</td>
<td>66.0</td>
<td>77.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>t−1</td>
<td>-</td>
<td>-</td>
<td>45.3</td>
<td>32.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>t</td>
<td>-</td>
<td>-</td>
<td>80.8</td>
<td>66.7</td>
<td>5.6</td>
<td>5.3</td>
</tr>
<tr>
<td>t−1</td>
<td>33.2</td>
<td>19.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Step 1. Compute the between-aircraft ellipsoid distances at the current and previous time points according to (1) (see Table 1).

Step 2. Compute the distance between aircraft and preset key waypoints and the distance between aircraft and preset route segments (key waypoints and route segments could both be acquired as per the flight plan) (see Table 2).
Table 3: Complexity relationships among different units.

<table>
<thead>
<tr>
<th></th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>P1</th>
<th>P2</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>-</td>
<td>0.09</td>
<td>0.20</td>
<td>0.14</td>
<td>0.06</td>
<td>-</td>
<td>0.21</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.09</td>
</tr>
<tr>
<td>F2</td>
<td>0.09</td>
<td>-</td>
<td>0.10</td>
<td>0.12</td>
<td>0.04</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>F3</td>
<td>0.20</td>
<td>0.10</td>
<td>-</td>
<td>0.26</td>
<td>0.06</td>
<td>-</td>
<td>0.43</td>
<td>-</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>F4</td>
<td>0.14</td>
<td>0.12</td>
<td>0.26</td>
<td>-</td>
<td>0.08</td>
<td>-</td>
<td>0.06</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>F5</td>
<td>0.06</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
<td>-</td>
<td>0.85</td>
<td>-</td>
<td>-</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SUM</td>
<td>0.48</td>
<td>0.36</td>
<td>0.62</td>
<td>0.59</td>
<td>0.23</td>
<td>0.85</td>
<td>0.71</td>
<td>0.00</td>
<td>0.32</td>
<td>0.00</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Step 3. Compute the between-aircraft approaching rate as per (2), compute aircraft-waypoint approaching degree and rate as per (4) and (5), and estimate aircraft-segment approaching degree and rate as per (7) and (8).

Step 4. Compute between-aircraft, aircraft-waypoint, and aircraft-segment complexity as per (3), (6), and (9), respectively (see Table 3).

Step 5. With five aircraft, two key waypoints, and four route segments in the current situation as nodes, the between-aircraft, aircraft-waypoint, and aircraft-segment complexity relationships as edges, and the data of complexity relationships in Table 3 as weights, we built a network model corresponding to this traffic scene (Figure 6).

As stated in Figure 6, this network model considers the complexity relationships not only between aircraft, but also between aircraft and key airspace units. Thus, by statistically analyzing the weights of all edges related to one unit, we can easily measure the contribution of this unit to the overall complexity. Cases such as acquisition of high-complexity aircraft in a sector, comparisons of complexity at different intersections or metering fixes, and comparison of yaw at different route segments are very important for the controller to rapidly form situation awareness and to improve the effectiveness of control strategies.

2.2.2. Sector Complexity. During the flight, aircraft frequently interact (e.g., two aircraft approach each other, or even conflict), and each interaction takes some energy from the controller to monitor or even interfere. Thus, between-aircraft interaction (e.g., proximity relationship) significantly affects the controller's workload and is the essence of traffic situation complexity [1, 2, 29]. Among all air traffic situations faced by the controller, in addition to the between-aircraft relationships, there are also relationships between aircraft and airspace structure (e.g., an aircraft is approaching a metering fix or deviating from the authorized airway). These relationships take much time and energy from the controller, and their aggregate complexity determines the control workload required by the air traffic situation. Thus, we define the complexity of air traffic situations as the total complexity relationships between-aircraft and between aircraft and waypoints or airways. This complexity is an objective reflection of air traffic situations from a global perspective. With greater complexity, the space-time relationships among different units in an air traffic situation are closer and thus result in greater difficulty for the controller.

Since the complexity index does not consider the controllers’ personal characteristics, it can be applied to different sectors. The model after simplification can also be applied to new-generation aviation systems, such as removal of the airway from the air traffic situation network.

Let the total amount of between-aircraft relationships in the air traffic situation at time $t$ be $C^A(t)$, the total amount of aircraft-waypoint relationships be $C^P(t)$, and the total amount of aircraft-segment relationships be $C^S(t)$, which can be computed from

$$C^A(t) = \sum_{i=1}^{N(t)-1} \sum_{j=i+1}^{N(t)} C^A_{i,j}(t)$$  \hspace{1cm} (10)

$$C^P(t) = \sum_{i=1}^{N(t)} C^P_{i,i}(t)$$  \hspace{1cm} (11)

$$C^S(t) = \sum_{i=1}^{N(t)} C^S_{i,i}(t)$$  \hspace{1cm} (12)
where $N(t)$ is the number of aircraft in the air traffic situation at time $t$. To comprehensively consider these three types of relationships, we used a Min–Max Normalization method to normalize $C^A(t)$, $C^P(t)$, and $C^S(t)$, and the results were $C^A_n(t)$, $C^P_n(t)$, and $C^S_n(t)$. In practical applications, the minimum/maximum of $C^A(t)$, $C^P(t)$, and $C^S(t)$ should be estimated from long-term historical statistics. Given $C^A_n(t)$, $C^P_n(t)$, and $C^S_n(t)$, and by using three dimensions (between-aircraft, aircraft-waypoint, and aircraft-segment complexity relationships), we can build a situation complexity vector $\vec{C}(t)$ at time $t$:

$$\vec{C}(t) = [\alpha_AC_n^A(t), \alpha_PC_n^P(t), \alpha_SC_n^S(t)], \quad (13)$$

where $\alpha_A$, $\alpha_P$, and $\alpha_S$ are the regulatory factors of between-aircraft, aircraft-waypoint, and aircraft-segment complexity relationships, respectively, and reflect the degrees of influence of different types of relationships; usually $\alpha_A + \alpha_P + \alpha_S = 1$. This weighted sum method gives air traffic controllers the flexibility to determine the relative importance of each local complexity metric to reflect more critical relationships that dictate their perceived workload. Let the situation complexity at time $t$ be $C(t)$, which can be computed from the 2-norm of the complexity vector.

$$C(t) = \|\vec{C}(t)\|. \quad (14)$$

3. Statistical Analysis

3.1. Data. Here, two air traffic control sectors S1, S2 in China were selected. In these two medium- and low-altitude controlled sectors, the flight level ranges were 6000–7800 and 5400–7800 km, respectively, the minimum horizontal separation is in both 10 km, the minimum vertical separation is in both 300 m, and the average flight velocity is about 780 km/h. We collected and statistically analyzed the radar data from 09:00 to 23:00 on February 1, 2015, from the two sectors. Figure 7 shows the airspace structure superimposed against the whole-day flight trajectory.

3.2. Complexity and Traffic Count. Although traffic count is the main feature of traffic situations, the complexity reflected by this index differs largely due to the differences in traffic organization. The comparison of complexity and traffic count is illustrated in Figure 8. Clearly, complexity increases with the rise of traffic count. The increasing rate of complexity increases with the rise of traffic count, or specifically, complexity is enhanced in a nonlinear way with the increase in traffic count. Meanwhile, when traffic count is constant, the situation complexity relationships differ significantly, which is consistent with reality. When the traffic count is 12, the complexity in sector S1 ranges from 0.33 to 0.53, while the complexity in sector S2 ranges from 0.29 to 0.56. Figure 8 demonstrates that, when traffic counts are the same, complexity is not necessarily the same. Currently, the threshold of traffic count in these two sectors is set as 12 each, and, in principle, when the aircraft number in each sector reaches 12, it is specified that the controller would not accept other aircraft from adjacent sectors. However, as shown in Figure 8(a), when the aircraft number was 12, the traffic situation complexity relationships were very different. Thus, many cases were “excessive restriction,” which may be the main cause for the very low efficiency of air traffic operations. In real-world operations, due to the pressure from traffic demand, the controller sometimes breaks through this overly conservative restriction and increases traffic volume in the sector to 16 or 17. However, such optional behavior would lead to an uncontrollable increase in workload and would also lead to numerous unpredictable risks, as shown by the circular areas in Figure 8(a). Note that the complexity index in this study not only involves the factor of traffic count, but also considers the factor of different traffic organization, which makes complexity-based management more efficient than traffic count-based management. For instance, the complexity threshold can be set as 0.7 (this threshold still needs further assessment). Namely, when the traffic situation complexity rises to 0.7, the controller does not accept entering aircraft for a brief period, or he or she may extend the separation between entering aircraft. We find under very simple traffic organization that the maximum aircraft number is improved to 15, largely improving operational efficiency. Meanwhile, since the complexity index explicitly considers traffic organization, it avoids some high-risk areas and ensures the safety level of air traffic.

3.3. Complexity and Trajectory Change. When the aircraft are very close to other aircraft or congestion areas, in order to avoid conflict, the controller has to issue some maneuvering instructions to these aircraft, including altitude change (AC), heading change (HC), and speed change (SC). Thus, the number of trajectory changes also reflects the difficulty of air traffic situations faced by a controller. In fact, the number of trajectory changes is a major index of dynamic density and also largely affects the controller’s workload [48]. Among all dynamic density models, AC, HC, and SC were all assigned with very large weights [28]. Here, the relationship of complexity with AC, HC, and SC were statistically analyzed for both S1 and S2 (Figures 9(a) and 9(b)). Clearly, complexity is aggravated with the increase of the HC number.

![Figure 7: Flight trajectory of a day of air traffic in S1 and S2.](image-url)
In comparison, the extent to which altitude changes with complexity is smaller in S1 than in S2, because S1 has relatively few available flight levels, which reduces the numbers of climbing or descending aircraft in S1. The change in the traffic situation is continuous, and sudden change rarely occurs. In addition to trajectory changes at the current time, complexity is also related to nearby trajectory changes at the current time. Here, we estimate the Pearson correlation coefficient between complexity at time $t$ and the trajectory change at time $t + \Delta t$ (here $\Delta t \in [-20 \text{min}, 20 \text{min}]$) [16]. The results for sectors S1 and S2 are shown in Figures 10(a) and 10(b), respectively. The results are consistent with our hypothesis. As for sector S1, the complexity at time $t$ is found to have a medium significant correlation with the AC at $t - 1 \text{min}$ and $t + 2 \text{min}$ and also to have a medium significant correlation with the HC and SC at time durations $t - 4 \text{min}$ and $t + 4 \text{min}$. Similar statistical results were also found in sector S2. In summary, a high-complexity traffic situation usually requires more tactical maneuvering, and, thus, more trajectory changes. Meanwhile, minutes before high complexity occurs, a large number of trajectory change might have already occurred, but minutes after the occurrence of high complexity, a large number of trajectory changes might still exist.

3.4. Complexity and Flight Conflict. During real air traffic operations, when the number of potential flight conflicts rises, the controller will spend more time in detection and resolution of conflicts, leading to a higher workload. If
the number of potential conflicts exceeds the ability of the conflict resolution system (e.g., the controller or automatic conflict resolution system), the safety level of ATM will decline. However, due to the impacts of uncertain factors (e.g., aircraft performance, weather), the precise time of occurrence and position of flight conflict are unpredictable. Thus, it is necessary to characterize an air traffic situation with high conflict risks. The flight conflict number and resolving difficulty are both closely related to traffic situation complexity. The dynamic density, which is commonly used to describe air traffic complexity, was preliminarily used to describe the influencing factors of the flight conflict rate [1]. Moreover, complexity was also described by the distance or time from the predicted conflict point to the current position [4]. Some researchers also studied the relationship between complexity (based on definition by fractal dimension) and conflict rate (number of conflicts within one hour) by using an air dynamic method [3]. In this section, we statistically analyze the correlations between complexity and conflict (predicted time is 5 min, horizontal separation minima = 10 km, and vertical separation minima = 300 m) (see Figure 11). The x-axis is complexity, the left y-axis is the number of potential conflicts per minute, and the right y-axis is the average number of potential conflicts per minute per flight. The number of potential conflicts per minute rises with the increase of complexity, and the increasing rate of the conflict number rises at a higher complexity level. On the other hand, the average number of potential conflicts per minute per flight also rises with the increase in complexity, but the increasing rate gradually declines. Particularly, in the high-complexity areas (e.g., complexity > 0.7), with the increase in complexity, the average number of conflicts per flight does not significantly increase, basically stabilizing around 0.65 (similar characteristics are found between the two sectors). It is indicated that 0.65 may be the bearable limit for controllers.

in these two sectors. When the average predicted number of conflicts per minute exceeds this limit, it is likely that the controller is unable to address the entire traffic situation. In this sense, it is possible to maintain air traffic at a safe level by controlling complexity to a reasonable extent. Thus, through the prediction of high-complexity situations, it is possible to identify critical situations that would require several tactical maneuvers to be solved along the planned trajectory of each single aircraft and identify highly congested regions that would require an entering aircraft to make too many adjustments to its flight plan to successfully execute them all [30]. On the other hand, a reasonable complexity threshold can be set by determining the average conflict number per min per flight or the total conflict number per min. When it is above this threshold, flow control is necessary. For instance, if the average predicted number of conflicts per min per flight should be below 0.65, the complexity threshold should be set at 0.7. Perhaps if the predicted number of conflicts per min should be below 6, the complexity threshold in sector S2 can be set at 0.64, while the complexity threshold in sector SI should be set at 0.57. In practical applications, a reasonable complexity threshold can be determined by balancing the risk of conflict and efficiency.

3.5. Complexity and Aircraft Acceptance Rate. In current ATM systems, airspace is divided into several sectors. As a result, an aircraft, after takeoff, has to pass through several sectors before arrival at the destination airport. In order to prevent traffic volume from exceeding the sector’s traffic capacity, the controller will restrict the number of entering aircraft from adjacent sectors according to the air traffic situation. The methods include Miles-In-Trail (MIT, to control the arriving aircraft at an appointed distance separation) and Time-Based-Metering (TBM, to control the arriving aircraft at an appointed time interval) [30]. Since it takes some time
4. Conclusion and Future Work

We propose a new index for measurement of air traffic situation complexity. This index objectively quantifies the complexity of air traffic situations by calculating the total number of complexity relationships among different nodes (aircraft, key waypoints, and airways) in a weighted air traffic situation network. We computed complexity using historical radar data from two air traffic control sectors in China and statistically analyzed the relationships of complexity with some commonly used indices, including traffic count, trajectory change, flight conflict, and aircraft acceptance rates. Comparisons of complexity and traffic count demonstrate that the complexity index is more precise than traffic count in reflecting the characteristics of air traffic situations. Thus, complexity-based management is more efficient than the traffic count-based management. The statistical results of complexity and trajectory changes indicate a significant positive correlation between them. The statistical results of complexity and flight conflicts indicate that timely control of high-complexity situations will help to resolve high operational risks, reduce unpredictable potential conflicts, and avoid excessive tactical maneuvers. The statistical results of complexity and aircraft acceptance rates indicate that the acceptance rate is most significantly correlated with complexity after several minutes of delay. Thus, the complexity index reflects both airspace structural characteristics and traffic flow characteristics and thus is a comprehensive measure of complexity. The complexity index is capable of computation and prediction by using data from flight positions and speeds and positions of waypoints and airways, and it is easy to use and comprehensive. This index helps with complexity-based management and is feasible for real-world ATM. On the one hand, our approach is relevant to many applications in the strategic stage for air traffic management, such as airspace design and flight schedule optimization. On the other hand, models developed in this study can be used as an effective tool for tactical traffic management. For example, dynamic modification of flight profiles can reduce the predicted complexities of the corresponding sector, and reorganization of traffic patterns can balance the complexities of several adjacent sectors. In fact, the ultimate goal of our future research is to predict and minimize the air traffic complexity based on the predicted flight trajectory, thereby improving the efficiency of tactical management. However, we used data from only one day. In the future, data from more sectors covering longer periods will be collected and applied to analysis of the complexity index. In addition, we will explore more scientifically rigorous methods of integrating local complexity metrics in future studies.
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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